CNC Manufacturing of Circular Faced Cylindrical Gears

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Spur cylindrical gears are usually cut using a hob and therefore present an essentially straight face to which crowning can be added to prevent edge contact. Rather than using a rack or hob, it is possible to cut cylindrical gears with a face mill cutter. In the following presentation, these gears are termed "spurved," i.e.—a contraction of "spur" and "curved."

Significant qualities of spurved gears include the convexo-concave contact in the face-width direction, which can result in improvements such as better lubricating conditions and lower contact stresses, increased bending stiffness (because of the circular face), and a self-centering characteristic in which only one member need be located axially with a small thrust bearing.

This paper presents the basic equations needed to model spurved gears. It is shown that cutter tilt can be used to control the lengthwise dimensions of the contact pattern, which conditions contact stresses and lubrication.

An application demonstrates actual spurved gear sets cut on a CNC machine using standard face mill cutters with modified roll.

Introduction

Spurved gears are essentially spur gears generated using a face mill cutter. Spurved gears can theoretically be cut on conventional spiral-bevel gear generators and multi-axis CNC machines.

Because of their convexo-concave contact in the face-width direction, spurved gears offer strength and contact characteristics, making them interesting for certain parallel shaft applications where increased tooth bending stiffness is required.

Liu (Ref.1)looks at the basic geometry of spur gears with a circular shape along the face-width, using different cutters for the pinion and gear so as to obtain the desired contact characteristic. Wu et al. (Ref. 2)., Zhang et al. (Ref. 3), Jiang et al. (Ref. 4) and Chen et al. (Ref. 5) all consider the profile as being curvilinear, thereby increasing the load carrying capacity and improving lubricating conditions at the expense of increased sensitivity to center distance change.

Fuentes et al. (Ref. 6) rather consider standard face mill cutters with a straight edge and look at generation either using completing or fixed-setting cycles.

In this paper, a simple and readily applicable approach is applied where off-the-shelf face mill cutters are used to cut cylindrical gears. The unified model (Ref. 7) is the basis for the tooth flank generator where the same face mill cutter is used to generate both the pinion and gear. Cutter tilt is introduced to control the width of the contact pattern in the face width direction, while modified roll is used to control transmission error (TE) in order to guarantee relief at contact entry and exit. An application example shows the excellent agreement obtained between the theoretical model and the actual part.

The Tooth Flank Generator

A tooth flank generator can be described as a group of software functions defining the shape of a tool and its movements relative to a work piece. Since the vast majority of gears are generated, the generating process should be the basis of any tooth flank generator.

The generating process (Ref. 7) describes one cutter blade, representing one tooth of a theoretical generating gear, meshing with the work piece. This is written as:

$$\overline{N}\cdot\overline{V}_r=0 \tag{1}$$

where the relative speed vector of the contacting tool to work surfaces is in a plane perpendicular to the common normal vector. When applied to the reference frames in Fig. 1, Eq. 1 yields an unbounded generated surface in a reference frame attached to the work piece. The surface is a function of the machine settings and three variables, respectively, cutter position α_c (angular or linear), work piece roll angle α_3 and S, the position of a point along the edge of the cutter blade:

$$S=f(\alpha_c,\alpha_3) \tag{2}$$

The solution of Equation 2 is a series of contact points between the cutter blade and the work that describes a line along the path of the cutter blade. The envelope of a series of such lines yields a generated tooth (Fig. 4). The tooth flank generator includes work and tool adjustments and movements found in gear cutting machines. In CNC machines, machine settings can be continuously altered during generation to allow improvements in gear kinematics. Figure 1 represents the most general case in the cutting process simulation and is therefore the basis for the general model. Vector $\vec{D}$ is the position of a point along a face mill or face hob cutter blade. Figure 2 shows the general representation of a face mill or face hob cutter blade from which vector $\vec{D}$ is obtained (Eq. 4).The implicit equation of the generated tooth surface is:

$$\vec{X}=\vec{D}[\alpha_c]_1[\tau]_3[J]_1[R][L_1m]_1[\Gamma]_2[P][\alpha_3]_3[R_c]_3 \tag{3}$$

$$\vec{D}=\begin{bmatrix} S\cos(\varphi) \\ 0 \\ (R\pm S\sin(\varphi)) \end{bmatrix} \tag{4}$$

Similarly, Equation 5 defines vector $\vec{N}$, the normal to the cutter blade at point S; Equation 6 gives the transformations between the cutter blade and the work piece.

