Calculation of the Tooth Root Strength of Worm Wheel Teeth Based on Local Stresses

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Calculating the root strength of worm wheel teeth is necessary to more accurately predict the formation of cracks. The starting point of this investigation was the difference between the analytical calculation of the tooth root strength based on the shear stress and fatigue tests. The strength calculated by the numerical analysis differed from that of the actual strength. An analytical method based upon the equivalent stress (von Mises stress) rather than the formerly used shear stress was investigated first. The results showed some improvement, but through a detailed investigation, additional deficiencies were identified. The best method for meeting the requirements was found in the use of local stresses. These stresses can be determined using a finite element analysis (FEA). According to the FKM guideline "Analytical strength assessment of components in mechanical engineering" the local stresses obtained from the FEA can be used to determine the fatigue strength of the worm wheel teeth. This paper explains the use of this algorithm and shows the improved results.

Introduction

Worm gear sets have a simple layout that creates a large transmission ratio in single stage. Because of the sliding rate, these types of gear drives are quiet in operation. The current technology uses a case-hardened worm and a bronze worm wheel to optimize friction and strength (Fig. 1, left). Because the worm gear is made of bronze, the teeth are the critical points of failure. The standard DIN 3996 (Ref. 1) provides design guidelines that include wear, pitting and fracture. In cases of high torque and low speed, the root strength of the teeth is the dominant design criterion (Fig. 1, right).

Reason for Investigation

To find the gear best suited for a particular application, it is important to know the maximum load that the gear can bear. The failure limits for wear and pitting are well known from many studies, so the calculations in these cases are sufficiently precise. The limits of the strength of the tooth root are less well known because there have been only a few investigations of this subject. The current practice is to calculate the tooth root strength based upon

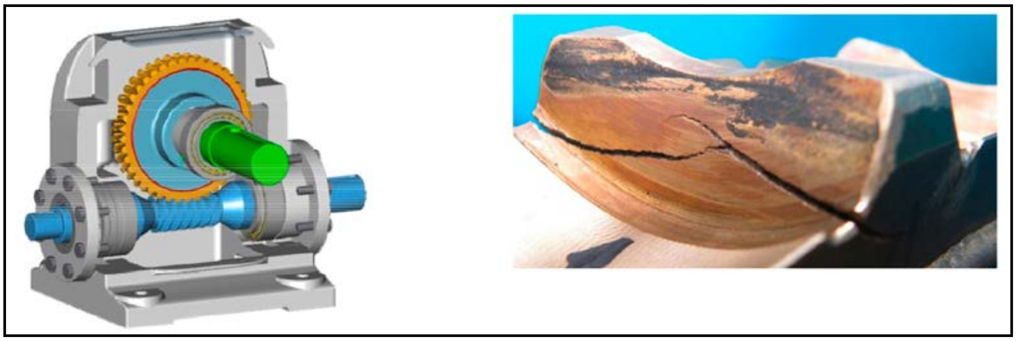

Figure 1 (Left): CAD model of the worm gear set; right: tooth with cracks at the tooth root notch.

