A Further Study on High-Contact-Ratio Spur Gears in Mesh with Double-Scope Tooth Profile Modification

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NOMENCLATURE	
O_1	Driving gear hub center
O_2	Driven gear hub center
HCRG	High-Contact-Ratio Gears
LCRG	Low-Contact-Ratio Gears
BTR	Bulk Tooth Rotation
T.E.	Transmission Error
LSR	Load-Sharing Ratio
Km	Combined torsional mesh stiffness, Nm/rad
P, P_1, P_2	Magnitude of profile modification, mm
$\Delta L_1, \Delta L_2$	Length of profile modification, mm
AT	Approach tooth (contact)
RT	Recess tooth (contact)
MT	Mid-tooth (between AT and RT)
SEQV	von Mises stress, MPa
S_1	First principal stress, MPa
S_3	Third principal stress, MPa
B	Base point
S, S_1, S_2	Relief starting point
t_0	Involute tooth tip
t_1, t_2	Modified tooth tip
σ	Profile rotation angle
e	Extra extension (the amount below base point).
ε_α	Profile contact ratio
P_b	Base pitch
T'_1, T'_2	Limits of the plane of action ($T'_1 T'_2 = \frac{1}{2}\ \varepsilon_\alpha\ P_b$)

Management Summary

This paper will demonstrate that, unlike commonly used low-contact-ratio spur gears, high-contact-ratio spur gears can provide higher power-to-weight ratio, and can also achieve smoother running with lower transmission error (TE) variations. The research presented in this paper concentrates on providing proofs and verifications on the topic by using modern numerical methods and comprehensive analysis. Additionally, a general bulk tooth rotation (BTR)-type tooth profile modification is introduced and applied to the high-contact-ratio spur gears in demonstration of improved tooth profile design.

Introduction

Research on high-contact-ratio spur gears with various tooth profile modifications was previously conducted using experimental (Refs. 1–2) and numerical methods (Refs. 2–3). Research by Yildirim (Ref. 2) has provided more details of the double-scope tooth profile modification, and his analytical results have shown the advantages of bringing the two reliefs together, resulting in an overall superior performance in terms of peak-to-peak transmission error value, maximum tooth load value, tooth load sharing ratio and smoother TE curves. More recently, Ajmi (Ref. 3) continued researching the double-scope tooth profile modification with modern numerical simulation methods. It was further confirmed that the relief with two slopes was superior to a conventional short or long, linear relief. Furthermore, the analysis presented here was conducted with the tooth loading conditions and dynamic conditions in order to assess some potentially interesting profile modifications for future consideration.

However, the above analyses can also be seen as having distinct shortcomings. First, Yildirim's analytical (or geometrical) results (Ref. 2) do not fully represent the characteristics of elastic bodies under load, though under very light loads it may be close enough to reality. Second, the predicted transmission error of Ajmi (Ref. 3) doesn't appear to satisfy the principle of solid mechanics, where the width of single, double and triple contact zone was shown to be far too rigid. In the case of common low-contact-ratio gears in mesh, the width of the single contact zone shrinks and the double contact zone expands while the load increases (Refs. 4–5). For the tip-relieved, high-contact-ratio gears in mesh, the width of single, double and triple contact zone will change dramatically while the load increases (Refs. 6–7). Obviously, it is vital to overcome those shortages in order to realize a unique, optimum design of high-contact-ratio spur gears. Based on the results of previous research (Ref. 6) and the use of advanced modeling techniques such as adaptive meshing and an element birth and death option, a satisfactory simulation result can be achieved on the high-contact-ratio spur gears in mesh with double-scope tooth profile modification. In order to compare previous results and published experimental tests, the original high-contact-ratio gear model (Ref. 8) was used, and for further reference, this study was also conducted using another classical high-contact-ratio spur gear model, which has been used by many researchers (Refs. 1, 7 and 9–11).

Table 1. Gear Parameters used in the Analysis.

Material	Steel
Friction coefficient	0.06
Number of teeth	54
Nodule M, mm	3.738
Pressure angle, deg	18
Addendum, mm	1.26
Dedendum, mm	1.66
Profile shift coefficient	0.143
Center distance, mm	202.9*
Face width, mm	11
Theoretical contact ratio ε_a	2.36
Tip fillet radius, mm	0.2
Hub radius, mm	25.4
Design (max) load, Nm	700

* When the profile shift coefficient is zero, the center distance should be the standard of 201.852 mm.