$$\vec{N}=\begin{bmatrix} \sin(\varphi) \\ 0 \\ \mp\cos(\varphi) \end{bmatrix} \tag{5}$$

$$\vec{N}_x=\vec{N}[\alpha_c]_1[\tau]_3[J]_1[R][L_1m]_1[\Gamma]_2[\alpha_3]_3[R_c]_3 \tag{6}$$

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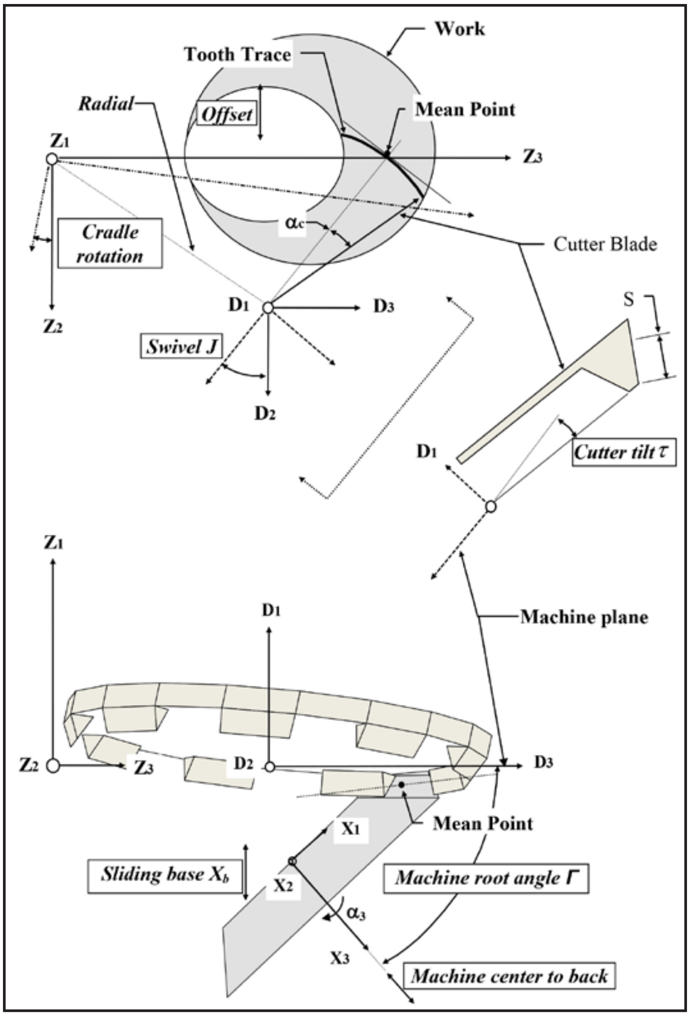


Figure 1 General reference frames.

