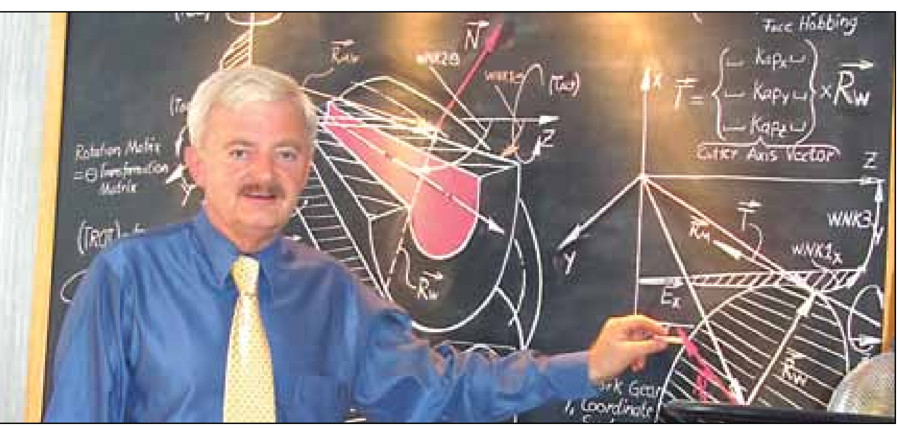
Tribology Aspects in Angular Transmission Systems

Part III: Zerol Bevel Gears

Dr. Hermann Stadtfeld

(This article is part three of an eight-part series on the tribology aspects of angular gear drives. Each article will be presented first and exclusively by Gear Technology*, but the entire series will be included in Dr. Stadtfeld's upcoming book on the subject, which is scheduled for release in 2011.)*



Dr. Hermann Stadtfeld *received a bachelor's degree in 1978 and in 1982 a master's degree in mechanical engineering at the Technical University in Aachen, Germany. He then worked as a scientist at the Machine Tool Laboratory of the Technical University of Aachen. In 1987, he received his Ph.D. and accepted the position as head of engineering and R&D of the Bevel Gear Machine Tool Division of Oerlikon Buehrle AG in Zurich, Switzerland. In 1992, Dr. Stadtfeld accepted a position as visiting professor at the Rochester Institute of Technology. From 1994 until 2002, he worked for The Gleason Works in Rochester, New York—first as director of R&D and then as vice president of R&D. After an absence from Gleason between 2002 to 2005, when Dr. Stadtfeld established a gear research company in Germany and taught gear technology as a professor at the University of Ilmenau, he returned to the Gleason Corporation, where he holds today the position of vice president-bevel gear technology and R&D. Dr. Stadtfeld has published more than 200 technical papers and eight books on bevel gear technology. He holds more than 40 international patents on gear design and gear process, as well as tools and machines.*

Design

If two axes are positioned in space and the task is to transmit motion and torque between them using some kind of gears, then the following cases are commonly known:

- Axes are parallel → cylindrical gears (line contact)
- Axes intersect under an angle → bevel gears (line contact)
- Axis cross under an angle → crossed helical gears (point contact)
- Axes cross under an angle (mostly 90°) → worm gear drives (line contact)
- Axes cross under any angle → hypoid gears (line contact)

Zerol bevel gears are the special case of spiral bevel gears with a spiral angle of 0°. They are manufactured in a single-indexing face milling process with large cutter diameters, an extra deep tooth profile and tapered tooth depth. The axis of Zerol bevel gears in most cases intersect under an angle of 90°. This so-called shaft angle can be larger or smaller than 90°; however, the axes always intersect, which means they have, at their crossing point, no offset between them (*Author's note: see also previous chapter, "General Explanation of Theoretical Bevel Gear Analysis" on hypoid gears*). The pitch