The FE Model and the Profile Modification

The FE model of the high-contact-ratio spur gear pair has parameters as shown in Table 1, based on the set ratio 1:1 of the test rig used by Munro for the transmission error measurements (Ref. 8). In the test rig, the gear center distance was set to 203.2 mm, which is slightly greater than the theoretical value of 202.9 mm. The FE model and details are shown in Figure 1, where the theoretical value 202.9 mm was used for the center distance.

The FE model was built with the following major considerations:

1) 2D plane stress quad elements were used for TE evaluations, combined torsional mesh stiffness and tooth load sharing ratio; and also for the primary calculations of tooth contact stress and tooth root stress. Over the mesh cycle, the majority of components were calculated with acceptable errors (Ref. 12), except for the higher-order components such as the tooth contact stress. 3D brick element models were then used for a better evaluation of the tooth contact stress over the flank face.

2) For computation efficiency and solution accuracy, adaptive meshing was used, and the necessary element sizing (Ref. 6) was performed.

3) When the tooth profile modification was applied, the element birth and death option was used, which has proven to be efficient for solving the model when rigid body motion was incorporated into the mesh cycle (Ref. 7). Finally, all considerations were incorporated into several looping programs, running automatically, that also included post-processing, as each mesh cycle contained over 100 calculation points. According to the previous research results (Ref. 2, 3 and 13), and the "MAAG Gear Book" (Ref. 13), the parameters of the profile modification—as shown in Figure 2—have the

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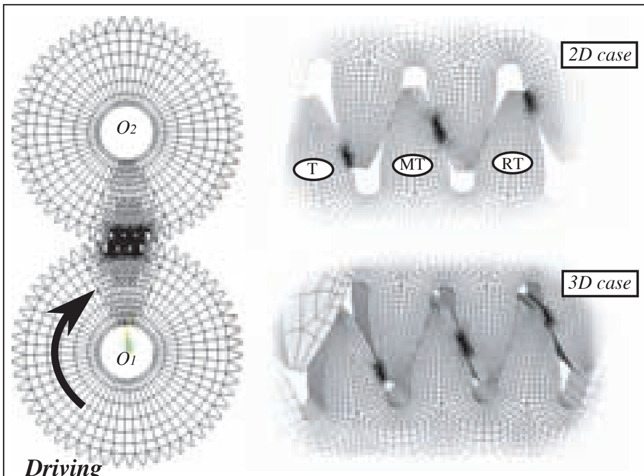

MT: mid-tooth (which will present the tooth load share and stresses).
AT: approach tooth.
RT: recess tooth.
Roll angle = 0: when MT contacts at the pitch point (unloaded).

Figure 1—FE model of the gears in mesh (ratio 1:1) and its mesh adaptation with contact.

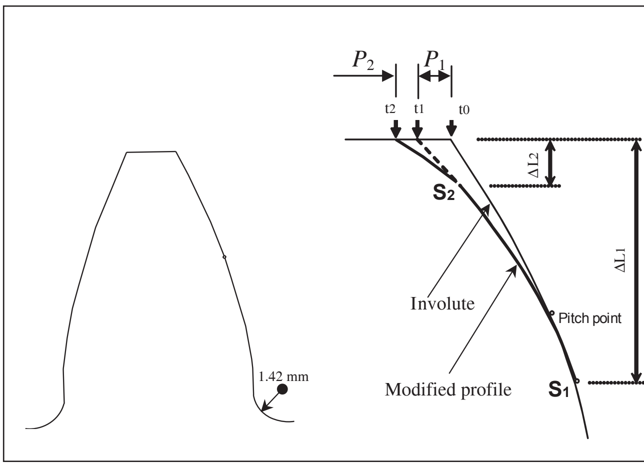

Figure 2—Details of profile relief.

following magnitude of linear type relief:

$$P_1 = |t_0 t_1| = 20\mu m, \tag{1}$$

$$P_2 = |t_1 t_2| = 10\mu m, \tag{2}$$

where: P_1 and P_2 are the amplitudes of relief, and t_0, t_1 and t_2 are the tooth tip positions.

