Optimal Flank Forms for Large Bevel Gears

Dr.-Ing. Hartmuth Müller

At first sight the appearance of 5-axis milling for bevel gears opens new possibilities in flank form design. Since in comparison to existing machining methods applying cutter heads no kinematic restrictions exist for 5-axis milling technology, any flank form can be machined.

Nevertheless the basic requirements for bevel gears did not change. Specifications and functional requirements like load carrying capacity and running behavior are still increasing demands for design and manufacturing. This paper describes the demands for gear design and gives an overview about different design principles in the context of the surrounding periphery of the gear set.

Free-Forming Tooth Forms

During the past years many papers had been published showing completely different approaches to flank shapes of gears. A so called S-type was presented on several machine tool shows worldwide claiming a significantly higher load carrying capacity compared to conventional spiral bevel gears (Ref. 1).

The S-type gear returns to mind the principle of herring bone gears (Fig. 1). The special lengthwise shape of the teeth will increase the contact ratio and reduce axial forces. Allegedly the load carrying capacity is to be 35% higher compared to a spiral bevel gear with same outer dimensions.

A detailed loaded tooth contact analysis of the S-type gear shows a significant disadvantage of such designs. Since the loaded contact is split into two contact patterns the possibility for spreading the contact with increasing load is rather limited. Therefore the Hertzian pressure distribution shows two excessively high maxima (Fig. 2). Another disadvantage is the course of the root bending stress. Conventional spiral bevel gears show the maximum in the middle and a decrease to the outer ends. The tooth root stress at the heel and the toe is very low. The S-type gear shows a lower maximum, but rather high values at the outer limits of the tooth. This is due to the very high stiffness of the tooth in the middle section caused by the S-type shape. In real applications the fear is that cracks will initiate in the root at the heel and toe area and propagate rather quickly.

Beside this rather exotic design of a spiral bevel gear set, old mathematical principles are considered for flank forms of bevel gears in another design study. A spherical involute tooth lengthwise in shape is considered (Ref. 2). Figure 3 shows a spherical involute bevel gear set.

The characteristic of any involute curve is its equidistance to itself. With cylindrical involute gears, a change in the axis distance will not affect the tooth mesh conditions. With spherical involute bevel gears, a change in the shaft angle will not change the contact conditions of meshing teeth. Any other displacement will be sensitive to the contact conditions.

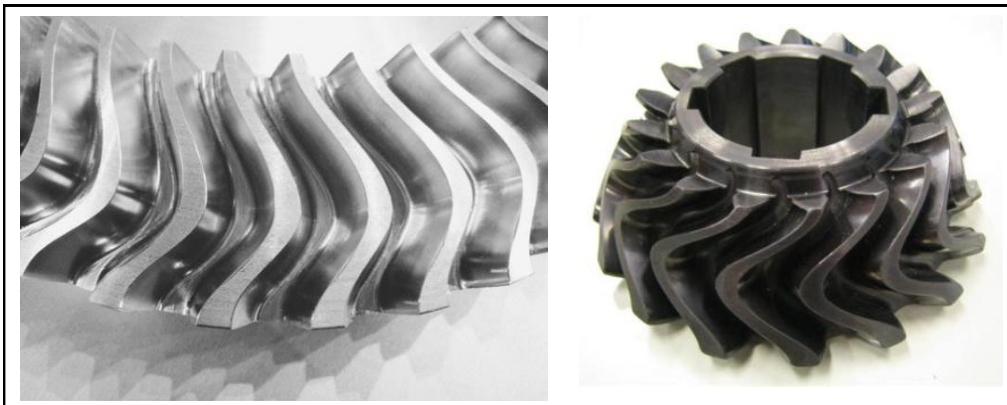

Figure 1 S-type gear (Courtesy Bierens).

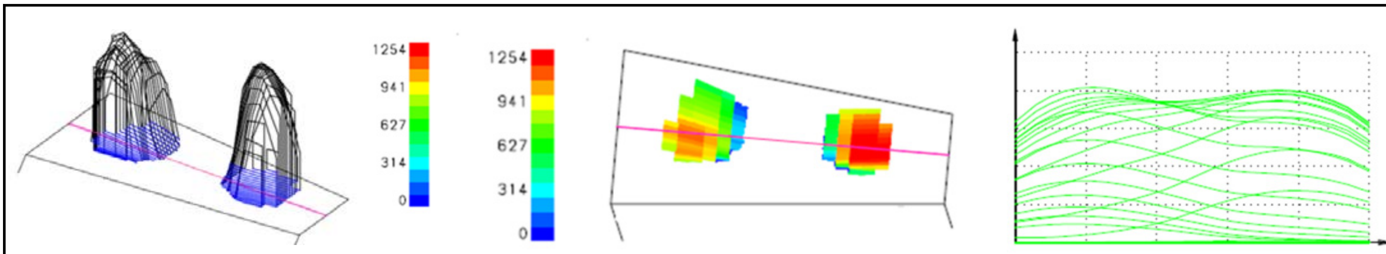

Figure 2 Loaded contact and root bending stress of S-type gear.

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Load Conditions

Typical gear stresses are always cyclical and are divided into bending stress, contact stress, compressive stress and shear stress. The material therefore needs to have a hard surface layer with residual, compressive stress, being well connected to a more ductile core handling the shear and bending stress. The larger the contact area, the lower will be the contact stress — and the more load that can be transmitted. Bending stress requires material properties like toughness and ductility. This is contradictory to the requirements for contact stress. Therefore carburizing and quenching offer the best conditions for designing a graded material.