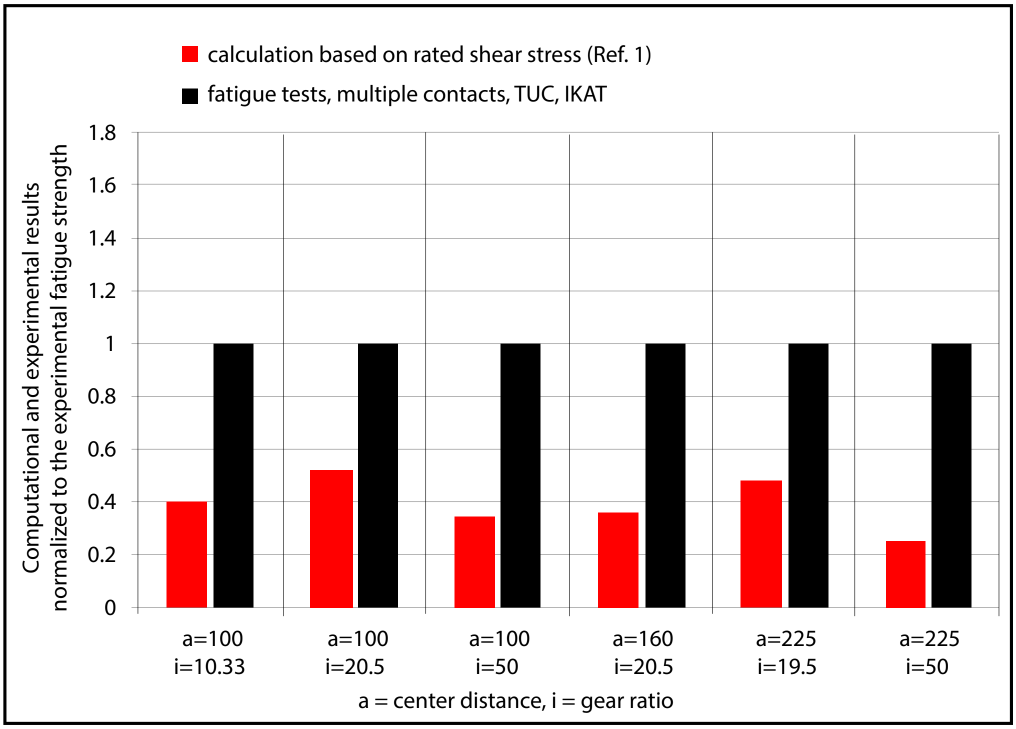

Figure 2 Comparison of calculated forces based on shear stress (red) and measured forces (black) normalized to the fatigue strength (Ref. 2).

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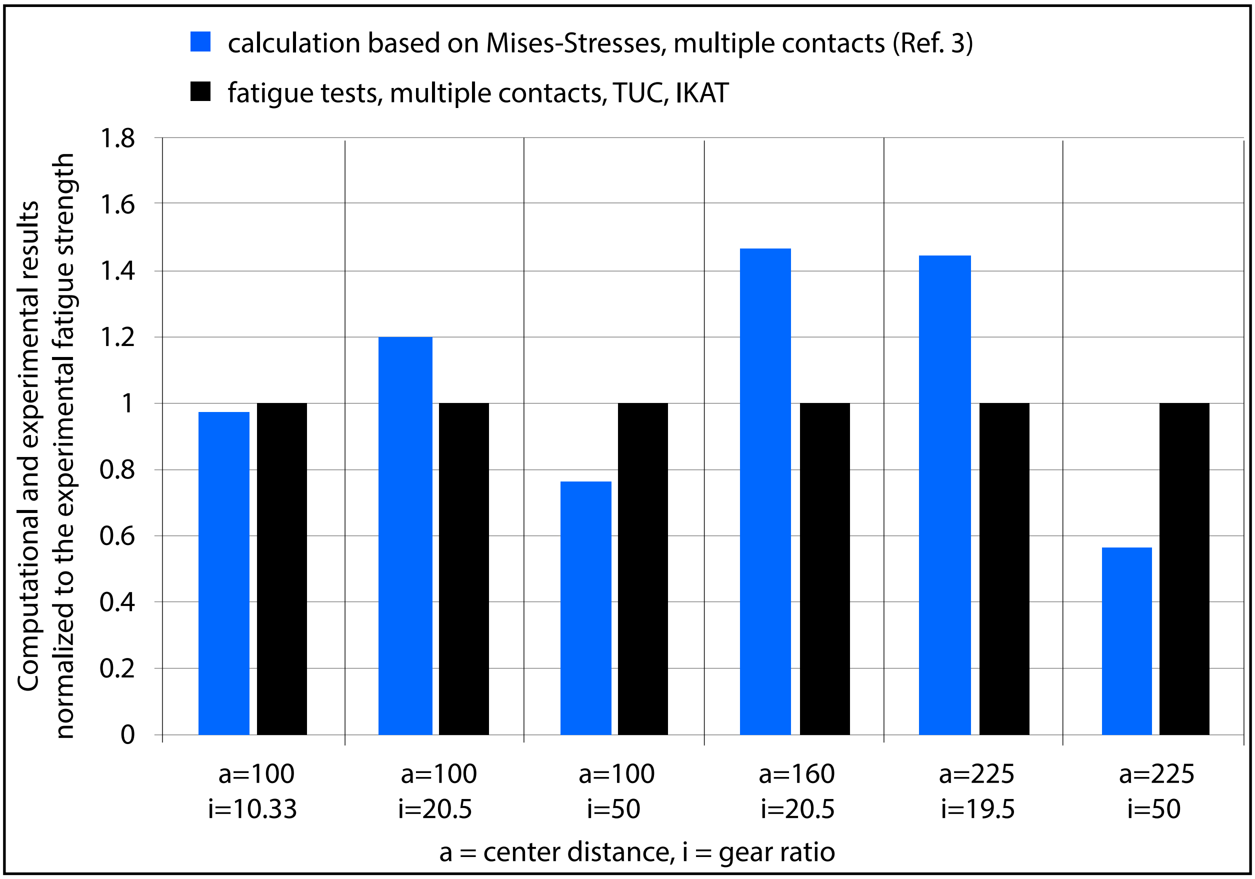

