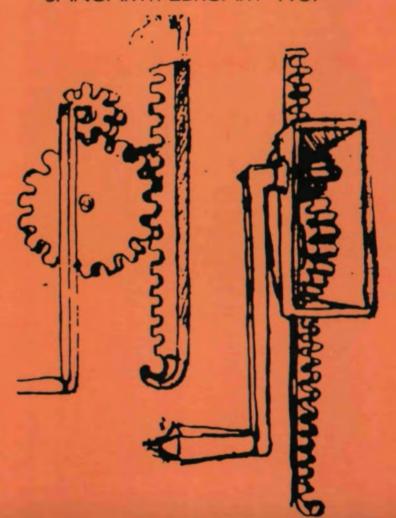
# GEAA R TECHNOLOGY

## The Journal of Gear Manufacturing

JANUARY/FEBRUARY 1987



Lubricant Jet Flow Phenomena
Gear Noise and the Sideband Phenomenon
Improvement in Load Capacity of Crossed Helical Gears
The Forming of Gear Teeth

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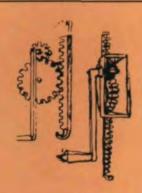
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THE ADVANCED TECHNOLOGY OF LEONARDO DA VINCI 1452-1519

**OUR COVER** 

This drawing illustrates a simple form of lifting jack, consisting of a cranked handle, reducing gears and a rack. Leonardo's sketch may be of an existing model or it may be of a proposed modification or a new design of the artist's own invention. Such a jack would have had many practical applications in Renaissance times, and the design is similar to one used on many modern automobile jacks.



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MANUSCRIPTS: We are requesting technical papers of every sort from manufacturers of gear making machinery and related equipment, universities, and engineers. Articles should be of an educational and training nature with general appeal to anyone having anything to do with the purchase of materials or machinery, or the design, manufacture, testing or processing of gears. Subjects sought are solutions to specific problems, explanations of new: technology, technology, technology technology. The processes, and alternative manufacturing methods. These can range from the "How to . . ." of gear cutting (BACK TO BASICS) to the most advanced technology. All manuscripts submitted will be carefully considered. However, the Publisher assumes no responsibility for the safety or return of manuscripts. Manuscripts was be accompanied by a self-addressed, self-stamped envelope, and be sent to GEAR TECHNOLOGY. The Journal of Gear Manufacturing, P.O. Box 1426, Elk Grove, IL 60007, (312) 437-6604.



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Topological Map

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## EDITORIAL

#### INDUSTRY SHOWS SHIFT EMPHASIS



A change has taken place within the industry that is going to have an enormous effect on the marketing, sales, and purchasing of gear manufacturing and related equipment. This change was the American Gear Manufacturers' Association, first biennial combination technical conference and machine tool minishow. The conference combined the presentation of technical papers with displays of a variety of gear industryrelated products. Although this first mini-show was small, and the timing right after the IMTS-86 was not the best, the enthusiasm and positive comments from the exhibitors and the attendees seem to promise a bright future for shows specifically for the gear industry. The AGMA conference in September was the

largest to date, with over 350 attendees. Future conferences, with an expanded exhibition, should far exceed these numbers.

AGMA is contemplating holding a show every two years to alternate with the IMTS. The planners of that show visualize much larger and more substantive displays. The next show is planned for the fall of 1987 at the 35,000 sq. ft. Cincinnati Convention Center in Cincinnati,

I would hope that all future shows would be held in smaller cities, thus keeping costs down for both exhibitors and attendees. These lower costs would encourage companies to send more of their employees to both the exhibition and the technical sessions. The combining of these two functions would give maximum benefits in terms of information received for dollars expended.

The careful selection of time, city and exhibition space alone, however, will not maximize the potential benefits these shows and conferences could have for our industry. Our industry requires a continuing effort to achieve and maintain technical leadership and excellence in order to remain competitive. AGMA leadership must work with contributors and their companies to improve the quality of the technical papers presented. Presentations devoted exclusively to extolling company products or annual sales talks disguised as technical papers should be eliminated from the program. GEAR TECHNOLOGY has given extensive exposure to authors who have written articles about important technical concepts, new techniques or important research results. I would hope that the incentive of having such papers published, thereby gaining a wider audience, would raise the quality and shift the emphasis of future conference

Providing an industry-wide forum for the sharing of research and ideas, as well as the opportunity to sell new products, is frequently a thankless and not particularly glamorous task. I think the foresight, hard work and commitment to technical excellence shown by the leadership of organizations like AGMA, SME, ASME, ASME—Gear Research Institute should be acknowledged. When we are able to look back on their contributions from the perspective of history, I think we will find that they were responsible for some of the most important changes in our industry.

## ATTENTION AUTHORS

GEAR TECHNOLOGY is requesting technical papers of every sort from its readers. We will consider offerings from manufacturers of gears, gear making machinery and related equipment or their employees, engineers, researchers and members of universities.

