Non-Standard Cylindrical Gears

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Abstract

This paper examines three gear geometries, each of which has special features not found on standard cylindrical gears. The first are high pressure angle spur gears where the pressure angle has been maximized at the expense of contact ratio. High pressure angles result in higher forces, but the stresses are less and are mostly compressive, leading to good fatigue resistance. The second design is a curved face width gear, giving a higher contact ratio compared to spur gears and lower contact and bending stresses. These gears are similar to double helical gears in that there are no axial forces. The third gear is a tapered face width gear, in which the addenda, dedenda and tooth width are all tapered. The benefits are that tooth engagement is gradual and compensation for center distance variations can be achieved by relative axial adjustment. All these gears were made of plastic materials.

Management Summary

Curved face width (CFW) spur gears are not popular in the gear industry. But these non-metallic gears have advantages over standard spur gears: higher contact ratio, higher tooth stiffness, and lower contact and bending stresses.

CFW gears also provide better operating features. Tooth height decreases in the sections away from the gear center, so a lower sliding friction is expected with consequences on the gear's thermal behavior. They axially locate each other, improving meshing when misaligned. There also are no axial forces inherent in helical gears.

For spur gears, load sharing follows the classic "top hat" shape with a sudden change when moving from partial load to full load. With a CFW gear, tooth contact and load changing are gradual like a helical gear. This should lead to smoother running, quieter gears compared to spurs.

But CFW gears are difficult to design and mount in gear trains. For certain tooth geometries, the gear train is sensitive to center distance variations.

Gears with high pressure angles (> 20°) are generally viewed as unsuitable due to lower contact ratios and higher noise levels. These gears, however, benefit from having stronger tooth forms and lower contact stresses and are more efficient due to shorter sliding distances. With dry-running plastic gears, the increased efficiency means lower running temperatures and, because of plastic's resilience, no noticeable noise difference. If the gears are fully lubricated, the high pressure angles result in higher entrainment velocities, leading to advanced lubrication. The authors tested 30° pressure angle gears and "extreme" 40° pressure angle gears with measureable improvements in strength and running temperatures.

Tapered face width gears can adjust backlash by axial movement of one of them. Such adjustment is much easier than backlash adjustment via radial center distance adjustment. The tapered face width principle can be applied to spur and helical internal and external gears and rack and pinion sets, so it is a versatile means of employing backlash control in, for example, automotive steering systems.

Introduction

The annual production of plastic gears now outnumbers the production of metal gears (more than 500 million per year for cars alone). However, the design of plastic gears relies heavily on the experience gained from many years of designing and manufacturing steel gears. Work in the United Kingdom and Romania has focused on increasing the transmissible power density of polymer gears. The basic geometry used for polymer gears is mainly determined from standard tooth proportions as recommended in steel gear standards. The work described sought to provide gears of superior performance designed to non-standard forms where pressure angles, addenda and dedenda are chosen for maximum efficiency and load carrying capacity. The forms suggested here are novel in concept and differ considerably from conventional practice.

The authors are primarily concerned with the design and practical aspects of plastic and polymer composite gears. The aim is to improve the performance of these gears in order that the transmissible power levels are raised so that plastic gears can be employed in a wider range of products. Historically, plastic gears have followed steel gear practice in terms of tooth proportions. However, since the primary manufacturing method is a molding process, almost any profile can be considered (i.e. tooth forms are not a function of the cutting process). This provides an opportunity for designing novel gear forms.

Wear is the predominant mode of failure for dry running gears, but root bending fatigue or pitch line fracture are also commonplace (Ref. 1). In the case of lubricated plastic gears, pitting can also arise. Plastic gears are particularly susceptible to temperature. In the case of dry running plastic gears, large friction forces will arise, and these can lead to high temperatures. It is not uncommon for the combined bulk and flash temperatures to exceed the melting temperature of the plastic, which will result in high wear rates (Ref. 2). Even if the temperatures are below the melting point, the mechanical properties of plastics are affected by heat and, for example, the elastic modulus decreases markedly with temperature, altering the contact ratio and load sharing. Thus, any design procedures and innovations which might reduce running temperatures and/or reduce tooth stresses in plastic gears are worth examining.

The gears described in this paper challenge the accepted wisdom of cylindrical gear design first by examining the effects of pressure angle, the benefits of a curved face width gear and the advantages of tapered face width gears, which permit gradual tooth engagement without the need for a helical tooth form.

