

Calculating SAP and TIF

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Question submitted by G. Lueck
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Q: Are there any simpler formulae for S.A.P. & T.I.F.? Are there any not involving the mating part?

Answer submitted by Dan Thurman

A: Start of Active Profile (SAP) is defined as the lowest point on the gear tooth where contact with the mating gear tooth tip can occur. On gears without tooth tip chamfers, it will be determined by the maximum outside diameter of the mating gear. It is usually expressed in degrees of roll above the base diameter. There is a diameter called the form diameter (D_f), associated with this roll angle, which can be calculated.

True Involute Form diameter (TIF) is defined as being the point on the gear tooth where the involute form must begin. It is not necessarily the same diameter as the form diameter determined by the SAP. For example, it is possible for a SAP to occur in an undercut area, but the TIF diameter to be at a higher point on the tooth profile.

SAP is dependent on the mating gear outside diameter and the operating center distance.

Figure 1 contains the equations for calculating SAP, Form Diameters, and mating gear outside diameters. Any combination

Terms and Definitions

N	Number of teeth in gear (specify as minus if an internal gear)
N_m	Number of teeth in mate (specify as minus if an internal gear)
P_n	Normal diametral pitch
D_o	Outside diameter of gear (inside dia. if gear is internal)
D_{om}	Outside diameter of mate (inside dia. if mate is internal)
D_b	Base circle diameter of gear
D_{bm}	Base circle diameter of mate
D_f	Form diameter
OD/ID	Outside diameter/Inside diameter
SAP	Start of active profile
TIF	True involute form
m_c	Profile contact ratio
ϕ_n	Normal pressure angle
ϕ_t	Transverse pressure angle
ϕ'_t	Operating transverse pressure angle
ψ	Helix angle (zero if spur gear)
θ_o	Roll angle at gear OD/ID
θ_{om}	Roll angle at mating gear OD/ID

Calculating Transverse Pressure Angle

$$\phi_t = \text{atan} \left(\frac{\tan(\phi_n)}{\cos(\psi)} \right) \quad (1)$$

Calculating Operating Transverse Pressure Angle

$$\phi'_t = \text{acos} \left[\frac{(N + N_m) \cdot \cos(\phi_t)}{2 \cdot P_n \cdot CD \cdot \cos(\psi)} \right] \quad (2)$$

Calculating Base Circle Diameter of Gear

$$D_b = \left[\frac{N \cdot \cos(\phi_t)}{P_n \cdot \cos(\psi)} \right] \quad (3)$$

Calculating Base Circle Diameter of Mating Gear

$$D_{bm} = \left[\frac{N_m \cdot \cos(\phi_t)}{P_n \cdot \cos(\psi)} \right] \quad (4)$$

Calculating Pressure Angle at OD/ID of Gear if Mating Gear OD/ID is Given

$$\phi_{om} = \text{acos} \left(\frac{D_{bm}}{D_{om}} \right) \quad (5)$$

Calculating SAP if Form Diameter is Given

$$SAP = 180 \cdot \frac{\sqrt{\frac{D_f^2}{D_n^2} - 1}}{\pi} \quad (6)$$

Calculating Pressure Angle at OD/ID of Mating Gear if SAP is Given

$$\phi_{om} = \text{atan} \left[\frac{(N + N_m) \cdot \tan(\phi) - \pi \cdot N \cdot \frac{SAP}{180}}{N_m} \right] \quad (7)$$

Calculating Start of Active Profile

$$SAP = 180 \cdot \frac{(N + N_m) \cdot \tan(\phi'_t) - N_m \cdot \tan(\phi_{om})}{\pi \cdot N} \quad (8)$$

Calculating OD/ID of Mating Gear

$$D_{om} = \frac{D_{bm}}{\cos(\phi_{om})} \quad (9)$$

Calculating Pressure Angle at SAP of Gear

$$\phi_f = \text{atan} \left(\frac{\pi \cdot SAP}{180} \right) \quad (10)$$

Calculating Form Diameter at SAP of Gear

$$D_f = \frac{D_b}{\cos(\phi_f)} \quad (11)$$

Calculating Pressure Angle at OD/ID of Gear

$$\phi_o = \text{acos} \left(\frac{D_b}{D_o} \right) \quad (12)$$

Calculating Roll Angle at OD/ID of Gear

$$\theta_o = 180 \cdot \frac{\tan(\phi_o)}{\pi} \quad (13)$$

Calculating Profile Contact Ratio

$$m_c = \frac{|\theta_o - SAP| \cdot |N|}{360} \quad (14)$$

* Note: quantities enclosed in vertical bars are absolute values.