Articles should have an educational, training or technical emphasis with general appeal to anyone having to do with the purchase of materials or machinery or the design, manufacture, testing or processing of gears. Solutions to specific manufacturing or production problems, research results, explanations of new technologies, manufacturing techniques, designs, processes or alternative approaches to manufacture are all appropriate subjects. Subject diffi-

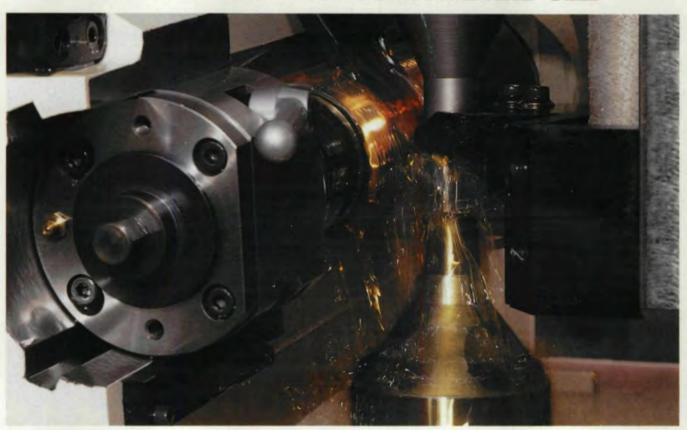
culty can range from pieces appropriate to our "Back to Basics" column to detailed discussions of advanced technologies. Advertising pieces, sales promotions or publicity releases are not accepted.

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## NOW A NEW HOBBER FROM MITSUBISHI, YOU CAN'T BEAT IT! COMPARE AND SEE!



In addition to the GA-series CNC gear hobbing machines which covers a range from 10" to 40" diameter gears, Mitsubishi is now announcing the new high performance GA15CNC gear hobber. The GA15CNC is a 6" machine with hob rotation of 1000rpm and table rotation of 150rpm as standard where it is an option with other manufacturers. This enables you to use multiple thread hobs to get higher production rates. Our results show that the cutting time can be reduced to about one half of the conventional machines. Needless to say, with the CNC control feature, there are no gears to change. Quick change hobs and quick change fixtures all adds up to quick changeover time. Setup time is reduced to about one third compared to conventional machines. "MENU" programming is another great feature. This relieves the operator from tedious calcula-



tions. Just input the gear and hob data. The built-in software will do all the calculations for you! You can also save floor space, hence money, with our machine. It takes only 50 sq. ft. of floor space! Compare it with the others. You'll be saving 1/2 to 3/4 of your valuable floor space! This is only the beginning! For further details, call or write us NOW!

#### Main Specifications

Maximum part diameter	6", optional 8"
Maximum pitch	6DP
Maximum hob diameter	4.7"
Maximum hob length	7"
Hob shift	5"
Hob speed	150 to 1000 rpm
Hob head swivel	+/-45 deg.
Table speed	150 rpm
Main motor	7.5 hp

## CNC GEAR HOBBER



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## **GUEST EDITORIAL**

#### STOPPING THE GREAT AMERICAN GIVEAWAY

Inviting an American shipbuilding industry official to discuss the subject of meeting foreign competition is like inviting Jackie Gleason to speak on dieting. I am painfully aware of the commercial shipbuilding industry situation. Let me tell you a little about it.

#### A Market Disappears

There was a four-month period in 1984 where not one single ocean-going commercial vessel was being built in the entire U.S.A. or Canada. That had not happened before in the 208-year history of the U.S. Except for naval ship construction and the construction of very complex merchant ships, North American shipyards cannot compete with foreign yards, expecially Japan and Korea. This is true even for shipyards, such as ourselves, which have made heavy investments in the most modern of facilities and use the most sophisticated computer aided engineering and manufacturing systems available.

Why such a dismal picture? Well, our critics are quick with several answers: noncompetitive labor costs, obsolete facilities, low productivity, poor quality, and questionable management—to name a few.

And I'd be the first to admit that there is some truth in some of these charges, but the real reasons we cannot compete are fourfold. First, the major foreign yards in the Far East do have labor rates which are only 10 to 30 percent of ours. Second, overseas yards are generally a part of highly integrated companies, which supply materials to their

#### **AUTHOR:**

#### MR. E. J. CAMPBELL

From time to time articles appear in journals of other industries that contain information we think will be of interest to our readers. Such a case is that of the address by Mr. E. J. Campbell, President and CEO of Newport News Shipbuilding Co. given at the Tenth Annual Meeting of the Iron Castings Society. Mr. Campbell's remarks about foreign competition and our trade policy have relevance to our industry as well as to the shipbuilding and iron casting businesses. Therefore, we reproduce here, with permission of Ironcaster Magazine, Mr. Campbell's remarks.

shipyard at low cost. Third, overseas yards enjoy government construction subsidies, and fourth, their customers receive government financing at well below market rates in that country or anywhere else.

What does this shipbuilding problem have to do with iron castings? The point is that our problem is also your problem. All four items I just presented also apply to your industry. In addition, your market with my industry in North America is rapidly decreasing as we lose ships to Korea and Japan. Why is that true? Well, ships, particularly tankers and other product carriers, contain a maze of pipes, valves, pumps, compressors, deck and other machinery, all of which use tons of castings. Approximately 12 percent of the cost of a ship is in your type of material, representing about \$18 million per ship for today's standard tanker of \$140 million.

It's a market which has disappeared for shipbuilders, and one that is also gone for you.

#### **Lopsided Trade Balance**

From the battered industrial giants of the Atlantic Coast to the high tech wizards of the Pacific Coast and almost everybody in between, where most of you come from, there are cries of alarm over the erosion of their North American market position by overseas competition.

Since 1970, the merchandise trade balance of payments showed only two winning years for the U.S. Significantly, the scores are getting even more lopsided in favor of our competitors. Our \$3 billion trade surplus in 1970 went to a \$26 billion deficit in 1980 and is trending to a \$150 billion deficit this year. Plainly, the balance is out of control, and will require dramatic nationwide action.

