# Tooth Modification and Spur Gear Tooth Strain

**Fred B. Oswald & Dennis P. Townsend**

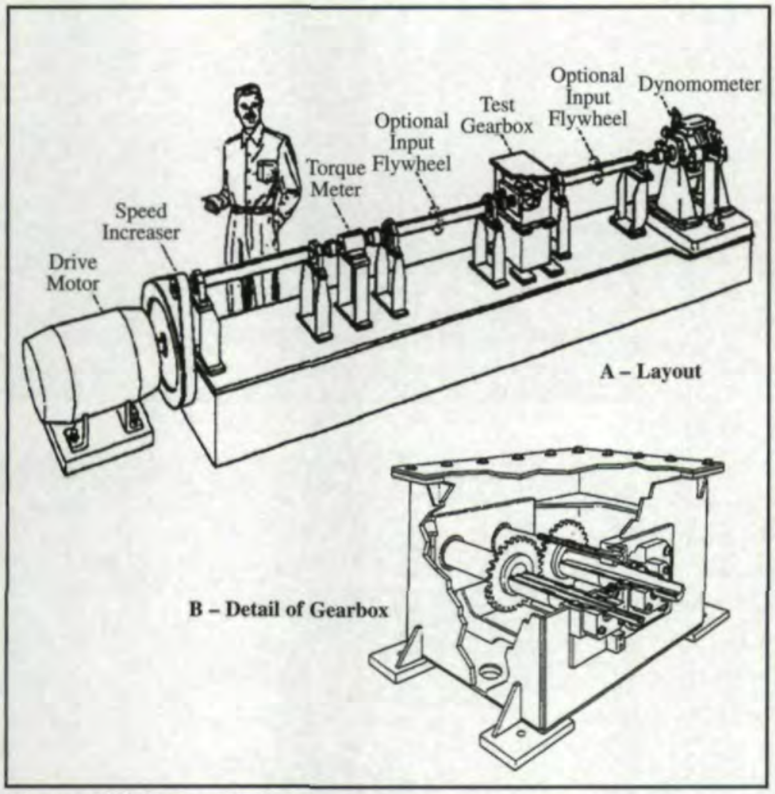


**Fig. 1 — NASA gear noise rig.**

**Fig. 2 — Strain gage installation on test gear.**

A major source of helicopter cabin noise (which has been measured at over 100 decibels sound pressure level) is the gearbox. Reduction of this noise is a NASA and U.S. Army goal.

Noise excitation in a transmission is caused by the load fluctuation as gear teeth enter and leave mesh. The cyclic variation in the numbers of teeth carrying the load causes a periodic change in the tooth stiffness and affects the relative position of the teeth. Any deviation in the angular position of the driven gear from its ideal position is called the transmission error. Transmission error arises from manufacturing and mounting errors and from tooth deflection under load.

High-quality gear designs often include modified tooth profiles (tip relief) to minimize transmission error. *Dudley's Gear Handbook* (Ref. 9) provides formulas for "first approximations" of tip relief based on the load and face width. Previous studies of spur gear profile modification include Refs. 2, 4, 5 and 8. Dynamic strain gage testing was reported in Refs. 6 and 7.

The goal for the research reported in this article was to examine the influence of tooth profile modification on dynamic tooth strain by means of controlled tests and to provide a database for further research. Data presented here include involute (tooth profile) charts for the test gears and time domain plots of static and dynamic gear tooth bending strain.

### Apparatus

Tests were performed on the NASA gear noise rig (Fig. 1). The rig features a single-mesh gearbox powered by a 150 kW (200 hp) variable speed electric motor. An eddy-current dynamometer loads the output shaft. The gearbox can operate at speeds up to 6,000 rpm. The rig was built to carry out fundamental studies of gear noise and the dynamic behavior of gear systems. It is designed to allow testing of various configurations of gears, bearings, dampers and supports. The gearbox is extensively instrumented for strain, noise and vibration measurements.