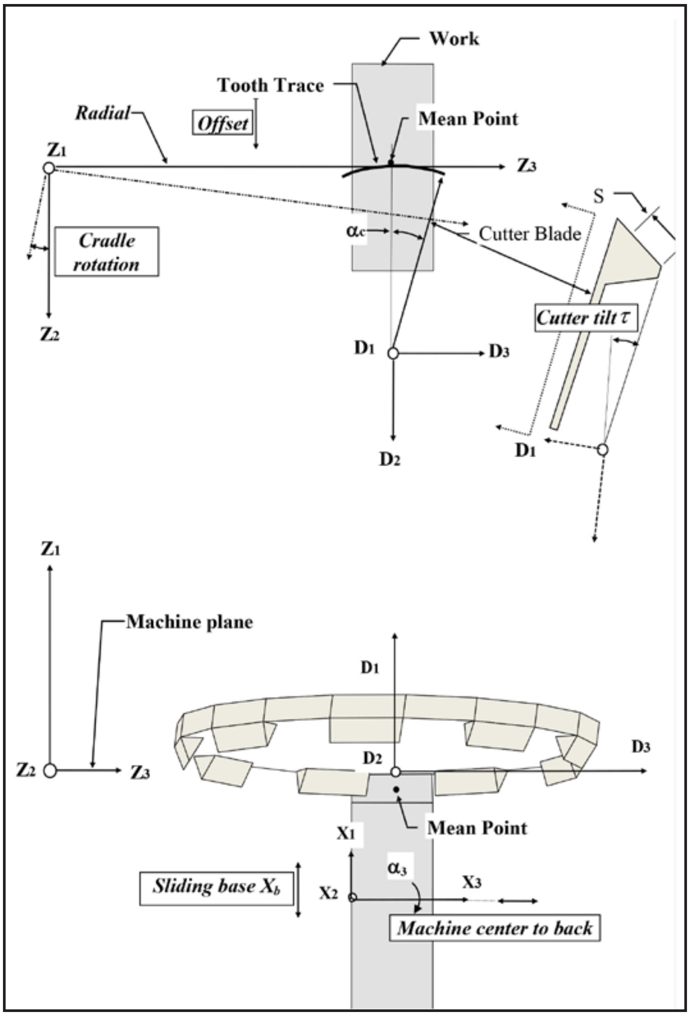


Figure 2 Spurved gear reference frames.

In Equations 3and 6, rotations and translations can be expanded in Taylor series to allow higher-order manufacturing flexibility on CNC machines. For example, the cradle rotation angle L_{1m} can be written as:

$$L_{1m} = a_3 R_r + A_2 (C_r - a_3 R_r)^2 - A_3 (C_r - a_3 R_r)^3 + A_4 (C_r - a_3 R_r)^4 - A_5 (C_r - a_3 R_r)^5 + A_6 (C_r - a_3 R_r)^6 \tag{7}$$

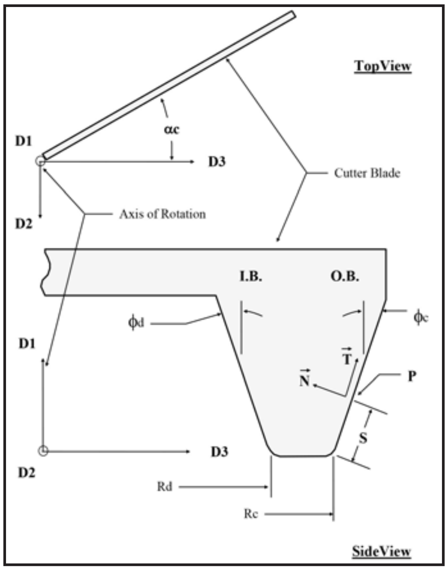


Figure 3 Cutter blade reference frame.

where α_3 is the roll angle of the work piece, R_r is the ratio of roll between the work piece and the cradle, C_r is the cradle angle given in the machine settings, and A_i are the coefficients for each term of the series.

For spurved gears, the terms involving cutter swivel J, root angle Γ and face hobbing rotation R_c are dropped from Equations 3 and 6, and the reference frames are simplified (Fig. 2); Figure 4 shows a spurved pinion model.

Tooth Contact Analysis of Spurved Gears

At any contact point, the coordinates on the pinion and gear teeth must be equal and the normal vectors must be collinear and opposed in a common reference frame:

$$\vec{Z}_P = \vec{Z}_G \qquad \vec{N}_P = \vec{N}_G \tag{8}$$

A true conjugate point of contact exists where 1) tooth to tooth separation is minimum on the Ease Off and 2) all components of the pinion and gear normal vectors are equal. The series of true conjugate contact points yields the path of contact (PoC). All other contact points yield the Ease-Off surface (Fig. 5), which emphasizes the deviations in conjugacy

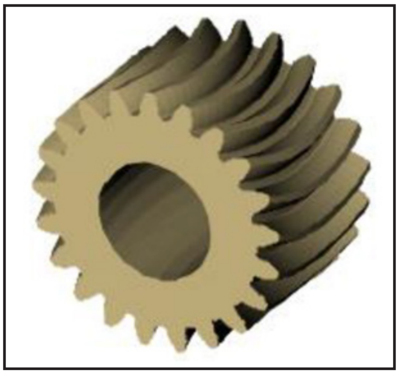


Figure 4 Spurved pinion.

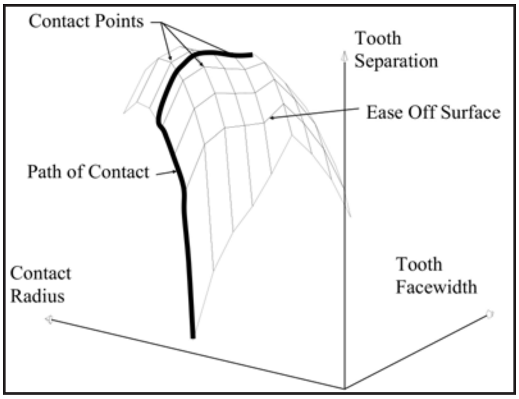


Figure 5 Ease-Off surface.

between the meshing pinion and gear teeth.