Much of the blame can be attributed to the natural generosity of the U.S.A. and Canada. We freely shared our resources in the late 1940's and 1950's with war-devastated nations, friend and foe alike. We generously exported our technology and management. We allowed our great inventions of the past forty years to be exploited by others, mindlessly exporting jobs and profits. Like happy children on Christmas morning, many nations have

(continued on page 9)

enjoyed the fruits of the great North American giveaway, but show little in the way of tangible

During the thirty years following the end of World War II, our economies were the strongest and most productive in the world. Our citizens enjoyed a rising standard of living, our governments financed a strong national defense, millions of new jobs were created, and developing nations of the world were helped by us in their economic growth.

The early multinational corporations were blessed with significant growth in sales and profits, both at home and overseas. They brought management methods, technology, and capital to both developed and undeveloped countries. This effort raised the standard of living for the countries in which they opereated, created jobs in North America, and sent millions in profits back home. Other American industries, although without significant overseas manufacturing facilities, were able to export products, thus sharing in an expanding world wide economy. Sure, there was growing competition from a recovering Europe, led first by the so-called "German economic miracle,"-but it was encouraged by the United States and Canada, and it provided funds for those foreigners to buy our goods. Detroit, for example, hardly noticed the Volkswagen invasion. But sometime in the early 1970's the cumulative effect of political change at home, slowing American productivity growth, and particularly competition from the Pacific rim countries, began to have a profound adverse effect on the competitive status of major North American industries.

#### **Industry Attacked**

There were also numerous politically inspired changes. Many were beneficial, concerning the health and safety of our citizens and employees, protection of the environment, and equal opportunity for all. The problem was that all were expected to be implemented almost immediately and paid for largely by industry. Not surprisingly, U.S. industry balked, and Congress, ever ready to exploit opportunities to be in the public's eye, jumped to the attack. The bruise brothers and sisters of the media, always delighted to spread the bad word, sank to the occasion. Ralph Nader, the man who brought us the \$17,000 Chevrolet, became a media

Industry was cast as the bad guys, unconcerned polluters, and bereft of social responsibility. Multinational corporations got extra special treatment.

They were cast as tax evaders, job exporters, payoff artists, and heartless exploiters of third world countries. International oil companies were cast as perversely making "indecent profits" from the Arab oil embargo. All of it was nonsense, but great fodder for irresponsible politicians and the media.

Since scapegoats are supposed to be sent into the wilderness after the sins of others are placed on their heads, no one in the government or elsewhere was particularly interested in listening to industry's side of the story. The result was the drawing up of even more government regulations imposed on top of the volumes already in force. Among these were restrictive laws on doing business in foreign countries. For example, changes in income tax regulations on U.S. citizens' foreign earnings made it almost impossible for them to work overseas.

Add to that Carter's grain embargo, his high inflation and interest rates, budget and trade deficits of historic proportions, and you have a scenario for some very difficult times for companies which must compete internationally-and, of course, that includes all of us.

Motorola recently estimated that the United States is losing 3500 jobs every day to foreign competition. Most people ignore this statistic until one day they discover "today's the day" for them. We really entered the "world economy" in 1973-1974, with the first oil crisis. But, even then, that event was considered by many as an isolated case, and they soon returned to business as usual. It really wasn't until the early 1980's, when problems surfaced on a broad front-autos, electronics, steel, machinery—that the magnitude of the problem became obvious.

#### Getting Back Into The Ballgame

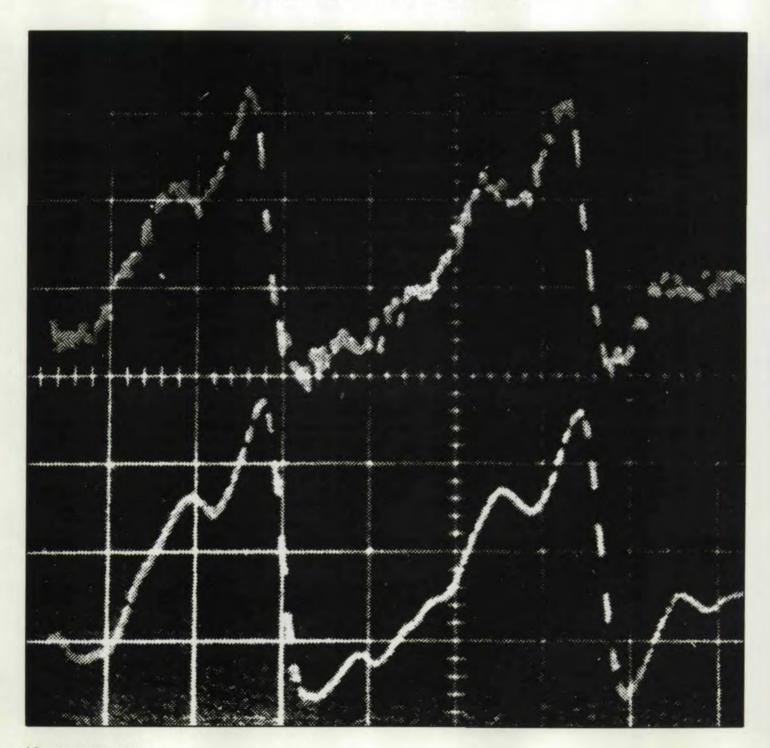
Considering the suffering that has occurred to many of you and your employees, I hate to say it, but the fact is that the massive problems of our manufacturing and basic industries over the past several years have been a "blessing in disguise." They "woke us up" to a changed world. And, to our credit, we're all taking measures to "get back in the ballgame."