Figure 3 **Comparison of calculated forces based on von Mises stress (blue) (Ref. 2) and measured forces (black) normalized to the fatigue strength.**

the nominal shear stress. This method is easy to use and is well suited for small gears up to a center distance of $a = 100$ mm because it was adjusted with tests on small gears. However, for all gears the prediction of lifetime given by this method underestimates the actual lifetime (Fig. 2).

Because the load on the tooth root consists of a shear stress, a bending stress and a compressive stress, focusing only on the shear stress underestimates the actual load. One attempt to improve the prediction of the tooth strength was to use the von Mises stress as a replacement for the shear stress (Ref. 6). The results of this calculation method showed an improvement for a wider variety of worm gears, but this method has the same weaknesses overall (Fig. 3).

Because the geometry is not considered, an optimization of the geometry is not possible. One reason for the shortfall is the improper consideration of the geometric details of the tooth. The analytical method is based on the geometry in the middle of the tooth, but the cracks start at the edges of the tooth. Therefore, the locations of the calculated and actual failures are not the same (Fig. 4).

When the von Mises stress is used, the tip circle diameter da2 is used as the lever to calculate the bending stress. The outside diameter de_2 is not taken into account like the embedding of the tooth because of the globoid, too. In contrast to the standard method, in the method based on the von Mises stress, the notch effect of the rounding of the tooth root is taken into account.

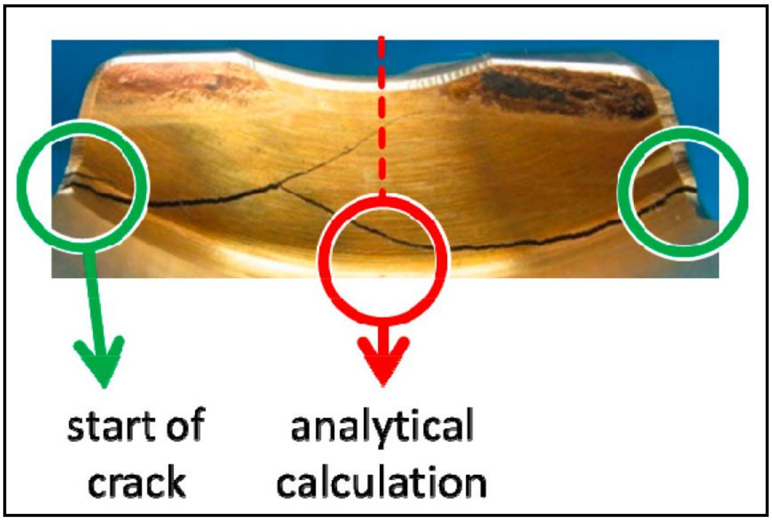

Figure 4 **Difference between the calculated and the actual point-of-failure (Ref. 3).**

Another reason is the older data in the standard for the material properties for bronze.

To solve these problems the complete geometry, the influence of the tooth contact, and the relative mounting locations of the worm and the wheel must be included. It would not be possible to include these effects using an analytical model based on rated stresses. The use of local stresses is more suitable than rated stresses, especially in the case of components with complex shapes. To determine the local stresses, a numerical method was chosen.

Figure 5 Framework of the method based on local stresses.

Research Program and Test Procedures

The task was to develop and to test a method based on the numerical evaluation of local stresses. This method includes an input for the geometry, a finite element program with a mesh generator, an input for boundary conditions, a solver and an algorithm to evaluate the fatigue life from the computed stresses (Fig. 5).