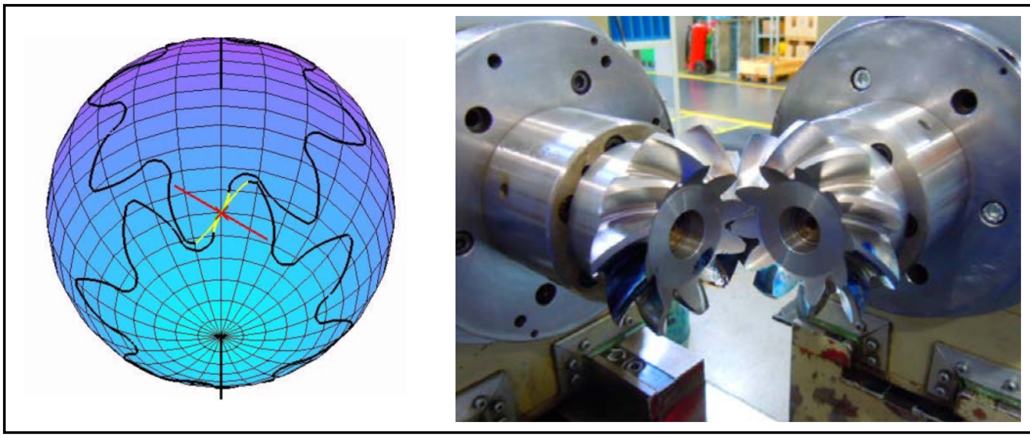

Figure 3 Spherical involute bevel gear set.

Design-relevant aspects of load handling properties are maximizing the tooth contact area and avoiding any stress maxima at the edges of the tooth; any tooth form being favorable to this aim will be reasonable. In addition, a high contact ratio is favorable since this will distribute the load to more teeth and consequently reduce the load-per-tooth pair.

Displacements

When transmitting torque the teeth will be deformed elastically. The higher the torque, the wider the loaded contact patter will be. Also, beyond tooth deflection, all components of a gearbox will be deflected. This load-induced deformation is caused by the tooth forces being perpendicular to the tooth flank surface when friction is negligible.

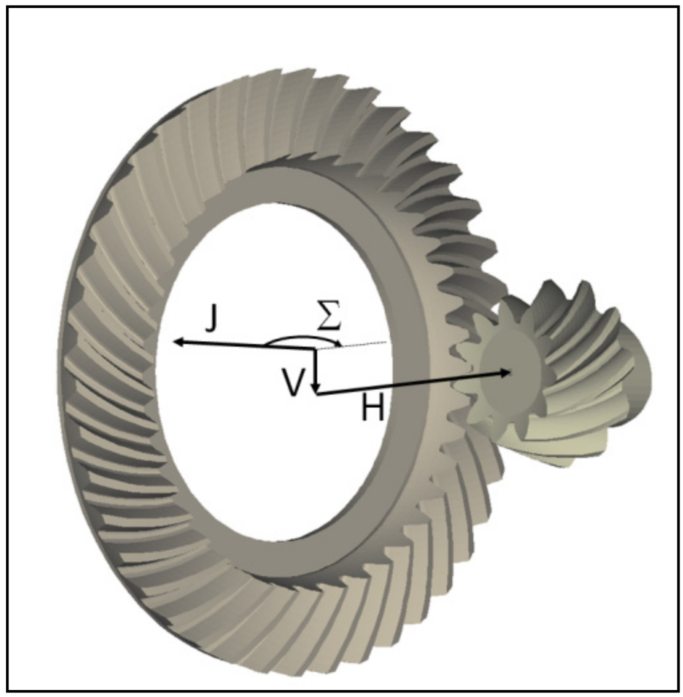

Figure 4 Load-induced displacements of a bevel gear set.

Balancing the safety factors for bending stress and contact stress is done by selecting the module and mean spiral angle. Any spiral angle other than zero will cause axial tooth forces. The general case of load-induced displacements of a bevel gear set is shown (Fig. 4). The tooth forces will move ring gear and pinion axially (*H* and *J*), in direction of the hypoid offset (*V*) and will change the shaft angle (Σ).

For achieving an optimal flank form for bevel gears there are two targets to meet:

- Using the complete flank area for the loaded contact pattern
- Insensitivity of contact pattern position under tooth-force-induced displacements

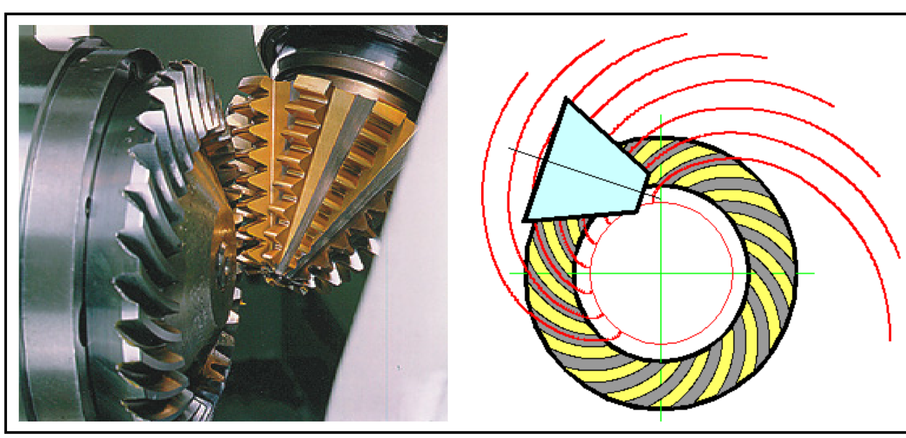

Figure 5 Palloid method with involute shape of tooth trace.