The first and second relief extent are given by the following,

$$\Delta L_1 = 0.5 T_1'T_2' \tag{3}$$

$$\Delta L_2 = 0.075 T_1'T_2' \tag{4}$$

and

$$T_1'T_2' = \varepsilon_\alpha P_b = 13.18 \text{ mm} \tag{5}$$

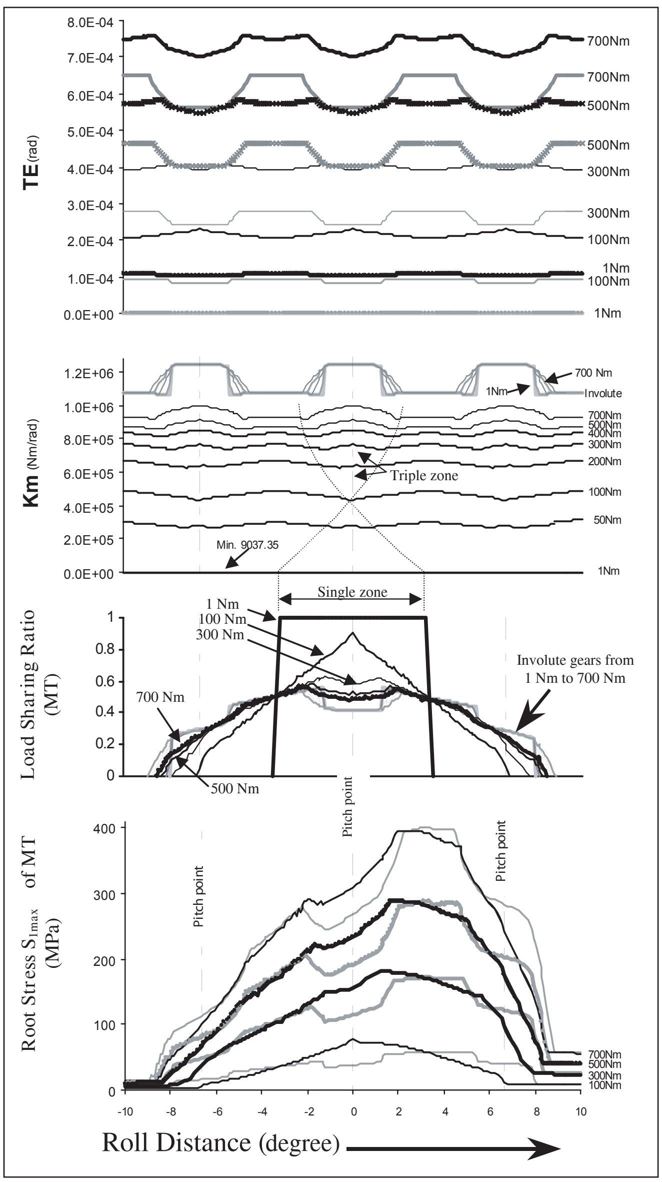

Figure 3—The changeover process under various input loads of the involute gears (light-color curves) and the profile-modified gears (dark-color curves).

Analysis with Tooth Profile Modifications

The analysis was conducted according to comprehensive methods (Refs. 4, 6 and 14) in which the techniques of adaptive mesh and element birth-death options were used to ensure accuracy and to compensate for the rigid body motion resulting from the profile modifications. The transmission error, combined torsional mesh stiffness (Km), load-sharing ratio and tooth root stress were produced over the complete mesh cycle simultaneously, as shown in Figure 3.

It can be seen that each of the components has also been compared with that of unmodified gears in order to clearly demonstrate the meshing characteristic variations. The contact stresses of unmodified and modified gears are presented separately (Fig. 4).

In summary, the double-scope profile modification achieved the following results:

- A primary rigid body motion occurred due to the long relief extent, which was causing a significant, unloaded transmission error.
- Transmission error variation was reduced, but no smooth TE curve was found near the design load of 700 Nm. The expectation of two smooth TE areas in moderate load range was not produced.
- Combined torsional mesh stiffness results were reduced significantly with lighter load and tended to be smoother.
- Tooth root stress was relatively high, with the lighter input load below the design load of 700 Nm.
- Contact stresses were significantly reduced, compared to unmodified gears, especially for loads under 300 Nm. Contact stresses tended to be smooth with only small stress irregularities at the relief starting point. However, the relatively high stress, especially for the contact recess case in Figure 4 (b), has shown that premature contact occurs near the tooth tip when the input load increases. The less significant, high stresses occurred at the contact approach case and at the relief starting point $\mathbf{S}_2$. Increasing the magnitude of profile modification P_1 or P_2 could avoid high stresses near the tooth tip, but it cannot avoid high stresses occuring at the relief starting point(s).