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Non-Standard Cylindrical Gears

Table 1			
Pressure angle (deg)	20	30	40
Contact ratio	1.65	1.35	1.12
Center distance extendibility (mm)	1.43	0.98	0.37

(30-tooth spur gear, module 2 mm, standard proportions)

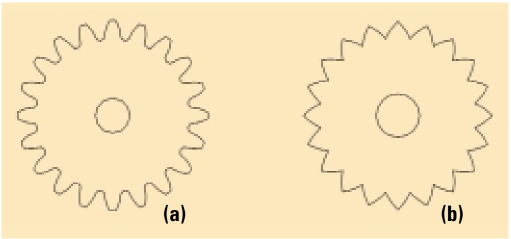

Figure 1—Showing (a) a standard 20° profile compared to (b) a 40° high pressure angle profile.

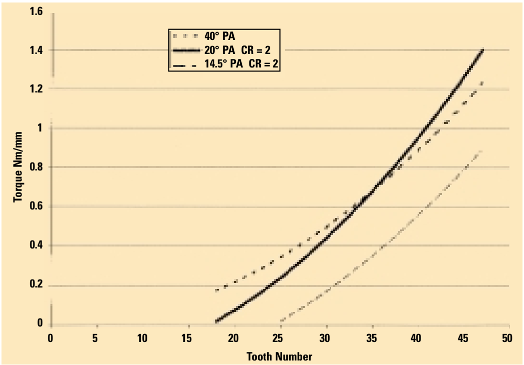

Figure 2—Comparison of torque capacity against tooth number for different pressure angles.

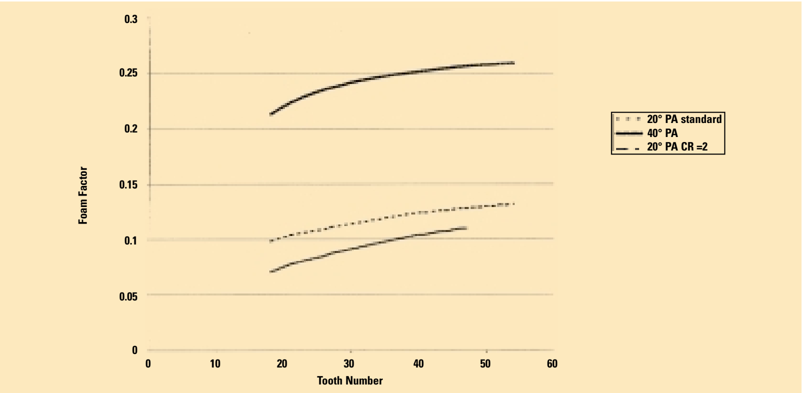

Figure 3—Showing Lewis form factor against tooth numbers for gears of varying pressure angles.

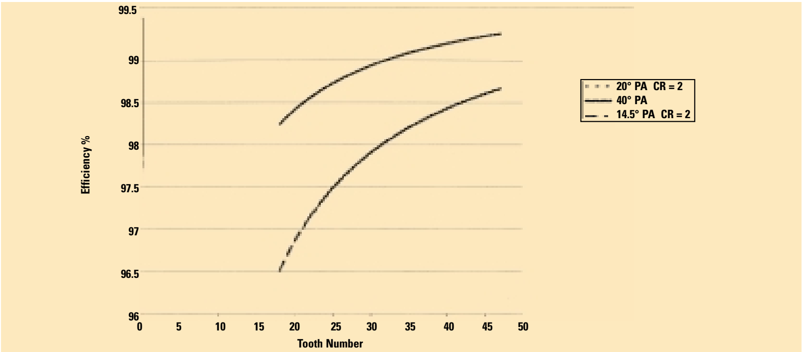

Figure 4—Showing variation of efficiency with tooth numbers for different pressure angles.

High Pressure Angle Gears

The aim: To investigate the potential of high pressure angle gear tooth profiles in polymer and polymer composite materials. A preliminary study showed that such tooth profiles could lower contact and bending stresses and increase efficiencies compared with gears of standard proportions. Studies on tooth forms showed that wear for gears molded with high 25° pressure angles was less than at standard (20°) or low (14.5°) pressure angles (Ref. 3). This raised questions as to what extent the pressure angle might be increased still further to reduce wear. A brief study was conducted for this application to investigate effects of pressure angle on sliding velocity, contact stress, tooth bending stress and efficiency. High pressure angle (30° and 40°) gears were injection molded from acetal (the benchmark material), a glass-filled nylon (high performance polymer composite) and PEEK (to permit high temperature operation). Tests were performed under dry and oil-lubricated conditions.