To evaluate the strength of the tooth root, the FKM guideline is an appropriate tool. Currently, however, the FKM guideline is valid only for steel and aluminum alloys. To include other metals, experiments on specimens including complete worm gears and finite element analyses were performed. For the method to be accurate, the strength limits must be known. Therefore, fatigue tests were performed on actual components using a hydraulic fatigue tester (Fig. 6). These results formed the basis for the comparison of the various methods.

Computation of stresses at tooth root. A simulation was required to compute the local stresses and to determine the parameters mentioned in the previous section. To evaluate the local stresses, the point at which the maximum occurs must be identified. This point is located on the tension side at the end face of the tooth (Fig. 7). The stresses obtained at this point were used to estimate the strength according to the FKM guideline (Ref. 4).

Using the parameters from actual worm gear sets, virtual models were created with CAD software. Next, with FEA software, meshes were generated and boundary conditions were defined for the model (Fig. 8). The stresses at the tooth root caused by applied forces were analyzed with the FEA software.

Calculation process. The calculation process was obtained from the FKM guideline (Ref. 4). To evaluate the worm wheel according to the FKM Guideline, both the static strength and the fatigue strength must be calculated; the procedures are shown in Figure 9.

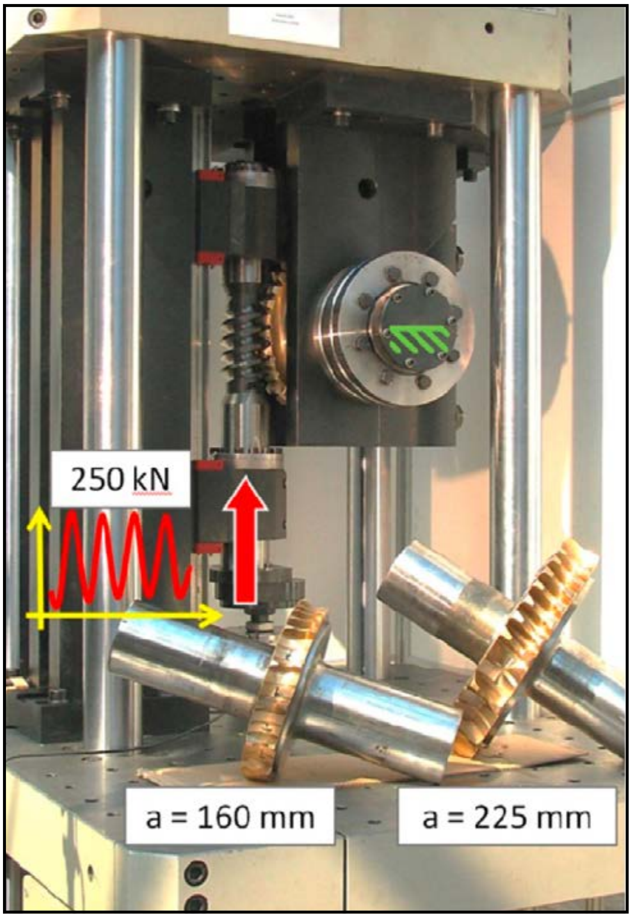

Figure 6 Hydraulic tester used for the fatigue tests.

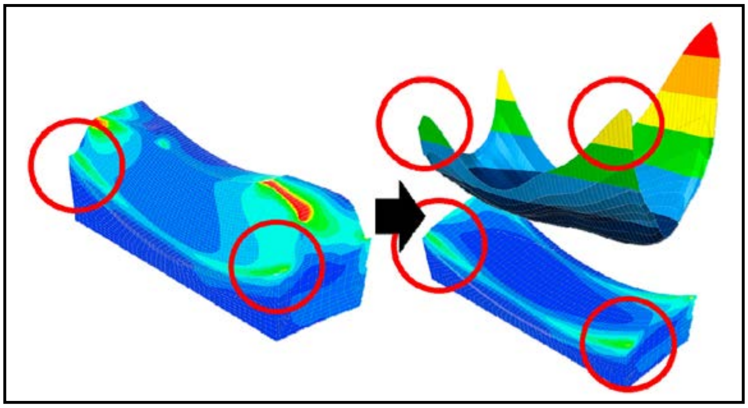

Figure 7 Points of maximum local stresses in the tooth root.