Considering the first aspect will exclude any flank shape that does not allow to loaded contact to spread over the complete flank area. S-type gears for example do not fulfill this requirement and will not help improving the load carrying capacity. The potential of spherical involute bevel gears with the insensitivity in shaft angle displacements will only bring improvements as long as displacements in *V*, *H* and

J are negligible in relation to Σ displacements.

Different Lengthwise Shapes

Even new flank forms are possible with 5-axis milling, while the advantages of classical spiral bevel gears still predominate. The loaded contact pattern can spread all over the flank area; the spiral angle increases the contact ratio and spreads the load to several pairs of teeth, and the root bending stress has its maximum in the middle of the face width. In the following, five different bevel gear principles are shown and compared to each other.

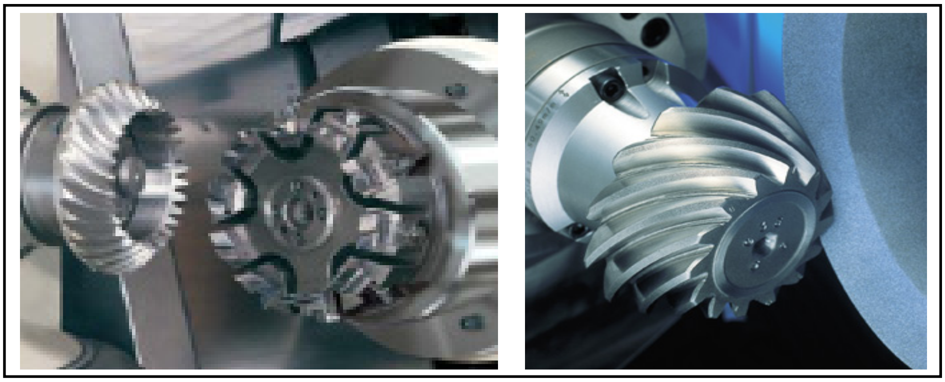

Figure 6 **Cyclo-Palloid and Wiener methods.**

Palloid. For the Palloid method a tapered hob is the cutting tool. The motion of the hob relative to the work piece creates an involute tooth lengthwise profile (Fig. 5). The involute tooth trace form is very insensitive to H and J displacements.

Cyclo-Palloid and Wiener. These gearing systems shown in Figure 6 use a face cutter-type tool to cut or grind the teeth of the bevel gear. The tooth trace shape for Cyclo-Palloid is an elongated epicycloid and an arc for Wiener.

Both systems show the same displacement behavior, depending on the tool diameter (Fig. 7).

The left part shows a large tool diameter and the right figure a small tool diameter. Since the tooth forces of a spiral bevel gear set will increase the horizontal position of the pinion $\Delta H > 0$ and the ring gear $AJ > 0$ and decrease the offset $\Delta V < 0$ the advantage of the small tool radius becomes obvious. If $\Delta H = -\Delta V$ the position of the contact pattern of the right part (Fig. 6) would not change at all. This situation is called the rectangular case (Ref. 3).

Comparison

The comparison of different gear designs is done for the gear data shown (Fig. 8). For all five designs the Ease-Off geometry is identical. Gear set No. 1 is the Palloid design; No. 2 is a Cyclo-Palloid design with a tool radius of 84 mm corresponding to the rectangular case; No. 3 is a Cyclo-Palloid design with a tool radius of 100 mm; No. 4 and No. 5 are Wiener designs in which No. 4 is a rectangular case. The differences in these designs are the different tooth trace shapes.

Figure 9 shows the course of the spiral angle over the face width. It can be seen that the involute lengthwise shape of the Palloid design has the biggest change in spiral angle, whereas

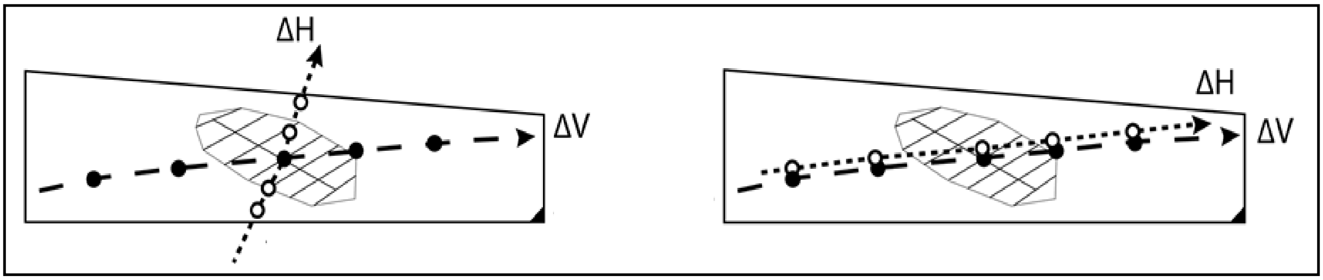

Figure 7 **Influence of the tool radius on V and H displacements.**

	Pinion	Gear	
Number of teeth	19	36	-
Shaft angle	90.0000		deg.
Hypoid offset	0.0000		mm
Mean normal module	5.4176		mm
Outer transverse module	8.0000	8.0000	mm
Mean spiral angle	36.8664	36.8664	deg.
Pitch cone angle	27.8241	62.1759	deg.
Nominal pressure angle	20.0000	20.0000	deg.
Profile displacement factor	0.0000	0.0000	-
Tooth thickness factor	0.0123	-0.0441	-
Mean whole depth	12.2121	12.2121	mm
Mean addendum	5.4276	5.4276	mm
Mean dedendum	6.7845	6.7845	mm
Outer normal backlash	0.3300		mm