The transmission error (TE) is derived from the PoC. It is the expression of the difference between the actual and theoretical angular positions of the gear member (Eq. (9)):

$$\delta\varphi_3 = \varphi_3 \theta_3 m_g \tag{9}$$

where $\delta\varphi_3$ is the TE, φ_3 is the calculated angular position of the gear and $\theta_3\ m_g$ is the theoretical angular position of the gear, based on the pinion angular position θ_3 and the gear ratio m_g. A negative result to Equation 9 means that the gear is late relative to the pinion, and is the desired situation in order to prevent premature contact entry.

If the gear tooth flank is coated with a marking compound, the succession of contact points as meshing proceeds leaves a trace indicating which part of the tooth flank comes in contact. This trace is called the contact pattern. The contact pattern is calculated as follows:

- The Ease-Off surface is calculated; from this, the path of contact and tooth profile separation are obtained for each point of the PoC;
- Each contact point of the Ease Off is checked to test whether separation is larger or smaller than the requested marking compound thickness. If separation is smaller or equal, the contact point is considered within the contacting area.

Design of Face Milled Spurved Gears

Similar to conventional spur gears, the basic design of spurved gears involves selecting the module, face width, and tooth numbers to achieve the desired gear ratio and tooth strength.

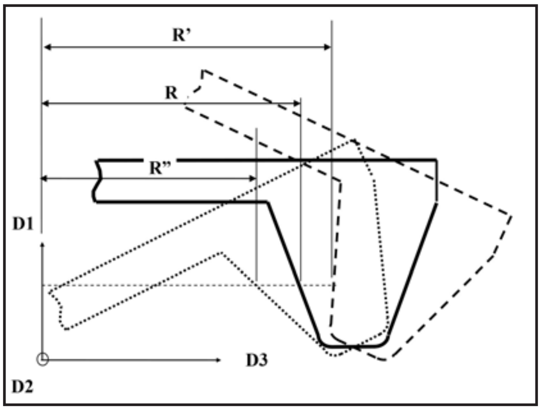


Figure 6 Effective cutter radius and cutter tilt.

However, the diameter and blade angles of the selected face mill cutter play an important role in the dimensions of the contact pattern, and therefore the load carrying capacity and sensitivity to positional changes of the gear set.

For a given operating pressure angle, selecting different angles for the convex and concave blades imposes cutter tilt. Cutter tilt changes the effective radius of curvature along the face width (Fig. 6) where the effective cutter radius R changes to R' or R" — depending on cutter tilt sign.

Therefore, changing the operating pressure angle and/or the cutter blade angles offers some control over the dimensions of the contact pattern. However, for a given cutter tilt angle, a smaller cutter diameter will generate a comparatively smaller contact pattern. Table 1 reflects this behavior where, for the given operating pressure angle and blade angles, a 4" cutter is shown to produce a significantly wider contact pattern than a 2" cutter. Module is 1.352 mm and face width 20 mm.

Table 1 Effect of cutter diameter on contact pattern size

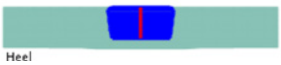
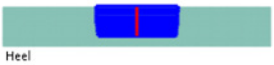
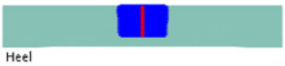
Characteristics	4" Cutter	2" Cutter
Oper. Pressure Angle	20"	20"
Convex Blade Angle	22"	22"
Concave Blade Angle	18"	18"
Contact Pattern	 Heel	 Heel

Table 2 Effect of cutter tilt on contact pattern size

Characteristics	4" Cutter	2" Cutter
Oper. Pressure Angle	20°	20°
Convex Blade Angle	18°	18°
Concave Blade Angle	22°	22°
Contact Pattern	 Heel	 Heel

Likewise, for a given operating pressure angle, inverting the values of the convex and concave blade angles reverses tilt and consequently modifies the effective cutter radius, thereby changing contact pattern size. Table 2 reflects this behavior where, for the same operating pressure angle as in Table 1, but with inverted blade angles, the contact patterns are seen to be significantly smaller than in Table 1.

