Determining Spline Misalignment Capabilities

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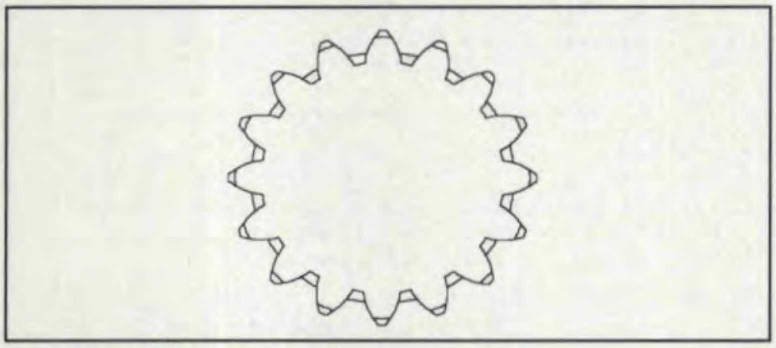

Fig. 1 — Inner and outer tooth profiles.

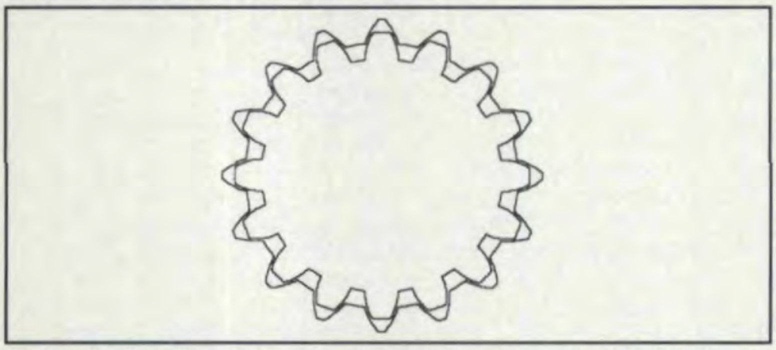

Fig. 2 — Tooth profiles where effective tooth thickness measured at the pitch diameter has been reduced by .015"

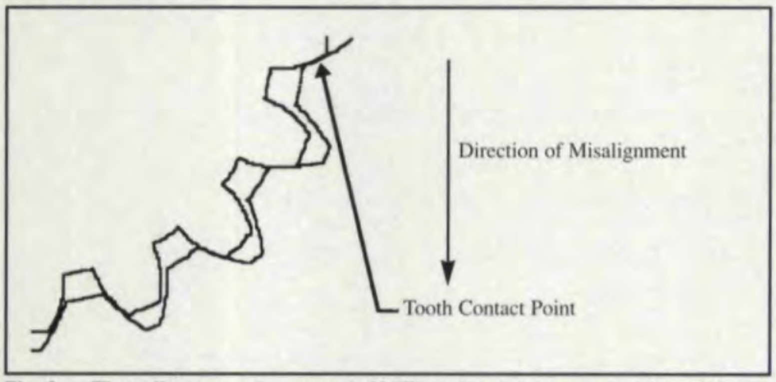

Fig. 3 — The spline center has moved .0065", and contact occurs on the tooth perpendicular to the direction of misalignment.

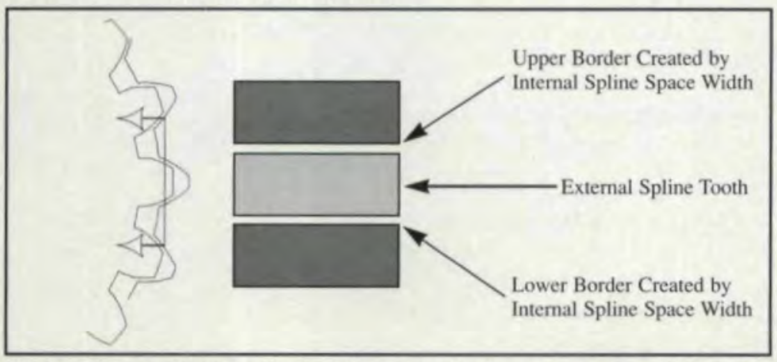

Fig. 4 — Cross section of one tooth on the external spline and one space width on the internal spline.