A poly-V belt drive was used as a speed increaser between the motor and input shaft. A soft coupling on the input shaft reduces the input torque fluctuations caused by non-uniformity of the belt splice.

The gearbox oil inlet temperature was maintained at 70 ±2°C for these tests. At the mean temperature of 70°, the viscosity of the synthetic turbine engine oil (MIL-L-23699B) used in the tests is 9.5 centistoke.

***Instrumentation***. General-purpose, constantan foil, resistance strain gages with gage length 0.38 mm (0.015") were installed in the tooth-root fillets on both the loaded (tensile) and unloaded (compression) side of two adjacent teeth on the output (driven) gear (Fig. 2). To measure maximum tooth bending stress, the gages were placed at the 30° tangency location (Ref. 1). Two methods of signal conditioning were used on strain gage signals: For static measurement, a strain gage (Wheatstone) bridge was used. For dynamic measurements, the strain gages were connected via a slip-ring assembly to constant-current strain gage amplifiers.

An 8-channel, 12-bit digital data acquisition system was used to record the dynamic strain data. The sample rate was varied from 6.6–50 kHz per channel to provide 500 samples per revolution for each channel. An optical encoder on the input gear shaft produced an accurate once-per-revolution pulse. The encoder was adjusted so the leading edge of its pulse occurred at a known roll angle of the gear. This allowed us to determine the roll angle at any point in the data record.

***Test Gears***. The test gears were identical spur gears (at 1:1 ratio) machined to master gear (AGMA Class 15) accuracy. Test gear parameters are shown in Table 1. Profile modifications were chosen to compensate for tooth deflection under load. No additional allowance was made for manufacturing errors, since these errors were not more than one-tenth of the computed deflection at the nominal load of 71.8 N-m (635 lb-in).

Six different gear profiles (Fig. 3) were tested. These include an unmodified profile, three combinations of linear profile modification (tip relief) and two different forms of parabolic modification. Linear modification is defined by two parameters: The amount of modification at the tip and the roll angle at the start of modification. For parabolic modification, a third parameter is needed. We identified two forms of this third parameter: In type-1 parabolic relief (see Fig. 3e), the modified profile blends smoothly into the involute trace. (It is tangent to the involute at the start of modification.) In type-2 parabolic relief (see Fig. 3f), the modified portion of the curve blends into the edge break at the end of the tooth. (It has an infinite slope at the tip.) These gears were made to the AGMA Class 15 quality level. Even so, there is not much apparent difference between the traces for set E (type-1) and set F (type-2) gears.