By definition, spurved gears are fully conjugate at mid-face. This means that in the absence of modifications to the profiles of the cutter blades, TE will be null. This is shown (Fig. 7), where three TE curves (blue, red and pink lines) are shown on the 0.0 line.

TE is desired in gear sets in order 1) to provide smooth motion transfer between meshing tooth pairs when subject to positional errors, and 2) to avoid excessive stresses at contact entry and exit. TE can be introduced in a number of ways. For example: 1) using blade deviations such as TopRem; 2) using circular cutter edges, or 3) using higher order changes, such as modified roll Eq. (7), on a CNC-controlled machine. The latter is the preferred method, as standard cutters can be used. Figure 8 shows the TE for the same gear set as that in Figure 7, but 2nd-order modified roll has been applied to provide relief at contact entry and exit.

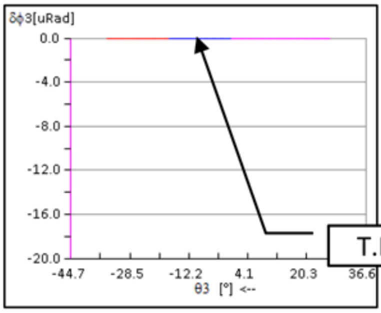


Figure 7 TE – basic gear set.

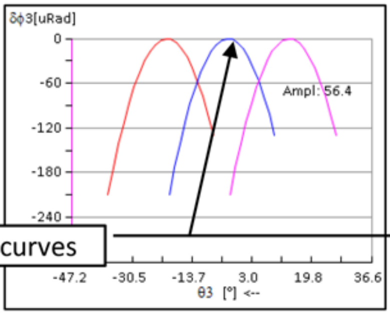


Figure 8 With modified roll.

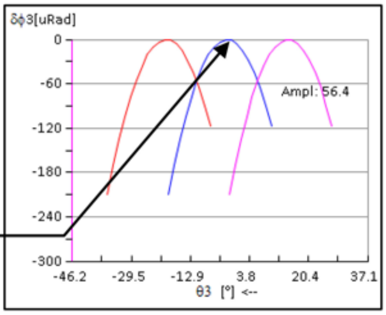


Figure 9 0.2 mm C.D. change.

Insensitivity to center distance change is a prime characteristic of spur gears. Spurved gears behave similarly, since they are fully conjugate in the mid-face section of the tooth where the PoC is centered. In Figure 9, the center distance of the gear set of Figure 8 was increased from 37.86 mm to 38.06 mm — thus a 0.2 mm increase for a tooth whole depth of 2.5 mm and yet, no change in the TE can be noticed.

Likewise, insensitivity to misalignment is essential in order to prevent toe and heel edge contact. Figure 10 shows the contact patterns of a spur and a spurved gear set having the same amount of lengthwise crowning, i.e. — 0.010 mm.

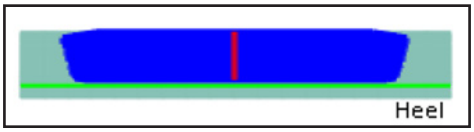


Figure 10 Contact pattern: spur gear.

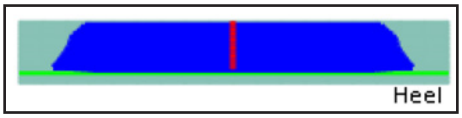


Figure 11 Contact pattern: spurved gear.

When subjected to the same amount of 2" misalignment (0.0333°), the contact patterns on both gear sets are shown to move in a similar manner (Figs. 12 and 13).

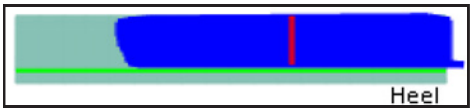


Figure 12 Contact pattern: spur gear.

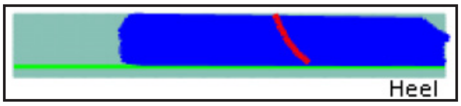


Figure 13 Contact pattern: spurved gear.

Likewise, when subjected to the same torque of 20 Nm, both the spur and spurved gear sets exhibit similar contact stresses if the Ease Off is comparable (Figs. 14 and 15).

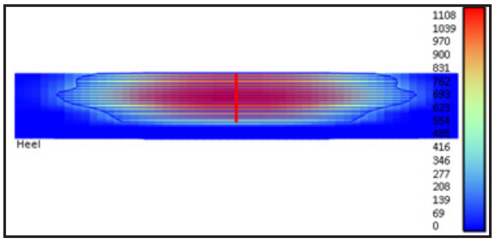


Figure 14 Contact stresses: spur gear.