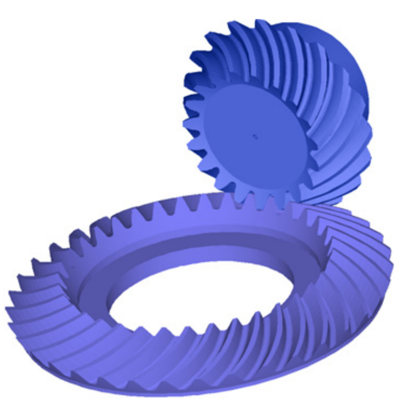

Figure 8 **Gear data for design comparison.**

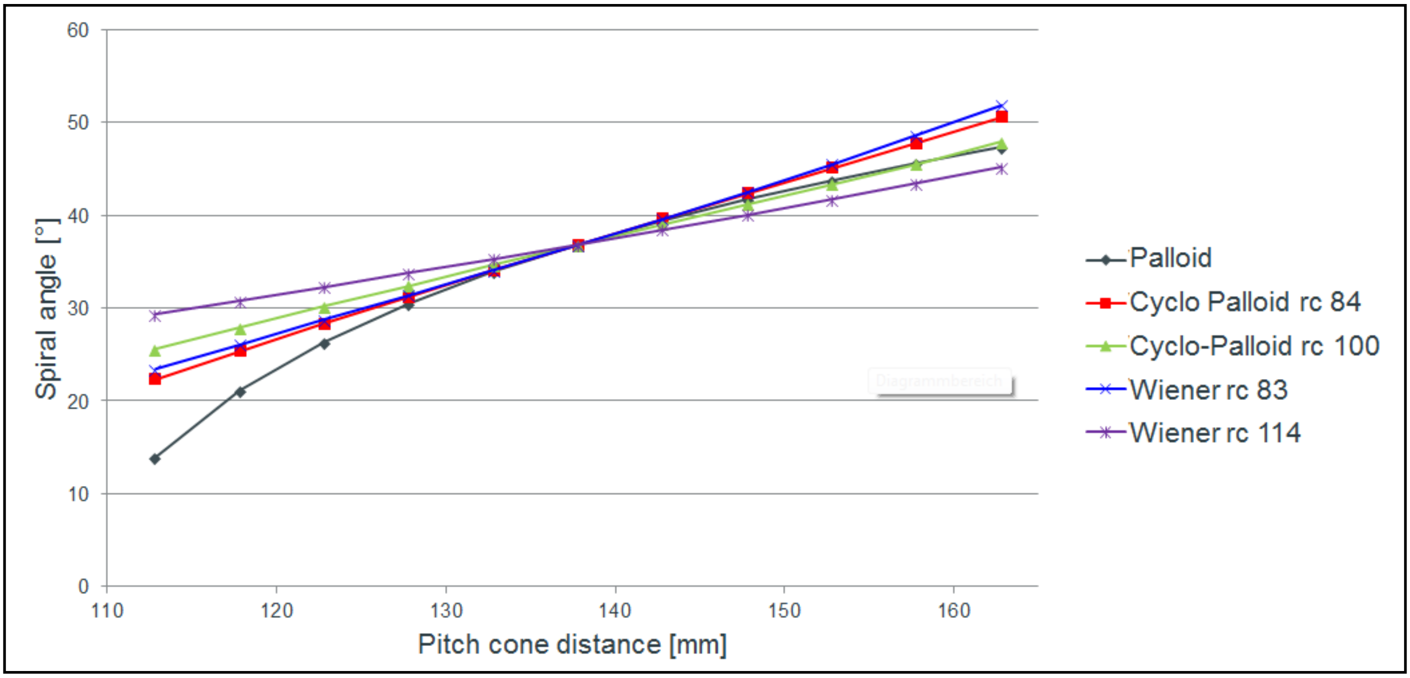

Figure 9 Spiral angle comparison.

