

Design Of Involute Gear Teeth

by
Fellows Corporation

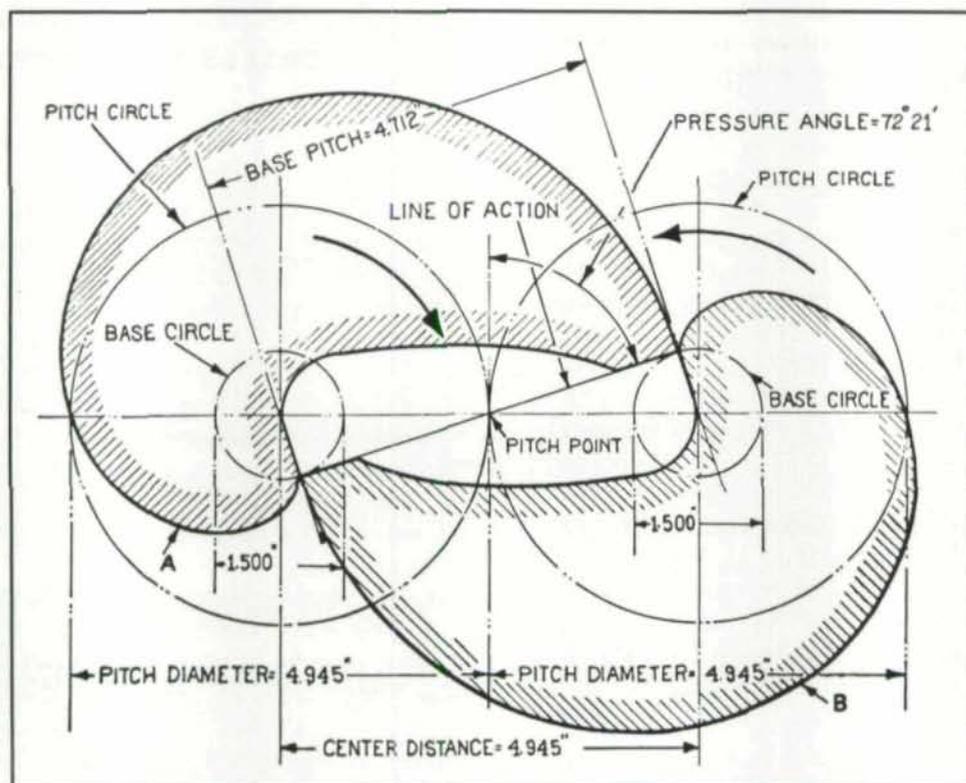


Fig. 1—Involute applied to two one-tooth gears, indicating that the involute has its origin at the base circle, but is not limited in length.

In designing involute gear teeth, it is essential that the fundamental properties of the involute curve be clearly understood. A review of "the Fundamental Laws of the Involute Curve" found in last issue will help in this respect. It has previously been shown that the involute curve has its origin at the base circle. Its length, however, may be anything from zero at the origin or starting point on to infinity. The problem, therefore, in designing gear teeth, is to select that portion of the involute, which will best meet all requirements.

Involute Curve Not Limited in Length

Fig. 1 shows a pair of one-tooth involute gears in theoretically perfect action. The application of the involute curve, as here presented, offers an interesting study. Although of little or no practical value as driving members, the extremities at which involute action may take place are here made plain and the nature of the involute curve made clearer.

It will be noted in Fig. 1 that each of the two involutes constitutes an unsymmetrical tooth. If B acts as a driver and rotates in the direction indicated by the arrow, A will rotate in the opposite direction. Contact will take place along the line of action which, in this particular case, is the circumference of the base circle and also the base pitch of the involute.

The circular pitch of these two involutes developed from a base circle of only $1\frac{1}{2}$ inches is 15.537 inches. The diametral pitch is 0.2022. It is of passing interest that the pressure angle must always be 72 degrees, 21 minutes, when a single involute effects complete rotation of a single engaging involute. The pressure angle of 72 degrees, 21 minutes is, of course, excessive, and these engaging involutes are obviously incapable of transmitting any but the lightest loads.

Factors to be Considered in Gear-Tooth Design

It has previously been shown that the transmission of smooth

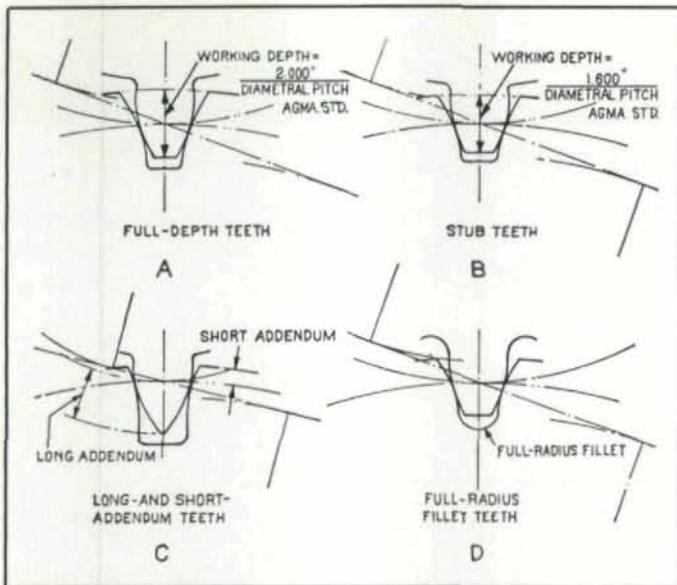


Fig. 2—Diagram illustrating various forms of involute gear teeth.

positive power requires that there must be a number of engaging teeth on driver and driven members. Also, that the following requirements must be satisfactorily met.

1. That there is no involute interference.
2. That there is no fillet interference.
3. That there is ample overlap of tooth action.
4. That a suitable pressure angle has been selected.
5. That excessive slippage is avoided.

Most of these requirements were briefly explained in the last issue, and we will now deal with their relationship to gear tooth design.

Gear-Tooth Shapes

There are in use today several forms of gear teeth; such as: $14\frac{1}{2}^\circ$ full-depth teeth, 20° full-depth teeth; and 20° stub teeth. Full-depth teeth, as shown at *A* in Fig. 2, have a working depth equal to:

$$\frac{2.000 \text{ inch}}{\text{*Diametral Pitch}}$$

There are also two stub-tooth forms, both having a pressure angle of 20 degrees. The so-called "Fellows" stub-tooth system, originated in 1906, is a combination of two diametral pitches. For example, $\frac{6}{8}$ pitch; in which the numerator of the fraction controls the number of teeth, circular pitch and pitch diameter; and the denominator of the fraction controls the working depth and the clearance. The American Gear Manufacturers Association has adopted a stub-tooth standard, see *B*, Fig. 2, in which the working depth is a constant proportion throughout the entire range and is equal to:

$$\frac{1.600 \text{ inch}}{\text{†Diametral Pitch}}$$

*Refer to Tables III to VI, inclusive.

†Refer to Table VII.

In those cases where a pinion having a relatively small number of teeth must operate with a gear having three or more times as many teeth, a condition known as interference is sometimes encountered. Several solutions to this problem have already been given. One method is to use long-and short-addendum teeth as indicated at *C* in Fig. 2. This form of tooth cannot be standardized as the lengths of the addendums on pinion and gear are governed by the ratio of the number of teeth in mesh, and the pressure angle used. For average applications, the maximum enlargement and reduction of the outside diameters of pinion and gear, respectively, seldom exceeds 10% of the standard outside diameters.

Still another form of tooth is indicated at *D* in Fig. 2. This is known as the "full-radius fillet" form. This form of tooth is used quite extensively for aircraft gears and pump gears. The advantages are that it provides not only a stronger tooth, but also one which is not so liable to have fracture cracks develop at the root of the tooth as a result of heat treatment.

Of course, other pressure angles, in addition to the standard $14\frac{1}{2}$ and 20 degree tooth shapes are used, but, a simple change in pressure angle, cannot in reality be considered as a different form of tooth. There are, however, other forms of teeth, but these are chiefly to meet special requirements, as will be discussed later.