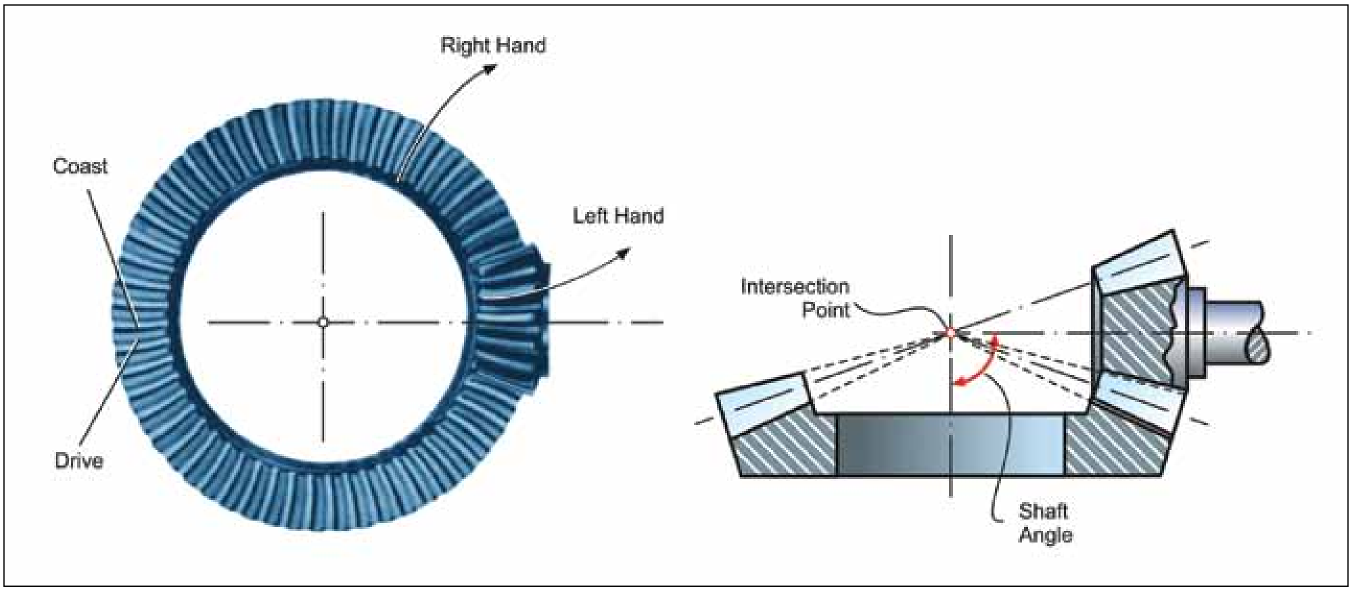


Figure 1—Zerol bevel gear geometry.

surfaces are cones, which are calculated with the following formula:

$$z_1/z_2 = \sin\gamma_1/\sin\gamma_2$$
$$\Sigma = \gamma_1 + \gamma_2$$
$$\text{in case of: } \Sigma = 90° \rightarrow \gamma_1 = \arctan(z_1/z_2)$$
$$\rightarrow \gamma_2 = 90° - \gamma_1$$

where:

z_1 Number of pinion teeth
z_2 Number of gear teeth
γ_1 Pinion pitch angle
Σ Shaft angle
γ_2 Gear pitch angle

The advantage of Zerol bevel gears is the low axial forces—like straight bevel gears—but their manufacturing process with completing face mill cutters is significantly faster, and grinding as a hard-finishing process with dressable grinding wheels leads to highly precise gear sets that are often used in aircraft applications.

Zerol bevel gear teeth follow in the face width direction a curve on the conical gear and pinion body that lies tangential to a cone element (zero spiral angle). The tooth lead function in the face width direction, if unrolled into a plane, is a circle. The tooth profile is an octoid. The tooth form with an octoid function will be associated with an initial "natural" profile crowning and, depending on the machining setup, some flank twist. Both effects are utilized together with certain corrective machine settings in order to generate the desired crowning (*see also "General Explanation of Theoretical Bevel Gear Analysis"*).

Figure 1 shows an illustration of a Zerol bevel gear set and a cross-sectional drawing. Per definition, Zerol gears are manufactured in a single-indexing process, applying a standard tooth taper, as shown in Figure 1. However, it is also possible to apply the face hobbing process with parallel-depth teeth in order to manufacture bevel gears with zero spiral angle. Those gears are not considered true Zerol gears, however, because the slot width taper—due to face hobbing—causes a crossover and fins at the root bottom that are often not acceptable as a production result because of top interference with the opposite member and increased root bending stress.

