A Rational Procedure for Designing Minimum-Weight Gears

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Abstract: A simple, closed-form procedure is presented for designing minimum-weight spur and helical gearsets. The procedure includes methods for optimizing addendum modification for maximum pitting and wear resistance, bending strength, or scuffing resistance.

Introduction

Gear design is a process of synthesis where gear geometry, materials, heat treatment, manufacturing methods, and lubrication are selected to meet the performance requirements of a given application. The designer must design the gearset with adequate pitting resistance, bending strength, and scuffing resistance to transmit the required power for the design life. With the algorithm presented here, one can select materials and heat treatment within the economic constraints and limitations of manufacturing facilities, and optimize the gear geometry to satisfy constraints on weight, size, and configuration. This article assumes that the gear ratio is known. Methods already exist[1] for choosing the ratio of each gearset in a multistage gearbox to minimize overall weight. The gear designer can minimize noise level and operating temperature by minimizing the pitch line velocity and sliding velocity. This is done by specifying high gear accuracy and selecting material strengths consistent with maximum material hardness, to obtain minimum-size gearsets with teeth no larger than necessary to balance the pitting resistance and bending strength.

Gear design is not the same as gear analysis. Existing gearsets can only be analyzed, not designed. While design is more challenging than analysis, current textbooks do not provide procedures for designing minimum-weight gears. They usually recommend that the number of teeth in the pinion be chosen based solely on avoiding undercut. This article will show why this practice or any procedure which arbitrarily selects the number of pinion teeth will not result in minimum-weight gearsets. Although there have been many technical papers on gear designs (see Refs. 2-3, for example) most advocate using computer-based search algorithms which are unnecessary. Tucker[4] came the closest to an efficient algorithm, but he did not show how to find the optimum number of teeth for the pinion.

Optimum Number of Pinion Teeth

The optimum number of pinion teeth maximizes the load capacity of a gearset. Fig. 1 shows that load capacity is limited by surface fatigue, bending fatigue, or scuffing, depending on the number of teeth in the pinion. Also, there is a lower limit to the number of teeth, below which undercut occurs. The shaded zone in Fig. 1 is bounded by all three failure-mode curves and the undercut limit. It applies to a homologous class of gears with a specific combination

Nomenclature

(N_p)**optimum** = optimum number of pinion teeth
(x1)min = minimum addendum modification coefficient to avoid undercut
ϕ_c = normal generating pressure angle
ϕ_s = transverse generating pressure angle
ψ_s = standard generating helix angle
C_1 = distance to SAP (See Fig. 2)
C_5 = distance to EAP (See Fig. 2)
C_6 = distance between interference points (See Fig. 2)
Ca = application factor
C_D = combined dreating factor
C_L = pitting resistance life factor
Cm = load distribution factor
C_p = elastic coefficient (Cp = 2300 for steel)
Cs = size factor
Cv = dynamic factor
d = operating pitch diameter of pinion
F = net face width
H_B = Brinell hardness
I = pitting resistance geometry factor
J = bending strength geometry factor
K_c = pitting resistance constant
K_D = combined dreating factor
K_L = bending strength life factor
K_t = bending strength constant
m_a = aspect (F/d) ratio
m_G = gear ratio ($m_G \geq 1$)
N = number of load cycles
n_c = pitting resistance safety factor
N_p = number of teeth in pinion
n_p = pinion speed (rpm)
n_t = bending strength safety factor
P = transmitted horsepower
P_n = normal diametral pitch
Sac = allowable (uncorrected) contact stress
Sat = allowable (uncorrected) bending stress
Snc = contact strength
Snt = bending strength
Subscripts/sign convention
p = pinion
1 = pinion, **2** = gear
(±) = upper sign external gearsets, lower sign internal gearsets

of gear geometry, material properties, and application requirements. The relative positions of the curves change as these parameters change. This is not a disadvantage to the gear designer because the algorithm presented here directly solves for the optimum number of pinion teeth, making it unnecessary to draw Fig. 1, which is shown strictly for demonstrating the concept of the optimum number of pinion teeth. The curve marked "Surface Fatigue", representing the pitting resistance of the gearset, is relatively flat, being only weakly influenced by the number of pinion teeth. In contrast, the curve marked "Bending Fatigue", representing the bending strength, depends strongly on the number of pinion teeth, and it drops rapidly as the number of teeth increases. Maximum load capacity occurs at point "A", where the pitting resistance and bending strength are balanced. For more pinion teeth (to the right of point "A"), load capacity is controlled by bending fatigue, while for fewer teeth (to the left of point "A"), load capacity is controlled by surface fatigue.