Comparison with previous transmission error research results can be made as shown in Figure 5, where it can be seen that the effect of primary rigid body motion has been removed in (a), (b) and (c) so that the absolute value of TE under each load was reduced. The results in (a) and (b) also show some zero TE under zero load. Experimental measurement (a) has shown large TE variations under zero and lighter loads. This usually is due to manufacturing or geometric errors, and corresponding errors larger than elastic deformation are common in many experimental tests.

Long tooth profile modification can also cause large zero (or lighter) load TE variations if the primary rigid body motion is comparatively small, or if the primary rigid body motion is not accounted for.

High-contact-ratio gears with long profile modification will result in single tooth contact under light loads. The single contact zone in the mesh cycle can quickly disappear when the load is increased to become a triple contact zone that will then expand in zone width as the load is further increased. However, these characteristics have not totally been shown in the results in Figure 5 (b) or 5 (c).

A Novel Profile Relief Based on Tooth Deformation Reconsideration

The numerical study of tooth deformation included the use of common low-contact-ratio gears in mesh (Ref. 7). For the complete gear pair in mesh, the tooth local deformation is shown in Figure 6.

Five significant deformations have been highlighted in Figure 6, where 1 and 2 represent the tooth root bulk rotation, 3 is the Herzian (contact) deformation, 4 is due to the local shear stress and 5 is due to tooth bending (and tension).

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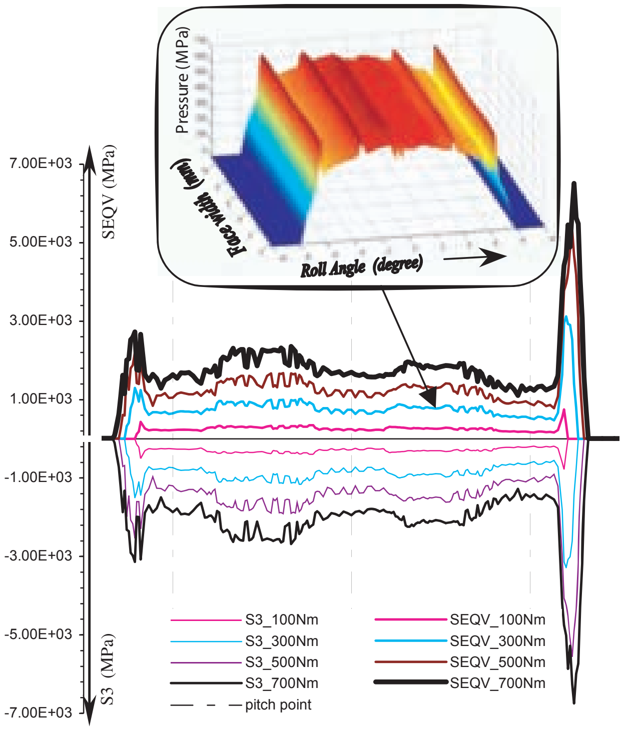
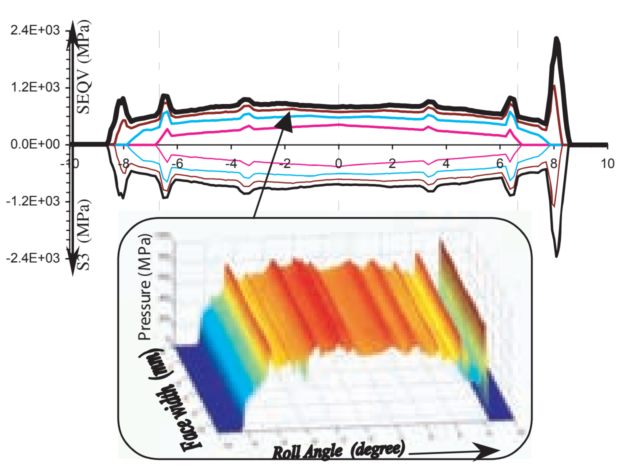
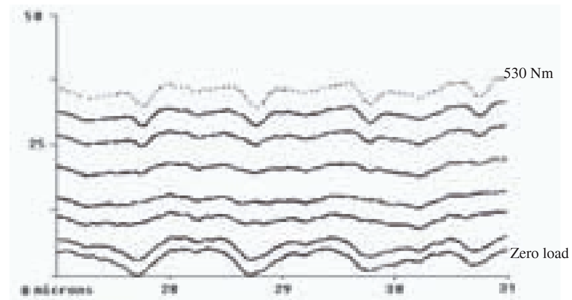


