The Effect of Straight-Sided Hob Teeth

Richard L. Thoen

Management Summary

It is well known that hobs with straight-sided teeth do not cut true involutes. In this paper, the difference between the straight side of a hob tooth and the axial profile of an involute worm is evaluated. It is shown that the difference increases as the diametral pitch increases, to the extent that for fine-pitch gearing, the difference is insignificant.

Introduction

The fact that hobs with straight-sided teeth do not cut true involutes has been known for a long time. Buckingham showed that the theoretical profile in the axial plane of an involute hob is not a straight line (Ref. 1). And Vogel portrayed an involute hob with curved—not straight-sided—teeth in the axial plane (Ref. 2).

The purpose of this article is to determine the space (gap) between the straight side of a hob tooth and the axial profile of an involute worm.

Nomenclature

r_b	Base radius
R	Reference radius (also known as pitch radius)
P	Transverse diametral pitch
P_n	Normal diametral pitch
r	Radial distance to involute and axial profiles
L	Lead
ϕ	Profile angle
Φ	Flank angle in transverse plane
Φ_n	Flank angle in normal plane
ψ	Helix angle at reference radius
ψ_b	Base helix angle
λ	Lead angle

Basic Geometry

The hob is really a worm with axial cutting gashes. A relatively simple method of computing the axial profile of an involute worm is based on an imaginary, triangular sheet tangent to the base cylinder—such as shown in Figure 1 (and in Ref. 2).

In Figure 2, the vertical line tangent to the base circle is of the same length as a string unwound from the base circle.

The length of the line is $r_b(\mathrm{inv}\phi + \pi/2)$

where, from well-known equations:

$r_b = R\cos\Phi$
$\tan\Phi = \tan\Phi_n / \cos\psi$
$R = N / 2P$
$P = P_n\cos\psi$

And $\mathrm{inv}\phi$ is obtained from $\cos\phi = r_b/r$, from which the $\mathrm{inv}\phi = \tan\phi - \phi$

where:

r is the radial distance to the involute and axial profiles (Figs. 1–2).

In Figure 1, the plane tangent to the base cylinder is an imaginary, triangular sheet that generates the helical tooth, in

continued

the same way that a point on a string generates a spur tooth.

The base helix angle (ψ_b) of the triangular sheet is found from:

$$\tan\psi = 2\pi R/L \text{ and } \tan\psi_b = 2\pi r_b/L$$

where:

$$L = 2\pi R/\tan\psi = 2\pi r_b/\tan\psi_b$$

where:

$$\tan\psi_b = \tan\psi\ (r_b/R)$$

Figure 1—Involute and axial profiles of an involute worm.

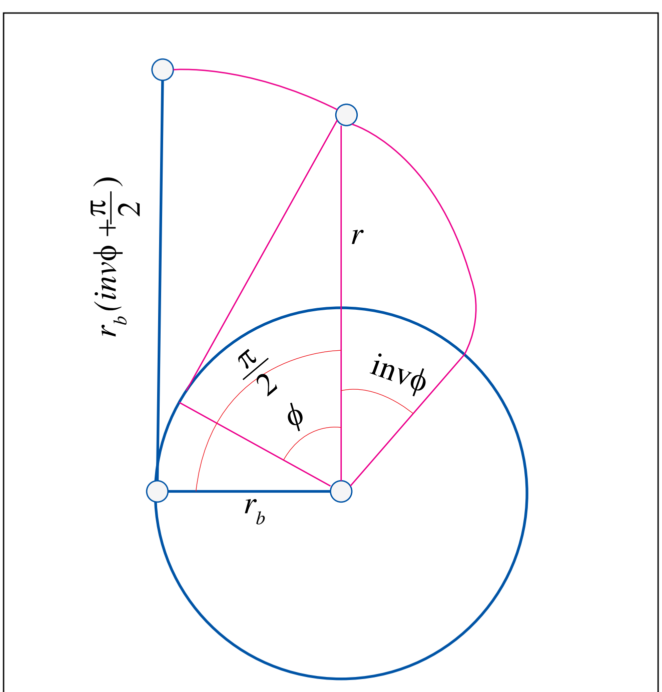

Figure 2—Transverse section of an involute worm.

where

$$r_b\ /\ R = R\cos\Phi\ /\ R = \cos\Phi$$

so,

$$\tan\psi_b = \tan\psi\cos\Phi$$

The horizontal distance (X) from the vertex of the base helix angle (ψ_b) to the transverse plane containing the r value is, from Figures 1 and 2:

$$\tan\psi_b = \frac{r_b\left(\text{inv}\phi + \frac{\pi}{2}\right)}{X} \quad (1)$$

$$X = \frac{r_b\left(\text{inv}\phi + \frac{\pi}{2}\right)}{\tan\psi_b} \quad (2)$$

This equation can be thought of as being that for X versus r in the axial plane.

To determine the gap between the hob and axial profiles for a specified value of r, the straight side of the hob tooth is superimposed on the axial profile at the reference radius ($r = R$), as shown in Figure 3. It should be noted that the hob profile is not tangent to the axial profile.

Numerical Example

It is not necessary to key in calculated values, because they can be stored in and recalled from a pocket calculator, such as the Hewlett Packard hp 33s, which can handle 12 digits.