Pressure Angle Depends on Portion of Involute Used

In designing gear teeth a primary consideration is to select that portion of the involute for the teeth which will best meet requirements. In Fig. 3 a series of parallel involutes, *A, B, C* and *D*, have been developed from the same base circle, and on these involutes, teeth of the same diametral pitch have been constructed. These teeth, as shown, have $14\frac{1}{2}$, 20, 25 and 30 degree operating pressure angles. It will be noted that as the pressure angle is increased, a different section of the involute is employed for that portion of the tooth above the base circle. It is also interesting to note the shift in the location of the pitch circles relative to the base circle as the operating pressure angles are increased. This would, of course, necessitate an increase in the center distance when mating with another gear.

In the case of the $14\frac{1}{2}$ degree pressure angle tooth, there is an undercut of the flank of the tooth which almost reaches to the pitch circle. This undercut condition naturally reduces

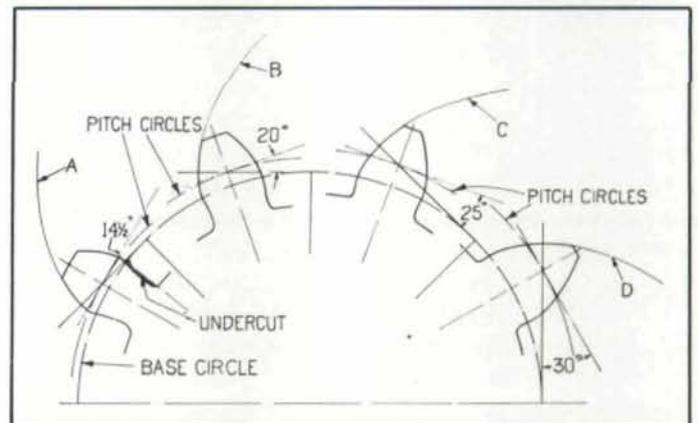


Fig. 3—Diagram illustrating that pressure angle is governed by portion of involute used for gear tooth.

gives $0.950 \div 2.868$, or approximately 33% overlap of action.

At C and D, Fig. 5, the addendums of the rack and pinions, respectively, have been shortened to 0.800 inch. In other words, the teeth are of stub-tooth form. At C, it will be noticed that contact between rack and pinion tooth does not start at the interference point, as was the case at A, but further along the line of action. At C the length of contact exceeds the base pitch by 0.670 inch. This gives $0.670 \div 2.868$, or approximately 23% overlap of action. At D, contact of the two 12-tooth pinions is well inside the interference points, and hence the line of contact is shorter than in all the previous cases. There is only 0.292 inch difference between the base pitch and the length of contact. Hence, the overlap of action is only $0.292 \div 2.868$, or approximately 10%. It will be seen from these diagrams that a pressure angle of $24^\circ 6'$ avoids interference and at the same time provides sufficient overlap of tooth action for the entire range of 12 teeth to a rack.

Relation of Pressure Angle to Interference

It has previously been explained that interference occurs when the involute of one tooth extends beyond the point of tangency of the line of action and the base circle. In Fig. 6, a 12-tooth pinion of 20 degrees pressure angle, full-depth tooth, is shown in engagement with a 40-tooth gear. It will be noted that the 40-tooth gear contacts the flank of the pinion tooth well in advance of the zone of contact. Therefore these gears will not operate satisfactorily, because the gear tooth "hooks" into the flank of the pinion tooth. The positions of the pinion and gear teeth, where involute interference commences, are indicated by the dotted outlines.

Involute interference between gear teeth can be determined graphically, as shown in Fig. 7, or by means of a simple calculation. In Fig. 7, $R_{(1)}$ represents the maximum permissible outside radius of the gear to avoid involute interference; R represents the base radius of the pinion; C is the center distance, and a the pressure angle.

Example: Assume that it is necessary to determine if involute interference will be present between a 10-pitch pinion of 12 teeth and a 30-tooth gear, the teeth to be $14\frac{1}{2}$ degree pressure angle and of full-depth. The pitch diameter of the pinion is 1.20 inches, and the base diameter is $1.20 \times \cos 14\frac{1}{2}$ degrees, or

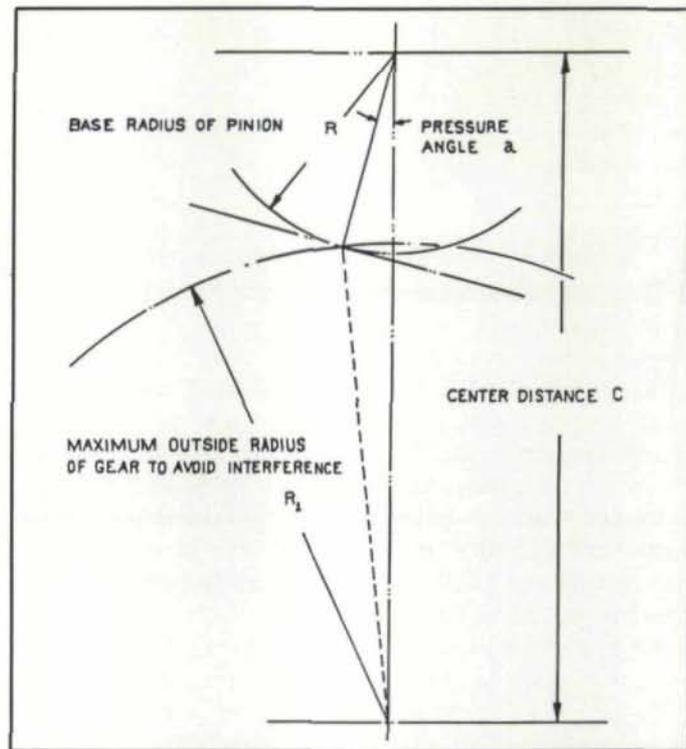


Fig. 7—Diagrammatical method for determining location of "natural" interference point.

1.1618 inches. The base radius of the pinion is therefore $1.1618 \div 2$, or 0.5809 inch. The center distance C is 2.10 inches. The maximum permissible outside radius of the gear to avoid involute interference is found as follows:

$$R_1 = \sqrt{C^2 + R^2 - 2(CR) \cos a}$$

$$R_1 = \sqrt{2.10^2 + 0.5809^2 - 2(2.10 \times 0.5809) \cos 14\frac{1}{2}^\circ}$$

$$R_1 = 1.544 \text{ inch, approximately}$$

The standard outside radius of the gear is 1.60 inch which is greater than R_1 by 0.056 inch, indicating that the outside radius of the gear would have to be reduced 0.056 inch to avoid involute interference; or other methods, previously explained in last issue, would have to be adopted.

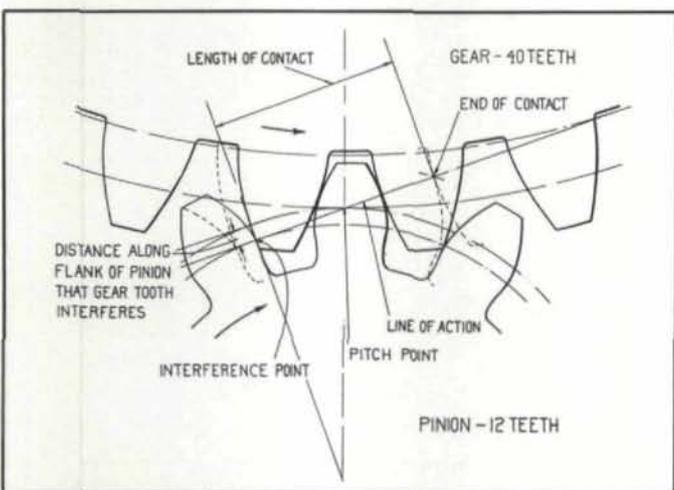


Fig. 6—Diagram illustrating interference of gear tooth with flank of pinion tooth.

Involute Interference between Gear and Rack Teeth

The previous example presented a method for determining involute interference between two gears. Fig. 8 illustrates a method for determining the smallest permissible number of teeth in a gear that will operate with a rack without involute interference. There are three controlling factors: diametral pitch, length of addendums and pressure angle.