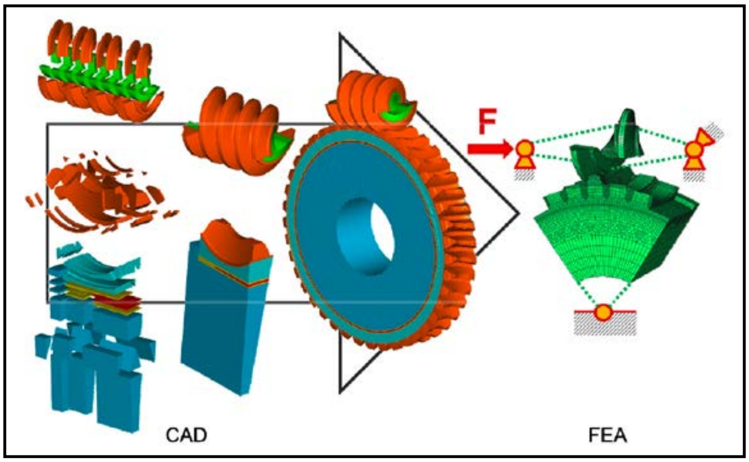

Figure 8 Procedure to construct the FEA model.

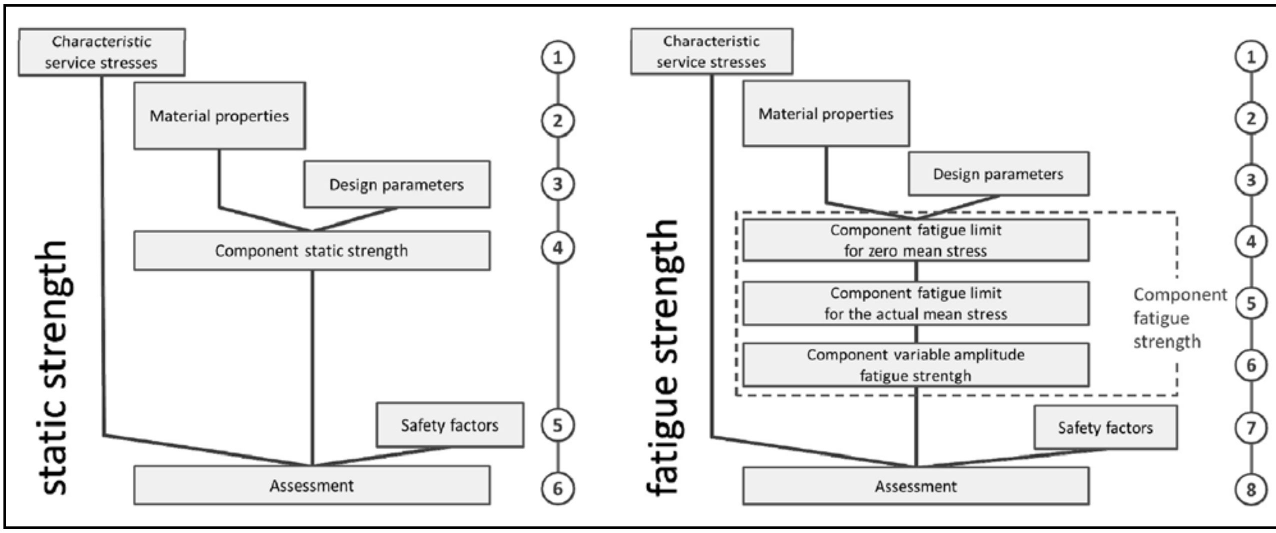

Figure 9 Calculation process for static strength and fatigue strengths (Ref. 4).

Both values are based on the determination of a degree of utilization. The utilization is determined by comparing the stress limit σ_{AK} with the working stress. The bearable stress σ_{AK} is evaluated on the basis of the tensile strength, the geometrical parameters and the material properties. The local stresses were determined from the finite element analysis, and the material properties were investigated through laboratory tests.

Determining parameters for bronze. The challenge was to determine the constants a_G and b_G to calculate the notch sensitivity n_σ and defining the constants a_M and b_M to identify the factor for the mean stress M_σ, which depends on the material. In addition, contributory factors that were required for the calculation were obtained. The tests shown in Figure 10 were performed to obtain these factors.