Analysis

Since the mentioned distortions in tapered-depth tooth systems are detected through comparison to conjugate mating flanks, it is possible to define potential contact lines that would apply in case of no distortions and conjugate flank surfaces. In order to allow deflections of tooth surfaces, shafts, bearings and gearbox housing without unwanted edge contact, a crowning in face width and profile direction is applied. A theoretical tooth contact analysis (TCA) previous to the gear manufacturing can be performed in order to observe the effect of the crowning in connection with the basic characteristics of the particular gear set. This also affords the possibility of returning to the basic dimensions in order to optimize them

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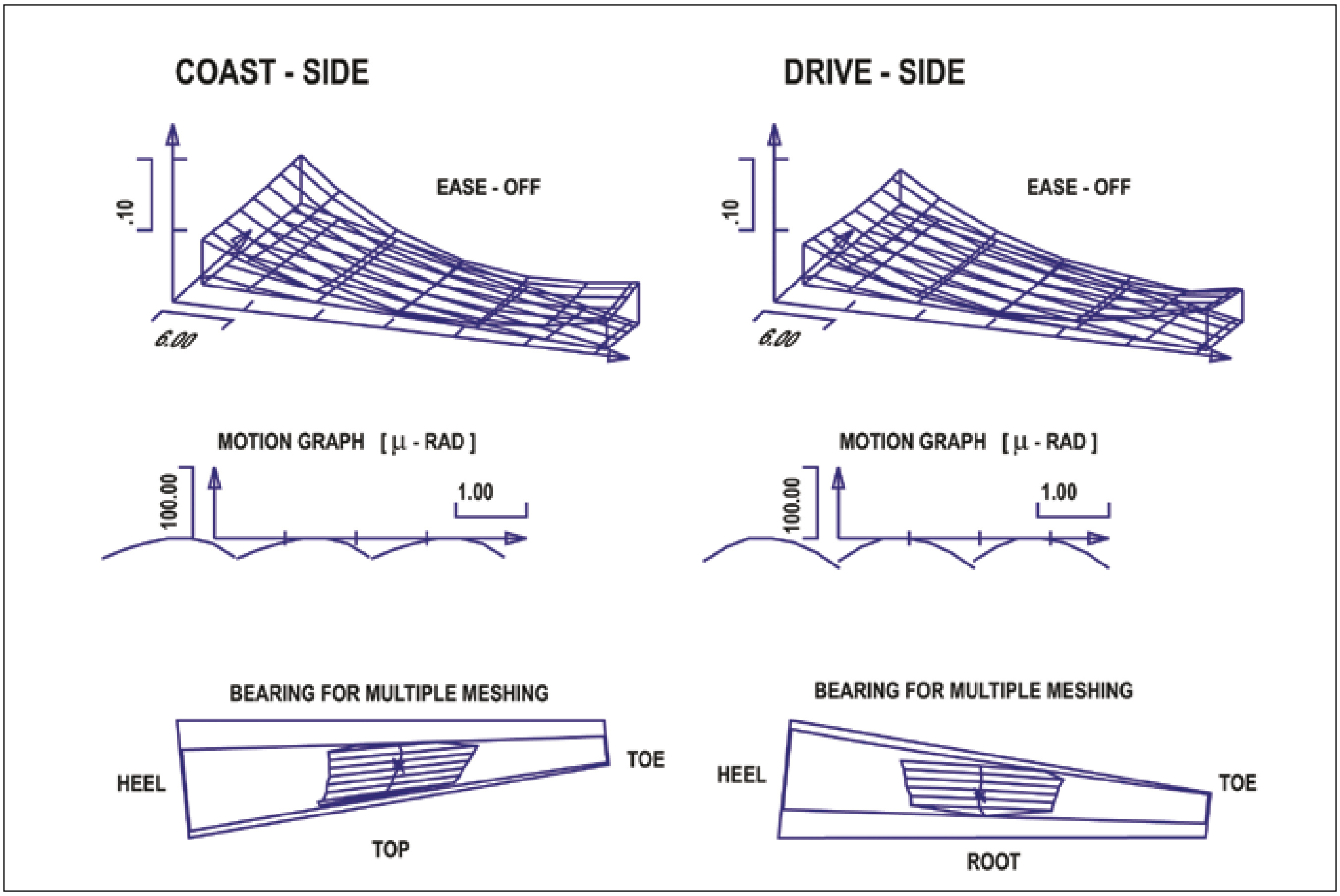

Figure 2—Tooth contact analysis of a Zerol bevel gear set.