When the pressure angle, diametral pitch, and addendums are known, the pitch radius and the smallest permissible number of teeth in the gear at which involute interference commences are determined as follows: Assume that it is necessary to find the smallest number of teeth in a gear of 10-diametral pitch, 0.100-inch addendum, and $14\frac{1}{2}^\circ$ pressure angle.

Referring to Fig. 8, distance $X = A \times \cot. 14\frac{1}{2}^\circ = 0.38667$ inch. Distance $Y = X \times \cot. 14\frac{1}{2}^\circ = 1.4951$ inches. Then, the pitch radius = $Y + A$, or $1.4951 +$

0.1000" = 1.5951 inches, and the pitch diameter = 1.5951" × 2 = 3.190 inches. The number of teeth = pitch diameter × diametral pitch, or 3.190" × 10 = 31.9, or 32 teeth.

The minimum number of teeth for 20° full-length is 18, and for 20° stub teeth, with 8/10 addendum, is 14.

Relation of Pressure Angle, Addendum and Pitch to Length of Contact

There is a limit to the amount that the tooth can be modified, the pressure angle increased, or the teeth shortened, if continuous action is to result. As shown in Fig. 9, the length of contact L must be greater than the base pitch B to avoid lack of continuous action. With this particular tooth ratio, pitch, pressure angle and tooth length, the theoretical length of the line of action extends from points e to f . The actual usable length of the line of action—or line of contact—is determined by the outside radii of both gear and pinion. If interference were present, however, this would not be the case. The starting point of action is at point h where the outside radius of the pinion cuts the line of action, and could extend to point e without interference. In this case, however, the other limit of contact is at point k , where the outside radius of the gear cuts the line of action.

In general practice, it is considered that for the best action, the length of the line of contact should be at least 1¼ times the base pitch (the base pitch is the circular pitch transferred to the base circle). The length of the line of contact can be determined graphically as shown in Fig. 9, or it can be calculated.

In those cases where a small pressure angle and long addendums are used, and especially in conjunction with a small number of teeth in the pinion and a high ratio, the length of the line of contact is not controlled by the length of the addendums of both gear and pinion due to interference. If for instance, the pinion has such a small number of teeth that the distance hf (f , representing the interference point) is less than the distance hk , interference would be present to reduce the effective length of the line of contact.

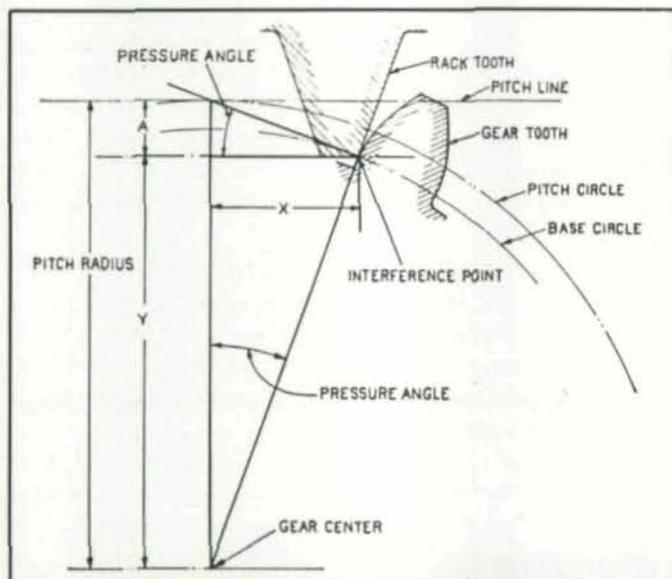


Fig. 8—Diagram illustrating method for determining permissible minimum number of teeth in a gear that will operate with a rack tooth without involute interference.

The length of the line of contact, ignoring the presence of interference, which should be determined separately, as previously explained, can be found by the following formula: (For notation see Fig. 9).

$$L = \sqrt{r_1^2 - r^2} + \sqrt{R_1^2 - R^2} - \sqrt{C^2 - (R + r)^2}$$

In which:

- L = Active length of line of contact
- r = Base radius of pinion
- r_1 = Outside radius of pinion
- R = Base radius of gear
- R_1 = Outside radius of gear
- C = Center distance

Example: Assume that it is necessary to determine the length of the line of contact L of a 10-pitch gear and pinion, the pinion having 15 teeth and the gear 30 teeth, the tooth form being 20 degrees full-depth.

Dimensions	Pinion	Gear
Pitch radii	0.750"	1.500"
Outside radii, r_1 and R_1	0.850"	1.600"
Base radii, r and R	0.7048"	1.4095"
Center distance, 2.250". Then:		

$$L = \sqrt{0.850^2 - 0.7048^2} + \sqrt{1.600^2 - 1.4095^2} - \sqrt{2.250^2 - (0.7048 + 1.4095)^2}$$

$$L = \sqrt{0.2258} + \sqrt{0.5733} - \sqrt{.5921}$$

$$L = 0.4752 + 0.7572 - 0.7695$$

$$L = 0.4629"$$

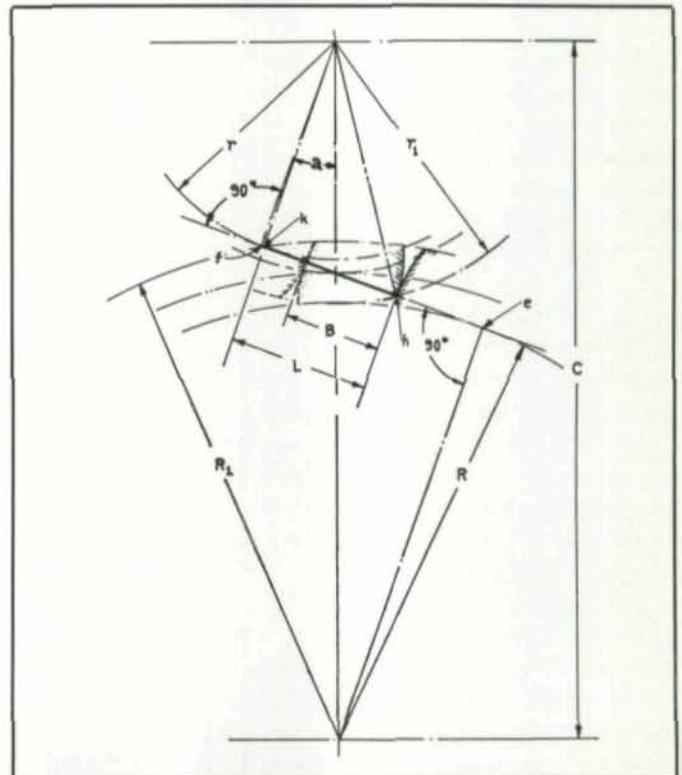


Fig. 9—Diagram illustrating mathematical method for determining actual length of tooth contact.

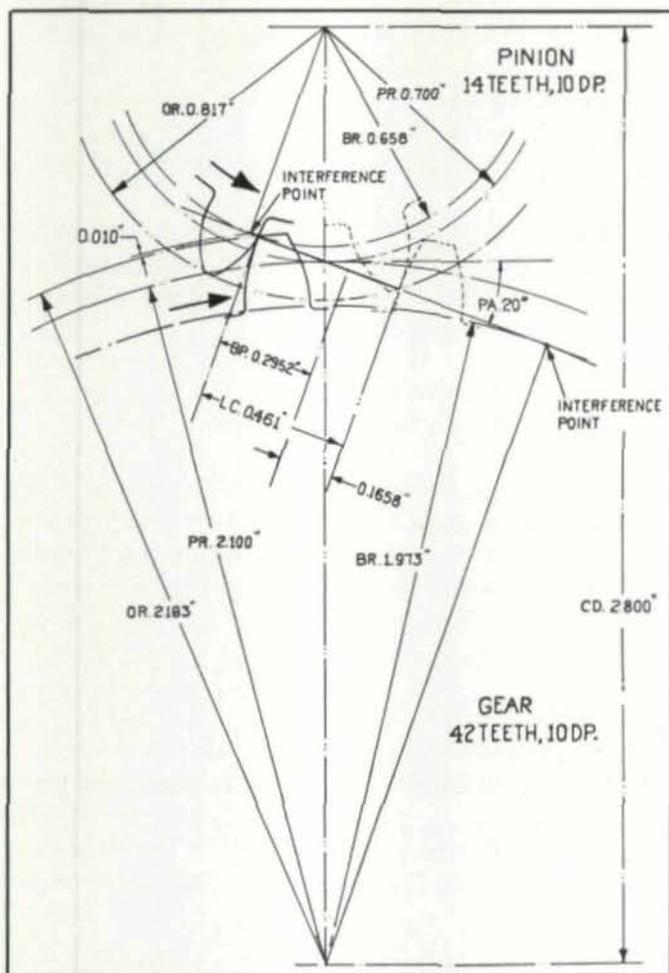


Fig. 10—Diagram illustrating method of designing gears for maximum efficiency by using long- and short-addendum teeth.