To determine the notch sensitivity n_σ in Equation 1 for $0.1 < G_\sigma < 1$, the stress gradient G_σ and the two factors a_G and b_G for bronze were required.

$$n_\sigma = 1 + \sqrt[4]{G_\sigma \cdot \mathrm{mm}} \cdot 10^{-\left(a_G + \frac{R_m}{b_G \cdot \mathrm{MPa}}\right)} \qquad (1)$$

The FKM guideline provides an equation to calculate the gradient G_σ, which is shown in Equation 2. The two required local stresses, the one at the surface of the tooth root $\sigma_{1,2}$ and the other at the next node under the surface in the vertical direction $\sigma_{1,2\_\Delta s}$, can be computed in the FEA. The distance Δs between the two points was measured (Fig. 11).

$$G_\sigma = \frac{1}{\Delta s} \cdot \left(1 - \frac{\sigma_{1,2\_\Delta s}}{\sigma_{1,2}}\right) \qquad (2)$$

To calculate a_G and b_G, the notch factor K_f was identified for a notch that was similar to the rounding of the tooth root used in four-point bending tests (Ref. 3) with a notched specimen. The form factor K_t was determined using the FKM guideline according to Reference 4. With these two factors the notch sensitivity n_σ can be calculated from the test results using Equation 3.

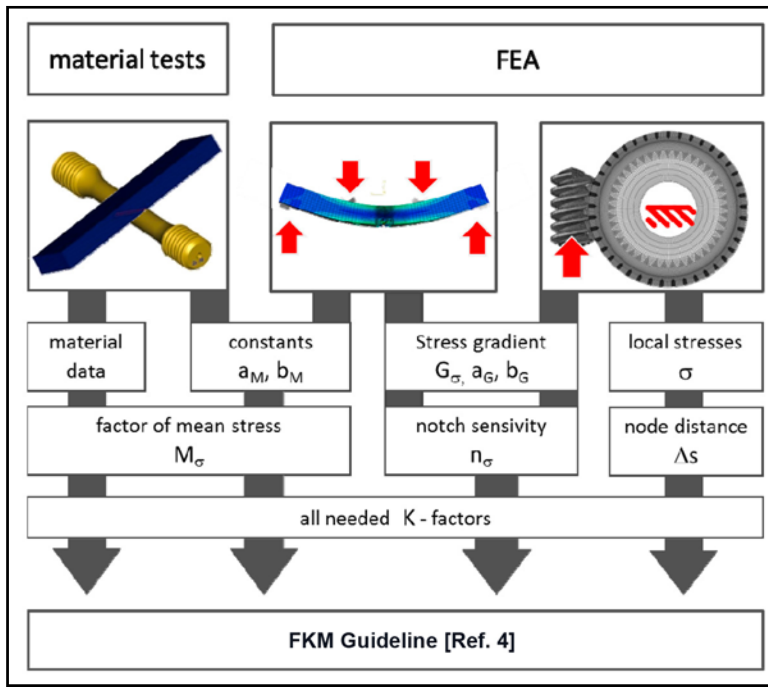

Figure 10 Adaption and validation of the FKM guideline.

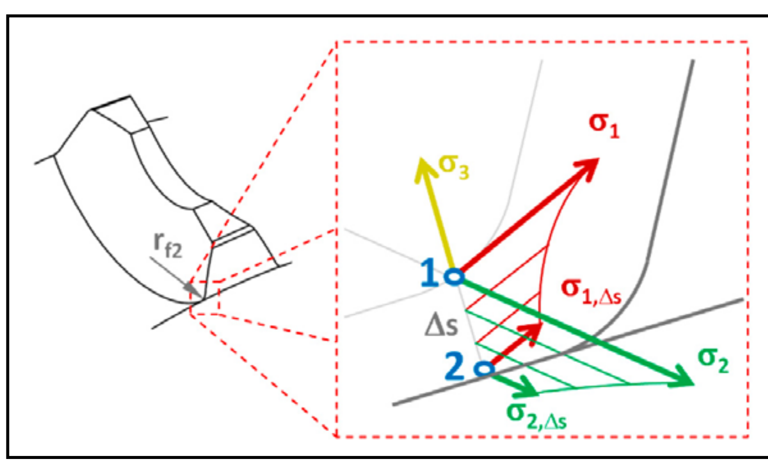

Figure 11 Locations of the principal stresses in the tooth root.