Fig. 1 — Equations.

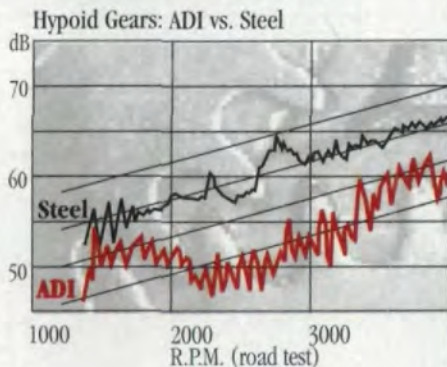
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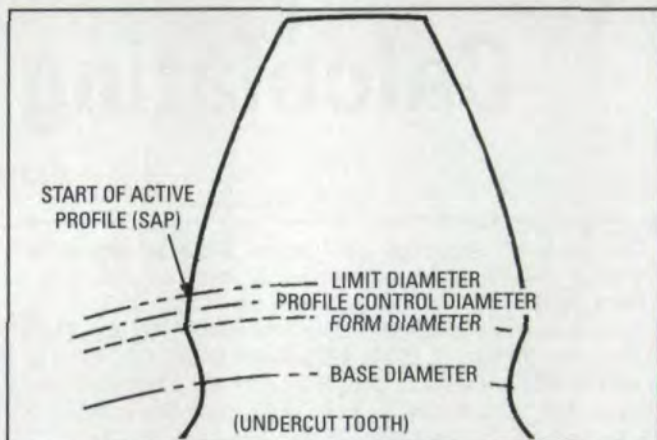


Fig. 2 — Start of Active Profile and Form Diameter.¹

of SAP, Form Diameter or mating gear outside diameter may be given. (Note: Use negative numbers to specify number of teeth on internal gears.)

Extended Example

The following example uses the formulas presented above and a sample gear set (see Fig. 3 for sample specifications) to calculate SAP.

Using formula (1), we can calculate the transverse pressure angle based on the values given for normal pressure angle and helix angle in the example:

$$\phi_t = \text{atan} \left(\frac{\tan(\phi_n)}{\cos(\psi)} \right)$$

$$\phi_t = \text{atan} \left(\frac{\tan(20)}{\cos(0)} \right)$$

$$\phi_t = 20 \text{ deg}$$

From this, we can calculate the operating transverse pressure angle using formula (2) and the given values for the center distance, the normal diametral pitch, the helix angle and the number of teeth in gear and mate:

$$\phi_t' = \text{acos} \left[(N + N_m) \cdot \frac{\cos(\phi_t)}{(2 \cdot CD \cdot P_n \cdot \cos(\psi))} \right]$$

$$\phi_t' = \text{acos} \left[(21 + 18) \cdot \frac{\cos(20)}{(2 \cdot 7.7915 \cdot 2.64 \cdot \cos(0))} \right]$$

$$\phi_t' = 27.0221166 \text{ deg}$$

N=21
Nm=18
CD=7.7915
Pn=2.64
 $\phi_n=20$ degrees
 $\psi=0$ degrees (spur gear)
D_o=9.177
D_{om}=8.159

Fig. 3 — Sample Gear Set for Extended Example.

