

# Design Formulas for Evaluating Contact Stress in Generalized Gear Pairs

David B. Dooner

## Introduction

A very important parameter when designing a gear pair is the maximum surface contact stress that exists between two gear teeth in mesh, as it affects surface fatigue (namely, pitting and wear) along with gear mesh losses. A lot of attention has been targeted to the determination of the maximum contact stress between gear teeth in mesh, resulting in many "different" formulas. Moreover, each of those formulas is applicable to a particular class of gears (e.g., hypoid, worm, spiroid, spiral bevel, or cylindrical—spur and helical). More recently, FEM (the finite element method) has been introduced to evaluate the contact stress between gear teeth. Presented below is a single methodology for evaluating the maximum contact stress that exists between gear teeth in mesh. The approach is independent of the gear tooth geometry (involute or cycloid) and valid for any gear type (i.e., hypoid, worm, spiroid, bevel and cylindrical).

## Relative Curvature

The contact stress between two gear teeth in mesh depends on the relative gear tooth curvature, material properties of the gear teeth, and the transmitted load between the gear teeth. Determination of the relative gear tooth curvature can be problematic for certain gear types. The relative gear tooth curvature between two gear teeth in mesh results in a contact that is either point-contact or line-contact. In general, the transverse contact ratio for two gears in mesh is greater than zero, and line-contact exists between the two gear teeth in mesh. Helical or spiral gears with line-contact experience both axial and transverse displacement during mesh. Such conditions are inherent for any tooth profile type (namely, involute or cycloid). Point-contact is the alternative scenario for two conjugate surfaces in mesh. That condition occurs when the transverse contact ratio is zero. That type of contact applies to circular-arc type profiles (namely, Novikov-Wildhaber or BBC).

Determination of the relative gear tooth curvature  $\Delta\kappa$  between two planar involute gear teeth is demonstrated prior to presenting the relative gear tooth curvature between two generalized gear teeth. Depicted in Figure 1 are two involute gear teeth in mesh. The radius of the input pitch circle is  $R_i$ , whereas  $R_o$  is the radius of the output pitch circle.  $\rho_i$  and  $\rho_o$  are the radii of curvature for the input and output gear teeth respectively. Projecting the pitch radii  $R_i$  and  $R_o$  onto the contact normal

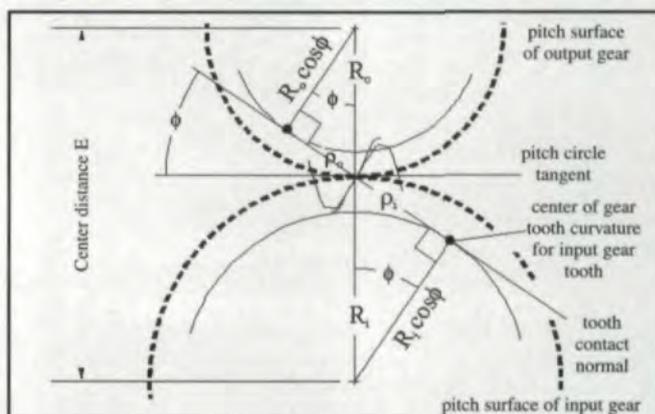


Fig. 1—Two pitch circles in contact.

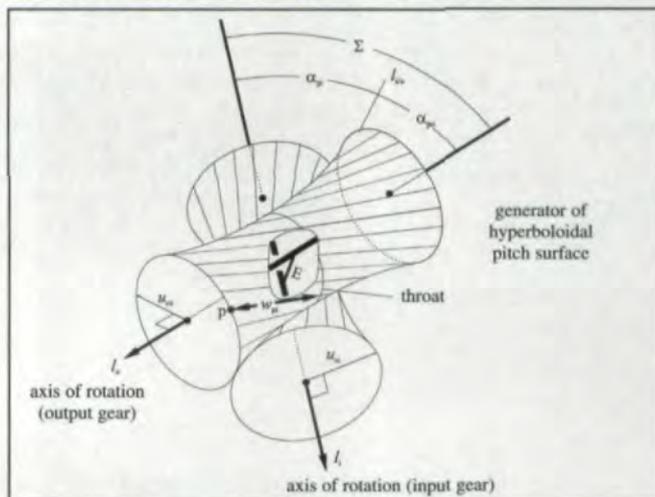


Fig. 2—Gear nomenclature.