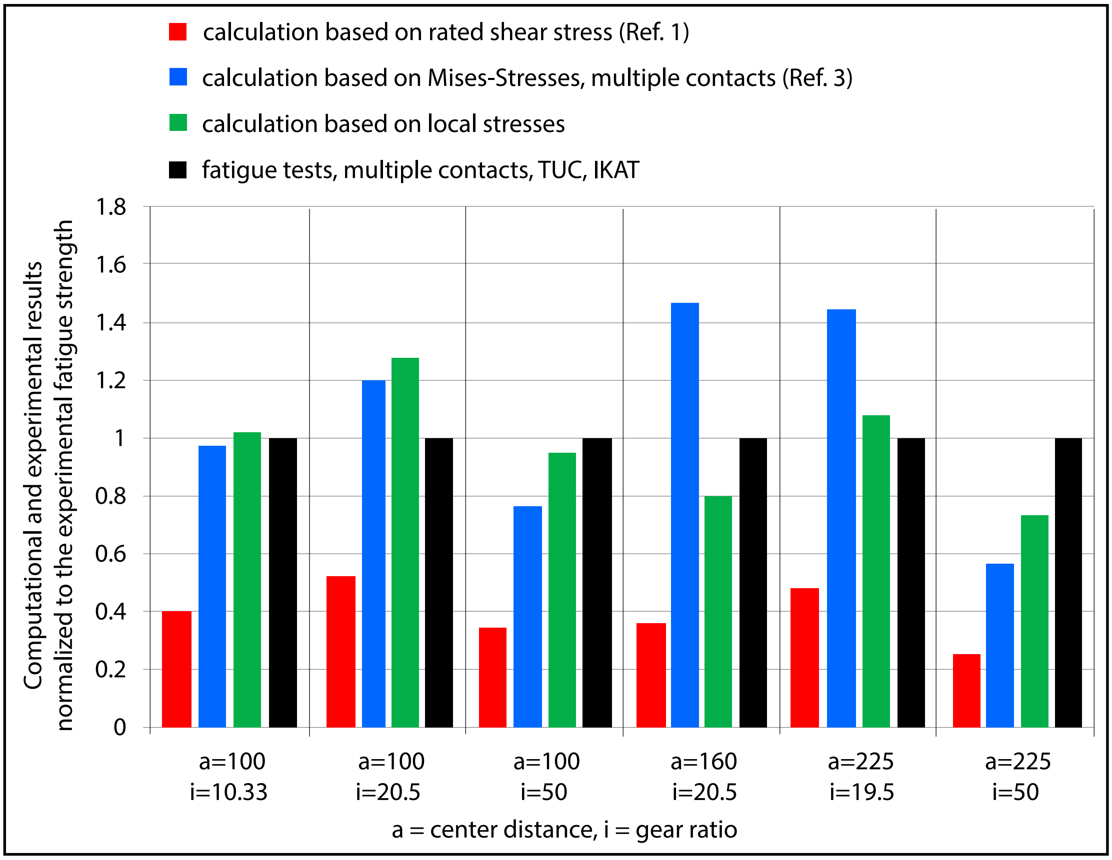

Figure 12 **Comparison of calculated force based on local stresses (green) and measured force (black) normalized to fatigue strength.**

The notch factor K_f can be calculated from the form factor K_t and the notch sensitivity n_σ:

$$K_f = \frac{K_t}{n_\sigma} \tag{3}$$

Given n_σ, the values of a_G and b_G can be determined. These two parameters, which are given in Table 1, are valid for the alloy CuSn12Ni.