The two failure modes are quite different. Surface fatigue usually progresses relatively slowly, starting with a few pits, which may increase in number and coalesce into large spalls. As the tooth profiles deteriorate with pitting, the gears generate noise and vibration, which warns of the surface fatigue failure. In contrast, bending fatigue may progress rapidly as a fatigue crack propagates across the base of a tooth, breaking the tooth with little or no warning. Hence, surface fatigue is often less serious than bending fatigue, which is frequently catastrophic.

Considering the differences between pitting fatigue and bending fatigue, it is prudent to select the number of pinion teeth somewhat to the left of point "A" (shown by the vertical line marked (N_p)optimum in Fig. 1), where surface

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fatigue controls rather than bending fatigue. With this design approach, not much load capacity is lost because the surface fatigue curve is relatively horizontal, while a margin of safety against bending fatigue is gained. This practice should not be carried to extremes, because pinions with few large teeth (with high specific sliding ratios) are prone to scuffing (See point "B" on curve marked "Scuffing Failure" in Fig. 1).

Some textbooks recommend using a number of teeth for the pinion equal to the minimum required to avoid undercut. This gives gearsets with less than optimum load capacity, which are prone to scuffing (see point "C" in Fig. 1). A pinion tooth number near (N_p)optimum provides a good balance between pitting resistance and bending strength, while good scuffing resistance is also obtained because the teeth are not larger than necessary.

Design Algorithm

There is no need for cut-and-try procedures for gear design if one exploits the near independence of pitting resistance and the number of pinion teeth. The following algorithm first solves for the diameter and face width of the pinion based on surface fatigue, and then solves for the optimum number of pinion teeth by simultaneously satisfying the surface fatigue and the bending fatigue constraints. It is derived from equations given in AGMA 218.01,[5] and is limited to steel. For alloys other than those shown, substitute allowable stresses, Sac and Sat from Ref. 5. Because it is necessary to approximate the geometry factors I and J, the final design must be verified using Ref. 5.

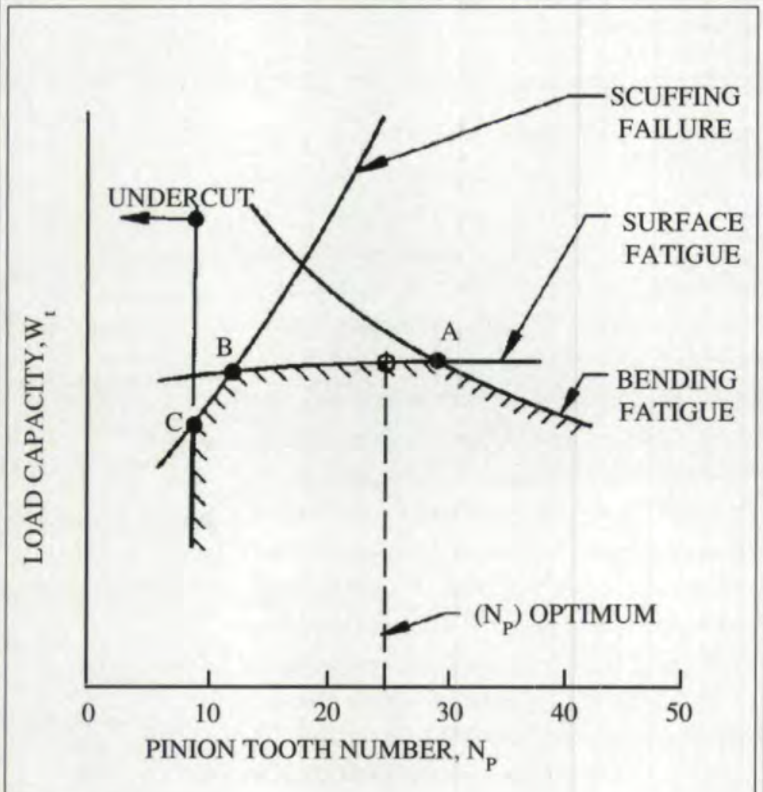

Fig. 1

Allowable (uncorrected) stresses for through-hardened steel:

$$Sac = 26000 + 327*H_B$$

$$Sat = -274 + 167*H_B - 0.152*H_B^2$$

Allowable (uncorrected) stresses for carburized steel:

$$Sac = 180{,}000$$

$$Sat = 55{,}000$$

Life factors:

$$C_L = 2.4660*N^{-0.0560}$$

$$K_L = 1.6831*N^{-0.0323}$$

Contact strength:

$$Snc = C_L*Sac$$

Bending strength:

$Snt = K_L*Sat$ (for reversed bending, multiply Snt by 0.7.)