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is an associate professor in the department of mechanical engineering at the University of Puerto Rico-Mayagüez. His research involves a universal approach for the concurrent design and manufacture of gears. That generalized approach can be used to produce gears with any number of teeth, any face width and any spiral angle. The approach applies to helical, bevel, worm, spiroid, hypoid and non-circular gears.

yields

$$\rho_i = R_i \sin \phi \quad (1a)$$

$$\rho_o = R_o \sin \phi \quad (1b)$$

where  $\phi$  is the angle between the pitch circle tangency and the tooth contact normal. For planar curves, the curvature  $\kappa$  and radius of curvature  $\rho$  are reciprocals (i.e.,  $\kappa = 1/\rho$ ). Thus, relative gear tooth curvature  $\Delta\kappa$  can be expressed as follows:

$$\Delta\kappa = \left( \frac{1}{\rho_i} + \frac{1}{\rho_o} \right) \quad (2a)$$

or

$$\Delta\kappa = \left( \frac{1}{R_i} + \frac{1}{R_o} \right) \frac{1}{\sin \phi} \quad (2b)$$

where

$R_i$  radius of input pitch circle  
 $R_o$  radius of output pitch circle  
 $\phi$  pressure angle.

The above expression establishes a unique relation between the pressure angle  $\phi$ , the pitch radii  $R_i$  and  $R_o$  and the relative gear tooth curvature  $\Delta\kappa$ . Regardless of the radii of tooth curvature  $\rho_i$  and  $\rho_o$ , the relative gear tooth curvature  $\Delta\kappa$  depends solely on pitch radii  $R_i$  and  $R_o$  and pressure angle  $\phi$ . The above relation for relative gear tooth curvature is for cylindrical gears with spur-type gear teeth. Furthermore, the relation is valid only for contact at the pitch point.

Prior to presenting a generalized relation for the relative gear tooth curvature, it is necessary to establish some nomenclature and introduce certain expressions. Depicted in Figure 2 are two pitch surfaces in mesh, two axes of rotation  $l_i$  and  $l_o$ , the perpendicular distance  $E$  between the two axes  $l_i$  and  $l_o$ , and the included angle  $\Sigma$  between the two axes  $l_i$  and  $l_o$ . The pitch surfaces in Figure 2 are hyperboloids. Notice in Figure 2 that each hyperboloidal pitch surface is determined by a series of straight lines. Hyperboloids can be envisioned as the surface produced by rotating a line or generator about a central axis. For example, rotating the common generator  $l_{i/o}$  between the input and output body about the axis of rotation  $l_i$  produces the input hyperboloidal pitch surface. The shape of the hyperboloidal pitch surface depends on an angle  $\alpha$  and distance  $u$ . The angle  $\alpha$  is the cone angle of the generator, and  $u$  is the radius of the hyperboloidal pitch surface at the throat. Introducing  $u_{pi}$  as the radius of the input hyperboloidal pitch surface and  $u_{po}$  as the radius of the output hyperboloidal pitch surface, then  $u_{pi} + u_{po} = E$  for two hyperboloidal surfaces in mesh. Similarly, defining  $\alpha_{pi}$  as the cone angle for the input hyperboloidal pitch surface and  $\alpha_{po}$  as the cone angle for the output hyperboloidal pitch surface, then  $\alpha_{pi} + \alpha_{po} = \Sigma$  for two hyperboloidal pitch surfaces in mesh. Also shown in Figure 2 is the distance  $w_{pi}$  between the throat and point  $p$ . As the two hyperboloids rotate, they are always in contact along the common generator.

Cylindrical gearing occurs when the angle  $\Sigma$  between the input axis of rotation and the output axis of rotation is zero (i.e.,



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$\Sigma = 0$  and hence the cone angles  $\alpha_{pi}$  and  $\alpha_{po}$  are also zero). In general, a gear type depends on both the center distance  $E$  (offset) and angle  $\Sigma$  between the input and output axes of rotation. When the distance  $E$  between two axes of rotation is zero, then the pitch surfaces become cones and the throat radii  $u_{pi}$  and  $u_{po}$  are zero. Alternately, when neither  $E$  nor  $\Sigma$  is zero, then the two pitch surfaces are hyperboloids. Equation 2b for relative curvature  $\Delta\kappa$  was derived in terms of cylindrical pitch surfaces, and consequently it is not valid for conical or hyperboloidal pitch surfaces.