High pressure angle gear geometry results in a reduction of the addendum and the elimination of the top land.

It also results in an increase in the radius of the involute at the pitch point, (see Fig. 1), which will manifest itself as a reduction in both the contact and bending stresses (the section modulus increasing with pressure angle) as well as a reduction in sliding velocities. Figure 2 shows the transmissible torque per mm face width against tooth number at constant Hertzian contact stress for a 40°, 20° and 14.5° pressure angle gears (18 teeth is about the minimum for this 40° pressure angle). The 20° and 14.5° pressure angle gears both have a contact ratio of 2. Also, Table 1 shows the effects of changing pressure angle on contact ratio and center distance extendibility.

The Lewis form factor for these gears is shown in Figure 3. This factor becomes questionable as an indication of tooth bending strength for the 40° pressure angle gear since radial load is not considered. The line of action of the force acting on the teeth for the majority of the contact period falls within the extent of the tooth root. This implies there is no bending of the tooth form and that both flanks of the tooth are in compression. This factor might be important for a number of brittle materials that have good strength in compression but break under moderate tension. Preliminary Finite Element Analysis (FEA) indicates that the profile of the 40° pressure angle gear teeth shows an improvement in tooth bending stress compared to the standard profile, even allowing for the difference in load sharing.

An unexpected result was the discovery that the apparent efficiency increased with pressure angle (Fig. 4). In nearly every case, contact and bending stresses and efficiency for the high pressure angle gear indicate improved performance. This higher efficiency mostly results from the reduced contact ratio of the high pressure angle design. If gears with low pressure angles were modified, such as by reducing their outside diameters, the claimed efficiency benefits would largely, possibly completely, disappear.

Still, high efficiency has two rewards for plastic gears. One is economy, and the second reward is a lower running temperature for a given transmitted power. Material properties deteriorate with increasing temperature so that, at low temperatures, the material will be stiffer and stronger. High pressure angle gears are not normally favored because of higher normal loads for a given transmitted torque, low contact ratios and higher noise. Our preliminary studies show that other advantages (such as low stresses, higher efficiency) might more than outweigh these considerations, resulting in a higher transmitted power for a given size of gear.

Finally, the higher resilience of plastics means that the operating noise levels are less than those for steel. Thus, noise considerations may not be a problem. During the subsequent tests, mesh temperatures, efficiency and noise were recorded. Average dynamic coefficients of friction will be back-calculated from measured efficiencies, so the results can be compared to gears made from the same materials, but of standard 20° pressure angle form. It will be possible to take the design a stage further and include high pressure angles with a helical tooth form. This would result in even higher transverse pressure angles as well as improved load sharing. Should the results prove promising, other materials might be investigated, such as ceramics. Thus, if high pressure angle gears are successful, there will be considerable future research and development potential.

Curved Face Width Gears

Curved face width gears (Fig. 8), have been developed in Russia (Ref. 4) and by Gleason in the U.S., but, due to their complex geometry, few engineers are aware of their existence. The particular advantages these gears have over standard spur gears are higher contact ratios and lower

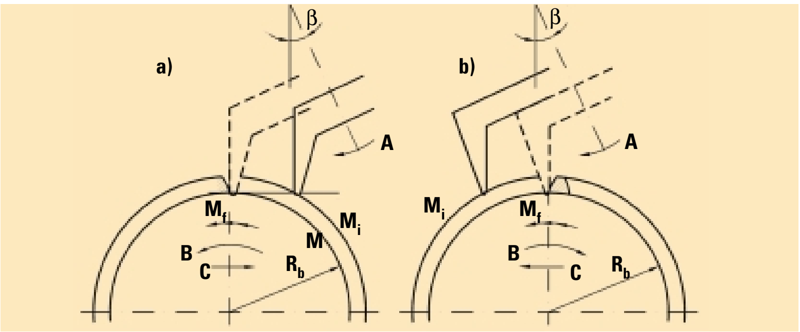

Figure 5—Gear tooth flank generation.