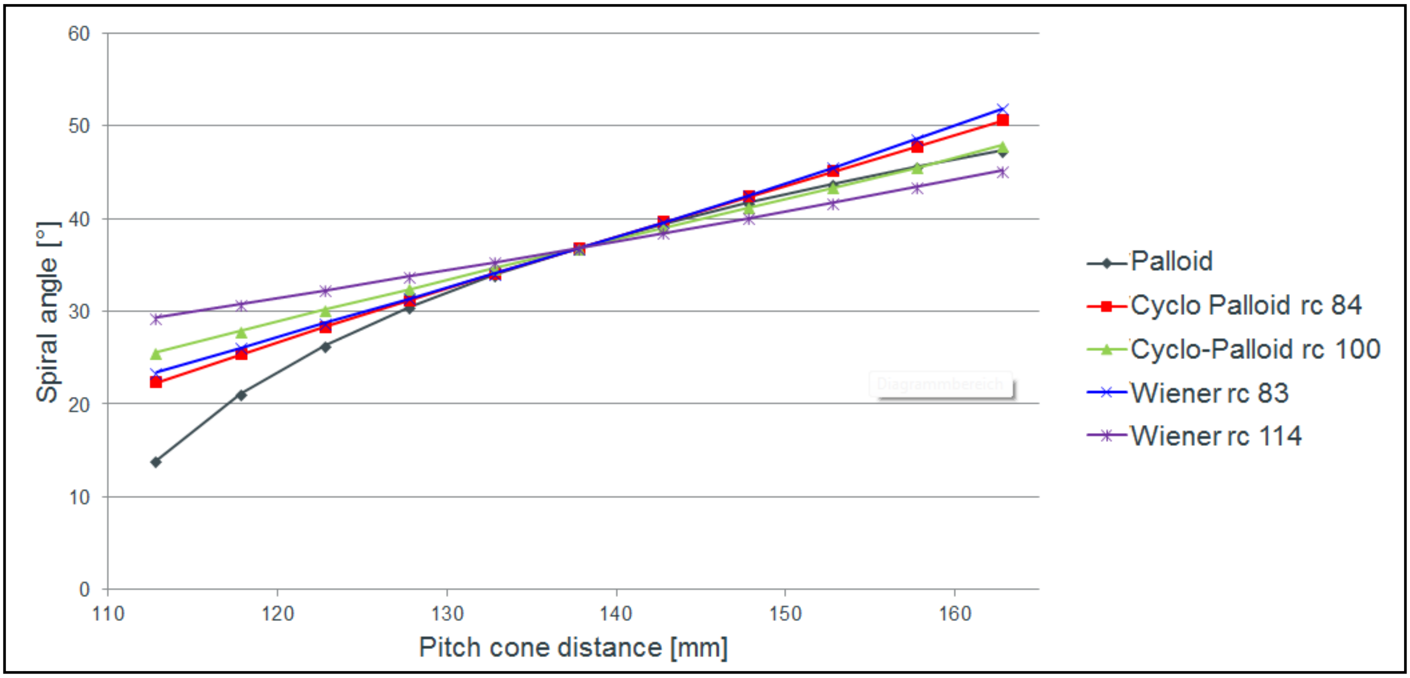

Figure 10 Lengthwise curvature comparison.

the Wiener designs have the smallest.

Comparing the course of the lengthwise curvature shows no change for the Wiener designs, a small change for the Cyclo-Palloid designs and a significant change for the Palloid design. Figure 11 shows the course of the normal module over the face width. It is interesting to see that for all rectangular case designs the normal module has its maximum in the middle of the teeth. For non-rectangular designs the maximum is at the heel or close to it. By changing the tool diameter it is possible to place the maximum normal module on nearly any position along the face width.

Another interesting characteristic can be seen for the Palloid design. The normal module does not change along the face width. Palloid designs have the unique feature of constant tooth height and constant slot width.

Since a bigger normal module gives a stronger tooth, it is obvious that the load carrying capacity will be influenced. Another effect influencing the load carrying capacity is the sensitivity of the contact pattern to load-induced displacements. This sensitivity is shown (Fig. 12) for the unloaded contact pattern and (Fig. 13) for the loaded contact pattern.

For the loaded TCA a torque of 2,000 Nm was applied.

The root bending stresses without displacements are very comparable to each design. The variation of the maximum bending stress for the pinions is from 446 MPa for the Wiener 114 design, up to 486 MPa for the Cyclo-Palloid 100 design. The ring gear shows maximum bending stresses from 440 to 452 MPa.

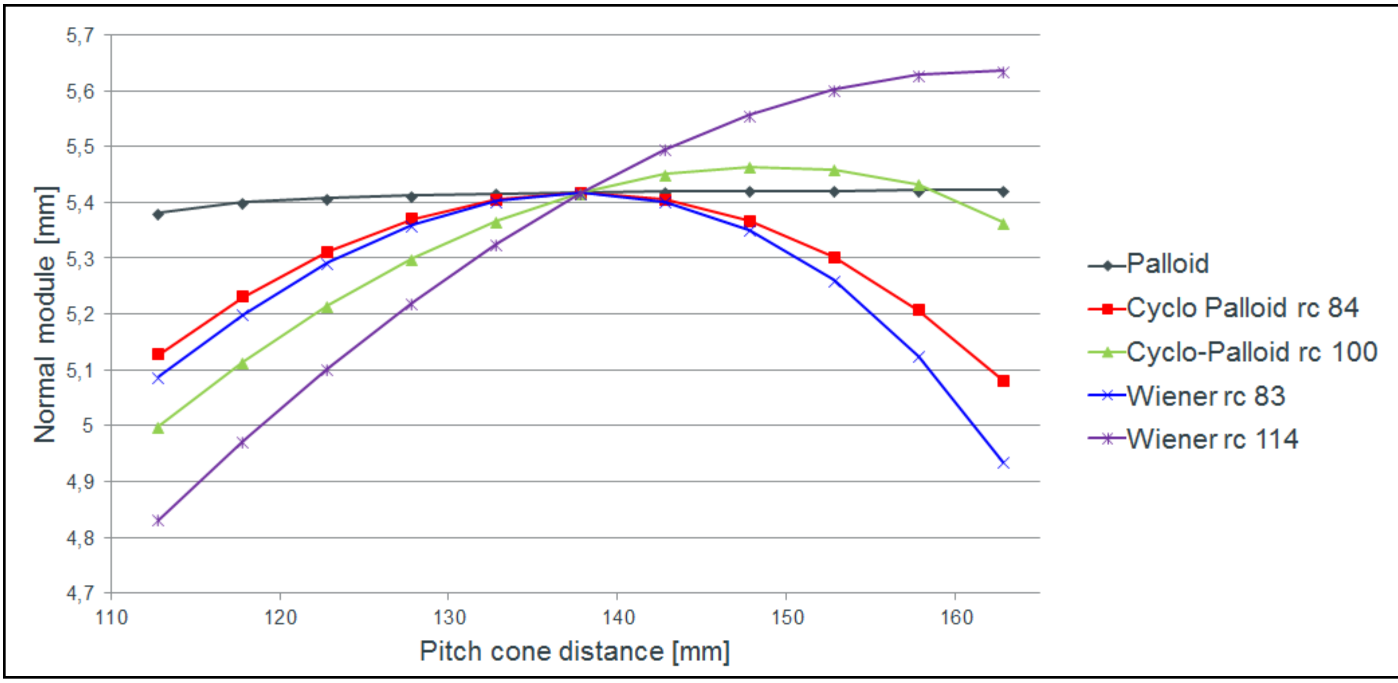

Figure 11 Normal module comparison.