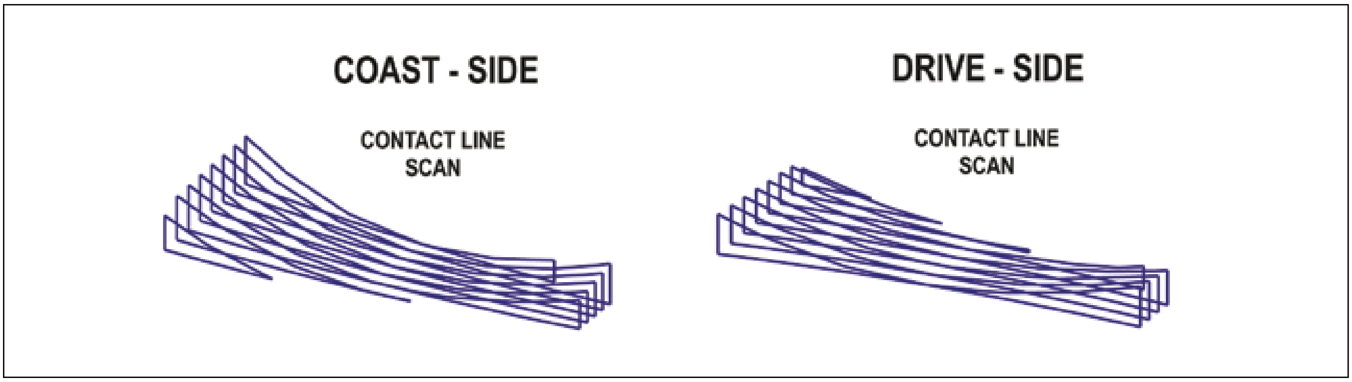

Figure 3—Contact line scan of a Zerol bevel gear set.

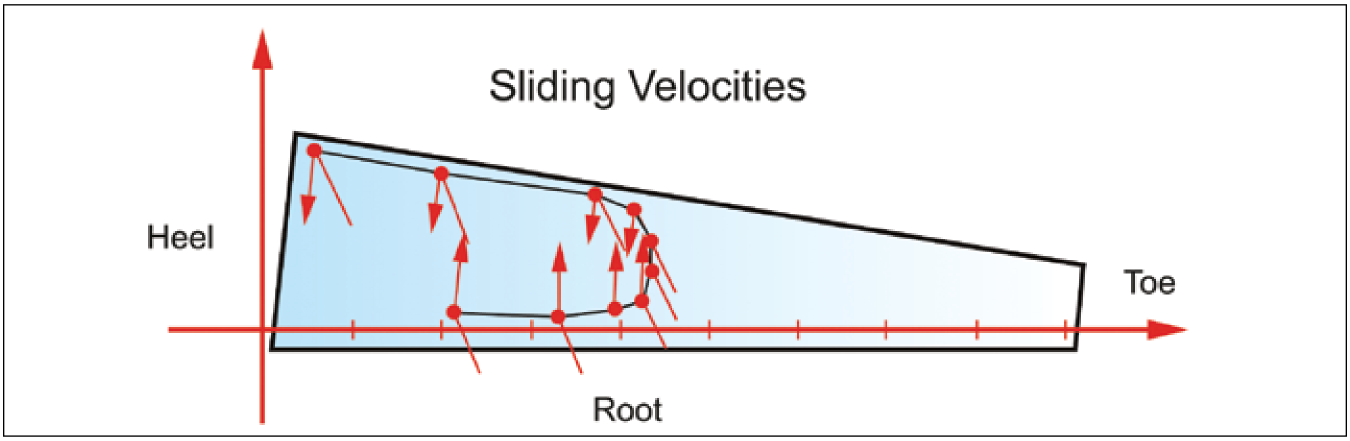

Figure 4—Rolling and sliding velocities of a Zerol bevel gear set along the path of contact.

if the analysis results show any deficiencies. Figure 2 shows the result of a TCA of a typical Zerol bevel gear set.

The two columns in Figure 2 represent the analysis results of the two mating flank combinations (*see also "General Explanation of Theoretical Bevel Gear Analysis"*). The top graphics show the ease-off topographies. The surface above the presentation grid shows the consolidation of the pinion and gear crowning. The ease-offs in Figure 2 have a combination of length and profile crowning, such that a clearance along the boundary of the teeth is established.