An important parameter for specifying relative gear tooth curvature is the effective radius. The effective radius is the distance between the point  $p$  on the pitch surface and the axis of rotation, as shown in Figure 2. For generalized gear pairs with a constant input/output gear ratio  $g$ , the effective radii  $u_{ei}$  and  $u_{eo}$  can be expressed

$$u_{ei} = \sqrt{u_{pi}^2 + w_{pi}^2 \sin^2 \alpha_{pi}} \quad (3a)$$

$$u_{eo} = \sqrt{u_{po}^2 + w_{po}^2 \sin^2 \alpha_{po}} \quad (3b)$$

where

- $u_{pi}$  radius of the input pitch surface (at the throat)
- $u_{po}$  radius of the output pitch surface (at the throat)
- $E$  shaft center distance between the two axes of rotation ( $u_{pi} + u_{po} = E$ )
- $w_{pi}$  axial position of tangent point on input pitch surface
- $w_{po}$  axial position of tangent point on output pitch surface ( $w_{pi} = -w_{po}$ )
- $\alpha_{pi}$  cone angle of input pitch surface
- $\alpha_{po}$  cone angle of output pitch surface
- $\Sigma$  the included shaft angle between the two axes of rotation ( $\alpha_{pi} + \alpha_{po} = \Sigma$ ).

It is also convenient to introduce the following relations

$$R_i = u_{ei} \cos(\gamma_{pi} + \psi_{pi}) \quad (4a)$$

$$R_o = u_{eo} \cos(\gamma_{po} + \psi_{po}) \quad (4b)$$

where

$$\tan \gamma_{pi} = \frac{u_{pi} \sin \alpha_{pi}}{\sqrt{u_{pi}^2 \cos^2 \alpha_{pi} + w_{pi}^2 \sin^2 \alpha_{pi}}} \quad (5a)$$

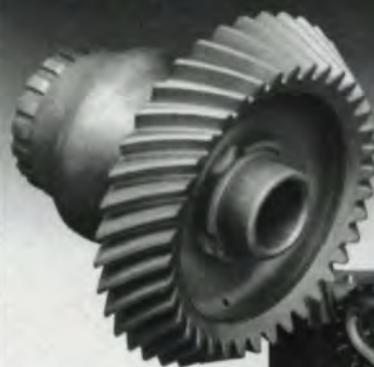
$$\tan \gamma_{po} = \frac{u_{po} \sin \alpha_{po}}{\sqrt{u_{po}^2 \cos^2 \alpha_{po} + w_{po}^2 \sin^2 \alpha_{po}}} \quad (5b)$$

to determine relative gear tooth curvature. It is of central importance to know that the gear ratio  $g$  is equal to the ratio of radii  $R_i$  and  $R_o$  (i.e.,  $g = R_i/R_o$ ). Cone angles  $\alpha_{pi}$  and  $\alpha_{po}$  are zero for cylindrical gears, and consequently  $\gamma_{pi}$  and  $\gamma_{po}$  are also zero. For bevel gears, the pitch radii  $u_{pi}$  and  $u_{po}$  are zero such that  $\gamma_{pi}$  and  $\gamma_{po}$  reduce to zero. For spur gears,  $\psi$  is zero.

In general, the extreme relative curvature between two gear teeth in mesh can be determined with the following expressions:

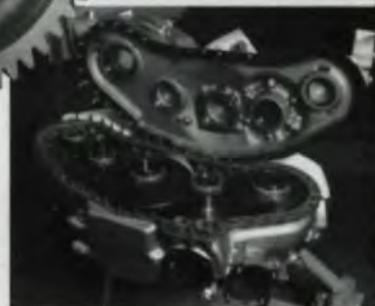
$$\Delta\kappa_{\min} = 0 \quad (6a)$$

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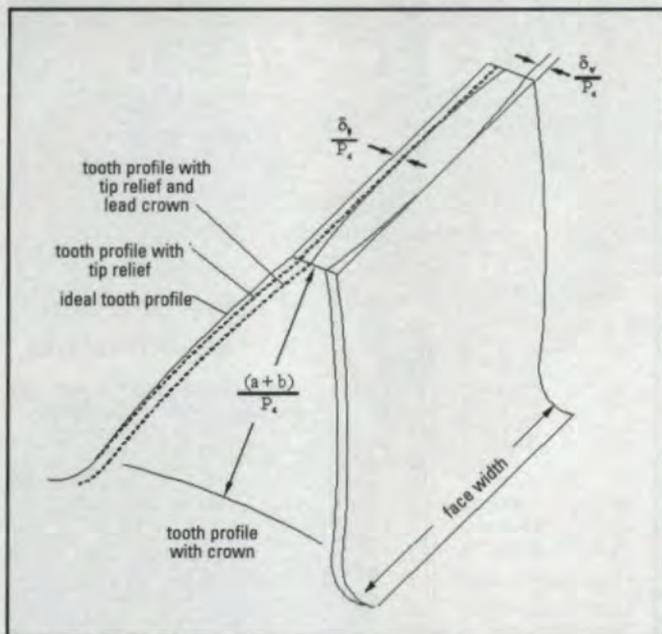


Fig. 3—Tooth profile modification.

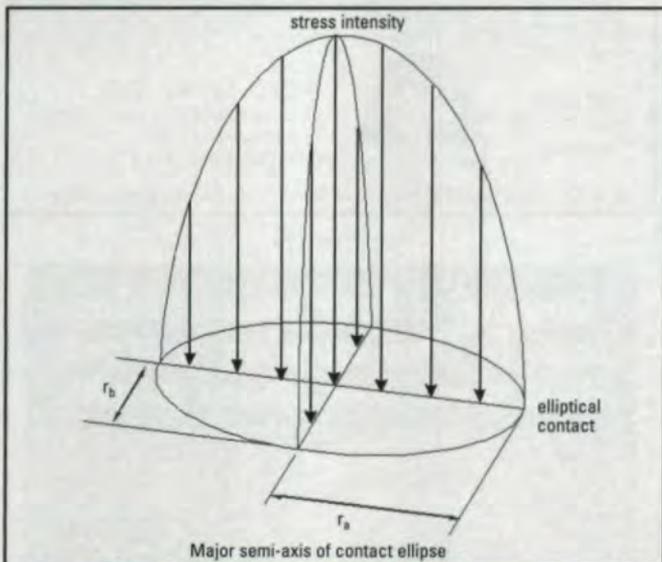


Fig. 4—Stress intensity for elliptical contact.

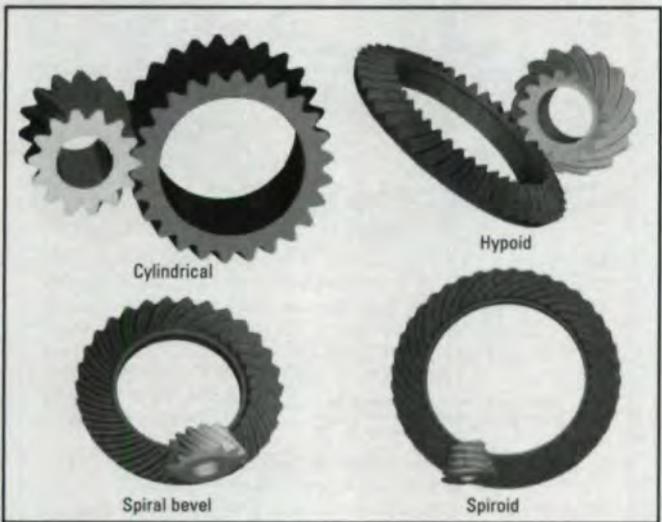


Fig. 5—Gear pairs.

$$\Delta\kappa_{\max} = \sqrt{\left[ \frac{1}{R_i^2} + \frac{2 \cos \Sigma}{R_i R_o} + \frac{1}{R_o^2} \right] (\cos^2 \psi_{pi} + \tan^2 \phi_n) \frac{\cos^2 \phi_n}{\sin \phi_n}} \quad (6b)$$

where

- $\Delta\kappa_{\min}$  minimum relative curvature between gear teeth
- $\Delta\kappa_{\max}$  maximum relative curvature between gear teeth
- $R_i$  virtual radius of the input hyperboloidal pitch surface
- $R_o$  virtual radius of the output hyperboloidal pitch surface
- $\psi_{pi}$  instantaneous spiral angle of the input gear ( $\psi_{pi} = -\psi_{po}$ )
- $\phi_n$  normal pressure angle
- $\Sigma$  included shaft angle.