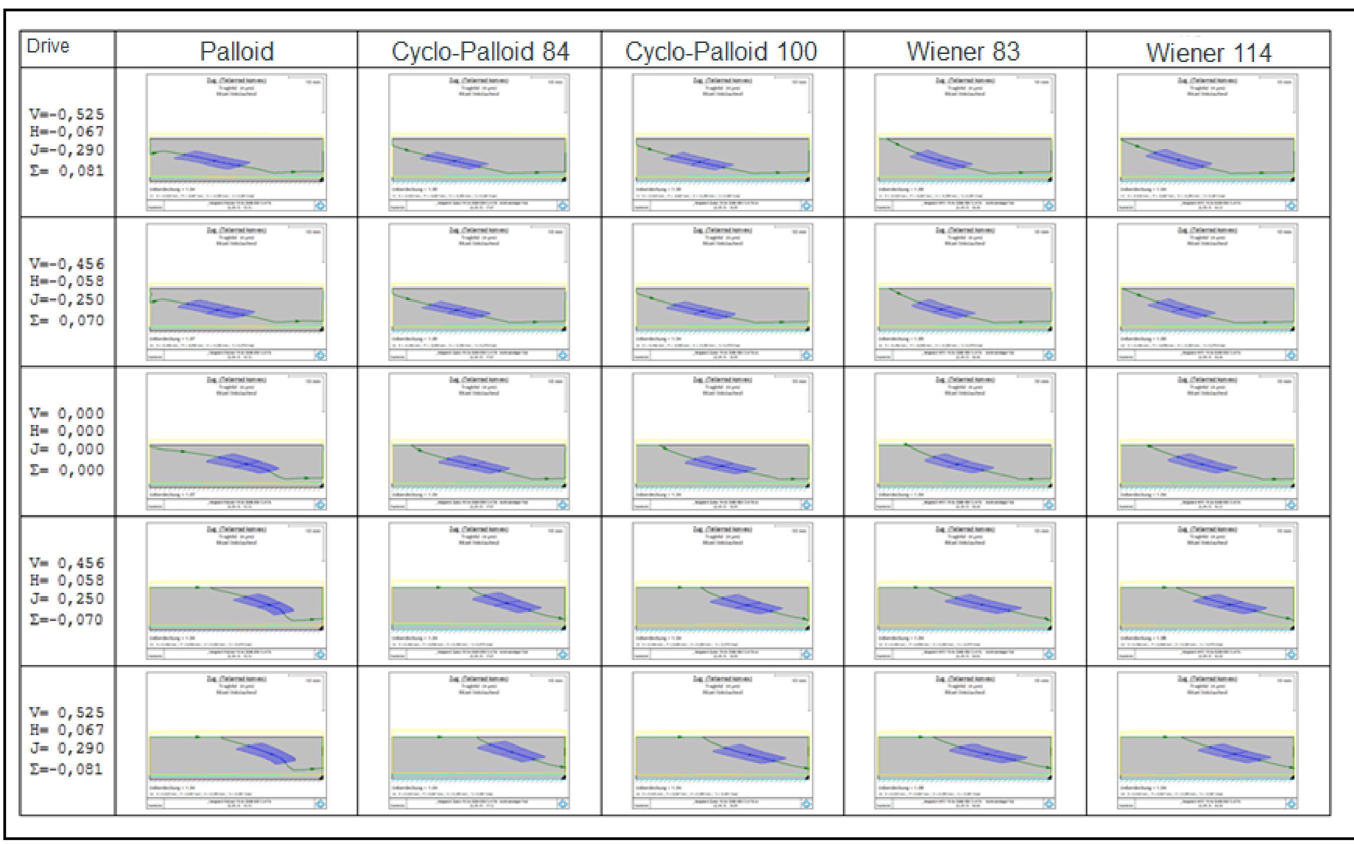

Figure 12 Unloaded contact pattern position with displacements.