Below each ease-off, the motion transmission graphs of the particular mating flank pair are shown. The motion transmission graphs show the angular variation of the driven gear in the case of a pinion that rotates with a constant angular velocity. The graphs are drawn for the rotation and mesh of three consecutive pairs of teeth. While the ease-off requires a sufficient amount of crowning in order to prevent edge contact and allow for load-affected deflections, the crowning in turn causes proportional amounts of angular motion variation of about 50 micro radians in this example.

At the bottom of Figure 2, the tooth contact pattern is plotted inside of the gear tooth projection. These contact patterns are calculated for zero load, and a virtual marking compound film of 6 μm thickness. This basically duplicates the tooth contact one could observe by rolling the real version of the analyzed gear set under light load on a roll tester, while the gear member is coated with a marking-compound-layer of about 6 μm thickness. The contact lines are oriented in the face width direction, depending basically on the 0° spiral angle. The path of contact connects the beginning and end of meshing, and its orientation is nearly perpendicular to the contact lines.

The crowning reflected in the ease-off results in a contact zone located inside of the boundaries of the gear tooth. A smaller tooth contact area generally results from large magnitudes in the ease-off and in the motion graph, and vice versa.

Figure 3 shows 10 discrete, potential contact lines with their individual crowning amounts along their length (contact line scan). The gap geometry in contact line direction can be influenced by a change in ease-off topography and optimized regarding the gap kinematic cases (*see also "General Explanation of Theoretical Bevel Gear Analysis," Fig. 8*). The gap geometry perpendicular to the contact line direction (not exactly the same as the path-of-contact direction) does not significantly depend on the ease-off topography, but is mainly dominated by the geometry of the mating tooth profiles.

Figure 4 shows the sliding and rolling velocity vectors of a typical Zerol gear set for each path of contact point for the 10 discussed roll positions. Each vector is projected to the tangential plane at the point-of-origin of the vector. The velocity vectors are drawn inside the gear-tooth-projection plane. The points-of-origin of both rolling- and sliding-velocity vectors are grouped along the path of contact, which is found as the connection of the minima of the individual lines in the contact-line-scan graphic (Fig.4). The velocity vectors can be separated in a component in contact-line direction and a component perpendicular to that—in order to investigate the hydrodynamic lubrication properties—by utilizing the information from the contact line-scan (curvature and curvature change) and the tooth surface curvatures perpendicular to the contact line direction (*see also "General Explanation of Theoretical Bevel Gear Analysis," Fig. 8, cases 1–6*).

In the example of the discussed Zerol bevel gear set, the sliding-velocity vectors are basically profile-oriented. In the top area, the sliding vectors point to the root. Moving along the path of contact from top to bottom, the sliding-velocity reduces its magnitude while attaining a magnitude of zero at the pitch line. Below the pitch line, the sliding-velocity develops, growing positive magnitudes (towards the root of the gear tooth). The maximal magnitude of the sliding-velocities (top-versus-root) is a result of the distance from the pitch line. In the present case, the distance between the lowest active flank line to the pitch line is larger than the distance from the pitch line to the top. The rolling-velocity vectors point to the root and have basically all the same orientation. The orientation is a result of the spiral angle (zero spiral angle delivers profile-oriented rolling). The shrinking magnitude of the rolling-velocity (moving from top to bottom) is caused by the decreasing circumferential speed towards the outer diameter.

The freedoms for optimizing the lubrication gap geometry and kinematics in Zerol bevel