The above formulas for extreme relative gear tooth curvature are applicable for hypoid, spiroid, worm, bevel and cylindrical gear pairs. Furthermore, those formulas are independent of the type of gear tooth profile. Recognize that when  $\Sigma = 0$  (i.e., planar gearing) and  $\psi = 0$  (i.e., spur gears), Equation 2b and the above relation for maximum relative gear tooth curvature,  $\Delta\kappa_{\max}$ , are identical. The mathematical development of the above expressions involves many mathematical relations, and only the results are presented. Additional insight into the mathematical derivations is provided in Reference 1.

**Profile Modification**

Ideally, two gear teeth in mesh are in line-contact for gear pairs with involute-type tooth profiles. However, gear designers introduce both profile relief and lead crown to accommodate errors in tooth spacing, runout, misalignment and deflections. Gear teeth with profile or tip relief have a reduction in tooth thickness in a particular transverse plane. The magnitude of the tip relief is usually restricted to micrometers ( $\mu\text{m}$ ) or a few thousandths of an inch. Crowned teeth have a reduction in tooth thickness in the lengthwise direction of the gear tooth. The magnitude of crowning is restricted to a few micrometers across the tooth face. Depicted in Figure 3 is a tooth profile with tip relief and lead crown. Such profile modification reduces theoretical line-contact to point-contact. Consequently, the above relations for extreme relative gear tooth curvature (i.e.,  $\Delta\kappa_{\max}$  and  $\Delta\kappa_{\min}$ ) must be modified to account for crown and profile relief.

There is no established standard for specifying tooth profile modification. Here, the deviation in ideal tooth profile is quadratic. Tip relief and lead crown are specified here in a manner analogous to the specification of addendum and dedendum. That is achieved by introducing a tip relief constant  $\delta_\phi$  and a lead crown constant  $\delta_\psi$ . Given the following tip relief constant  $\delta_\phi$  and lead crown constant  $\delta_\psi$ , the changes in curvature  $\delta\kappa_\psi$  and  $\delta\kappa_\phi$  are

$$\delta\kappa_\psi = \frac{8}{F_i^2 + F_o^2} \frac{\delta_\psi}{P_d} \quad (7a)$$

$$\delta\kappa_\phi = \left( \frac{2P_d}{a+b} \right)^2 \frac{\delta_\phi}{P_d} \quad (7b)$$

where

- $\delta_\psi$  lead crown constant
- $\delta_\phi$  tip relief constant
- $a$  addendum constant
- $b$  dedendum constant

$F_i, F_o$  face width  
 $P_d$  normal diametral pitch.

$\delta_{\psi}$  and  $\delta_{\phi}$  are the same for both the input and output gear elements such that the modified curvatures become

$$\delta\kappa_{min} = \frac{\delta\kappa_{\psi} + \delta\kappa_{\phi} (\sin^2\phi_n \tan^2\psi_{pi})}{1 + \sin^2\phi_n \tan^2\psi_{pi}} \quad (8a)$$

$$\delta\kappa_{max} = \frac{\delta\kappa_{\phi} + \delta\kappa_{\psi} (\sin^2\phi_n \tan^2\psi_{pi})}{1 + \sin^2\phi_n \tan^2\psi_{pi}} \quad (8b)$$

where

- $\delta\kappa_{min}$  change in minimum relative tooth curvature
- $\delta\kappa_{max}$  change in maximum relative tooth curvature
- $\delta\kappa_{\phi}$  change in relative tooth curvature in profile direction
- $\delta\kappa_{\psi}$  change in relative tooth curvature in lengthwise direction
- $\phi_n$  normal pressure angle
- $\psi_{pi}$  spiral angle.