The contact ratio is then equal to the length of the line of contact L divided by the base pitch, and the base pitch is equal to the circular pitch times the cosine of the pressure angle. The circular pitch is $0.3142''$, and the base pitch is $0.3142'' \times \cos 20^\circ = 0.2952''$. Then the contact ratio equals $0.4629'' \div 0.2952 = 1.57$ approximately, which is greater than the theoretical minimum required for the best action.

Designing Gears for Maximum Efficiency

Mention has already been made of the relationship of tooth ratio, tooth length, pressure angle, etc., to interference, undercut, length of line of contact, etc. When so-called standard tooth proportions, pressure angles, and interchangeability, are neither necessary nor desirable, it is possible to so proportion the addendums of gear and pinion, respectively, and select a pressure angle which will provide the best possible operating conditions to meet the requirements.

The first step, after the tooth ratio has been decided upon, is to select a diametral pitch, which, with standard tooth proportions, will provide the tooth strength necessary to carry the assumed load. The tooth ratio and pitch, of course, will establish the center distance and pitch diameters. The tooth ratio will have a bearing on the pressure angle selected, as a starting point in determining the proportions of the teeth.

As a concrete example, assume that the ratio is 3 to 1, and

that 14 and 42 teeth of 10-diametral pitch have been selected. The calculated pitch diameters would then be: 1.400 inches for the pinion, and 4.200 inches for the gear. The center distance would then be:

$$\frac{4.200 + 1.400}{2}, \text{ or } 2.800 \text{ inches.}$$

To proceed, we lay out a diagram, on an enlarged scale, as shown in Fig. 10, space off the center distance, and assume a pressure angle of 20 degrees. This establishes the line of action and the interference points. In cases of an unequal ratio, it is always the larger of the two gears that is liable to cause interference. Hence, the outside circle of the gear should not extend beyond the interference point on the pinion tooth.

Assuming that the pinion is the driver, we can now proceed to lay out the teeth, and in order to be on the safe side, and avoid possible interference, we draw a circle representing the outside circle of the gear 0.010 inch inside the interference point. If the outside diameter of the gear, thus determined, is less than the standard diameter for a 10 pitch 42-tooth gear, then the outside diameter of the 14-tooth pinion would be enlarged a similar amount. The next step is to decide whether standard or special cutters will be used. Assume in this case that it is decided to use standard cutters, 20 degree pressure angle, full-depth teeth. The whole depth of a 10-pitch gear is 0.2250 inch. This distance for gear and pinion, respectively, is laid out on the center line, and circles drawn representing the root circle of the gear, and outside and root circles of the pinion. Where the outside circle of the gear cuts the line of action is one extremity of the line of contact, and where the outside circle of the pinion cuts the line of action is the other extremity.

We can now measure (or calculate), the actual length of contact, and by comparing this with the base pitch, can determine the overlap of action. If this meets the requirements, the problem is solved. It will be noticed in Fig. 10 that the normal pitch is 0.2952 inch, and the length of the line of contact is 0.461 inch. The overlap of action then equals $(.461 \div .2952) - 1$, or 56%, approximately. The use of long- and short-addendums for pinion and gear, respectively, have avoided involute interference, and provided a sufficient overlap of tooth action.

Referring to Fig. 10, it will be seen that the outside diameter of the pinion has been increased from 1.600 inch to 1.634 inch, an increase of 0.034 inch. The outside diameter of the gear has been reduced from 4.400 to 4.366 inches, a decrease of 0.034 inch, the same amount as the pinion. In effect, long- and short-addendums for pinion and gear, respectively, have solved our problem.

If on the other hand, the ratio had been such that a sufficient overlap of action could not be obtained and interference avoided, other pressure angles could be used until the desired results had been obtained. It also might be necessary to use special cutters—this would especially be the case if it was desired to balance the teeth in gear and pinion—, respectively, for strength. In most cases, the pinion teeth would be weaker than the gear teeth; therefore, the thickness of the teeth on the gear would be reduced, and the thickness of the pinion teeth increased.

Pitch Diameter and Its Relation to Center Distance

The pitch circles of a pair of gears are the imaginary circles

on which the gear teeth "roll" without slippage. These circles are tangent to each other at the pitch point. The radii of the pitch circles of a pair of gears are determined by dividing the center distance into the same proportion as the numbers of teeth in the two mating members. Thus, for a given center distance and tooth ratio, the pitch circle diameters are fixed. In some cases, in order to indicate backlash, the pitch diameters are dimensioned a slight amount undersize. This procedure is incorrect. Backlash is obtained by decreasing the thickness of the teeth, and should be indicated by a chordal tooth thickness dimension. Backlash in a pair of gears can be determined by the thickness of a feeler, which can be placed between the teeth, or by a change in center distance, see Fig. 11.

When gears are cut by the generating method, backlash between the teeth can be obtained either by using a cutter with teeth thicker than standard, or by feeding the cutter in to a sufficient depth to reduce the thickness of the gear teeth the necessary amount.

Backlash between Gear Teeth

Theoretically speaking, gear teeth should run together without appreciable backlash. From a practical standpoint, however, this is impossible due to the following reasons: 1. Perfection in cutting and mounting is an impossible achievement because of manufacturing tolerances, which should be as wide as possible to reduce costs. 2. Space between the teeth must be provided to aid lubrication. 3. Temperature changes due to speed and other causes affect sizes of gears and spacing of shafts on which gears are mounted. In view of these conditions, it is necessary to provide a certain amount of freedom between the teeth, so that they will not bind when operating together.

The term "backlash" can be defined as the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the pitch circles, as actually indicated by measuring devices. Backlash may be determined in the plane of rotation or normal plane, and along the line of action.

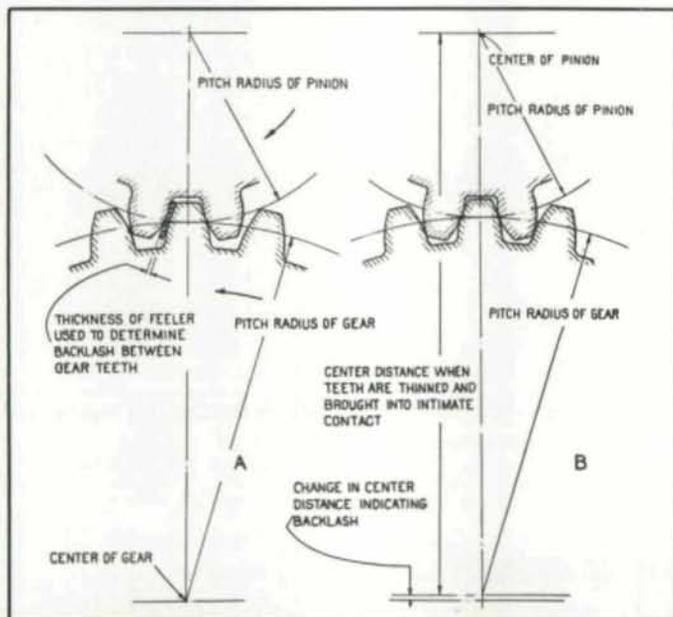


Fig. 11—Diagram illustrating two methods of determining backlash between gear teeth.