(a) Contact Stress of MT - Involute gears

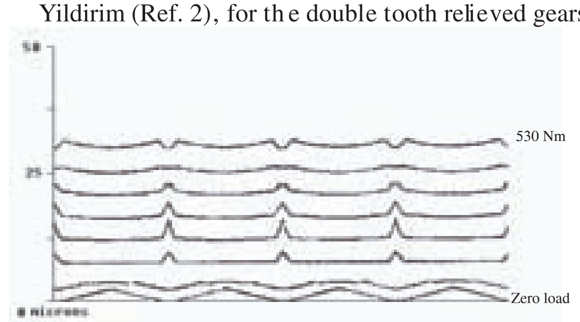


(b) Contact Stress of MT - Profile relieved gears

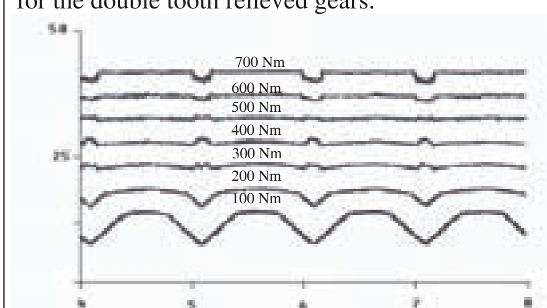
Figure 4—Contact stresses of mid-tooth over the mesh cycle under various loads.



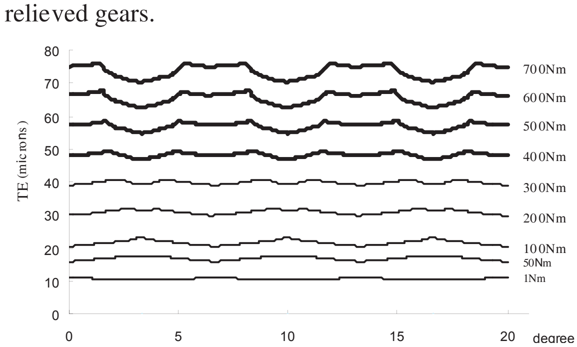
(a) Experimental measurement of the static TE by Yildirim (Ref. 2), for the double tooth relieved gears.



(b) Analytical prediction of the static TE by Yildirim (Ref. 2), for the double tooth relieved gears.



(c) Simulated static TE by Ajmi (Ref. 3), for the double tooth relieved gears.



(d) Simulated static TE of the double tooth relieved gears in this research.

Figure 5—Experimental and analytical results of TE predictions.

The rotation of the unloaded tooth profile to the deformed tooth demonstrates that the tooth bulk rotation is the largest amount out of the total tooth deformation. Based on the above information, a bulk rotation tooth profile (about tooth root) type relief was tested, called the bulk tooth rotation (BTR)-type tooth profile relief or modification. The modified tooth profile starts from point S, which is below the base point B with a (small) distance e called extra extension. The modified tooth profile S t_1 was produced by rotating the involute profile S t_0 through an angle σ with a minor profile extension near t_1. Compared to the current application recommendations—BS (Ref. 15) and ISO (Ref. 16)—the tooth bending-based standards, bulk tooth rotation-type tooth profile relief is production-friendly, especially for the double relief.