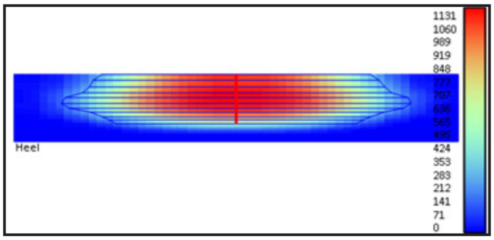


Figure 15 Contact stresses: spurved gear.

Application: CNC Manufacturing of Spurved Gear Sets

In order to illustrate the method, two spurved gear sets, $m = 1.352$ mm, were designed with the parameters listed in Tables 3 and 4. The spurved sets were manufactured on a 5-axis Doosan CNC machine. Figures 16 and 18 show the predicted contact patterns on the gear, and Figures 17 and 19 show the actual contact patterns obtained on the VH tester.

Table 3 Test #1: spurved gear set

Characteristics	Pinion	Gear
#Teeth	21	35
Cutter Diameter	2"	
Convex Blade Angle	22.5°	22.5°
Concave Blade Angle	17.5°	17.5°
Cutter Tilt	2.5°	–2.5°

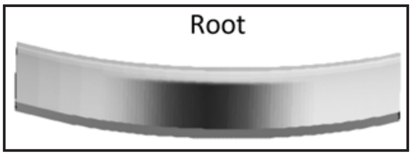


Figure 16 21x35 contact pattern simulation

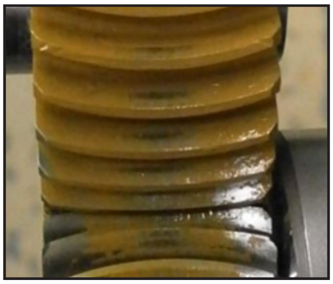


Figure 17 21x35 actual contact pattern.

Table 4 Test #2: spurved gear set

Characteristics	Pinion	Gear
#Teeth	41	41
Cutter Diameter	3.5″	
Convex Blade Angle	21.667°	21.667°
Concave Blade Angle	18.333°	18.333°
Cutter Tilt	1.667°	1.667°

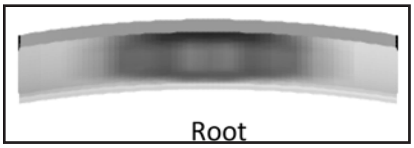


Figure 18 41x41 contact pattern simulation.

Figure 19 41×41 actual contact pattern.

Figures 16 to 19 show the excellent agreement between the simulation and the actual contact patterns. Figure 20 shows the CMM output, for the $Z_1 = 41$ pinion. Slight pressure angle error can be seen on the convex flank, which can easily be corrected by closed loop.

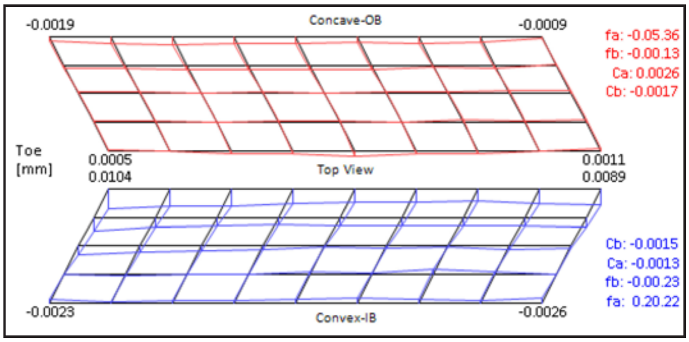


Figure 20 CMM output: $Z_1 = 41$.

Conclusion

The term "spurved gears" refers to spur gears with a curved face cut using a face mill cutter. In this paper, a general method is presented to design, analyze and manufacture spurved gears on a 5-axis CNC machine. Cutter tilt is used to control the size of the contact pattern, and modified roll is used to control transmission error. When the Ease Off and size of the contact pattern are comparable, spur and spurved gears behave in a similar way when subjected to the same center distance and misalignment errors. However, because of their circular face, spurved gears are expected to be stiffer than spur gears, and thereby may offer an advantage where sudden torque is applied. Examples show that model prediction for contact pattern shape and location correlate very well with that of actual parts.

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