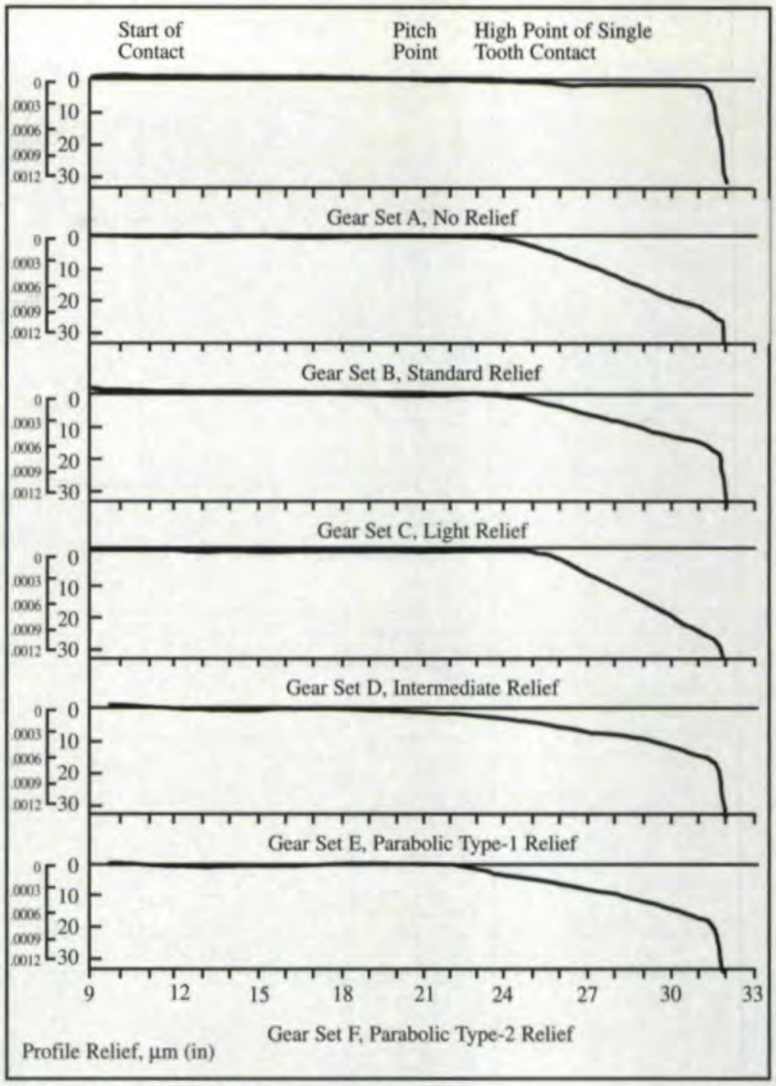


**Fig. 3 — Profile charts for the six sets of test gears.**

**Table 1 — Test Gear Parameters**

| **Gear Type** | **Standard, Full-Depth Tooth** |
|---|---|
| No. Teeth | 28 |
| Module, mm (Diametral Pitch, $in^{-1}$) | 3.175 (8) |
| Face Width, mm (in) | 6.35 (0.25) |
| Pressure Angle, deg | 20 |
| Theoretical Contact Ratio | 1.64 |
| Tooth Root Radius, mm (in) | 1.35 (0.053) |
| Max. Tooth Spacing Error, µm (in) | 1.8 (0.00007) |
| Max. Profile Error, µm (in) | 1.3 (0.00005) |

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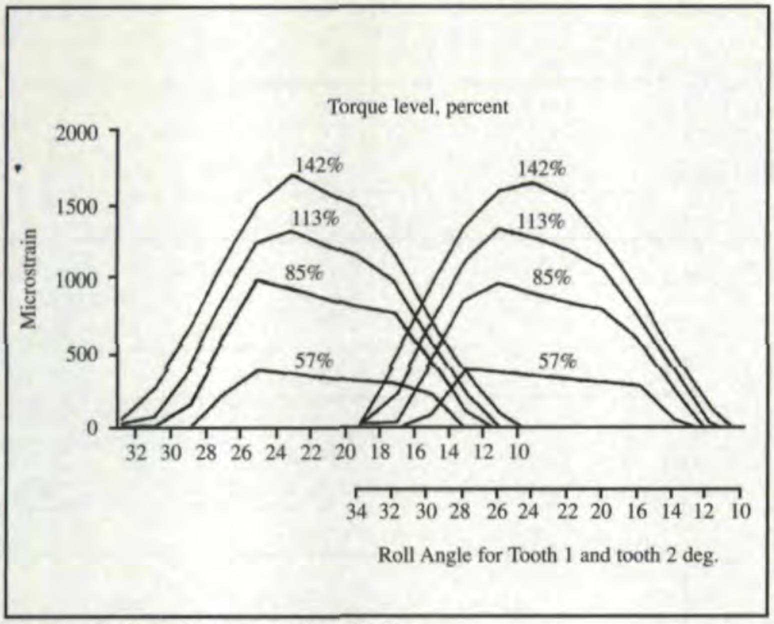