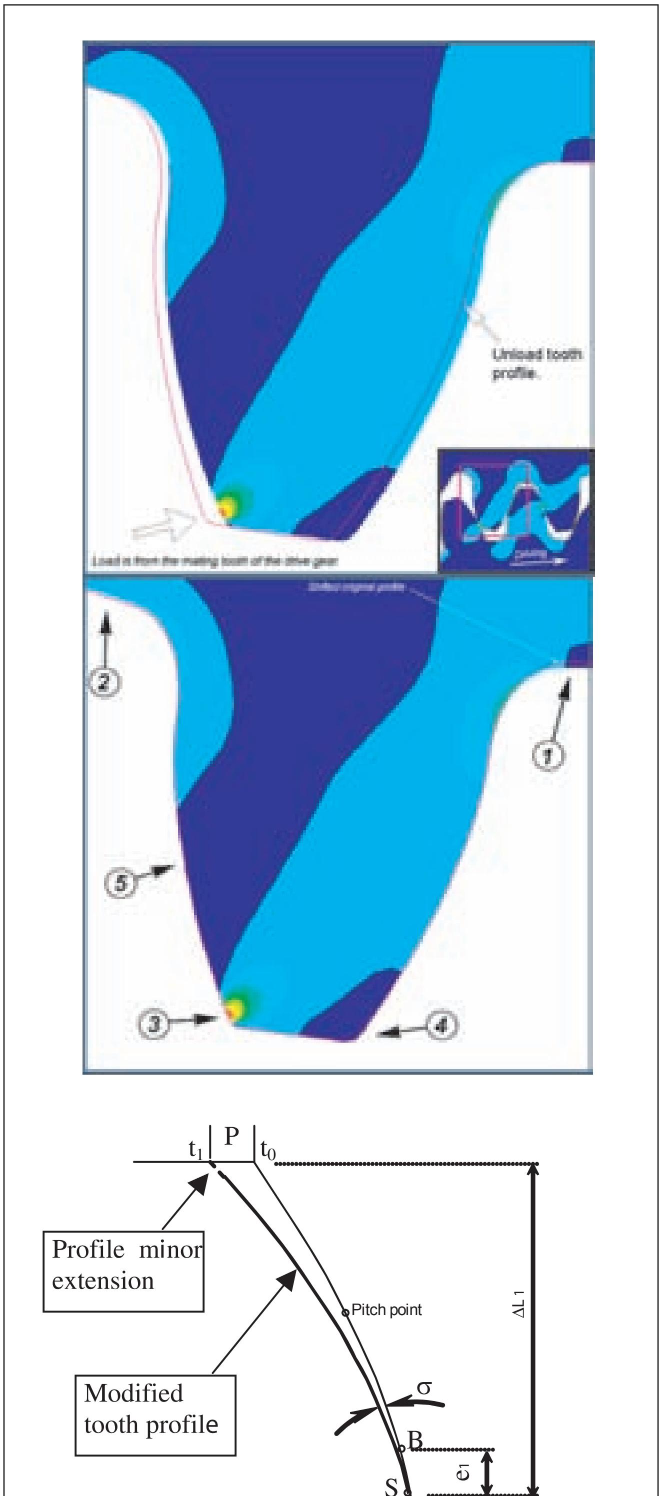


Figure 6—Local tooth deformation and profile modification.

Analysis with Bulk Tooth Rotation (BTR)-Type of Tooth Profile Modification

Bulk tooth rotation-type tooth profile modification was applied on the high-contact-ratio gears with a relief magnitude of 70 µm . The relief starting point was positioned at the profile rotating point, just under the base point. Thus the modified tooth profile simulated 1.6 degrees of bulk tooth rotation about the root. The results of FE simulation of BTR-type tooth profile modified gears over the mesh cycle are shown in Figure 7.

It can be seen that a smoother transmission error has been achieved for both the lighter load of 50 Nm and the higher load (design load) of 700 Nm, and these characteristics are also shown in the curves of combined torsional mesh stiffness. More significant is that the tooth contact stresses have been optimized over the load range. To confirm the 2D analysis results, the analysis was also confirmed with 3D modeling. The 3D results are compared as shown in Figure 8. Significant differences between 2D and 3D results were found in the tooth contact stresses, where the 2D stress for SEQV (von Mises stress) may be overstated compared to the 3D results, and the 2D third principal stress may be underestimated. However, the contact stress optimizations have been achieved. The major disadvantage of the BTR-type relief has been found in the tooth root stress as shown in Figure 9, where the tooth root stress could be increased up to 28% from that of involute gears when the input load is 300 Nm. However, there is only a minor increase of 4% at the input design load of 700 Nm.