For spur gears (i.e.,  $\psi = 0$ ), the face widths  $F_i$  and  $F_o$  are identical and equal to the distance between the heel and toe. The above change in relative gear tooth curvature for modified gear teeth must be added to the theoretical value. Thus, extreme gear tooth curvature can be expressed

$$\kappa_{min} = \delta\kappa_{min} \quad (9a)$$

$$\kappa_{max} = \Delta\kappa_{max} + \delta\kappa_{max} \quad (9b)$$

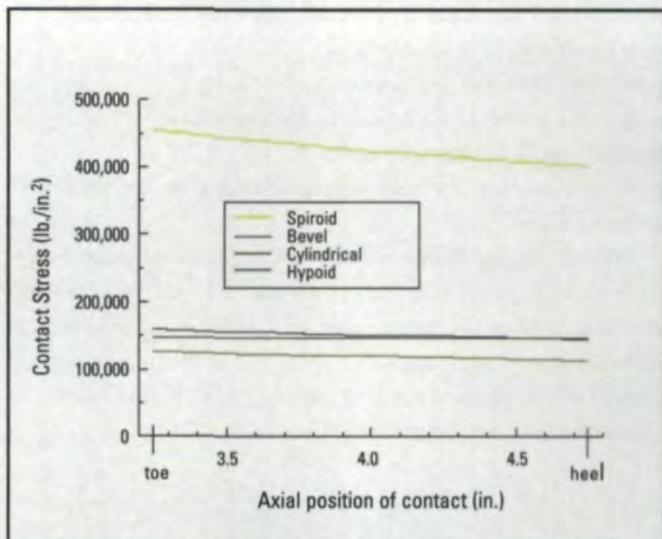


Fig. 6—Maximum contact stress across the face of cylindrical, hypoid, bevel and spiroid gear pairs.

Contact Stress

The transmitted load between two gear teeth is non-uniformly distributed over the surface area of contact. Depicted in Figure 4 is an elliptical contact area with semi-axes  $r_a$  and  $r_b$  along with a parabolic stress intensity. The sum of the pressure distribution over the area of contact results in the net force applied to the gear mesh interface. Determination of the maxi-

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maximum contact stress is based on the following assumptions:  
 gear tooth materials are elastic,  
 gear tooth materials are isotropic,  
 gear tooth material properties are homogeneous,  
 contact area is frictionless, and  
 radii of curvature are very big compared with the semi-axes contact ellipse.

The maximum compressive stress is evaluated using a matrix-based formula (see Ref 2). Hence, it is advisable that the elastic moduli of the foundations are similar in magnitude. It is recommended that Hertz's formulas for predicting maximum contact stress are used for gear elements with highly dissimilar elastic foundations. Introducing the constant

$$C = P \left( \frac{4\sqrt{6}}{3\pi^3} (5\pi - 4) \right) \left( \frac{1 - \mu_i^2}{E_i} + \frac{1 - \mu_o^2}{E_o} \right) \quad (10)$$

where

- P normal contact force
  - E<sub>i</sub> modulus of elasticity for input gear
  - E<sub>o</sub> modulus of elasticity for output gear
  - μ<sub>i</sub> Poisson's ratio for input gear
  - μ<sub>o</sub> Poisson's ratio for output gear,
- the semi-axes of the contact ellipse become

$$r_a = \left[ C \left( \frac{\kappa_{max}}{\kappa_{min}} \right)^{1/4} \frac{1}{\kappa_{min}} \right]^{1/3} \quad (11a)$$

$$r_b = \left[ C \left( \frac{\kappa_{max}}{\kappa_{min}} \right)^{1/4} \frac{1}{\kappa_{min}} \right]^{1/3} \quad (11b)$$

where

- r<sub>a</sub> major semi-axis of contact ellipse
- r<sub>b</sub> minor semi-axis of contact ellipse
- C constant
- κ<sub>max</sub> maximum relative curvature with profile modification
- κ<sub>min</sub> minimum relative curvature with profile modification.