**Fig. 4 — Static strain traces for Gear Set B.**

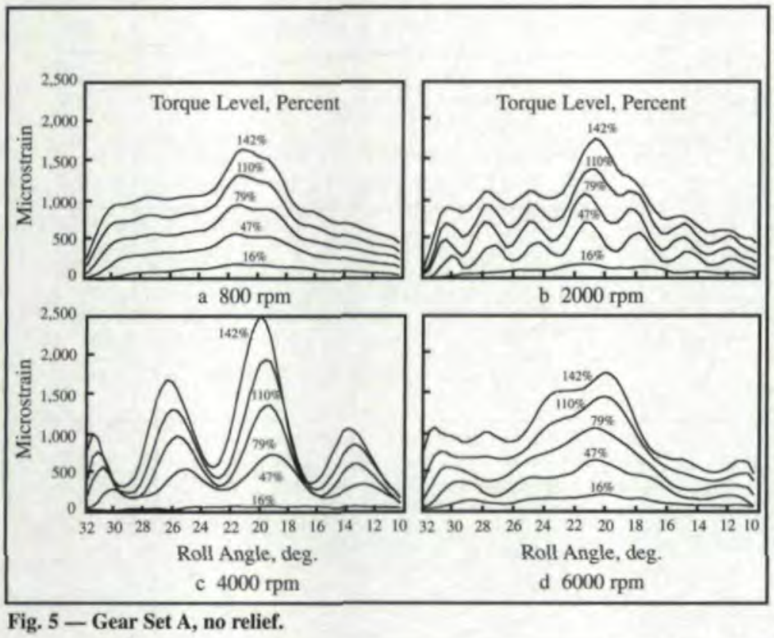


**Fig. 5 — Gear Set A, no relief.**

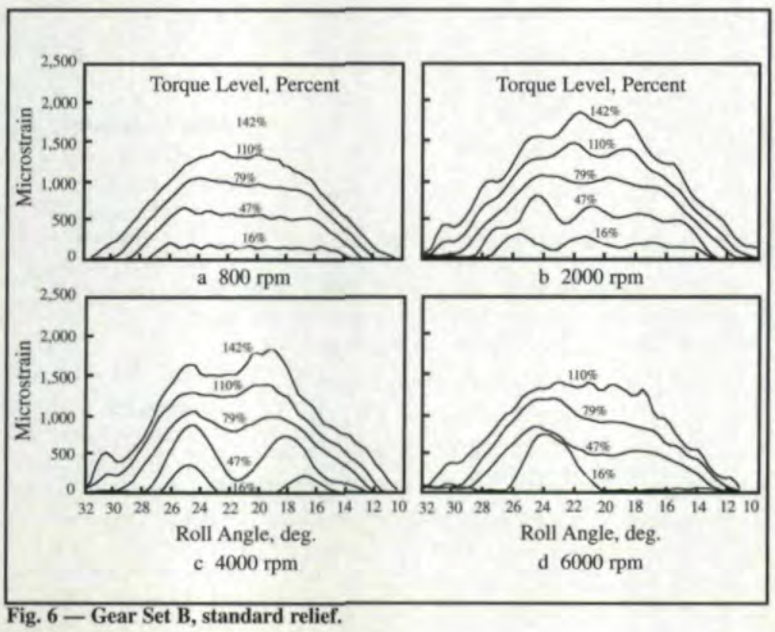


**Fig. 6 — Gear Set B, standard relief.**

## Test Procedure

***Static Strain Data***. Strain data was recorded under static (non-rotating) conditions. Static measurements provide information on load sharing characteristics of the gear pairs. Strains were recorded for roll angles from 32° to 10° roll angle for each tooth. (Note: the strain gages are on the driven gear, hence contact starts near the tip and proceeds towards lower roll angles at the root of the tooth.) Readings were taken at torque levels of 57%, 85%, 113% and 142% of the nominal torque of 71.8 N-m (635 lb-in).

***Dynamic Strain Data***. Dynamic strains were recorded for each of the six gear pairs over a matrix of load-speed test conditions: 4 speeds (800, 2,000, 4,000, 6,000 rpm) and torque levels of 16%, 47%, 79%, 110% and 142% of the nominal torque of 71.8 N-m (635 lb-in). The data were then digitally resampled using linear interpolation at 1,000 samples per revolution and synchronously averaged. Time domain synchronous averaging is a technique now in wide use in gear diagnostics (see Ref. 3), and was used here to reduce random "noise" effects (such as torque fluctuation caused by the belt drive). Its implementation requires at least two channels of data—a timing signal plus the data of interest. The timing signal provided the resample intervals needed for exactly one revolution of the gear.