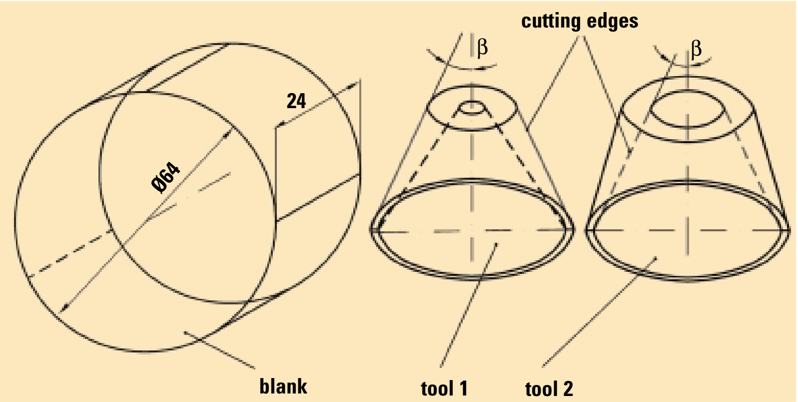

Figure 6—Showing the solid primitives used to simulate the curved face width gear teeth generation.

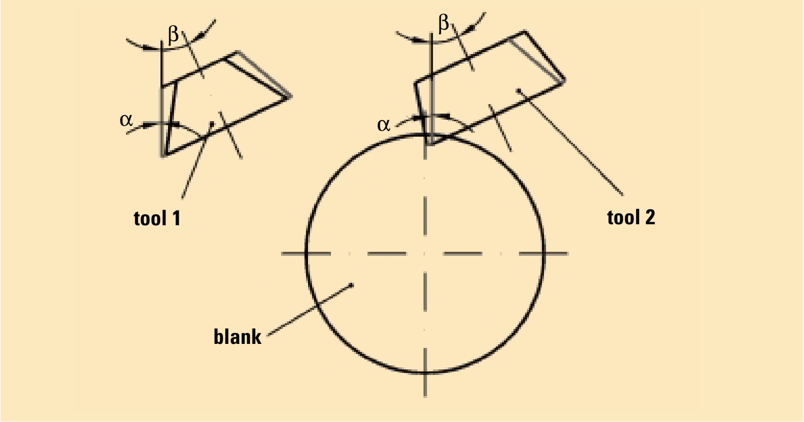

Figure 7—Simulation of the tooth cutting process.

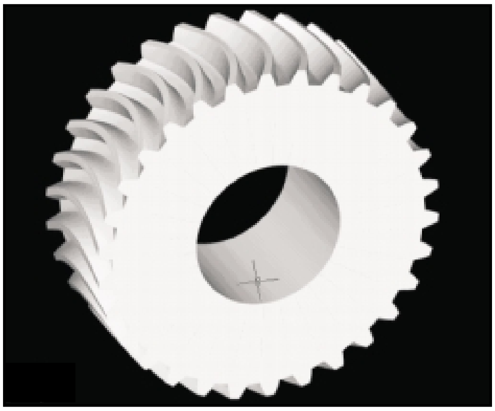
Figure 8—Pictorial representation of the curved face width spur gear, with modified geometry.

contact and bending stresses, due to the double curvature of the tooth flanks. They axially locate each other, offering better meshing in plane misalignment conditions, and there are no axial forces, which are inherent in helical gears. The difficulty in gear train mounting and, for certain tooth geometries, a sensitivity to center distance variations are the only two serious disadvantages.

The curved face width spur gear described in this article differs from previous curved face width gears in that they have a modified geometry with a variable tooth height and width along the gear face width. Reduced bending stresses, lower sliding velocities and, when run in oil, enhanced lubrication conditions were expected. The shape of these gears makes the design of a molding die commercially impractical. Hence this proposed design is suitable only for machined gears, plastic or metal. Such gears must be mounted with close control of relative axial position, generally not required for spur or helical parallel-axis gears (Ref. 5).

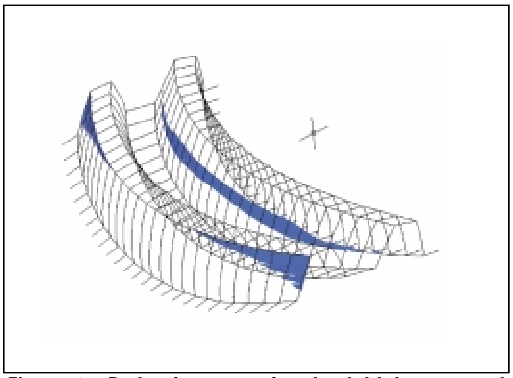

Figure 9—Path of contact for the initial gear tooth contact.