Conclusions

Double-scope tooth profile modifications are attracting interest, as they show great potential for improving gear designs for applications including reduced noise and vibration. However, the research in this paper shows that neither previous nor current results can provide smooth transmission error curves at both design and light loads. Also, the tooth loading condition— especially the tooth contact stress—was still subject to high fluctuations over the mesh cycle. Tooth-loading condition is a critical parameter in high-contact-ratio gear applications, especially the tooth contact stress (or pressure) which is critical to the wear, pitting and operating temperature which was studied by Cornell (Ref. 9). For now, the application of double-scope tooth profile modifications still requires extensive study to ensure its effectiveness and its manufacturing possibilities.

An alternative tooth profile modification for high-contact-ratio gear applications—the bulk tooth rotation-type—has been introduced. Documentation in this paper has shown that BTR-type tooth profile modification can provide smooth transmission error pass in both light-load (cruise running) and full, heavy-load conditions (design load). Moreover, the tooth

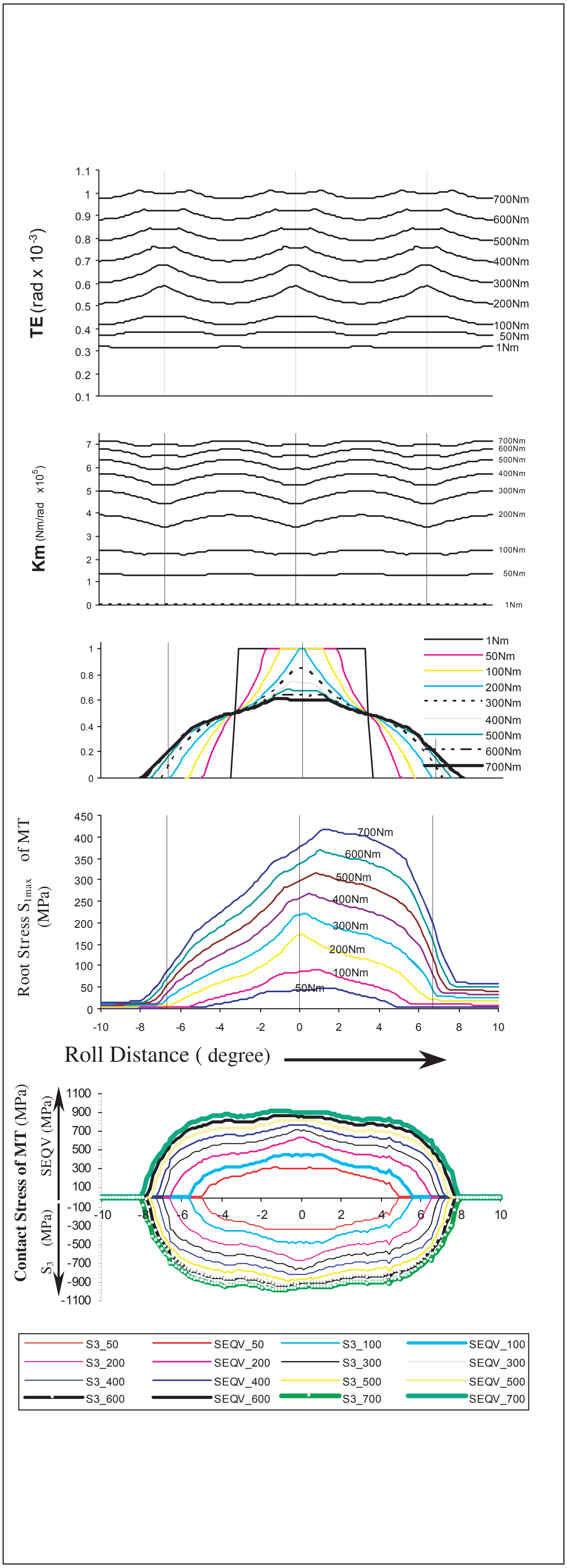


Figure 7—The changeover process under various input loads of bulk tooth rotation-type, relieved HCR gears (2D model).