Consider, for example, the new strategies of General Motors—such as the Saturn car plans announced in January for meeting the competition of the next decade. And Ford and Chrysler have made monumental gains in regaining competitiveness and are committed to maintaining their (continued on page 21)

## Lubricant Jet Flow Phenomena in Spur and Helical Gears

L. S. Akin California State University Long Beach, CA

D. P. Townsend National Aeronautics and Space Administration, Lewis Research Center, Cleveland, Ohio



#### Abstract:

The work reported is an extension from a previous study which was limited to standard centers and tooth proportions only. This article includes long and short addendums and modified center distances. The analysis develops the equations for the limit values of variables necessary to remove prior severe limitations or constraints necessary to facilitate computer analysis. A new computer program, IMPOUT2, has been developed using these newly established "Limit Formulas" to prevent negative impingement on the pinion. The industrial standard nozzle orientation usually found where the offset S = 0 and inclination angle  $\beta = 0$  will often cause the pinion to be deprived of primary impingement, which can be an important cause of incipient scoring failure in high-speed drives.

#### Introduction

In the gearing industry, gears are lubricated and cooled by various methods. At low to moderate speeds and loads, gears may be partly submerged in the lubricant which provides lubrication and cooling by splash lubrication. (1) With splash lubrication, power loss increases considerably with speed. (1) This is partially because of churning losses. It is shown that gear scoring and surface pitting can occur when the gear teeth are not adequately lubricated and cooled. (2) The results of spur gear oil jet lubrication(3) show that as the gear pitch line velocity increases at a constant into-mesh lubrication condition, the limiting tooth load that will cause gear scoring is drastically reduced. This is primarily because the method of lubrication does not provide adequate cooling at the increased pitch line velocity. In a study of high-speed, heavy-duty gears, (4) the authors showed that the oil jet location and amount of oil flow to the gears varies nearly linearly with gear pitch line velocity and also with tooth load. The authors(5) showed that with radial jet lubrication, the gear tooth temperature at various speeds and loads can be considerably reduced by increasing the lubricant jet pressure and flow rate to obtain better cooling. It was shown (5,6) that the oil jet lubrication is the most effective method when the jet

#### **AUTHORS:**

MR. D.P. TOWNSEND is a resident NASA gear consultant for gear systems for NASA and the military along with numerous industrial companies. Townsend earned a BSME from the University of West Virginia. During his career at NASA he has authored over fifty papers in the gear and bearing research area. For the past several years, he has served in active committee roles for ASME. Presently he is a member of the ASME Design Engineering Executive Committee.

DR. LEE S. AKIN has been working in mechanical engineering since 1947, specializing principally in technologies related to rotating machinery. About half of this time has been spent in the gear industry and the other half in the aerospace industry, concentrating on mechanisms involving gears and bearings as well as friction, wear, and lubrication technologies.

Since 1965, when he received his Ph.D. degree in mechanical engineering, he has been extensively involved in gear research especially related to the scoring phenomena of gear tooth failure. In 1971 he joined forces with Mr. Dennis Townsend of NASA Lewis Research Center, and together they have produced numerous papers outlining aspects of their research on technologies related to gear scoring.

is directed in the radial direction with adequate pressure and flow. Many gears are lubricated by directing the oil jet at the engaging side of the mesh (into-mesh lubrication) or at the disengaging sides of the gear mesh (out-of-mesh lubrication). The authors analyzed oil jet lubrication when the oil jet is directed at the engaging side of the gear mesh, and it was shown that there is an optimum oil jet velocity and oil jet location to obtain the best lubrication and cooling for intomesh lubrication. (7,8)

The oil jet lubrication for out-of-mesh lubrication was analyzed, (9) and the oil jet impingement depth was determined for standard gearing dimensions. Also in the same reference, the oil jet location and direction were limited to a no-offset condition (directed at the pitch point only) and in a direction normal to the line of centers. This method will give good results for standard gear dimensions with gear ratios close to unity. However, when nonstandard dimensions, spread center distance, etc. and large gear ratios are used, the oil jet direction and location should be changed to provide the optimum oil jet impingement depth and maximum cooling conditon.

The objective of the work reported herein is to analyze the out-of-mesh jet lubrication with most of the simplifying constraints removed. (9) Since most high-performance gears require addendum modifications and sometimes spread centers in addition, the analysis presented herein set out to include these and other related conditions. One practical constraint is added: A nozzle orientation that allows a pinion or gear to be missed provides no primary cooling to the missed member. Such solutions are not permitted in this analysis since they are not of practical value and can mislead an inexperienced gear design engineer.

#### Brief Description of the Impingement Cycle

The beginning of the pinion impingement cycle is about to start as the leading edge of the top land of the gear is passed as shown in Fig. 1. Gear tooth rotation continues toward the jet stream until the jet reaches the trailing edge of the gear tooth as shown in Fig. 2. At this position, the time t is set at zero (t=0) and the geometrical position of the gear  $\theta_{el}$  is calculated. Also the initial position of the pinion  $\theta_{p1}$  is calculated at (t=0). Then, the geometry of the lowest impingement point on the pinion is established by setting the time of flight of the jet stream head equal to the time of rotation on the pinion from (t=0) until impingement on the pinion takes place at  $\theta_{p2}$ , see Fig. 3, (@ $t_f = t_w$ ). Fig. 4 shows the initial position of the jet head (@t=0) when the impingement flow toward the lowest impingement point on the gear is initiated. Here the initial position of the pinion is  $\theta_{p4}$  and  $\theta_{g2}$  for the gear. Again the times of flight and rotation are equated  $(t_f = t_w)$ . Impingement at the lowest point on the gear  $(L_g)$  is shown in Fig. 5. The gear position is  $\theta_{g3}$  at this lowest point of impingement.