## Results and Discussion

Static strain data was collected primarily for use in an effort to compute normal and frictional loads between gear teeth (see Ref. 7). Fig. 4 shows static strain data from two consecutive teeth on gear set B (standard "long relief" profiles) taken at four different torque levels. Static strain readings were recorded at two-degree increments, except an extra reading was taken at 21°, which is near the pitch point (20.85°). Comparing readings from the strain gages mounted on adjacent teeth provides an indication of the accuracy and consistency of the strain gage installations. For all the gears tested, the maximum difference (worst case) in measured static strain between the two tensile gages was 4.6%.

The dynamic strain data for the six test gear designs is shown in Figs. 5–10. These figures show strains measured by one of the load-side gages as sets of parametric curves, each set for a single gear design, at a single speed, but at several torque levels.

***Set A—Unmodified Gears***. The gears designated Set A have essentially a true involute profile (see Fig. 3) up to the edge break at about 31° roll angle. Ref. 4 shows that the transmission error of a perfect involute gear is zero at no load (torque), but increases with the load. Therefore, we would expect these gears to show increasing dynamic action as torque increases. This is indeed the case. Low-speed measurements (Fig. 5a) show very smooth operation with little dynamic excitation. Tooth contact extends from about 32° to 10°. Two pairs of teeth are in contact except in the single contact zone, which extends from about 22° to 18°.

As the speed increased to 2,000 rpm (Fig. 5b), dynamic effects become apparent, especially at the higher torques. A regular pattern of waves can be seen in the strain curves. At 1,000 rpm, one tooth pitch period occupies about 13° of roll angle (360° divided by 28 teeth). In this span there are about four cycles of strain. This indicates that the dynamic loading frequency is four times the tooth mesh frequency. At 4,000 rpm (Fig. 5c), the dynamic action is much stronger, and there are two strain cycles per tooth pitch. At 6,000 rpm, the dynamic effects are not as strong and there is no regular wave pattern.

***Set B—Standard Relief*.** The strain data from Set B gears is shown in Fig. 6. Because of dynamometer control problems at the time these data were taken, the highest torque (142%) curve is missing in Figs. 6a and 6d.

Set B gears have linear profile modification (Fig. 3) extending from the high point of single tooth contact to the tip. Munro calls this "long relief." (In contrast, "short" relief has a modification zone one-half as long.) The amount of relief at the tooth tip corresponds to the tooth deflection expected from a torque level of 115% of the nominal 71.8 N-m.

The low speed traces (Fig. 6a) show that the single contact zone is longer than in the Set A gears. The profile modification has apparently decreased the contact ratio. At higher speeds (Figs. 6b to 6d), there was smooth operation near the design torque, but much rougher operation away from design torque, especially at high-speed, low-torque conditions. At 4,000 rpm and 16% torque, there are two strain "spikes" at the beginning and end of tooth contact, and the strain is zero at the pitch point. This indicates that the teeth bounce out of contact. At 6,000 rpm and 16% torque, there is a single large strain spike. This indicates relatively high dynamic loading.

***Set C—Light Relief*.** The gears from Set C are similar to Set B except the amount of relief is much less, corresponding to the deflection for 70% torque. As would be expected, these gears operate more smoothly at light torque, but with increased dynamics at higher torques.