Introduction

Introducing backlash into spline couplings has been common practice in order to provide for component eccentric and angular misalignment. The method presented here is believed to be exact for splines with even numbers of teeth and approximate for those with odd numbers of teeth. This method is based on the reduction of the maximum effective tooth thickness to achieve the necessary clearance. Other methods, such as tooth crowning, are also effective.

Tooth Thickness and Its Relationship to Misalignment Capability

It is important to understand what feature of the spline provides the misalignment capability. The approach presented here focuses on the spline tooth and space width that is perpendicular to the direction of misalignment. As shown in the following set of figures, this set of teeth is the geometry that controls the misalignment.

Fig. 1 shows the inner and outer tooth profiles of a 16-tooth, 24/48-pitch, 30° pressure angle, fillet root, side-fit spline at maximum material conditions. In this state, no allowance for misalignment exists. The effective tooth thickness of the external spline (inner profile) can be reduced during manufacturing to provide clearance (see Fig. 2).

In Fig. 2, the external spline has been reduced. In shaping or hobbing, this is accomplished by increasing the feed of a standard cutter. In this illustration, the effective tooth thickness measured at the pitch diameter has been reduced by .015".

The resulting clearance allows the spline centerlines to be offset to provide for component misalignment. This appears in Fig. 3, where an enlarged section has been shown for clarity. Note that the amount the spline center moved is .0065" [(Tooth Thickness Reduction/2) • cos30°]. When the spline tooth in Fig. 3 is rotated 30° about the shaft centerline, the normal to the involute profile at the pitch diameter is in the misalignment direction. At that point, the profile of the internal and external splines are colinear. This is the limiting geometry, and it is compensated for by the above trigonometric relationship to the horizontal tooth position.

To calculate the allowable misalignment (angular and eccentric), it is necessary to consider

$$t1 = \frac{-(s - 2 \cdot \cos\beta \cdot HK \cdot \sin\theta + 2 \cdot \cos\beta \cdot \tan\lambda \cdot HK \cdot \cos\theta - 2 \cdot \cos\beta \cdot GZ \cdot \tan\lambda - 2 \cdot \cos\beta \cdot GH)}{(-\cos\beta \cdot \cos\theta - \cos\beta \cdot \tan\lambda \cdot \sin\theta)}$$

$$t2 = \frac{-(s + 2 \cdot \cos\beta \cdot GZ \cdot \tan\lambda + 2 \cdot \cos\beta \cdot GH + 2 \cdot \cos\beta \cdot HK \cdot \sin\theta - 2 \cdot \cos\beta \cdot \sin\theta \cdot Ls - 2 \cdot \cos\beta \cdot \tan\lambda \cdot HK \cdot \cos\theta + 2 \cdot \cos\beta \cdot \tan\lambda \cdot \cos\theta \cdot Ls)}{(-\cos\beta \cdot \cos\theta - \cos\beta \cdot \tan\lambda \cdot \sin\theta)}$$

Fig. 5 — Equations to find the tooth thickness required by tip 1 and tip 2 criteria.

one tooth on the external spline and one space width on the internal spline. The set is perpendicular to the direction of misalignment. Viewing a cross section of this from the side in Fig. 4, one can see the tooth of the external spline as well as the borders of the internal spline space width. The values for the space width and tooth thickness at the pitch diameter are used in the calculations.

Coupling

In the example shown in Fig. 6, we shall consider the external spline on the end of a pump shaft and the internal spline on the end of a motor shaft. The pump pivots around point *H*, while the motor pivots around point Z. Point Z is offset vertically by distance *HG* and horizontally by distance *GZ*. Distance *HK* describes the spline length beyond pivot point *H*, and *Ls* is the spline length. The length of the motor's spline teeth must be sufficient to ensure engagement with the pump's spline. The pump angular misalignment is angle θ, and the motor's angular misalignment is angle β. The intercepts *b1*, *b2* and *b3* for contact at Tip 1 and Tip 2 are calculated, but distances *b1–b2* and *b2–b3* are probably not equal to one another. The formulas using *t*1 as the required tooth thickness to meet the geometric constraints that evolve from tip 1 contact with a space width are as follows:

$\gamma = 180 - \beta$

$\text{tip } 1X = HK \cdot \cos\theta - \frac{t1}{2} \cdot \sin\theta$

$\text{tip } 1Y = HK \cdot \sin\theta + \frac{t1}{2} \cdot \cos\theta$

$b1 = \text{tip } 1Y - \tan\lambda \cdot \text{tip } 1X$

$b2 = -GZ \cdot \tan\lambda - GH$

$s = 2 \cdot (b2 - b1) \cdot \cos\beta$

The additional formulas using *t*2 as the required tooth thickness to meet the geometric constraints that evolve from tip 2 contact with a space width *s* are

$\gamma = 180 - \beta$

$\text{tip } 2X = (HK - Ls) \cdot \cos\theta + \frac{t2}{2} \cdot \sin\theta$

$\text{tip } 2Y = (HK - Ls) \cdot \sin\theta - \frac{t2}{2} \cdot \cos\theta$

$b2 = -GZ \cdot \tan\lambda - GH$

$b3 = \text{tip } 2Y - \tan\lambda \cdot \text{tip } 2X$

$s = 2 \cdot (b2 - b3) \cdot \cos\beta$

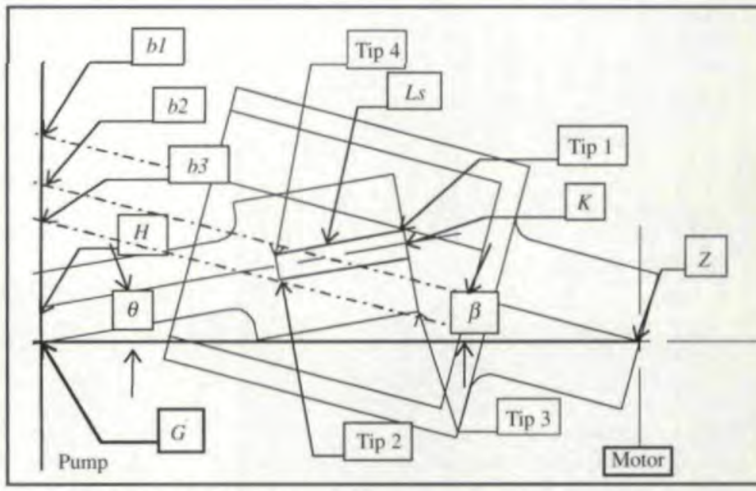

Fig. 6 — Diagram for a direct coupling.

These equations can be rearranged to find the tooth thickness required by the tip 1 and tip 2 criteria. The smaller value for tooth thickness must be chosen (see Fig. 5).

By making the appropriate substitutions, the following expressions are derived for the tooth thickness at tip 1 and tip 2.

$a = 2 \cdot \cos\beta \cdot GH$

$b = 2 \cdot \cos\beta \cdot HK \cdot \sin\theta$

$c = 2 \cdot \cos\beta \cdot \tan\lambda \cdot HK \cdot \cos\theta$

$d = 2 \cdot \cos\beta \cdot GZ \cdot \tan\lambda$

$e = -\cos\beta \cdot \cos\theta - \cos\beta \cdot \tan\lambda \cdot \sin\theta$

$f = 2 \cdot \cos\beta \cdot \sin\theta \cdot Ls$

$g = 2 \cdot \cos\beta \cdot - \tan\lambda \cdot \cos\theta \cdot Ls$

$t1 = \frac{-(s - b + c - d - a)}{e}$

$t2 = \frac{-(s + d + a + b - f - c + g)}{e}$

The fixed angular displacement of the geometry forces consideration of both *t*1 and *t*2 as independent solutions. The spline tooth thickness that will fit within space width *s* is the smaller of these two values. A second set of equations must be derived in the same manner for contact at tip 3 and tip 4. All permutations of $\pm\beta$ and $\pm\theta$ must be tried to evaluate the smallest tooth thickness.

The final result should be multiplied by the cosine of the pressure angle as shown in Fig. 3 to obtain the required tooth thickness. ⭘

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