Thus the depth of primary impingement on the gear is  $d_g$ as shown in the figure. When the jet velocity Vg is reduced below Vi(min), the pinion is missed, and the impingement depth on the gear is not increased as expected, but reduced.

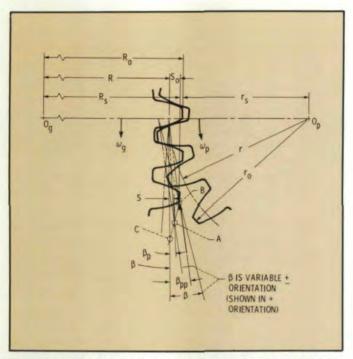


Fig. 1 – Jet coordinate origins for impingement on pinion: A = General case where  $0 < S < S_0$ ; B = special case where  $S = S_0$ ; C = classic case where S = 0 deg.

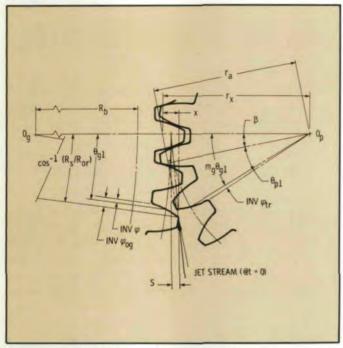


Fig. 2-Impingement on pinion (@t = 0)

#### Model Controls and Restraints

Development of the mathematical model used in reference 1 was restricted to spur gears, and the nozzle position was restricted to the arbitrary offset distances S=0 and arbitrary inclination angle  $\beta=0$  orientation. The geometric definition of S and  $\beta$  are described below and shown in Fig. 1. The foregoing restrictions have been removed in this mathematical development. Also the origin for the jet stream trajectory is defined as the position where the jet crosses the gear outside diameter (O.D.) at A, B, and C. (Fig. 1) Position A shows

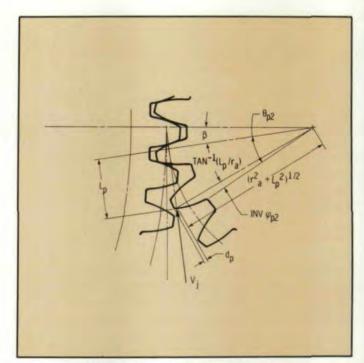


Fig. 3-Impingement on pinion (@ $t_f = t_w$ )

the general case where  $0 < S < S_0$ ; B is a special case where  $S = S_0$ ; and C is the classic case where S = 0 pointing at the pitch point and perpendicular to the line of centers.

In this article, the value of the arbitrary offset S where the jet line crosses the gear I.D. is restricted to  $S_{(min)} \leq S \leq S_0$  where

$$S_0 = \frac{(a_{gr}m_g - a_{pr})}{m_g + 1} + \frac{a_{gr}^2 - a_{pr}^2}{2r_r(m_g + 1)} \tag{1}$$

as shown in Fig. 1. (See Nomenclature for variable definitions not described in text). Thus the operating (or running) offset  $S_0$  to the crotch or common intersection of the outside diameters is the maximum value allowed for the offset S to remain within the geometric definitions described in this article. Further when the addendum modification  $\Delta a_{pr}$  and  $\Delta a_{gr}$  are unequal and extreme enough to cause  $S_0$  to be negative, then  $S_0 \leqslant S \leqslant 0$ . Also, when  $\Delta a_p = \Delta_g = \Delta a = 0$  with standard centers,  $S_0$  reduces to

$$S_0 \text{ (std)} = \frac{(1/(P_n \cos \psi))((N_g/N_p) - 1)}{/((N_g/N_p) + 1) = a(m_g - 1)/(m_g + 1)}$$

Also the inclination angle  $\beta$  is confined to a point at the line of centers between the confines of the outside diameters of the pinion and gear, respectively, as shown in Fig. 1.

The inclination angle  $\beta$  is considered positive when slanted from right to left through the point of origin A, B, or C as shown in Fig. 1. At  $\beta$ =0 the jet is pointed perpendicular to the line of centers, and when  $\beta$  is negative, it is slanted from left to right through the point of origin on the O.D. of the gear, not shown in Fig. 1. It is not usually considered wise to use negative  $\beta$  angles with near-standard proportion gears, lest we starve the gear of adequate coolant.

The mathematical definition of the arbitrary inclination

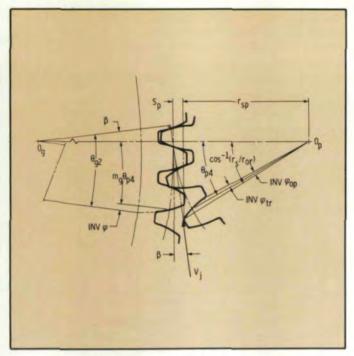


Fig. 4-Impingement on pinion (@t = 0)

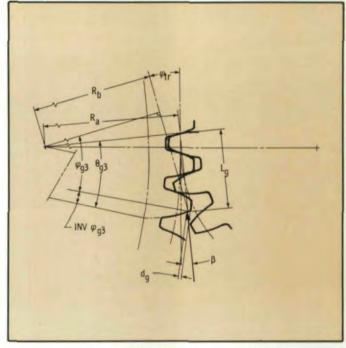


Fig. 5-Impingement on gear (@ $t_f = t_w$ )

angle  $\beta$  is:

$$\beta = \tan^{-1}(x/R_i) \tag{2}$$

where:

$$R_i = (R_{or}^2 - R_s^2)^{1/2}$$
 and

x = an arbitrary offset distance from S position to where the jet stream crosses the common line of centers.