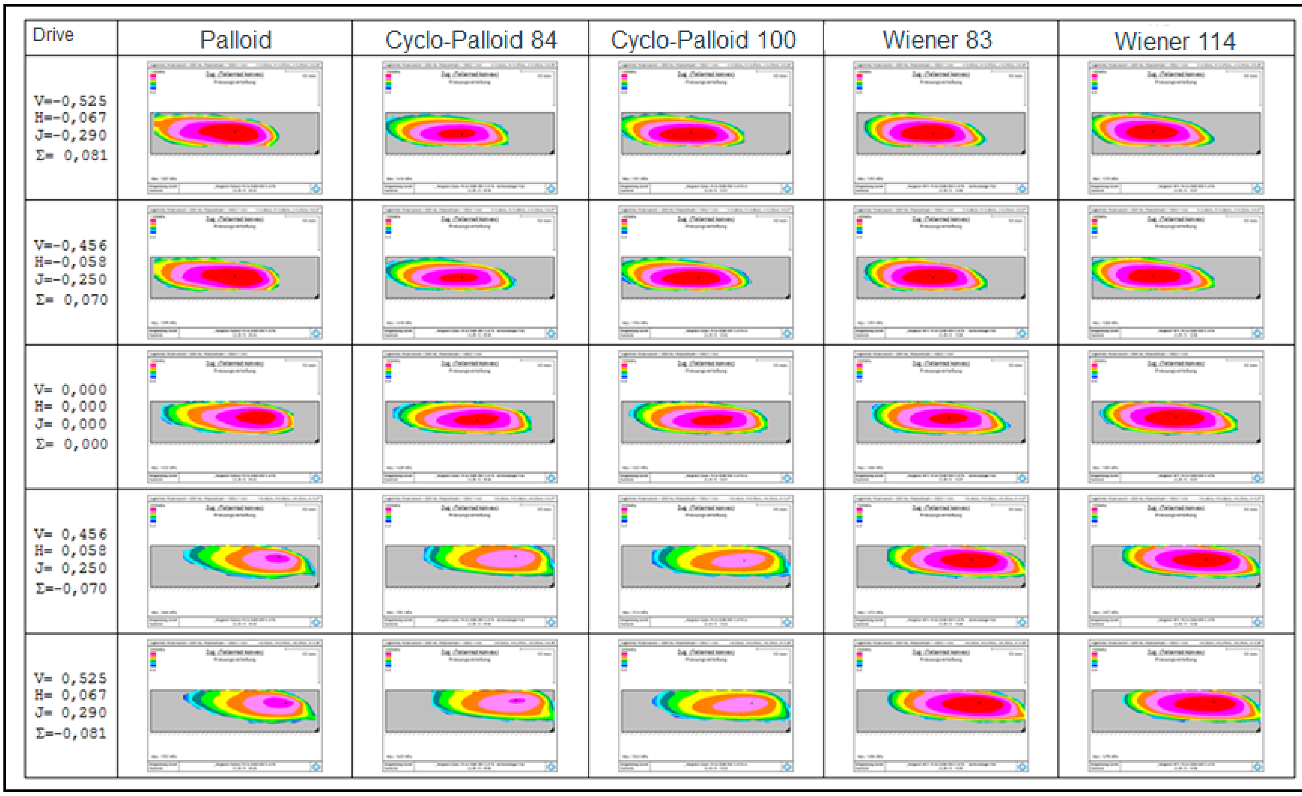

Figure 13 Loaded contact pattern position with displacements.

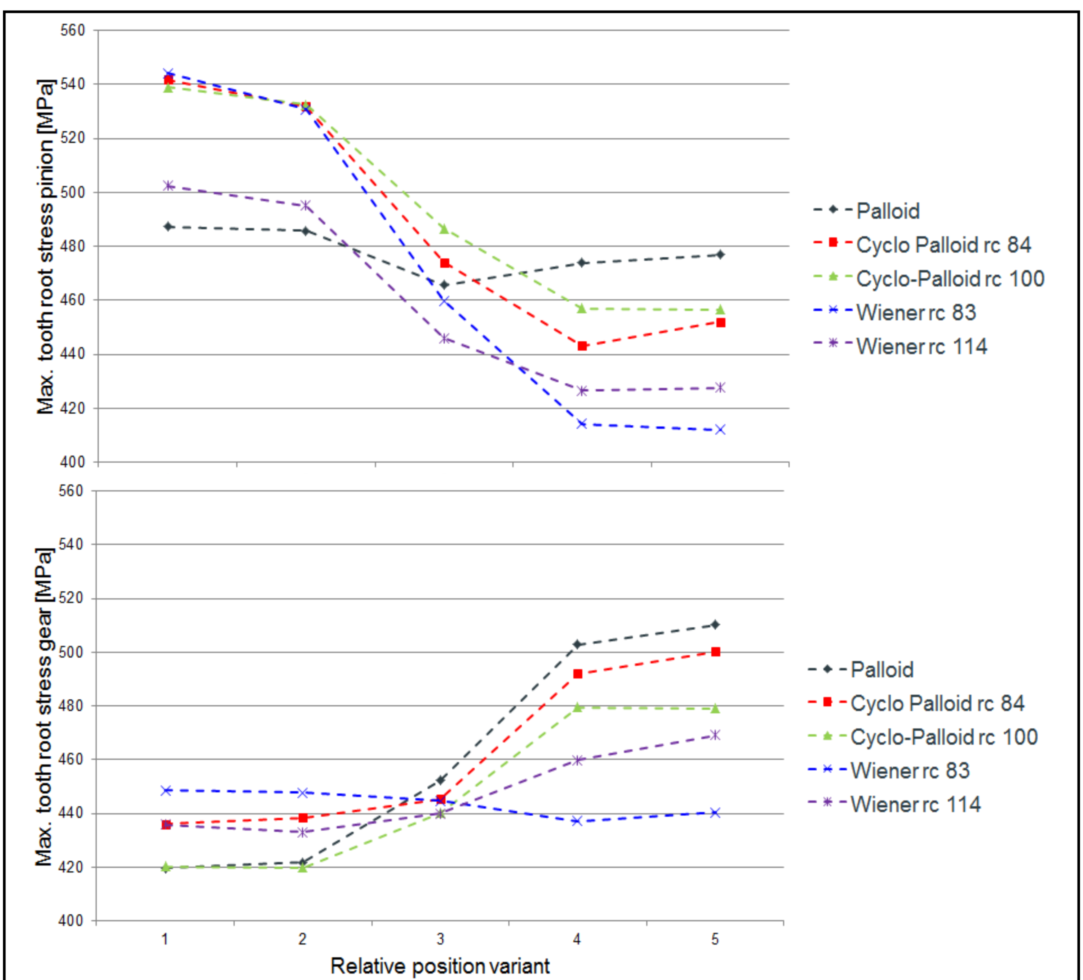

Figure 14 Maximum root bending stress with displacements.