The maximum contact stress σ<sub>c</sub> is obtained by integrating the pressure distribution over the area of contact A<sub>c</sub> and equating to the transmitted load P. The maximum stress σ<sub>c</sub> is 3π/4 times the average stress for an elliptical contact area and a proportional stress intensity, hence

$$\sigma_c = \frac{3\pi}{4} \frac{P}{\pi r_a r_b} \quad (12)$$

That formula neglects common loading factors that instantaneously increase the transmitted load P. One such loading factor that results in an instantaneous increase in transmitted load is the dynamic load that results from transmission error. A second loading factor that increases the transmitted load is a distribution factor. Shaft misalignment can result in a concentrated load for gears with high contact ratios. A third type of loading factor that gives an instantaneous increase in transmitted load is an application factor. Such factors are inherent in "rough" operating machinery, like internal combustion engines and crushing mechanisms. Additional insight into those factors is provided by AGMA. Contact stress is further affected by tractive or shear forces that result from the relative sliding and friction at the mesh and residual stresses in the tooth sub-surface. The magnitude of the shear load on the gear surface depends on the type of lubricant and its thickness. Simultaneously, the gear designer should be aware that relative gear tooth sliding at the contact

Table 1—Gear Pair Parameters.

	Cylindrical	Hypoid	Bevel	Spiroid
Shaft center distance E (in./mm)	2.00 50.8	2.00 50.8	0.00 0.00	2.00 50.8
Included shaft angle Σ (deg./rad.)	0.0 0.0	90 1.571	90 1.571	90 1.571
Axial position of toe w <sub>toe</sub> (in./mm)	3.25 82.55	3.25 82.55	3.25 82.55	3.25 82.55
Axial position of heel w <sub>heel</sub> (in./mm)	4.75 120.65	4.75 120.65	4.75 120.65	4.75 120.65
Number of teeth on input gear N (integer)	14	15	12	2
Number of teeth on output gear N (integer)	26	41	31	41
Nominal spiral angle ψ <sub>p</sub> (deg./rad.)	17.42 0.304	35.54 0.620	37.13 0.648	81.21 1.417
Nominal pressure angle (normal) φ <sub>n</sub> (deg./rad.)	30.00 0.524	55.94 0.976	65.07 1.136	47.80 0.834
Input shaft torque T (in.-lb/N-m)	900 101.7	900 101.7	900 101.7	120 14.36
Axial contact ratio m <sub>a</sub> (dimensionless)	1.5	1.5	1.5	4
Addendum constant (dimensionless)	1	1	1	1
Dedendum constant (dimensionless)	1	1	1	1
Tip relief constant δ <sub>t</sub> (dimensionless)	0.01	0.01	0.01	0.01
Lead crown constant δ <sub>v</sub> (dimensionless)	0.02	0.02	0.02	0.02

Table 2—Contact Stress Calculations.

	Cylindrical	Hypoid	Bevel	Spiroid
E <sub>i</sub> = E <sub>o</sub> (lb./in. <sup>2</sup> /N/m <sup>2</sup> )	30X10 <sup>6</sup> 207X10 <sup>9</sup>	30X10 <sup>6</sup> 207X10 <sup>9</sup>	30X10 <sup>6</sup> 207X10 <sup>9</sup>	30X10 <sup>6</sup> 207X10 <sup>9</sup>
μ <sub>i</sub> = μ <sub>o</sub> (dimensionless)	0.287	0.287	0.287	0.287
R <sub>i</sub> (in./mm)	0.6672 16.95	1.388 35.25	1.151 29.24	0.1180 3.00
R <sub>o</sub> (in./mm)	1.239 31.47	3.793 96.34	2.974 75.54	2.418 61.42
Σ (deg./rad.)	0 0	90 1.571	90 1.571	90 1.571
P <sub>d</sub> (in./mm)	10.49 0	5.405 26.74	5.212 21.16	8.477 13.4
α <sub>pi</sub> (deg./rad.)	0 0	0.467 63.26	0.369 68.84	0.234 76.6
α <sub>po</sub> (deg./rad.)	0 0	1.104 5.666	1.201 0	1.337 1.53
γ <sub>i</sub> (deg./rad.)	0 0	0.099 21.35	0 0	0.027 25.18
γ <sub>o</sub> (deg./rad.)	0 0	0.373 0.0063	0 0.025	0.439 0.1061
δκ <sub>min</sub> (1/in./1/m)	0.248 0.1324	0.886 0.0538	0.984 0.0477	4.177 0.005
δκ <sub>max</sub> (1/in./1/m)	5.252 0.0063	2.118 0.0225	1.878 0.0250	0.189 0.1061
κ <sub>min</sub> (1/in./1/m)	0.248 4.425	0.886 0.8818	0.984 1.008	4.177 6.413
κ <sub>max</sub> (1/in./1/m)	174.21 1.552	34.72 1.158	39.69 1.855	252.5 1.514
P (lb./N)	6.903 1.71X10 <sup>-4</sup>	5.151 0.874X10 <sup>-4</sup>	8.247 1.399X10 <sup>-4</sup>	6.734 1.143X10 <sup>-4</sup>
C (in. <sup>2</sup> /mm <sup>2</sup> )	0.076 0.4570	0.056 0.2133	0.0903 0.2418	0.074 0.1443
r <sub>a</sub> (in./mm)	11.607 0.1073	5.418 0.0341	6.142 0.0380	3.665 0.0186
r <sub>b</sub> (in./mm)	0.439 0.439	0.866 0.866	0.965 0.965	0.472 0.472
σ <sub>c</sub> (ksi/MPa)	149 1.018	119 823	151 1.043	424 2.925