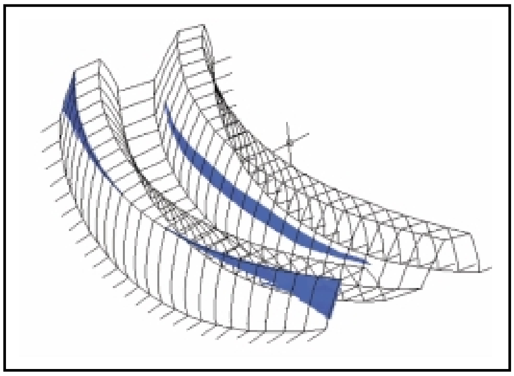

Figure 10—Path of contact after 1° rotation.

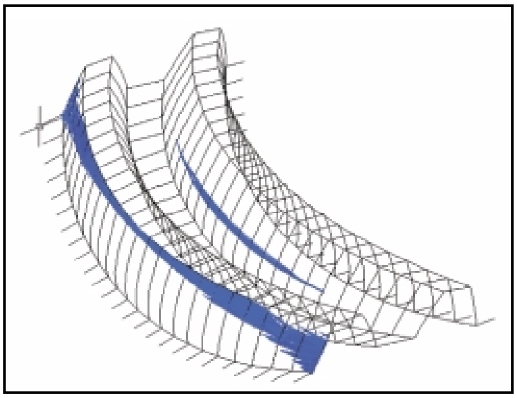

Figure 11—Path of contact after 3° rotation.

The kinematics of the gear generation process. Two different cutters were used for the concave and convex tooth flank generation. Figure 5 illustrates the kinematics of the generation process, where the tools are working in the half-width plane of the blank. The normal sections of the cutters show straight lines for the imaginary rack-cutter flanks, with a zero pressure angle. The designed geometry for the tooth flanks is based on the following kinematic principles:

- the cutting tool performs rotational motion (A) about its inclined axis, leading to a curved tooth along the gear face width and a variation in tooth height;
- the gear being generated is rotated about its axis (B) and is provided with translational motion (C), tangential to the base circle of the gear (R_b), in order to obtain the rolling motion required for involute tooth form generation.

Two phases are necessary in order to generate the entire gear. The cutting process starts with the tooth-by-tooth concave flank generation (Fig. 5a) followed by the convex flank generation (Fig. 5b), where a new tool is used, properly positioned relative to the already cut concave flank.

Simulation of the gear generation process. The modified geometry of the curved face width spur gears described here requires a theoretical investigation before the gears can be made. A numerical simulation of the gears' generation and mesh, based on the traditional conjugate surface generation theory, was developed by one of the authors (Ref. 6). The complexity of the gear tooth geometry implied complex mathematical calculations and large computer programs. The authors used the advantages of solid modeling techniques in order to obtain a complete representation of the gear. The gear design model can be further used for the gear generation error analysis, for finite element analysis and for rapid prototyping.

To simulate the kinematic generation of the curved face width gear flanks, the blank and the tools are modeled by cylindrical and conical solid primitives using solid modeling methods with conveniently chosen dimensions. Work previously done by the authors on measuring and comparing the performance of plastic gears led to a "standard" gear being used that has a 20° pressure angle, 30 teeth of module 2 mm and addendum equal to the module. In order that the curved face width gear could be compared to standard geometries, the first curved face width gears were made the same size. Thus, a gear blank of 64 mm diameter and 24 mm face width (Fig. 6) was made. The rotational motion of the two cutters about their own axes was replaced by the solids "Tool 1" and "Tool 2," obtained by rotating the tool's normal sections around the same axis, using the REVOLVE command. The rotational motion of the blank was performed incrementally by using a ROTATE3-D command, with an angular increment; its translational motion was also done incrementally.

The tool successively intersects the blank until the generation of a tooth flank is completed (Fig. 7). The removal of the "material" is done using a Boolean subtraction operation, performed by the SUBTRACT command. The automatic development of the procedure enables a dynamic view of the process and a representation of the gear to be made (Fig. 8).

Gear meshing simulation. A theoretical analysis of the gears in mesh shows that the contact line depends on the magnitude of the transmitted torque and on the gear alignment. When the concave side of the pinion tooth enters mesh, contact starts at the tooth dedendum, at the end of the gear face width and extends to the midpoint of the face width, leading to a concave curve of contact. Similarly, contact on a tooth's convex flank determines a convex curve of contact, starting at the tooth dedendum, at the gear's mid-section. The facilities offered by the solid modeling technique enabled the gear mesh to be simulated, in order to investigate the path of contact. Using the virtual models and the dynamic simulation of the gear meshing, the theoretical path of contact was obtained from a virtual model of the ideal gear train.