The contact strengths and bending strength are calculated for both the pinion and gear, and the minimum values of Snc and Snt are used in the following equations.

Combined derating factor:

$$C_D = K_D = \frac{Ca*Cs*Cm}{Cv} \qquad (1)$$

Geometry factors for spur gears:

$$I = \frac{\sin\phi_c*\cos\phi_c}{2}\left(\frac{m_G}{m_G \pm 1}\right) \qquad (2)$$

$$J = 0.45$$

Geometry factors for helical gears:

$$I = \frac{1 + 0.00682*\phi_c}{4.0584}\left(\frac{m_G}{m_G \pm 1}\right) \qquad (3)$$

$$J = 0.50$$

Pitting resistance constant:

$$Kc = \frac{126000*P*C_D}{I*n_p}\left(\frac{C_p*n_c}{Snc}\right)^2 \qquad (4)$$

Bending strength constant:

$$Kt = \frac{126000*P*K_D}{J*n_p}\left(\frac{n_t}{Snt}\right) \qquad (5)$$

Aspect (F/d) ratio:

$$m_a = \frac{m_G}{m_G + 1} \quad \text{(for spur and single helical)} \qquad (6)$$

$$m_a = \frac{2*m_G}{m_G + 1} \quad \text{(for double helical)}$$

Pinion diameter:

$$d = \left(\frac{Kc}{m_a}\right)^{1/3} \qquad (7)$$

Face width:

$$F = d*m_a \qquad (8)$$

Optimum number of pinion teeth:

$$(N_P) \text{ optimum} = \frac{Kc}{Kt} \qquad (9)$$

(Round to integer)

Addendum Modification

Once the diameter, face width, and optimum number of teeth for the pinion are determined with the design algorithm, routine methods are used to select the number of teeth in the gear, diametral pitch, and operating center distance. However, the gear design is not complete until the addendum modification has been selected considering the following criteria:

- •avoiding undercut
- •balanced specific sliding
- •balanced bending fatigue life
- •balanced flash temperature
- •avoiding narrow toplands.

Avoiding undercut. The design algorithm usually gives a number of pinion teeth considerably larger than the number to avoid undercut. Conditions which lead to small numbers of teeth are high material hardness, short design life, large gear ratios, and high bending fatigue safety factors. With reasonable selections of these parameters, (N_p)optimum is usually greater than 20. In any case, the minimum addendum modification coefficient (to avoid undercut) for the pinion is given by:

$$(x_1)_{min} = 1.1 - N_p\left(\frac{\sin^2 \phi_s}{2* \cos \psi_s}\right) \qquad (10)$$

Balanced Specific Sliding. Maximum pitting and wear resistance is obtained by balancing the specific sliding ratio at the ends of the path of contact. This is done by iteratively varying the addendum modification coefficients of the pinion and gear until the following equation is satisfied:

$$\left(\frac{C_6}{C_1} \mp 1\right)\left(\frac{C_6}{C_5} \mp 1\right) = m_G^2 \qquad (11)$$

where:

C_1 = distance to SAP (See Fig. 2)
C_5 = distance to EAP (See Fig. 2)
C_6 = distance between interference points (See Fig. 2)

Balanced bending fatigue life. Maximum bending fatigue resistance is obtained by iteratively varying the addendum modification coefficients of the pinion and gear until the ratio