Methods for Determining Amount of Backlash

Several methods are used for indicating and checking backlash. The common method, particularly with spur gears, is to place the mating gears on pins located at the correct center distance, and then measure the backlash by the use of a feeler gage inserted between the teeth, as shown at A in Fig. 11.

Another method is to place the gears on pins and bring the teeth into intimate contact, and then determine the difference between the standard and measured center distance, as shown at B, Fig. 11. This last check does not indicate backlash directly, but the amount of backlash can be determined by the following formula:

$$B = 2 \tan a \times d$$

In which:

B = Backlash in inches

a = Pressure angle

d = Difference between standard and measured center distances

A third method, shown diagrammatically in Fig. 12, is to use a dial indicator in connection with a fixture for holding the gears on studs. One of the gears should be fixed so that it cannot rotate on the stud. The gears are set at standard center distance. The "free" gear is rotated so that its profile (away from the indicator plunger) is in intimate contact with the other gear. The indicator plunger is then set in contact with the profile of one tooth, and the needle set at zero. The "free" gear is then rotated to bring the opposite sides of the teeth in contact, and the reading on the dial indicator noted. This reading indicates the relative "rotary" movement of the "free" gear, and, hence, the backlash or freedom that exists between the teeth.

In checking helical gears, the backlash is measured in the normal plane, instead of in the plane of rotation, as is the case with spur gears. The method just described can be applied satisfactorily to both spur and helical gears. In providing for backlash, it is customary, when a small pinion is to operate with a larger gear, to reduce the thickness of the teeth on the larger gear to provide the necessary backlash, leaving the pinion teeth of standard tooth thickness.

Effect of Center Distance Change and Pressure Angle on Amount of Backlash

The chart in Fig. 13 is presented to illustrate how a change

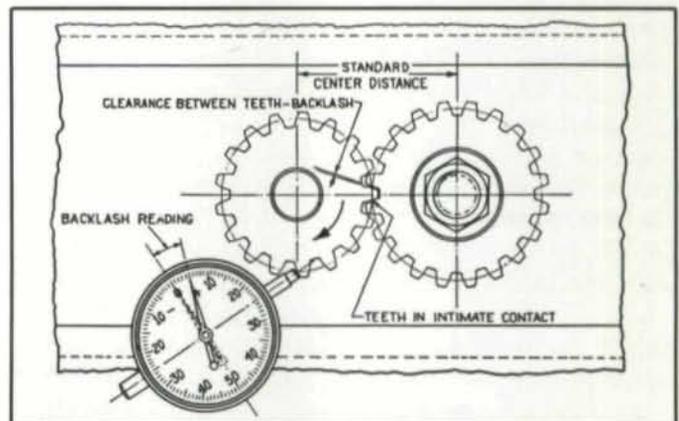


Fig. 12—Diagram illustrating method of measuring backlash with dial indicator—gears held on pins at standard center distance.

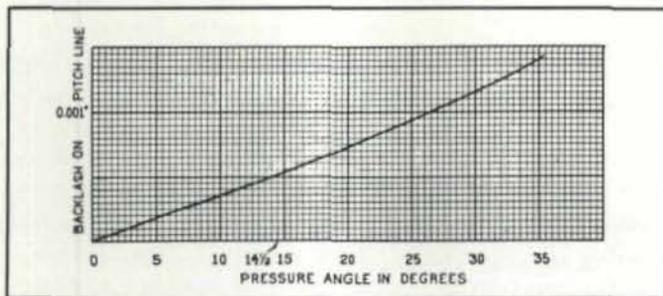


Fig. 13—Chart illustrating how pressure angle affects change in backlash for each 0.001 inch change in center distance.

in pressure angle affects the amount of backlash between gear teeth when the center distance is increased. As shown on this chart for each 0.001 inch increase in center distance, the backlash between the teeth increases as the pressure angle is increased. This increase in backlash for the various pressure angles listed for each 0.001 inch change in center distance is as follows:

Pressure Angle in Degrees	Increase in Backlash in Inches
5	0.00017
10	0.00035
14½	0.00052
15	0.00054
20	0.00073
25	0.00094
30	0.00115
35	0.0014

For example, the difference in the amount of backlash for each 0.001 inch change in the center distance between 14½ and 20 degrees pressure angles is: $0.00073 - 0.00052 = 0.00021$ inch more backlash for 20 than 14½ degrees pressure angle.

Other Factors Affecting Amount of Backlash between Mating Gear Teeth

In addition to changes in center distance, several other factors affect the actual amount of backlash between mating gear teeth such as: tooth spacing errors, runout, and errors in lead of helical gears. Obviously, the amount should be sufficient to permit the gears to rotate freely when cut to prescribed manufacturing tolerance. Table I lists minimum, maximum, and average backlash for spur and helical gears.

It should be understood that gears, which are operated at high speeds, require more backlash than gears operating at slower speeds. The values in Table I should prove satisfactory for gears operating at speeds up to 1500 surface feet per minute. For gears operating at speeds in excess of 1500 surface feet per minute, the values should be increased slightly over those listed in the maximum column. Gears which are to operate at higher speeds should also be more accurately cut and mounted than slower operating gears.

In precision gears, where close tolerances on backlash are demanded, several methods are employed. One method is to use selective assembly; the other is to mount the gears so that

the center distance can be adjusted, such as by the use of eccentric bushings.

TABLE I
BACKLASH BETWEEN TEETH OF MATING SPUR AND HELICAL GEARS

DIAMETRAL PITCH	BACKLASH IN INCHES		
	MINIMUM	AVERAGE	MAXIMUM
1	0.025	0.0325	0.040
1½	0.018	0.0225	0.027
2	0.014	0.0170	0.020
2½	0.011	0.0135	0.016
3	0.009	0.0115	0.014
4	0.007	0.0090	0.011
5	0.006	0.0075	0.009
6	0.005	0.0065	0.008
7	0.004	0.0055	0.007
8 and 9	0.004	0.0050	0.006
10 to 13	0.003	0.0040	0.005
14 to 19	0.003	0.0040	0.005
20 to 40	0.002	0.0030	0.004
41 to 60	0.0015	0.0022	0.003
61 to 120	0.0010	0.0015	0.002
121 and Finer	0.0005	0.0007	0.001

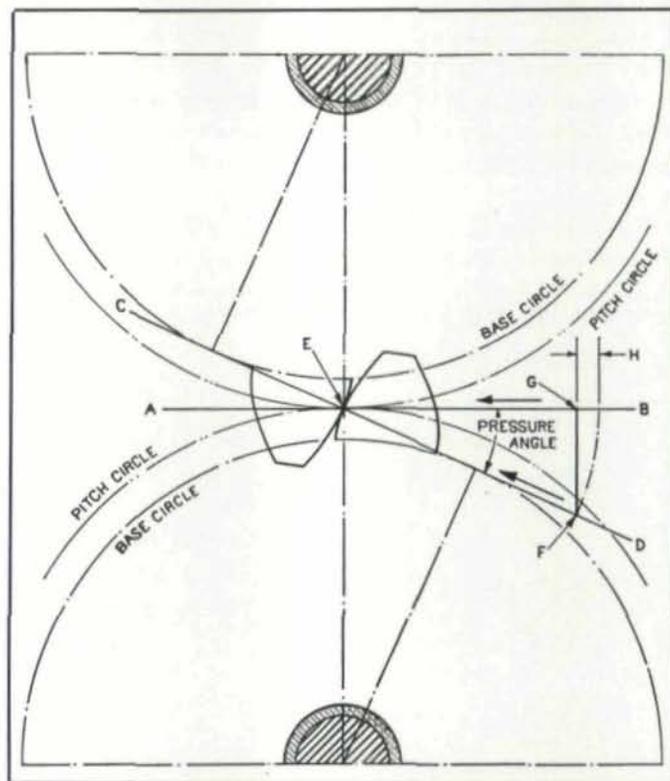


Fig. 14—Diagram illustrating effect of an increase in the pressure angle on the load transmitted to the supporting bearings.