Given $P_n = 20$, $\Phi_n = 20°$, $\lambda = 3°17'$:

Thus:

λ $= 17'/60 + 3° = 3.283°$

ψ $= 90° - \lambda = 86.71°$

R $= N/2P_n\cos\psi = 1/2•20\cos\psi = 0.4365$

$\tan\Phi$ $= \tan\Phi_n\ /\ \cos\psi$

Φ $= 81.05°$

$\cos\Phi$ $= 0.1554$

r_b $= R\cos\phi = 0.0678$

$\cos\phi$ $= r_b/R = R\cos\Phi/R = \cos\Phi$

$\text{inv}\phi$ $= \tan\Phi - \Phi = 4.940$

$\tan\psi_b$ $= \tan\psi\cos\Phi = 2.709$

$$X = \frac{r_b\left(\text{inv}\phi + \frac{\pi}{2}\right)}{\tan\psi_b} = 0.1630 \quad (3)$$

In Figure 3, for the hob profile:

$$y = 1/P_n = 0.05,$$

or $z = 0.05 \tan 20° = 0.0181$

so,

$$X_R + z = 0.184242$$

For the axial profile:

$r \quad = R + y = 0.4865$

$\cos\phi \quad = r_b / r,$

$\phi \quad = 81.98°$

$\mathrm{inv}\phi \quad = \tan\phi - \phi = 5.669$

$$X_{R+y} = \frac{r_b \left(\mathrm{inv}\phi + \frac{\pi}{2}\right)}{\tan\psi_b} \begin{array}{rl} = & 0.181295 \\ - & 0.181242 = X_R + z \\ \hline \text{Gap} = & 0.000053 \text{ inches} \end{array} \quad (4)$$

Conclusion

The gaps for various hobs are shown in Figure 4, wherein it is seen that there is an excess of material on the hob addendums (lack of material on gear addendums) and a lack of material on the hob dedendums (excess of material on gear addendums).

Figure 4 also shows that the gaps decrease as the diametral pitch increases. Indeed, for an 80-diametral pitch hob with 1°10′ lead angle, the largest gap is only one micro-inch. Conversely, for a 3.6-diametral pitch hob with 4°43′ lead angle, the largest gap is 0.0007 inches.

In the fine-pitch field, therefore, the effect of straight-sided hob teeth is negligible. As Louis Martin, chairman of the AMGA Fine-Pitch Committee from its inception in 1941 until 1953, stated (Ref. 3):

"The glaring mistake that has been made by the gear industry is to try to relate fine-pitch requirements with experience gathered from the coarse-pitch field."

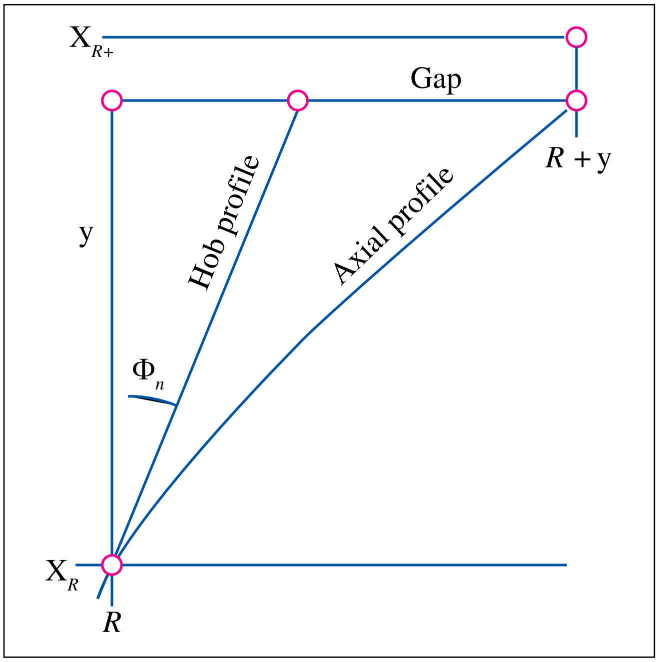

Figure 3—Determination of gap between hob and axial profiles.

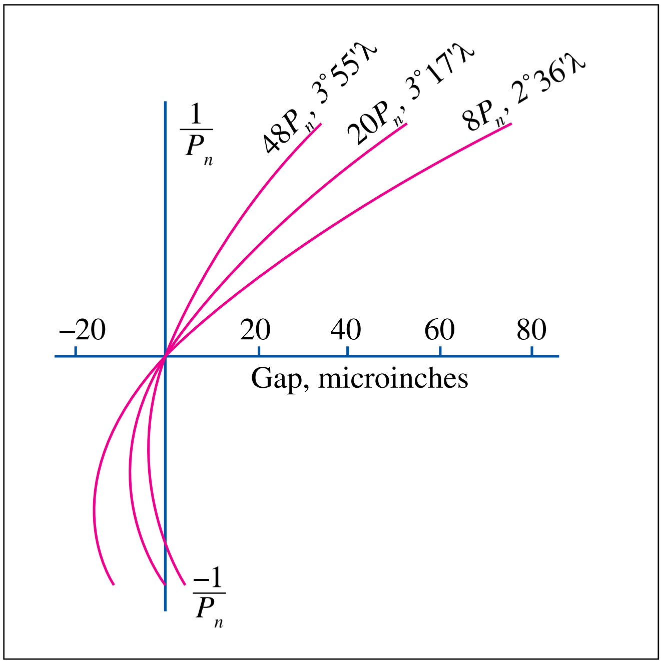

Figure 4—Gaps between hob and axial profiles for various hobs.

References

1. Buckingham, Earle. *Spur Gears*. McGraw Hill, 1928, pp. 274–375.
2. Vogel, Werner F. *Involutometry and Trigonometry*, Michigan Tool Co., 1948, Figure 26, p. 314.
3. Martin, Louis. Discussion at an AGMA convention, May 30, 1960.

Richard L. Thoen *is a consultant specializing in medium- and fine-pitch gearing. He is the author of several articles and papers on measurement, involute mathematics, statistical tolerancing and other gearing subjects.*