Figures 9–11 show several representations of the path of contact on the pinion teeth's convex flanks (a 0.1 mm interference was used for the path representation). The initial contact shown in Figure 9, obtained by hand positioning of the virtual gear, corresponds to an arbitrary point along the theoretical line of contact. The gear train was next rotated by 1° (Fig. 10) and then 3° (Fig. 11), to obtain the contact path during gear meshing. Analyzing the tooth contact, it is obvious the "contact line" is a spatial curve, which changes in curvature during the mesh cycle.

Bending stresses and deflections in curved face width gears. The authors designed a modified geometry for the plastic curved face width spur gears in order to enhance the load capacity and to avoid excessive tooth deflection. The database obtained from the gear generation simulation was used to analyze the gear tooth resistance, using finite element analysis procedures. Stresses and stiffness of a curved face width gear were compared to those of a standard spur gear.

Curved and straight spur gears were modeled using COSMOS/M set to the following geometrical parameters: modulus 2 mm, 30 teeth and 24 mm face width. Poisson's ratio for the plastic material used in the gear's manufacture, Ertalon 66SA, was 0.3; and the tensile modulus of elasticity was 3,450 MPa. The analysis was developed for two teeth in mesh under the applied loads given in Table 2 and shown in Figure 12.

Table 2

Torque [Nm]	Von Mises stresses [N/mm^2]	Maximum deflection [mm]
4	1.18	0.014
6	1.77	0.021
8	2.36	0.027
10	2.96	0.035
12	3.55	0.041

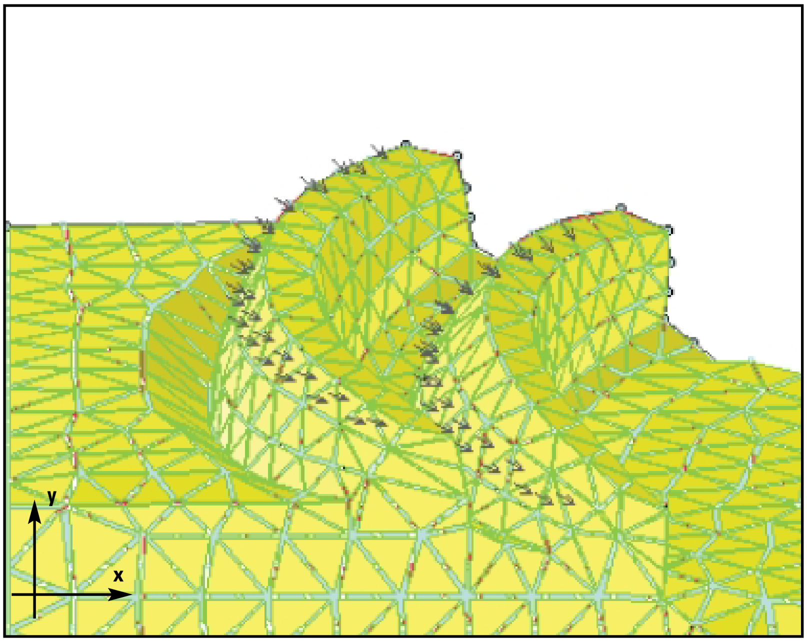

Figure 12—Showing the applied loads.

Non-Standard Cylindrical Gears

Table 3

Torque [Nm]	Von Mises stresses [N/mm^2]	Maximum tooth deflection [mm]
4	10.78	0.017
6	16.47	0.022
8	21.55	0.033
10	27.59	0.043
12	32.64	0.051

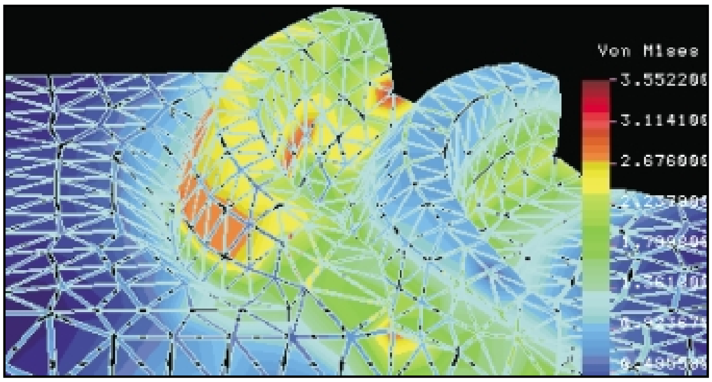

Figure 13—Von Mises stress distribution.