Relation of Pressure Angle to Load on Supporting Bearings

An increase in the pressure angle does not have a marked effect on the resultant load on the supporting bearings as is (Continued on page 45)

Double Enveloping Worm Gears . . .

(Continued from Page 16)

(2) reductions at approximately 92%-93% overall efficiency or three (3) reductions at about 89%-90% efficiency. The worm gearbox with a 20:1 ratio will have about 85%-87% efficiency. A 30:1 ratio helical reducer will generally require three (3) meshes with approximately 89%-90% efficiency. The 30:1 wormgear speed reducer will have an efficiency of approximately 83%-84%. You can see the helical box is more efficient, but certainly not to the degree often claimed.

There are other inherent advantages in worm gearing which must be considered in evaluating the application and the type of gearing intended for that application. Double enveloping worm gearing will take a momentary overload of 300%, whereas helical gearboxes are only designed for 200%, momentary overload. Helical gearboxes restrict motor starting capacity to 200%, whereas double enveloping worm gearboxes permit 300%. Generally speaking, worm gearboxes are smaller in overall size and weight, and in terms of horsepower capacity, generally less expensive. In addition, with compactness of the double enveloping wormgear principle, double enveloping gearboxes are more compact and weigh less, horsepower for horsepower, than cylindrical gear reducers.

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E-2 ON READER REPLY CARD

Design of the Involute . . .

(Continued from page 44)

generally supposed. In other words, bearing pressures are not greatly affected by an increase in the pressure within the usual limits. This condition is graphically presented in Fig. 14. To construct this diagram, draw a line *AB* at right angles to the line of centers and tangent to both pitch circles. Then draw a line *CD* tangent to the base circles and passing through the pitch point *E*; this line representing the pressure angle. Now drop a perpendicular at any point *G* on line *AB*, passing through line *CD* at point *F*. With *E* as a center and *EF* as a radius scribe an arc. Increases in the load on the supporting bearings due to changes in pressure angle can be determined graphically by noting the changes in distance *H*, as the pressure angle changes. It is apparent that the load-increase is the ratio of lengths *EG* to *EF*, and is, therefore, proportional to the secant of the pressure angle.

The second column in Table II gives the secants of various pressure angles listed in the first column, and ranging from 14½ up to and including 30 degrees.

The last column lists in terms of percentage, the increase in the load as compared with 14½ degrees. It will be noticed that an increase in the pressure angle from 14½ to 20 degrees, results in an increased load on the supporting bearings of only 3 percent.

(Continued on the next page)

Scoring Load Capacity . . .

(Continued from page 30)

Conclusion

A new method for scoring load capacity rating, based on the calculation of a mean, weighted flank temperature, the integral temperature, has been described. The limiting temperatures necessary, for the definition of a scoring safety factor, can be obtained from any available gear oil test. The method is valid for all types of oils as straight mineral, mild and EP-oils, as well as, synthetic oils where gear scoring tests are available. The method was checked with more than 300 scoring tests on test rigs and more than 100 practical gears with and without scoring damages. A good correlation was found for the Integral Temperature Criterion, and it was obviously superior to the Total Temperature Method, as well as, to the Scoring Index Method.

The method has been modified for bevel and hypoid gears(10) and even in this field of application a good correlation between calculated scoring factors and field experience was achieved.

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E-4 ON READER REPLY CARD

TABLE II

RELATION BETWEEN PRESSURE ANGLE AND LOAD ON BEARINGS

PRESSURE ANGLE DEGREES	SECANT	COMPARISON OF INCREASED LOAD ON SUPPORTING BEARINGS IN TERMS OF PERCENTAGE
14½	1.0329	
17	1.0457	1½%
20	1.0642	3%
22½	1.0824	4.8%
24-6	1.0955	6%
27½	1.1274	9.1%
30	1.1547	11.3%

TABLE III

GEAR TOOTH PARTS
Standards in Bold-Face Type. For Full-Length Tooth Generated Gears
Dedendum = 1.250 ÷ D. P. Up to and Including 19 D. P.

Diametral Pitch (D P)	Circular Pitch (C P) (Inches)	Module (M)	DIMENSIONS IN INCHES				
			Circular Thickness (CTh.)	Addendum (A)	Dedendum (D)	Whole Tooth Depth (WD)	Double Tooth Depth (DD)
3.1416	1	8.0851	0.5000	0.3183	0.3979	0.7162	1.4324
3.1750	0.9895	8	0.4947	0.3150	0.3937	0.7087	1.4174
3.2774	0.9586	7½	0.4793	0.3052	0.3814	0.6866	1.3732
3.3510	15/16	7.5798	0.4687	0.2984	0.3730	0.6714	1.3428
3.3867	0.9276	7½	0.4638	0.2953	0.3691	0.6644	1.3288
3.5034	0.8967	7¼	0.4484	0.2854	0.3568	0.6422	1.2844
3.5904	¾	7.0744	0.4375	0.2785	0.3482	0.6267	1.2534
3.6286	0.8658	7	0.4329	0.2756	0.3445	0.6201	1.2402
3.7630	0.8349	6¾	0.4174	0.2657	0.3322	0.5979	1.1958
3.8666	13/16	6.5691	0.4062	0.2586	0.3233	0.5819	1.1638
3.9077	0.8040	6½	0.4020	0.2559	0.3199	0.5758	1.1516
4	0.7854	6.3500	0.3927	0.2500	0.3125	0.5625	1.1250
4.0640	0.7730	6¼	0.3865	0.2460	0.3076	0.5536	1.1072
4.1888	¾	6.0638	0.3750	0.2387	0.2984	0.5371	1.0742
4.2333	0.7421	6	0.3711	0.2362	0.2952	0.5314	1.0628
4.4174	0.7112	5¾	0.3556	0.2264	0.2830	0.5094	1.0188
4.5696	11/16	5.5585	0.3437	0.2188	0.2735	0.4923	0.9846
4.6182	0.6803	5½	0.3401	0.2165	0.2707	0.4872	0.9744
4.7124	¾	5.3900	0.3333	0.2122	0.2653	0.4775	0.9550
4.8381	0.6493	5¼	0.3247	0.2067	0.2584	0.4651	0.9302
5	0.6283	5.0800	0.3142	0.2000	0.2500	0.4500	0.9000
5.0266	½	5.0531	0.3125	0.1989	0.2487	0.4476	0.8952
5.0800	0.6184	5	0.3092	0.1968	0.2461	0.4429	0.8858
5.3474	0.5875	4¾	0.2938	0.1870	0.2338	0.4208	0.8416
5.5851	9/16	4.5478	0.2812	0.1790	0.2238	0.4028	0.8058
5.6444	0.5566	4½	0.2783	0.1772	0.2215	0.3987	0.7974
5.9765	0.5257	4¼	0.2628	0.1673	0.2091	0.3764	0.7528
6	0.5236	4.2333	0.2618	0.1667	0.2083	0.3750	0.7500
6.2832	½	4.0425	0.2500	0.1591	0.1989	0.3580	0.7160
6.3500	0.4947	4	0.2473	0.1575	0.1968	0.3543	0.7086
6.7733	0.4638	3¾	0.2319	0.1476	0.1845	0.3321	0.6642
7	0.4488	3.6286	0.2244	0.1429	0.1786	0.3215	0.6430
7.1808	7/16	3.5372	0.2187	0.1393	0.1741	0.3134	0.6268
7.2571	0.4329	3½	0.2164	0.1378	0.1722	0.3100	0.6200
7.8154	0.4020	3¼	0.2010	0.1279	0.1599	0.2878	0.5756
8	0.3927	3.1750	0.1964	0.1250	0.1563	0.2813	0.5626
8.3776	¾	3.0319	0.1875	0.1194	0.1492	0.2686	0.5372
8.4667	0.3711	3	0.1855	0.1181	0.1477	0.2658	0.5316
9	0.3491	2.8222	0.1745	0.1111	0.1389	0.2500	0.5000
9.2364	0.3401	2¾	0.1701	0.1082	0.1353	0.2435	0.4870
9.4248	½	2.6950	0.1667	0.1061	0.1326	0.2387	0.4774
10	0.3142	2.5400	0.1571	0.1000	0.1250	0.2250	0.4500
10.0531	5/16	2.5266	0.1562	0.0995	0.1244	0.2239	0.4478
10.1600	0.3092	2½	0.1546	0.0984	0.1230	0.2214	0.4428
11	0.2856	2.3091	0.1428	0.0909	0.1137	0.2046	0.4092
11.2889	0.2783	2¼	0.1391	0.0886	0.1107	0.1993	0.3986
12	0.2618	2.1167	0.1309	0.0833	0.1042	0.1875	0.3750
12.5664	¼	2.0213	0.1250	0.0796	0.0995	0.1791	0.3582