zone can cause a rise in temperature at the mesh, resulting in a temperature gradient in the gear tooth and thus further affecting localized tooth contact stress.

**Examples**

Four examples are presented to illustrate the determination of contact stress between gear teeth in mesh. The first example is a helical cylindrical gear pair, the second example is a hypoid gear pair with non-zero spiral angle, the third example is a spiral bevel gear pair, and the last example is a spiroid gear pair (i.e., a hypoid gear pair with high spiral angle). Each gear pair has a 1.5-inch face width. The nominal gear parameters for each gear pair are provided in Table 1. Graphical illustrations of the gear pairs are provided in Figure 5. Intermediate calculations and final contact stress are presented in Table 2 for the face midpoint. Values for maximum contact stress are based on a single concentrated load and neglect load sharing resulting from high contact ratios, tooth deflections or wheel body deflections. A computer program has been written, and the variation in contact stress across the face of the gear pairs is depicted in Figure 6.

**Summary**

Simplified design formulas for evaluating the maximum contact stress between two gears in mesh are presented. The methodology is summarized as follows:

- demonstrated that relative tooth curvature for planar gears depends on pitch radii and pressure angle,
- presented a generalized formula for extreme relative curvature between gear teeth in mesh that is valid for any tooth type (involute or cycloid) and any gear type (cylindrical, bevel, hypoid, spiroid or worm),
- presented a generalized formula for relative gear tooth curvature for arbitrary tooth profile modification (tip relief and lead crown),
- presented explicit expressions for semi-axes of elliptical contact based on mattress formula,
- presented formula for maximum contact stress between gear teeth, and
- presented four examples to illustrate the use of formulas to determine maximum contact stress.

**References**

1. Dooner, D.B. and A. A. Seireg, *The Kinematic Geometry of Gearing: A Concurrent Engineering Approach*, John Wiley and Sons Inc., New York, 1995.
2. Dooner, D.B. "On the Generalized Contact Stress," *The International Conference on Mechanical Transmissions*, Chongqing, P.R. China, April 5-10, 2001.

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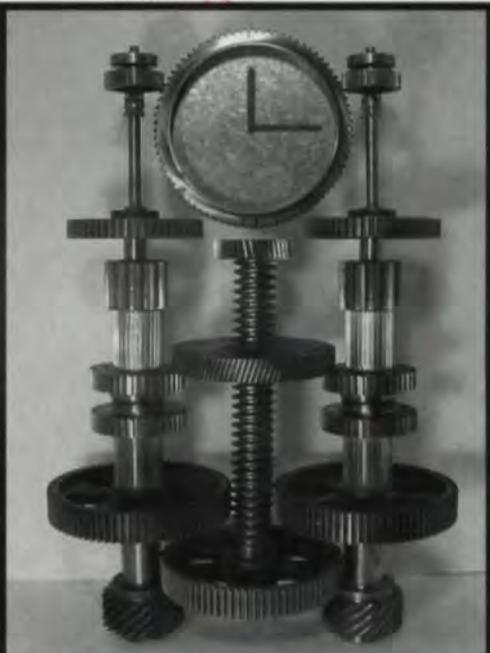


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