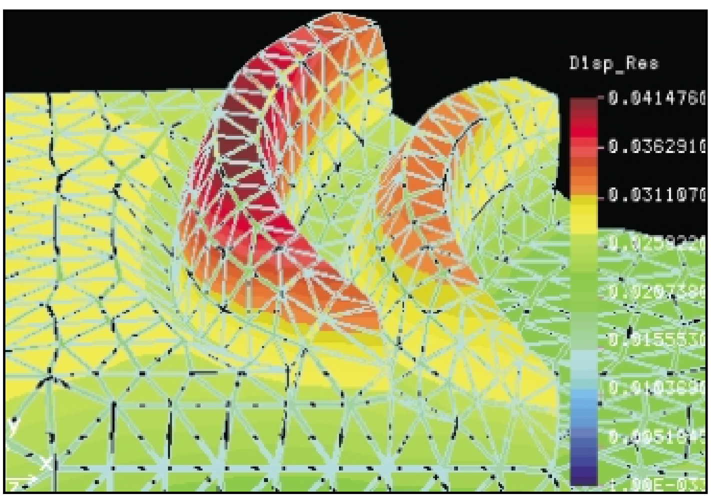

Figure 14—Distribution of curved face width tooth deflections.

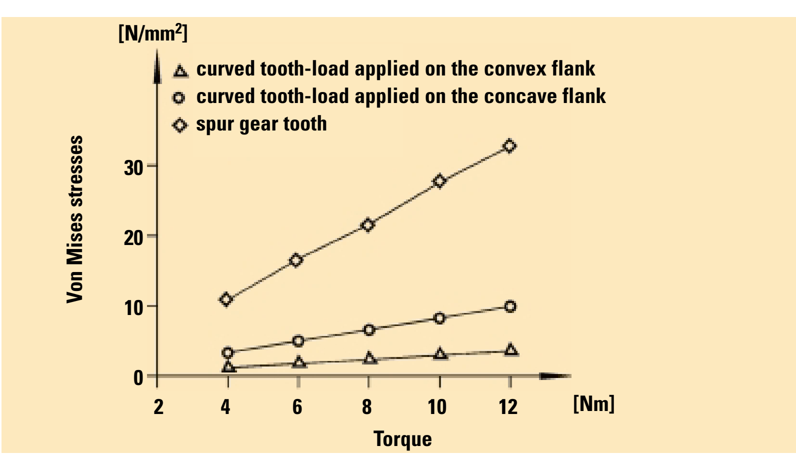

Figure 15—Comparison of maximum stresses in curved face width and standard spur gears.

Table 2 lists the results of the tooth bending stresses and deflections for a standard region of the curved face width gear—a continuum divided into 18,348 linear tetrahedral elements interconnected at 3,721 nodes. For the maximum load applied (12 Nm), Figures 13 and 14 illustrate the distribution of the Von Mises stresses and the tooth deflections, respectively.

Figure 13 shows that maximum stresses are concentrated on the dedendum of the tooth with load applied close to its addendum. Figure 14 shows that maximum tooth deflection appears in the gear mid-face width, at the tooth addendum. Due to variable tooth height and width, the tooth stiffness can differ, as shown by the variation of tooth deflection along the gear face width. It can be seen that, close to the gear face width ends, the tooth deflection decreases by approximately 20% compared to the gear mid-section tooth deflection. A similar analysis, done for the curved face width gears with the load applied on the concave flanks, showed higher bending stresses concentrated on the tooth dedendum and similar maximum displacements of the tooth addendum, at the gear's mid-face width.

The analysis of the spur gear tooth bending resistance was also developed, for the same applied loads. The results are given in Table 3 and show the high bending stresses developed compared to the curved face width gears; the bending stresses developed in standard spur gears being approximately 10 times higher than the curved gear tooth stresses. The result of the finite element analysis on the bending resistance is summarized in Figure 15.