TABLE IV

GEAR TOOTH PARTS
Standards in Bold-Face Type. For Full-Length Tooth Generated Gears
Dedendum = 1.250 ÷ D. P. Up to and Including 19 D. P.
Dedendum = 1.200 ÷ D. P. + 0.002" for 20 D. P. and Finer

Diametral Pitch (D P)	Circular Pitch (C P) (Inches)	Module (M)	DIMENSIONS IN INCHES				
			Circular Thickness (CTh.)	Addendum (A)	Dedendum (D)	Whole Tooth Depth (WD)	Double Tooth Depth (DD)
12.7000	0.2474	2	0.1237	0.0787	0.0984	0.1771	0.3542
13	0.2417	1.9538	0.1208	0.0769	0.0962	0.1731	0.3462
14	0.2344	1.8143	0.1122	0.0774	0.0892	0.1607	0.3214
14.5143	0.2164	1½	0.1082	0.0689	0.0861	0.1550	0.3100
15	0.2094	1.6933	0.1047	0.0667	0.0833	0.1500	0.3000
16	0.1964	1.5875	0.0982	0.0625	0.0781	0.1406	0.2812
16.7552	¾	1.5159	0.0938	0.0597	0.0746	0.1343	0.2686
16.9353	0.1855	1½	0.0928	0.0591	0.0738	0.1329	0.2658
17	0.1848	1.4941	0.0924	0.0588	0.0735	0.1323	0.2646
18	0.1745	1.4111	0.0873	0.0556	0.0694	0.1250	0.2500
19	0.1663	1.3368	0.0827	0.0526	0.0658	0.1184	0.2368
20	0.1571	1.2700	0.0785	0.0500	0.0620	0.1120	0.2240
20.3200	0.1546	1¼	0.0773	0.0492	0.0611	0.1103	0.2206
21	0.1486	1.2095	0.0748	0.0476	0.0591	0.1067	0.2134
22	0.1428	1.1546	0.0714	0.0455	0.0565	0.1020	0.2040
23	0.1366	1.1043	0.0683	0.0435	0.0542	0.0977	0.1954
24	0.1309	1.0583	0.0655	0.0417	0.0520	0.0937	0.1874
25	0.1257	1.0160	0.0628	0.0400	0.0500	0.0900	0.1800
25.1328	¾	1.0106	0.0625	0.0398	0.0497	0.0895	0.1790
25.4000	0.1237	1	0.0618	0.0394	0.0492	0.0886	0.1772
26	0.1208	0.9769	0.0604	0.0385	0.0482	0.0867	0.1734
27	0.1164	0.9407	0.0582	0.0370	0.0464	0.0834	0.1668
28	0.1122	0.9071	0.0561	0.0357	0.0449	0.0806	0.1612
29	0.1083	0.8759	0.0542	0.0345	0.0434	0.0779	0.1558
30	0.1047	0.8467	0.0523	0.0333	0.0420	0.0753	0.1506
31	0.1013	0.8194	0.0506	0.0323	0.0407	0.0730	0.1460
32	0.0982	0.7938	0.0490	0.0313	0.0395	0.0708	0.1416
33	0.0952	0.7697	0.0476	0.0303	0.0384	0.0687	0.1364
33.8667	0.0928	¾	0.04638	0.0295	0.0374	0.0669	0.1338
34	0.0924	0.7471	0.04620	0.0294	0.0373	0.0667	0.1334
35	0.0898	0.7257	0.04488	0.0286	0.0363	0.0649	0.1298
36	0.0873	0.7056	0.04368	0.0278	0.0355	0.0631	0.1262
37	0.0849	0.6865	0.04245	0.0270	0.0344	0.0614	0.1228
38	0.0827	0.6684	0.04134	0.0263	0.0336	0.0599	0.1198
39	0.0806	0.6513	0.04028	0.0256	0.0328	0.0584	0.1168
40	0.0785	0.6350	0.03927	0.0250	0.0320	0.0570	0.1140
41	0.0766	0.6195	0.03831	0.0244	0.0313	0.0557	0.1114
42	0.0748	0.6048	0.03740	0.0238	0.0306	0.0544	0.1088
43	0.0731	0.5907	0.03653	0.0233	0.0299	0.0532	0.1064
44	0.0714	0.5773	0.03570	0.0227	0.0293	0.0520	0.1040
45	0.0698	0.5644	0.03491	0.0222	0.0287	0.0509	0.1018
46	0.0683	0.5522	0.03415	0.0217	0.0281	0.0498	0.0996
47	0.0668	0.5404	0.03342	0.0213	0.0275	0.0488	0.0976
48	0.0655	0.5292	0.03272	0.0208	0.0270	0.0478	0.0956
49	0.0641	0.5184	0.03205	0.0204	0.0265	0.0469	0.0938
50	0.0628	0.5080	0.03141	0.0200	0.0260	0.0460	0.0920
50.2656	1/16	0.5053	0.03130	0.0199	0.0258	0.0457	0.0914
50.8000	0.0618	½	0.03092	0.0197	0.0256	0.0453	0.0906

TABLE V

GEAR TOOTH PARTS
Full-Length Teeth Fine-Pitch Gears
Dedendum = 1.2000 ÷ D. P. + 0.002"
Whole Depth = 2.2000 ÷ D. P. + 0.002"

Diametral Pitch (D P)	DIMENSIONS IN INCHES				
	Circular Thickness (CTh.)	Addendum (A)	Dedendum (D)	Whole Tooth Depth (WD)	Double Tooth Depth (DD)
51	0.03080	0.0196	0.0255	0.0451	0.0902
52	0.03021	0.0192	0.0251	0.0443	0.0886
53	0.02964	0.0189	0.0246	0.0435	0.0870
54	0.02909	0.0185	0.0242	0.0427	0.0854
55	0.02856	0.0182	0.0238	0.0420	0.0840
56	0.02805	0.0179	0.0234	0.0413	0.0826
57	0.02756	0.0175	0.0231	0.0406	0.0812
58	0.02708	0.0172	0.0227	0.0399	0.0798
59	0.02662	0.0169	0.0223	0.0393	0.0786
60	0.02618	0.0167	0.0220	0.0387	0.0774
61	0.02575	0.0164	0.0217	0.0381	0.0762
62	0.02534	0.0161	0.0214	0.0375	0.0750
63	0.02493	0.0159	0.0210	0.0369	0.0738
64	0.02454	0.0156	0.0208	0.0364	0.0728
65	0.02417	0.0154	0.0205	0.0358	0.0716
66	0.02380	0.0152	0.0202	0.0353	0.0706
67	0.02344	0.0149	0.0199	0.0348	0.0696
68	0.02310	0.0147	0.0196	0.0344	0.0688
69	0.02277	0.0145	0.0194	0.0339	0.0678
70	0.02244	0.0143	0.0191	0.0334	0.0668
72	0.02182	0.0139	0.0187	0.0326	0.0652
74	0.02123	0.0135	0.0182	0.0317	0.0634
76	0.02067	0.0132	0.0178	0.0309	0.0618
78	0.02014	0.0128	0.0174	0.0302	0.0604
80	0.01964	0.0125	0.0170	0.0295	0.0590
82	0.01916	0.0122	0.0166	0.0288	0.0576
84	0.01870	0.0119	0.0163	0.0282	0.0564
86	0.01827	0.0116	0.0160	0.0276	0.0552
88	0.01785	0.0114	0.0156	0.0270	0.0540
90	0.01745	0.0111	0.0153	0.0264	0.0528
92	0.01707	0.0109	0.0150	0.0259	0.0518
94	0.01671	0.0106	0.0148	0.0254	0.0508
96	0.01636	0.0104	0.0145	0.0249	0.0498
98	0.01603	0.0102	0.0142	0.0244	0.0488
100	0.01571	0.0100	0.0140	0.0240	0.0480
102	0.01540	0.0098	0.01		