***Set D—Intermediate Relief*.** Set D gears have a shorter relief zone than Sets B or C. Munro in

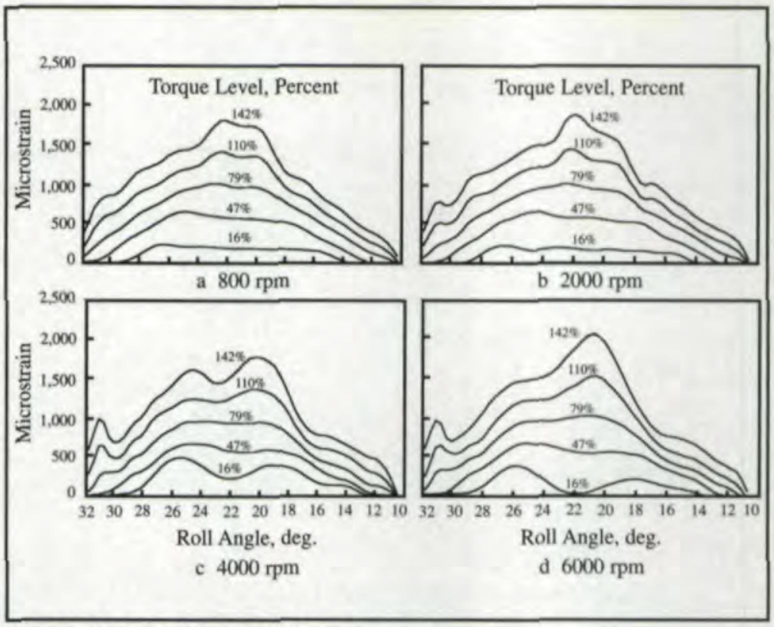


**Fig. 7 — Gear Set C, light relief.**

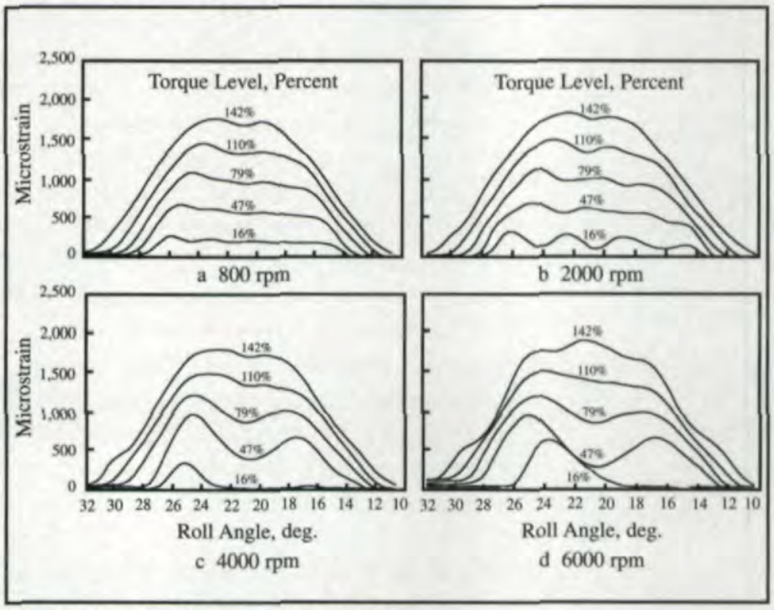


**Fig. 8 — Gear Set D, intermediate relief.**

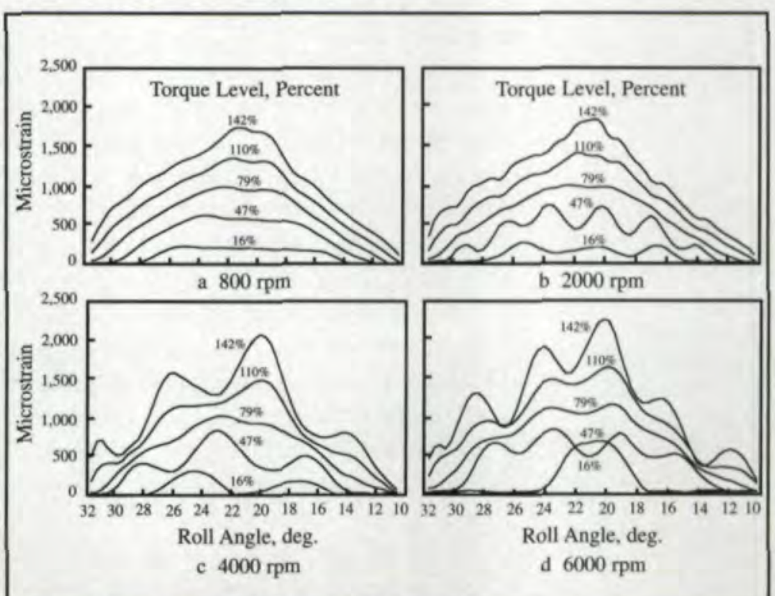


**Fig. 9 — Gear Set E, parabolic type-1 relief.**

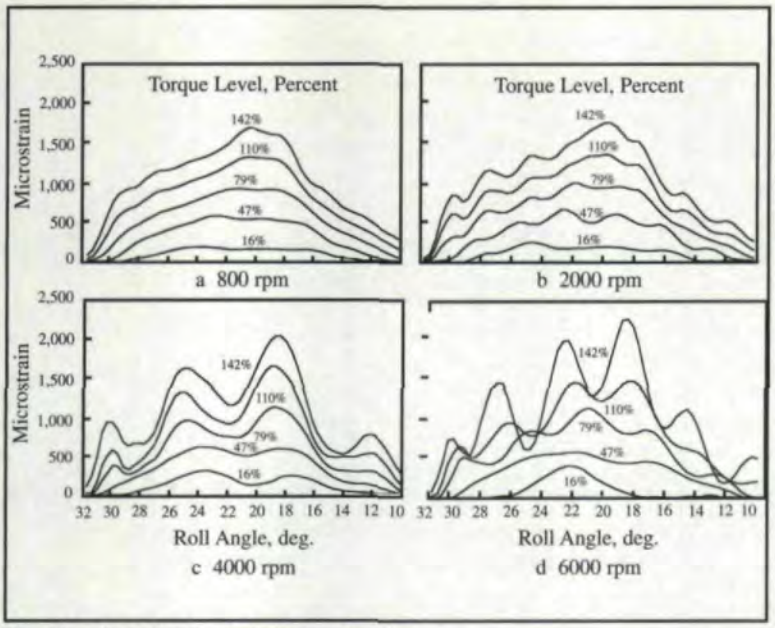


**Fig. 10 — Gear Set F, parabolic type-2 relief.**

Ref. 4 calls this "intermediate relief" and recommends it for gears which operate under a range of loads. The amount of relief at the tooth tip corresponds to 135% torque. The results (Fig. 8) show very smooth operation at high torque, and light torque operation is comparable to Set B. Since they show improvement at high torque and no worse operation at low torque (compared to Set B), intermediate relief is an improvement over long relief.

***Set E—Parabolic Type-1***. These gears are similar to Set C gears except the modification zone extends to the pitch point. (Munro calls this "extra long relief.") The modification at the tip corresponds to torque of 65%. These gears were difficult and expensive to manufacture. Their performance was disappointing. The dynamic strains were large at both high and low torques. Results may have been better if the relief zone were much shorter.

***Set F—Parabolic Type-2***. These gears are similar to Set E except the modification amount at the tip corresponds to the tooth deflection at 85% torque, and the modification zone is slightly shorter. Like Set E, these are difficult and expensive to manufacture. The performance was similar to that of Set E.

## Summary and Conclusions

Low contact ratio spur gears with various profile modifications were tested in the NASA gear noise rig. Dynamic tooth bending strains were recorded for each gear design at 36 operating conditions. The experimental results were compared to examine the influence of the tooth profile on the dynamic behavior of the gears under various operating conditions. The following conclusions were drawn from the data:

1. The proper type and amount of tooth profile modification can significantly reduce dynamic loads in spur gears, especially for gears that operate at high speed and under high torque.

2. The parabolic modification gears tested here seem not to offer any advantage. This may be because the modification zone is too long.

3. Profile modification increases dynamic loads in gears that operate significantly below their design torque. This is especially so for a long modification zone.

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