Tapered Gears

The tapered gear design utilizes the principle of access and recess action within a single gear. At the mid-plane, the geometry of the tooth form is substantially standard. The gear has a linear taper so that the profile is modified to access action at one end of the gear and recess action at the other. The matching meshing gear must have a similar taper and the access region must mesh with the recess portion of the mating gear. This condition can be satisfied when the direction of the taper is inverted (Fig. 16). For spur gears, this profile results in the flanks of the teeth being slightly helical and a small end thrust will be generated when torque is transmitted. Interestingly, the direction of the axial force is the same for both drive and overrun conditions.

Another feature of the gear profile is the ability of the gears to compensate for center distance variation or backlash by adjusting the relative axial position of the gears. This results from the taper in the tooth width. The taper of the gears is in all proportions linear with axial position, so that the addendum, dedendum and tooth width all change proportionally (Fig.17). This produces a shape that has a natural draft angle on all external surfaces, facilitating the manufacture of these gears by some molding process. This process might be cavity molding for plastic gears or sintering or forging for metal gears.

The contact ratio varies across the plane of the meshing gears, and the slight helical profile of the flanks provides a small phase difference between the ends of the gear. The important feature is that the contact ratio changes smoothly as the mesh progresses so that the teeth are gradually loaded and unloaded. This feature should help to make these gears quiet in operation. The beam strength of the tooth profile should be comparable with conventional gears' tooth profiles. The second moment of area is that the tooth root is larger for the tapered gear profile than for a standard tooth.

The tapered gear can also be made helical (Fig. 18) with a different helix angle on the opposite flanks of the teeth. In this case, left- and right-handed helical gears are required. A special case is where the helix angle is made so that one flank of the tooth is straight while the other is helical. For example, an axial reaction force might be generated while the gear pair is driving and no such force is generated during overrun.

Conclusions

Three novel gear types are examined in this paper, each offering some benefits over conventional cylindrical gears. High pressure angle gears of the same materials have withstood higher loads than gears of standard geometries. Noise has not been a problem. Curved face width gears are shown to provide lower stresses compared to spur gears of the same dimensions. These gears are manufactured by cutting the plastic, and though this form of production results in less accurate gears, these gears run satisfactorily and demonstrate their capacity to operate at higher torque levels than conventional spur gears. Tapered face width gears are modeled and manufactured using rapid prototyping techniques. The main advantages of the tapered gears are the ease of manufacture by molding techniques, including sintering and forging. Kinematically, they offer a degree of action that effectively provides mesh overlap. The gears provide tolerance to mold shrinkage due to the adjustability of the mesh by axial displacement.

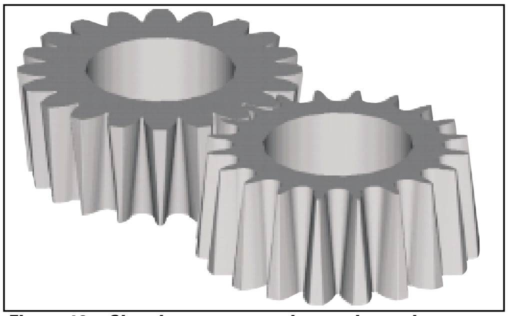

Figure 16—Showing two tapered gears in mesh.

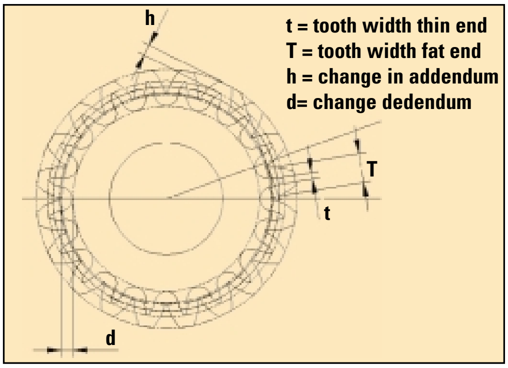

Figure 17—Comparison of the extremes of profile for the tapered gear.

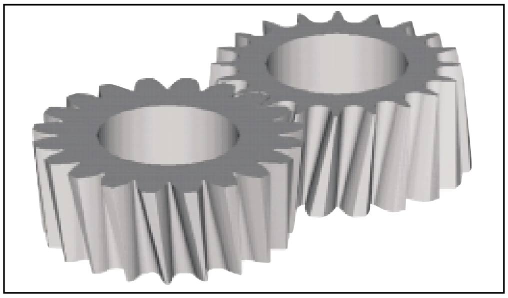

Figure 18—Showing a pair of helical tapered gears.

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