 $R_{or} = R_o + \Delta a_g$  = operating outside radius

$$R_0 = N_g/(2P_n\cos\psi) + 1/P_n = \text{std. O.D., gear}$$

 $R_s = R_r + S = \text{radius to offset from gear center}$ 

 $R_r = C_r N_g / (N_g + N_p) = \text{operating pitch radius, gear}$ 

 $\Delta a_g = \Delta N_p/(2P_n \cos \psi) = \text{gear addendum modification}$ 

When  $\beta$  is given then,

$$x = R_i \tan \beta$$
, where  $\beta$  is arbitrary

Thus to remain within the confines established for  $\beta$ ,  $\beta_{max}$  and  $\beta_{min}$  are defined as:

$$\beta_{\text{max}} = \tan^{-1} \left[ (S + a_{pr}) / R_i \right]$$
 and  $\beta_{\text{min}} = \tan^{-1} \left[ (S + a_{gr}) / R_i \right]$ 

Also, for any given offset S, the angle  $\beta_p$  to the pitch point (used as a normalizer) is

$$\beta_{\nu} = \tan^{-1}(S/R_i) \tag{3}$$

and, further, if  $S=S_0$ , then the angle to the pitch point is

$$\beta_{\nu\nu} = \tan^{-1}(S_0/R_i)$$

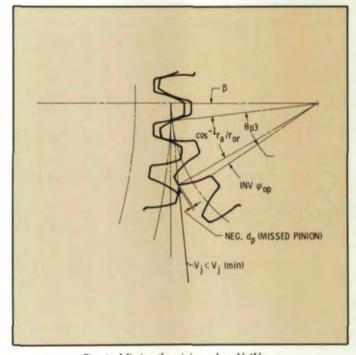


Fig. 6-Missing the pinion when  $V_j < V_{j(min)}$ 

Since  $\beta$  can be arbitrarily selected, it is necessary to provide the user with a normalized (dimensionless) input value for  $\beta$  that will not be out of bounds of the solvable geometry. This is done here using the  $\beta_i$  input parameter where  $-1 \le \beta_i \le 1$  and is defined in the following way. When  $\beta_i$  is positive (+):

$$\beta_i = \frac{\beta - \beta_p}{\beta_{\text{max}} - \beta_p}, + \beta = \beta_i (\beta_{\text{max}} - \beta_p) + \beta_p$$
 (4)

and when  $\beta_i$  is negative (-):

$$\beta_i = \frac{\beta - \beta_p}{\beta_p - \beta_{\min}}, + \beta = \beta_p - \beta_i (\beta_{\min} - \beta_p)$$
 (5)

so that when

 $\beta_i = 0$ :  $\beta = \beta_p$  (pointing at pitch point)

 $\beta_i = 1 : \beta = \beta_{\text{max}}$  (pointing at O.D. pin.)

$$\beta_i = -1: \beta = \beta_{min}$$
 (pointing at O.D. gr.)

Further if the user desires to know the value of  $\beta_i$  that will make  $\beta = 0$ , he calculates  $\beta_{io}$  from:

$$\beta_{io} = \frac{\beta_p}{\beta_{\min} - \beta_p}$$
 if  $\beta_{io}$  is negative as is usual.

Similarly, when S is normalized we get

$$S_i = S/S_o \tag{6}$$

where the value of  $S_i$  is confined to  $S_{i(min)} \leq S_i \leq 1$ .

Thus the use of the computer program, IMPOUT2, can be entered using  $S_i$  and  $\beta_i$  without knowing  $S_i$  or  $\beta$  with some relative feel for where the jet nozzle is pointing.

A further constraint of paramount importance is  $S_{i(\min)}$  so defined, given  $\beta_i$  or  $\beta$  and  $\Delta a_p$  and  $\Delta a_g$ , so that  $d_{p(\max)} = 0$ , thus making the pinion O.D. barely reachable when the jet

velocity  $V_i$  approaches infinity. This further confines  $S_i$  so that  $S_{i(min)} < S_i \le 1$  to maintain impingement at least on the top land of the pinion, where  $S_{(min)}$  is found by iterating:

$$abs[r_{or}^2 - r_o^2 + L_{p(min)}^2] \le 10^{-3}$$
 (7)

as a function of  $S_{(\min)}$  with given  $\beta$  and  $\Delta a_{p,g}$  until the inequality is satisfied when the abovementioned restraints on  $S_i$ ,  $\beta_i$ ,  $\Delta a_p$ , and  $\Delta a_g$  are given, the user of the program IMPOUT2 is assured of impingement on the pinion. The definition of  $d_{p(\max)}$  is given later in this article.