We will also need the base circle diameter of the mating gear, which can be determined using formula (4):

$$D_{bm} = N_m \cdot \frac{\cos(\phi_t)}{P_n \cdot \cos(\psi)}$$

$$D_{bm} = 18 \cdot \frac{\cos(20)}{2.64 \cdot \cos(0)}$$

$$D_{bm} = 6.4069951$$

Finally, we'll use formula (5) to calculate the pressure angle at the OD of the mating gear based on the given outside diameter:

$$\phi_{om} = \text{acos} \left(\frac{D_{bm}}{D_{om}} \right)$$

$$\phi_{om} = \text{acos} \left(\frac{6.4069951}{8.159} \right)$$

$$\phi_{om} = 38.2546016$$

Using the calculated and given values, we can now calculate SAP using formula (8):

$$SAP = 180 \cdot \left[\frac{(N + N_m) \cdot \tan(\phi_t) - N_m \cdot \tan(\phi_{om})}{\pi \cdot N} \right]$$

$$SAP = 180 \cdot \left[\frac{(21 + 18) \cdot \tan(27.0221166) - 18 \cdot \tan(38.2546016)}{\pi \cdot 21} \right]$$

$$SAP = 15.55 \text{ deg}$$

The calculated SAP is given in degrees of roll above the base circle diameter. It can be converted to a corresponding form diameter (D_f) as follows:

$$D_b = N \cdot \frac{\cos(\phi_t)}{P_n \cdot \cos(\psi)}$$

$$D_b = 7.4748277$$

$$D_f = D_b \cdot \sqrt{\left(\pi \cdot \frac{SAP}{180} \right)^2 + 1}$$

$$D_f = 7.7451$$

Many times we will be starting with an existing gear where we already know the form diameter (D_f) or the corresponding roll angle (θ_f) where the involute begins, and we want to find the outside diameter of the mating gear which will reach that point on the involute. In that case, the following equations apply:

Given the Form Diameter (D_f) from the example above:

$$D_f = 7.7451$$

$$SAP = 180 \cdot \frac{\sqrt{(D_f^2 - D_b^2)}}{\pi \cdot D_b}$$

$$SAP = 15.55$$

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$$\phi_{om} = \text{atan} \left[\frac{(N + N_m) \cdot \tan(\phi_t) - \pi \cdot N \cdot \frac{SAP}{180}}{N_m} \right]$$

$$\phi_{om} = 38.2546462 \text{ deg}$$

$$D_{om} = \frac{D_{bm}}{\cos(\phi_{om})}$$

$$D_{om} = 8.159$$

By definition, calculation of start of active profile requires knowledge of the mating gear. However, it is possible to calculate SAP without knowing anything about the mating gear except for the profile contact ratio.

$$\theta_o = 180 \cdot \frac{\sqrt{D_o^2 - D_b^2}}{\pi \cdot D_b}$$

$$\theta_o = 40.8088321 \text{ degrees roll angle at outside diameter of gear}$$

$$m_c = 1.4736 \text{ (Contact Ratio Between Gear and Mate)}$$

$$SAP = \theta_o - \frac{360 \cdot m_c}{N}$$

$$SAP = 15.55 \text{ degrees}$$

It is sometimes helpful to be able to calculate the corresponding roll angle (θ_{xm}) on a mating gear, given a known roll angle on gear one. For example, from our original external mesh:

$$SAP = 15.55 \text{ degrees}$$

$$\theta_{xm} = 180 \cdot \frac{(N + N_m) \cdot \tan(\phi_t) - \pi \cdot N \cdot \frac{SAP}{180}}{\pi \cdot N_m}$$

$$\theta_{xm} = 45.175 \text{ degrees}$$

Any roll angle can be converted to a diameter according to the following relationship:

$$D_x = D_b \cdot \sqrt{\left(\pi \cdot \frac{\theta_x}{180}\right)^2 + 1}$$

For more information about SAP and TIF, see ANSI/AGMA 115.01 *Basic Gear Geometry* and ANSI/AGMA 1012-F90 *Gear Nomenclature, Definitions of Terms with Symbols*. ⚙

References

1. Fig. 2 extracted from ANSI/AGMA 1012-F90, *Gear Nomenclature, Definitions of Terms with Symbols*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, Virginia 22314.

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