Table 1 Parameters a_G and b_G

a_G	see [Ref. 3]
b_G	see [Ref. 3]

The next factor to consider is the mean stress, M_σ, which describes the dependency between the mean stress and the stress amplitude and depends on the material. To determine M_σ as in Equation 4, the material constants a_M and b_M were required.

$$M_\sigma = a_M \cdot 0.001 \cdot \frac{R_m}{\text{MPa}} + b_M \tag{4}$$

Both constants were reversely specified using the mean stress factor $K_{AK\sigma}$ (Eq. 5). It is possible to calculate this factor from the fatigue bending strength σ_{AK} obtained from the bending tests and the alternating bending strength σ_{WK} (Ref. 5). The values for the bronze alloy CuSn12Ni were determined to be $\sigma_{AK} = 280$ MPa and $\sigma_{WK} = 140$ MPa. The factor for residual stresses $K_{E\sigma}$ was set to 1 in accordance with the FKM guideline.

$$K_{AK\sigma} = \frac{\sigma_{AK}}{\sigma_{WK} \cdot K_{E\sigma}} \tag{5}$$

With the value of $K_{AK\sigma}$ and Equation 6, M_σ could be specified iteratively. The mean stress σ_m and the stress amplitude σ_a were obtained from the four-point bending tests, whereas σ_a was determined from the stresses obtained from the FEA and Equation 7. In knowing M_σ, the parameters a_M and b_M for the bronze alloy tested could be identified (Table 2).

Table 2 Mean stress coefficients a_M and b_M for bronze

a_M	see [Ref. 3]
b_M	see [Ref. 3]

$$K_{AK\sigma} = \frac{\dfrac{1 + M_\sigma/3}{1 + M_\sigma}}{1 + \dfrac{M_\sigma}{3} \cdot \dfrac{\sigma_m}{\sigma_a}} \tag{6}$$

$$\sigma_a = \frac{1}{2} \cdot \sigma_{1,2FEA} \tag{7}$$

Because the tension-compression fatigue stress σ_{Wzd} was unknown, Equation 8 was invoked using the tensile strength R_m and the tension-compression fatigue stress factor $f_{W\sigma}$, for which guidelines recommend a value of 0.3.

$$\sigma_{Wzd} = f_{W\sigma} \cdot R_m \tag{8}$$

All other required factors, including the K factors, were determined according to the guideline. The factor $K_{NL,E}$, which describes the non-linear, elastic range of the stress-strain behavior and depends on the material, was determined (Ref. 3) for the bronze alloy used. The roughness factor R_z was measured to determine the coefficient $K_{R\sigma}$.

The factor for the edge layer K_V and the factor K_S, which include the influence of a protective layer, were set to 1.0. With all these factors, the parameter $K_{WK\sigma}$ is calculated as:

$$K_{WK\sigma} = \frac{1}{n_\sigma} \cdot \left(\left(1 + \frac{1}{K_f} \cdot \left(\frac{1}{K_{R\sigma}} - 1 \right) \right) \cdot \frac{1}{K_V \cdot K_S \cdot K_{NL,E}} \right) \quad (9)$$

Validation of results. Figure 12 shows the comparison between the fatigue strength determined from the fatigue tests and the calculated strength based on the various prediction methods. The investigated force limits were normalized to the experimental values. The solid black columns show the fatigue force, which was determined from fatigue tests on actual components. This was the reference by which the results of the various methods were evaluated.

The columns with the checkerboard pattern show the fatigue force calculated on the basis of the rated shear stress. These forces are much lower than the forces from the tests, so this method underestimates the actual strength. The striped columns show the fatigue force calculated based on the von Mises stress. The results from this method are closer to those from the fatigue tests, but the strength is overestimated in certain cases. This problem can be solved by introducing a factor based on the geometry of the worm wheel teeth. At this point, no factor was included to adjust the calculated results to match the fatigue tests.

The dotted columns show the forces from the method based on local stresses. In comparison with the fatigue forces from the tests, this method shows the best results of the three computational methods. These results show the importance of computing the root strength of the worm wheel teeth. In addition, the usefulness of the FKM guideline for bronze materials was shown.

Summary and Outlook

To choose the appropriate worm gearing for a specific application, the strength of the root of the worm wheel tooth is important, especially in cases with low speed and high torque. This paper compared the results of the calculation of worm wheel stresses based on local stresses as well as on nominal stresses. It was demonstrated that the analytical method is not very precise due to the specific geometry of the worm wheel. It was then shown that a method based on local stresses fits well with the test results. This method was developed by adapting the FKM guideline for the material properties of bronze.

A further improvement is possible as an improvement of the FEA model; to include the operational stability, the amount of damage D should be taken into account (Ref. 6).

This will be the subject of future research.

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