Once  $S_{(min)}$  is found from equation (7),  $S_{i(min)}$  can be calculated from

$$S_{i(\min)} = S_{(\min)} / S_o \tag{8}$$

#### Development of the Geometric Model for the Pinion

The problem to be solved here depends on what is given. If the jet velocity is given, then we solve implicitly for the impingement depth  $d_p$  so that subsequently this depth can be used to determine the cooling effect on the pinion and gear, respectively. On the other hand, if the desired depth of impingement is given, and  $d_p < d_{p(\max)}$ , then the desired jet velocity  $V_j$  can be calculated explicitly from the equation:

$$V_{ip} = (((R_{or}^2 - R_s^2)^{3/2} \sec \beta - p_x \sin \beta - L_p)\omega_p)/(\theta_{p2} - \theta_{p1})$$
 (9) where

 $\omega_v = \text{angular velocity (rad/s)}$  $\psi$  = helix angle  $N_p$  = number teeth on pinion  $N_g$  = number of teeth on gear wheel  $\Delta a_p = \Delta N_p/(2P_p \cos \psi)$  = pinion addendum modification  $r_x = r_r - S + x = \text{radius to jet stream intersection}$ on line of centers from pinion center (see Fig. 2)  $r_r = C_r N_p / (N_g + N_p)$  = pinion operating pitch radius (in.)  $C_r = C + \Delta a_p + \Delta a_g = \text{operating center distance (in.)}$   $L_p = ((r_{or} - d_p)^2 - r_a^2)^{12} = \text{impingement distance (in.) (see Fig. 13)}$  $r_{or}r_{o} + \Delta a_{p} =$  operating outside radius of pinion (in.)  $r_0 = N_p/(2P_n \cos \psi) + 1/P_n$  = outside radius of pinion (see Fig. 1)  $r_a = r_x \cos \beta$  = perpendicular distance from pinion center (see Fig. 2)  $d_p$  = impingement depth from pinion O.D. (in.) (see Fig. 3)  $\theta_{p1} = m_g \theta_{g1} + \text{inv}\phi - \beta = \text{jet head position } t = 0 \text{ (see Fig. 2)}$  $m_g = N_g/N_{p1} = \text{gear ratio (see Fig. 2)}$  $\phi_{tr} = \cos^{-1}(N_i \cos \phi_t / (N_i + (2p_n \Delta a_i)))$  = operating pressure angle (see Fig. 5)  $N_i = N_p + N_g$  $\Delta a_i = \Delta a_p + \Delta a_g$  $\phi_t = \tan^{-1}(\tan\phi_n/\cos\psi) = \cos^{-1}(R_b/R) = \text{transverse pressure angle}$  $\phi_n$  = normal pressure angle at pitch radius  $inv\phi_{tr} = tan\phi_{tr} - \phi_{tr} = radians$  (see Fig. 4)  $\theta_{g1} = \cos^{-1}(R_s/R_{or}) - \mathrm{inv}\phi_{og} + \mathrm{inv}\phi_{tr}$  (see Fig. 1)  $\phi_{og} = \cos^{-1}(R_b/R_{or}) = \mathrm{pressure}$  angle at gear O.D.  $inv\phi_{og} = tan\phi_{og} - \phi_{og} = radians$  (see Fig. 2)  $\theta_{p2}$ =tan<sup>-1</sup>  $(L_p/r_a)$ +inv $\phi_{p2}$  = radians  $\phi_{p2}$ =cos<sup>-1</sup>  $(r_b/(r_a^2+L_p^2)^{1/2})$  = pressure angle at impingement point  $inv\phi_{p2} = tan\phi_{p2} - \phi_{p2} = radians$  (see Fig. 3)

As will be seen by reviewing the input parameters, a substantial amount of calculation is required before equation (7) for Vip can be solved. Thus a computer program is required to do the job reliably, such as the program IMPOUT2 mentioned above.

Generally solving for the required velocity to obtain a desired depth  $d_v$  on out-of-mesh cooling would be unusual, mainly because of the limit imposed by  $d_{p(max)}$  which is the limiting depth of impingement when  $V_i \rightarrow \infty$  so that  $0 < d_p < d_{p(max)}$ . We calculate  $d_{p(max)}$  from

$$d_{p(\text{max})} = r_{or} - (r_a^2 + L_{p(\text{min})}^2)^{1/2}$$
(10)

where

$$L_{p(\min)} = r_a \tan(\theta_{p1} - \text{inv}\phi_{p2})$$

Note that  $\phi_{\nu 2} = f_n(L_{\nu})$  and  $L_{\nu} = f_n(S + \beta)$  as  $V_i \rightarrow \infty$ 

In real geared systems, the jet velocity  $V_g$  is already specified by the system lubricant pressure so that we solve for  $d_p$  within the range  $0 < d_p < d_{p(max)}$  which is usually quite narrow. When V<sub>g</sub> is given, it is necessary to solve for d<sub>p</sub> implicitly using an iterative technique. This can be accomplished by solving for the impingement distance  $L_p$  (See Fig. 3) implicitly, using the equation

$$L_{p} = \frac{(R_{or}^{2} - R_{s}^{2})^{\nu_{i}}}{\cos\beta} - \frac{V_{j}(\theta_{p2} - \theta_{p1})}{\omega_{p}} - r_{x}\sin\beta$$
 (11)

with  $V_i$  given and noting that  $\theta_{p2} = f_n(L_p)$ .

Then the depth of impingement on the pinion is

$$d_p = r_{or} - (r_a^2 + L_p^2)^{1/2} \tag{12}$$

where  $0 < d_p < d_{p(max)}$  and a negative  $d_p$  means that the jet missed the pinion.