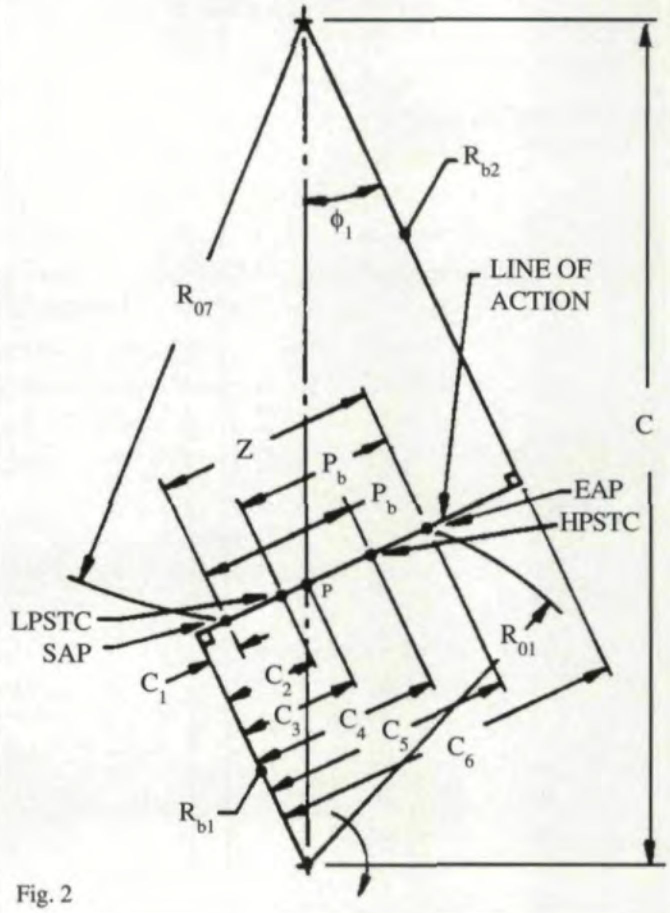

Fig. 2

of the bending strength geometry factors equals the ratio of bending strengths, i.e.,

$$\frac{J_1}{J_2} = \frac{Snt_2}{Snt_1} \qquad (12)$$

Balanced flash temperature. Maximum scuffing resistance is obtained by minimizing the contact temperature. This is done by iteratively varying the addendum modification coefficients of the pinion and gear, while calculating the flash temperature by Blok's equation, until the flash temperature peaks in the approach and recess portions of the line of action are equal. The flash temperature should be calculated at the points SAP, LPSTC, HPSTC, EAP, and at several points in the two pair zones (between points SAP and LPSTC and between points HPSTC and EAP. (See Fig. 2.)

Avoiding narrow toplands. The maximum permissible addendum modification coefficients are obtained by iteratively varying the addendum modification coefficients of the pinion and gear until their topland thicknesses are equal to the minimum allowable (usually $0.3/P_n$).

Design Audit

With the addendum modification selected, the gear design is complete. It is necessary to audit the design by analyzing the stresses and lives (using Ref. 5) because approximate values were used for I and J. The only change that is usually required to meet the design life is a small adjustment of the face width. Although it is beyond the scope of this article, the selection of the lubricant type and viscosity should be verified by calculating the film thickness and flash temperature to ensure that they are within allowable limits.

Example

Using the data from Ref. 3, Example 1:

$Snc = 200{,}000$ psi
$Snt = 60{,}000$ psi
$P = 20$ hp
$n_p = 1260.5$ rpm
$\phi_c = 20°$
$m_G = 5$
$m_a = 0.25$
$n_c = n_t = 1.0$
$C_D = K_D = 1.0$

The design algorithm gives;

$I = 0.134$
$J = 0.450$
$K_c = 1.973$
$K_t = 0.074$
$d = 1.991''$
$F = 0.498''$
(N_p)optimum = 27
$P_n = 13$

Ref. 3 obtained essentially the same results after an extensive computer search.

Conclusions

1. Maximum load capacity or minimum-weight gearsets are obtained by selecting the optimum number of teeth for the pinion. (N_p)optimum, which balances the pitting resistance and the bending strength.

2. (N_p)optimum is easily found from a simple, closed-form design algorithm.

3. Addendum modification is designed to obtain maximum pitting and wear resistance, bending strength, or scuffing resistance by balancing specific sliding, bending strength, geometry factors, or flash temperature.

4. Any design procedure that selects the number of teeth in the pinion based solely on avoiding undercut or which arbitrarily selects the number of pinion teeth, will not result in gearsets with optimum load capacity. Such procedures usually give gearsets with low pitting resistance and low scuffing resistance. ■

References:

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