TABLE VI
GEAR TOOTH PARTS

Full-Length Teeth Fine-Pitch Gears
Dedendum = $1.2000 + D.P. + 0.002''$
Whole Depth = $2.2000 + D.P. + 0.002''$

Diametral Pitch (D.P.)	DIMENSIONS IN INCHES				
	Circular Thickness (C.Th.)	Addendum (A)	Dedendum (D)	Whole Tooth Depth (W.D.)	Double Tooth Depth (D.D.)
106	0.01482	0.0094	0.0133	0.0228	0.0456
108	0.01454	0.0093	0.0131	0.0224	0.0448
110	0.01428	0.0091	0.0129	0.0220	0.0440
112	0.01402	0.0089	0.0127	0.0216	0.0432
114	0.01378	0.0088	0.0125	0.0213	0.0426
116	0.01354	0.0086	0.0123	0.0210	0.0420
118	0.01331	0.0085	0.0122	0.0206	0.0412
120	0.01309	0.0083	0.0120	0.0203	0.0406
122	0.01288	0.0082	0.0118	0.0200	0.0400
124	0.01267	0.0081	0.0117	0.0197	0.0394
126	0.01247	0.0079	0.0115	0.0195	0.0390
128	0.01227	0.0078	0.0114	0.0192	0.0384
130	0.01208	0.0077	0.0112	0.0189	0.0378
132	0.01190	0.0076	0.0111	0.0187	0.0374
134	0.01172	0.0075	0.0110	0.0184	0.0368
136	0.01155	0.0074	0.0108	0.0182	0.0364
138	0.01138	0.0072	0.0107	0.0179	0.0358
140	0.01122	0.0071	0.0106	0.0177	0.0354
142	0.01106	0.0070	0.0105	0.0175	0.0350
144	0.01091	0.0069	0.0103	0.0173	0.0346
146	0.01076	0.0068	0.0102	0.0171	0.0342
148	0.01061	0.0068	0.0101	0.0169	0.0338
150	0.01047	0.0067	0.0100	0.0167	0.0334
152	0.01033	0.0066	0.0099	0.0165	0.0330
154	0.01020	0.0065	0.0098	0.0163	0.0326
156	0.01007	0.0064	0.0097	0.0161	0.0322
158	0.00994	0.0063	0.0096	0.0159	0.0318
160	0.00982	0.0063	0.0095	0.0158	0.0316
170	0.00924	0.0059	0.0091	0.0149	0.0298
180	0.00873	0.0056	0.0087	0.0142	0.0284
190	0.00827	0.0053	0.0083	0.0136	0.0272
200	0.00785	0.0050	0.0080	0.0130	0.0260
210	0.00748	0.0048	0.0077	0.0125	0.0250
220	0.00714	0.0045	0.0075	0.0120	0.0240
230	0.00683	0.0043	0.0072	0.0116	0.0232
240	0.00655	0.0042	0.0070	0.0112	0.0224
250	0.00628	0.0040	0.0068	0.0108	0.0216

TABLE VII

GEAR TOOTH PARTS
ANSI Standard Stub-Tooth Gears
Addendum = $1 + D.P.$
Dedendum = $0.7 + D.P.$
Whole Depth = $1.8 + D.P.$

Diametral Pitch (D.P.)	DIMENSIONS IN INCHES				
	Circular Thickness (C.Th.)	Addendum (A)	Dedendum (D)	Whole Tooth Depth (W.D.)	Double Tooth Depth (D.D.)
3	0.5236	0.2667	0.3333	0.6000	1.2000
3½	0.4488	0.2286	0.2857	0.5143	1.0286
4	0.3927	0.2000	0.2500	0.4500	0.9000
5	0.3142	0.1600	0.2000	0.3600	0.7200
6	0.2618	0.1333	0.1667	0.3000	0.6000
7	0.2244	0.1143	0.1428	0.2571	0.5142
8	0.1963	0.1000	0.1250	0.2250	0.4500
9	0.1745	0.0889	0.1111	0.2000	0.4000
10	0.1571	0.0800	0.1000	0.1800	0.3600
11	0.1428	0.0727	0.0909	0.1636	0.3272
12	0.1309	0.0667	0.0833	0.1500	0.3000
13	0.1208	0.0615	0.0769	0.1384	0.2768
14	0.1122	0.0571	0.0714	0.1285	0.2570
15	0.1047	0.0533	0.0667	0.1200	0.2400
16	0.0982	0.0500	0.0625	0.1125	0.2250
17	0.0924	0.0471	0.0588	0.1059	0.2118
18	0.0873	0.0444	0.0556	0.1000	0.2000
19	0.0827	0.0421	0.0526	0.0947	0.1894
20	0.0785	0.0400	0.0500	0.0900	0.1800
22	0.0714	0.0364	0.0454	0.0818	0.1636
24	0.0654	0.0333	0.0417	0.0750	0.1500
26	0.0604	0.0308	0.0384	0.0692	0.1384
28	0.0561	0.0286	0.0357	0.0643	0.1286
30	0.0524	0.0267	0.0333	0.0600	0.1200
32	0.0491	0.0250	0.0312	0.0562	0.1124
34	0.0462	0.0235	0.0294	0.0529	0.1058
36	0.0436	0.0222	0.0278	0.0500	0.1000
38	0.0413	0.0211	0.0263	0.0474	0.0948
40	0.0393	0.0200	0.0250	0.0450	0.0900

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Items for this column should be sent to **GEAR TECHNOLOGY**, The Journal of Gear Manufacturing, P.O. Box 1426, Elk Grove Village, IL 60007 or phone (312) 437-6604. We welcome news announcements concerning new employees, retirement, or other announcements of interest to the gear industry. Announcements must be received by the 25th of the month, two months prior to the date of the next publication. Items received after this date will be held for the following publication.

M & M Precision & Reishauer Marketing Agreement

M & M Precision Systems have recently completed a marketing agreement with **Reishauer Corp.** The Reishauer staff have completed training at the M & M Precision Systems plant in Dayton, Ohio, and they are available immediately to assist the M & M network.

American Pfauter Adds New Distributor

American Pfauter Ltd., manufacturer of gear production and measuring systems, has expanded its distributor organization by naming Machine Tool Systems, Inc., a subsidiary of the Robert E. Morris Co. of Hartford, Connecticut. Under the management of President David Yaged and V.P. of Sales Dick Long, the new distributorship will serve Alabama, Georgia, Florida, Virginia, North and South Carolina from its offices in Charlotte and Greenville.

Gleason Opens Detroit Office

The Detroit Sales Office for **Gleason** is now open at: 370 Franklin Center, 29100 Northwestern Highway, Southfield, MI 48034. Geoffrey Ashcroft, Regional Sales Manager for this office can be reached by calling (313) 353-5205.

Acme-Cleveland Purchases Two British Firms

Spline Gauges Ltd. and its affiliate, **Piccadilly Precision Engineering Ltd.** of Tamworth England have been purchased by **Acme-Cleveland**. These companies make gauges and other products used to control the accuracy in the production of high precision gears. They will continue to be managed by Keneth Foster, who is the current managing director.

Cadillac Machinery Sales Program

Cadillac Machinery, Elk Grove, Illinois announces a new sales program to allow gear manufacturers to convert their under utilized machinery into the capital dollars needed to upgrade to the latest technology that new machinery has to offer. Call Richard Goldstein (312) 437-6600 for details.

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