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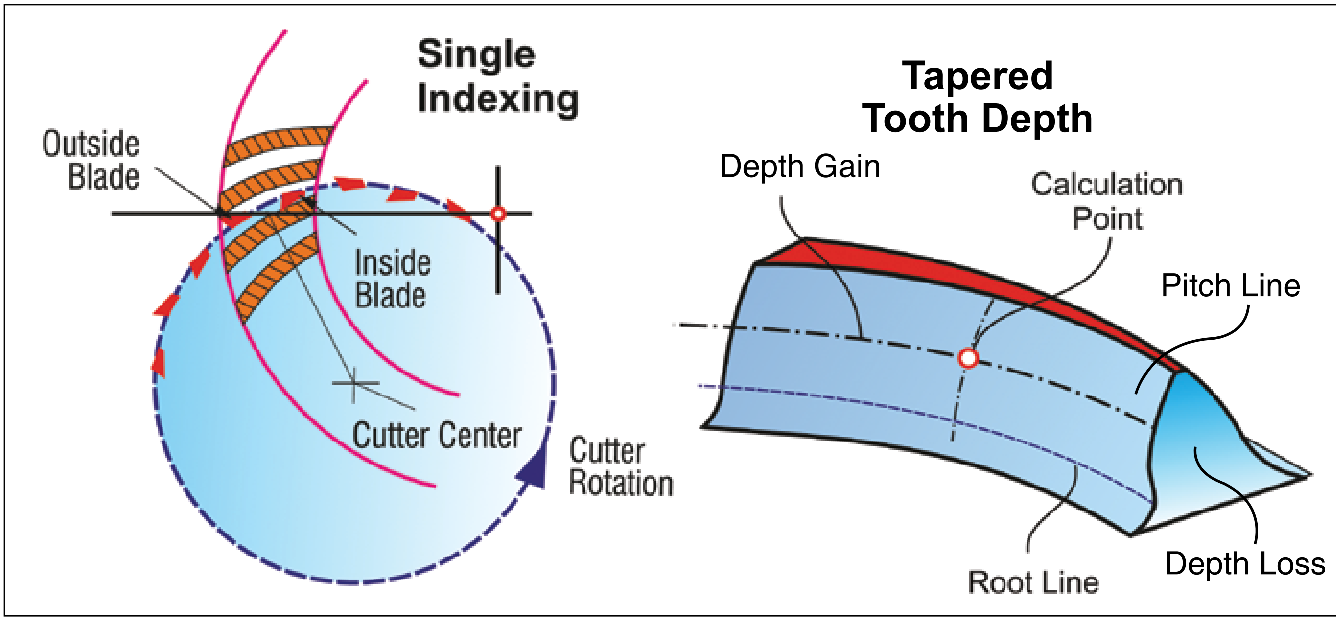

Figure 5—Face milling process and tapered tooth depth.

Figure 6—Zerol bevel gear cutting.

gears are limited to the relocation of the pitch line and a change in crowning in length direction.

Manufacturing

Zerol bevel gears are manufactured in a single-indexing face milling process. In the face milling process, the blades are oriented around a circle and pass through one slot (while they plunge or generate the flanks of that particular slot), as illustrated in Figure 5. The work is not performing any indexing rotation. At the blade tip and in equidistant planes (normal to the cutter head axis), the slot width produced has a constant width between toe and heel. In order to achieve a proportionally changing slot width (and tooth thickness), the root line of face milled bevel gears is inclined versus the pitch line (Fig. 5, right). This modification has to be implemented in both members, which is why the face angle requires the same modification as the root angle of the mating member.

Figure 6 is a photo of the view into the work chamber of a free-form bevel and hypoid gear cutting machine during the high-speed dry-cutting of a Zerol bevel gear. The face cutter head has coated carbide stick blades that are arranged in blade groups of one inside and one outside blade oriented around a circle.

Hard-finishing after heat treatment, if required by the particular application, is generally done by grinding. The grinding wheel resembles the cutter-head geometry, while the grinding machine uses the same set-up geometry and kinematics as the cutting machine for the previous soft machining.

Application

Most Zerol bevel gears to be used in power transmissions are manufactured by carburizing steel and undergo a case-hardening to a surface hardness of 60 Rockwell C (HRC) and a core hardness of 36 HRC. Because of the higher pinion revolutions, it is advisable to give the pinion a higher hardness than the ring gear (e.g., pinion 62 HRC, gear 59 HRC).

Regarding surface durability, Zerol bevel gears are very similar to straight bevel gears. At the pitch line, the sliding-velocity is zero, and the rolling-velocity under certain loads cannot maintain a surface-separating lubrication film. This might in certain cases of high load or speed result in pitting along the pitch line that can destroy the tooth surfaces and even result in tooth flank fracture. However, it is possible that the pitting can be stabilized if the damage-causing condition is not often represented in the duty cycle. Figure 7 is a photograph of typical pitch line pitting on a Zerol bevel ring gear flank surface.

Zerol bevel gears have axial forces that can be calculated by applying a normal force vector at the position of the mean point at each member (*see also "General Explanation of Theoretical Bevel Gear Analysis"*). The force vector normal to the transmitting flank is separated in its X, Y and Z component (Fig. 8).