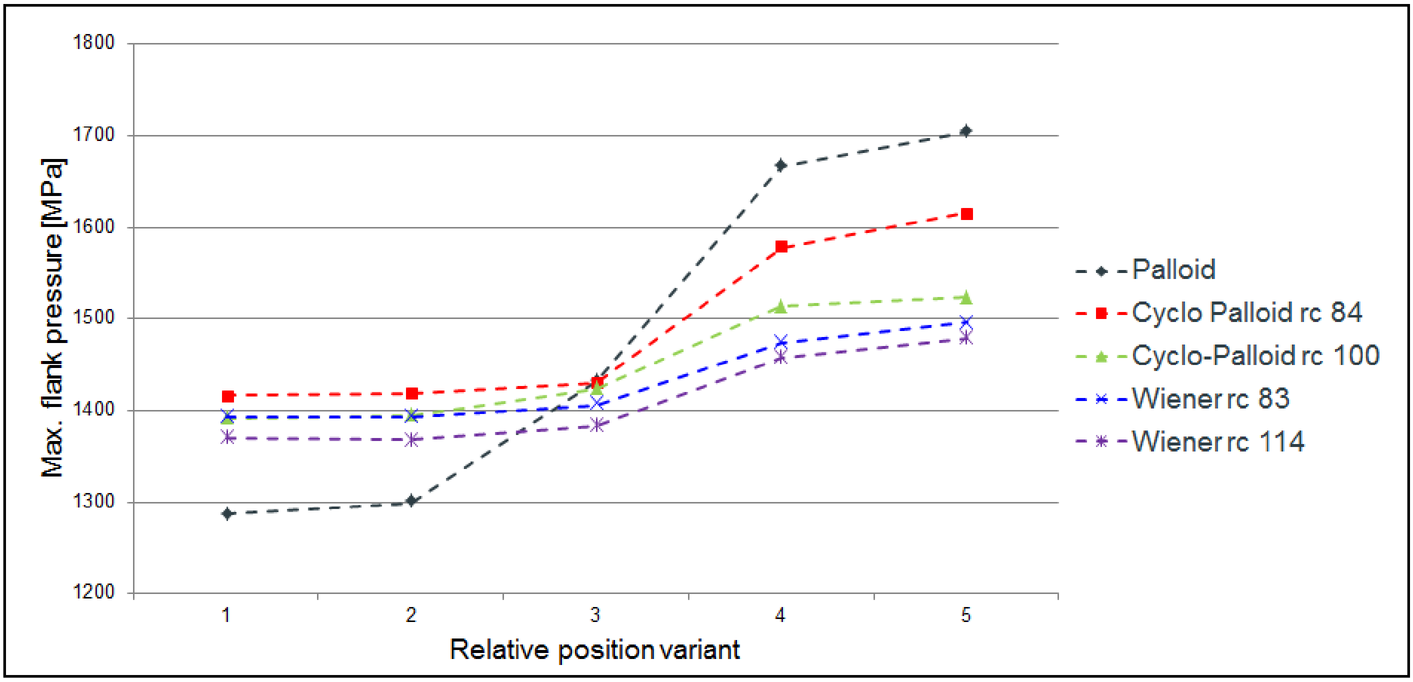

Figure 15 Maximum contact stress with displacements.

Similar to the root bending stress, the maximum of the Hertzian pressure without deflections is very similar. The Wiener 114 design has the lowest value with 1,390 MPa, the Palloid design the highest at 1,430 MPa.

The main differences in load carrying capacity can be seen when displacements are taken into account. Figure 14 shows the maximum of the root bending stresses of pinion and gear for the displacements used in Figures 12 and 14. Since the displacements move the contact pattern in tooth height and tooth lengthwise direction, the bending stresses will change significantly for the different designs. The most variation shows the Palloid design for the ring gear with a variation from 420 to 510 MPa.

The variation of the contact stress under displacements is shown (Fig. 15). For the Palloid design the values vary from 1,280 to 1,700 MPa. This is caused by the extremely localized loaded contact pattern near the tip of the ring gear. This also explains the high root bending 3-9 WZL stress. The Wiener 114 design has the lowest variation in contact stress, changing from 1,380 to 1,480 MPa.

Summary and Outlook

Gear design requires tooth forms that are able to handle contact stress as well as bending stress.

- This can be achieved by flanks allowing the contact pattern to grow with increasing torque.
- The other aspect is the insensitivity to load-induced displacements; the smaller the contact pattern movement under load, thus less of an increase in root bending stress can be seen.
- A similar effect includes stress; i.e. — better centering means less variation in maximum pressure.
- For designing higher-rated gears in terms of load carrying capacity, flank forms are required that guarantee the above-mentioned behavior.
- Whether entirely different shapes of gear teeth have this potential seems very questionable.

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Dr. Hartmuth Müller is head of technology & innovation for Klingelnberg. Upon earning both his degree in electrical engineering (1980) and his Ph.D. (1990) at University Karlsruhe, he joined Klingelnberg Ettlingen (1990) as head of software development and was later (1997) named head of the Klingelnberg Ettlingen Technical Center. Mueller in 2001 became Klingelnberg GmbH managing director and subsequently (2003) Klingelnberg Group CTO.