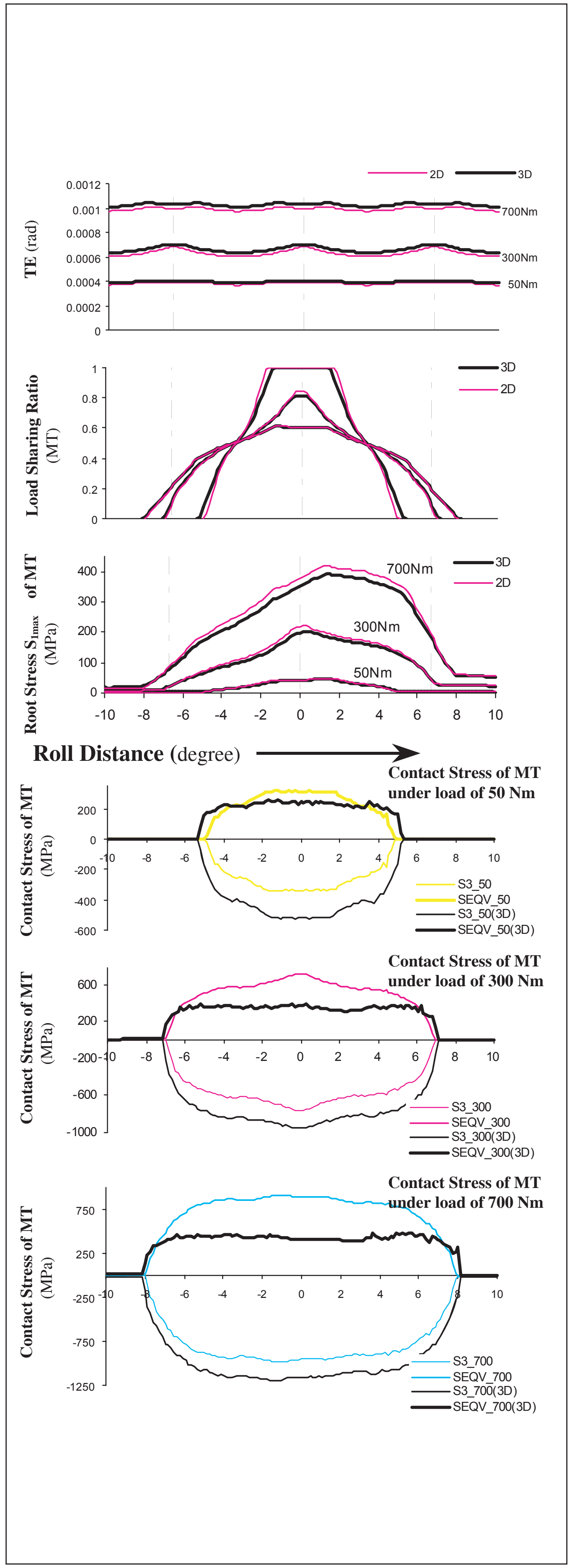


Figure 8—The changeover process under various input loads of bulk tooth rotation-type, relieved HCR gears (2D and 3D model).

pressure (contact stresses) has been smoothed out over the mesh cycle.

BTR-type tooth profile modification has general benefits for both low-contact-ratio gears and high-contact-ratio-gears. However, its major drawback—especially for high-contact-ratio gear applications—is slightly increased tooth root stress when the input load is below the design load. To that extent, this could affect the gears' fatigue life.

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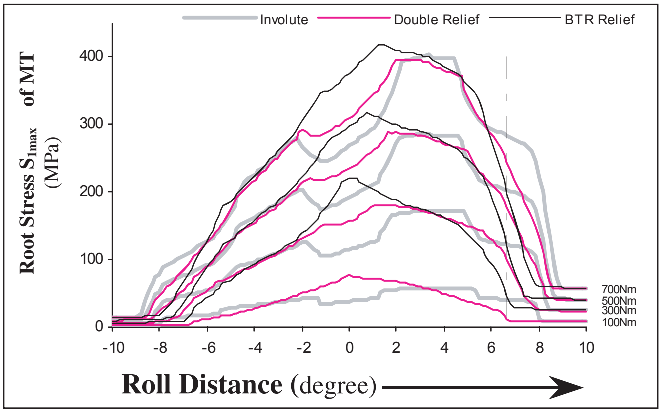

Figure 9—Tooth root stress comparisons.

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