The relationship in Figure 8 leads to the following formulas, which can be used to calculate bearing force components in a Cartesian coordinate system and assign them to the bearing load calculation in a CAD system:

$$F_x = -T \,/\, (A_m \bullet \sin\gamma)$$

$$F_y = -T \bullet (\cos\gamma \bullet \sin\alpha \,/\, (A_m \bullet \sin\gamma \bullet \cos\alpha)$$

$$F_z = T \bullet (\,\sin\gamma \bullet \sin\alpha) \,/\, (A_m \bullet \sin\gamma \bullet \cos\alpha)$$

where:

T	torque of observed member
A_m	mean cone distance
γ	pitch angle
α	pressure angle
F_x, F_y,F_z	bearing load force components

The bearing force calculation formulas are based on the assumption that one pair of teeth transmits the torque with one normal force vector in the mean point of the flank pair. The results are good approximations that reflect the real bearing loads for multiple-tooth meshing within an acceptable tolerance. A precise calculation is for example possible with the Gleason bevel and hypoid gear software.

Zerol bevel gears have lesser axial forces than spiral bevel gears. The axial-force component due to the spiral angle is zero. Zero spiral angle minimizes the face contact ratio to zero but results in the maximal tooth root thickness.

As a rule, bevel gears that are not ground or lapped after heat treatment show the highest root strength with the lowest spiral angles. This explains why in those cases Zerol and straight bevel gears are still the bevel gears of choice.

Zerol bevel gears can operate with regular

Figure 7—Pitch line pitting on a Zerol bevel gear surface.

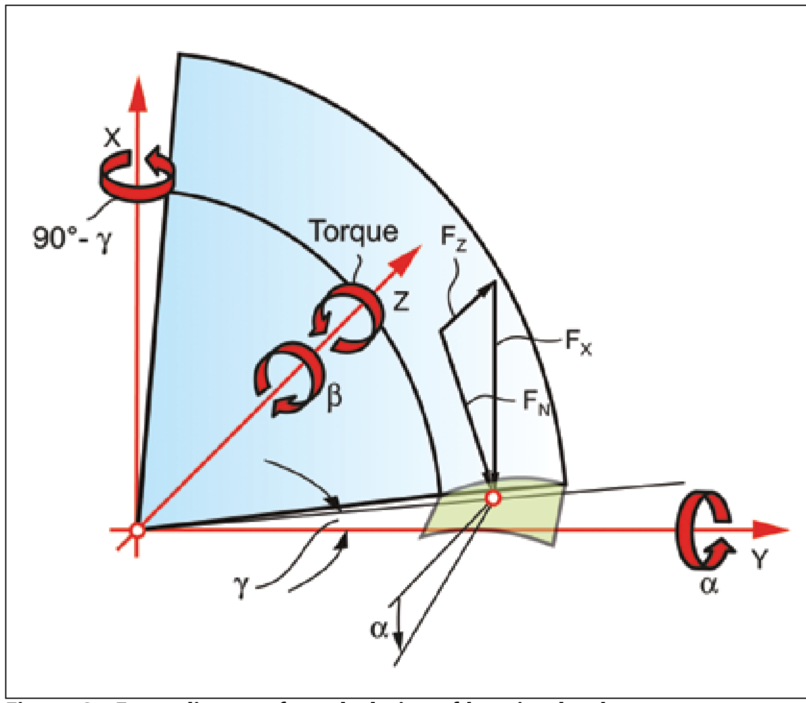

Figure 8—Force diagram for calculation of bearing loads.

transmission oil or, in the case of low RPMs, with a grease filling. With circumferential speeds above 10 m/min., a sump lubrication with regular transmission oil is recommended. The oil level has to cover the face width of the teeth that are the lowest in the sump. More oil causes foaming, cavitations and unnecessary energy loss. There is no requirement for any lubrication additives. The preferred operating direction of Zerol bevel gears is the drive side, where the convex gear flank and the concave pinion flank mesh together. In the drive direction (Fig. 8), the forces between the two mating members bend the pinion sideways and axially away from the gear, generating the most backlash. Coast-side operation reduces the backlash in extreme cases to zero, which interrupts any lubricant flank separation and leads to immediate surface damage—which is often followed by tooth fracture.

Next issue: Spiral Bevel Gears.