Missing the pinion (or neg.  $d_p$ ) can be avoided by proper placement of the nozzle offset S where S(min) < S < So as established above and/or  $V_{i(min)} < V_i < \infty$  where (see Fig. 6).

$$V_{j(\min)_p} = \frac{((R_{or}^2 - r_s^2)^{\frac{1}{2}} - (R_{or}^2 - r_a^2)^{\frac{1}{2}} - r_x \sin\beta)\omega_p}{(\theta_{p3} - \theta_{p1})}$$
(13)

where

$$\theta_{p3} = \cos^{-1}(r_a/r_{or}) + \text{inv}\phi_{op}(\text{rad})$$

$$\theta_{op} = \cos^{-1}(r_b/r_{or}) \text{ and}$$

$$\text{inv}\phi_{op} = \tan\phi_{op} - \phi_{op}(\text{rad})$$

Equation (13) provides the minimum velocity to reach the top land of the pinion for the selected offset S when  $S_{(min)} < S$ and  $d_p = 0$ , assuming that  $S_0$  is positive.

#### Development of the Geometric Model for the Gear

In the case of the gear meshing with the pinion, the jet velocity of the gear is common with that for the pinion; so that even if the velocity  $V_{ip}$  is found explicitly to provide a desired or specified depth of impingement on the pinion  $d_p$ , the jet velocity for the gear is always provided by  $V_{ip} = V_{gp} = V_i$  so that the impingement depth for the gear is always found for the implicit solution for Lp from the equation:

$$L_{g} = \frac{(r_{or}^{2} - r_{sp}^{2})^{\nu_{1}}}{\cos \beta} + r_{x} \sin \beta - \frac{V_{j}(\theta_{g3} - \theta_{g2})}{\omega_{g}}$$
(14)

where we note that

$$\theta_{g3} = f_n(L_p) \text{ (see Fig. 5)}$$

$$r_{sp} = r_r - S_p = \text{offset radius from pinion center (see Fig. 4)}$$

$$S_p = ((r_{or}^2 - r_x^2 \cos^2 \beta)^{\frac{1}{2}} + r_x \cos \beta) \sin \beta - (x - S) \text{ (see Fig. 4)}$$

$$R_x = R_r + S - x$$

$$\theta_{g2} = \theta_{p4} / m_g + \text{inv} \phi_{tr} + \beta \text{ (see Fig. 4)}$$

$$\theta_{g3} = \tan^{-1}(L_g / R_a) + \text{inv} \phi_{g3} \text{ (see Fig. 5)}$$

$$\theta_{p4} = \cos^{-1}(r_{sp} / r_{or}) - \text{inv} \phi_{or} + \text{inv} \phi_{tr}) \text{ (see Fig. 4)}$$

$$R = R_x \cos \beta \text{ (see Fig. 5)}$$

$$\phi_{g3} = \cos^{-1}(R_b / R_a^2 + L_g^2)^{\frac{1}{2}} \text{ (see Fig. 5)}$$

$$\text{inv} \phi_{g3} = \tan \phi_{g3} - \phi_{g3} \text{ (rad) (see Fig. 5)}$$

$$\phi_{or} = \cos^{-1}(r_b / r_{or})$$

$$\text{inv} \phi_{or} = \tan \phi_{or} - \phi_{or} \text{ (rad)}$$

$$R_b = R \cos \phi_t \text{ (see Figs. 3 and 5)}$$
Then the impingement depth on the gear is calculated from

#### Computerized Parametric Study

 $d_{e} = R_{or} - (R_{o}^{2} + L_{e}^{2})^{1/2}$ 

A rather intricate computer program has been developed which should be useful to the design engineer as well as the researcher performing parametric studies. This program, IMPOUT2, was used in the study for this section of the

The out-of-mesh nozzle orientation imposes severe impingement depth problems especially when the gear ratio is larger than one-to-one. This is demonstrated in Fig. 7 where it can be seen that when the gear ratio  $m_g$  is equal to 1.0 the depth of jet oil impingement is equal on pinion and gear for a perpendicular jet pointed at the pitch point. However, when m, is larger than unity, the impingement depth on the pinion is very dependent on the offset S. This is shown in Fig. 7 using the dimensionless offset Si. As can be seen when  $S_i = 1.0$ , the position at the intersection of the pinion and gear O.D.'s the jet is pointed at the pitch point  $\beta_i = 0$ . Both pinion and gear have near equal impingement depth, but as the offset S<sub>i</sub> is lowered or the gear ratio is increased, the impingement on the pinion rapidly disappears. In the figure at  $S_i = 0.96$ , the depth disappears at  $m_e = 6.0$ , and at  $S_i = 0.863$ it disappears at 2.5. At  $S_i = 0$  it disappears at 1.2. Obviously, when  $S_i = 0$ , the pinion receives jet impingement only when  $m_o \le 1.2$ . This illustrates the idea that for a given pinion/gear tooth combination we need to know Si(min) where no impingement is possible, even when the jet velocity  $V_i$  approaches infinity  $(V_i \rightarrow \infty)$ . This then allows solutions for  $d_p$  and  $V_{ip}$  when  $S_{i(min)} < Si < 1.0$ . The gear depths  $d_g$  are also shown for teeth with a working depth of 0.25 in. and, therefore, the primary impingement can only reach about 1/10th the working depth as shown. Fig. 8 shows the effect of the inclination angle  $\beta_i$  on pinion impingement depth  $d_p$ . As expected, when the jet pressure is increased, so is the impingement depth, and as the  $\beta_i$  ratio is decreased, the depth is increased up to the maximum depth  $d_{p(max)}$  as a function of  $S_i$  and  $\beta_i$  per Equation 10.

Fig. 9 shows the effect of offset  $S_i$  on impingement depth  $d_p$  and  $d_g$  on a mesh with long and short addendums and a spread center distance. The mesh has been somewhat overcompensated so that  $S_0$  is negative, which is rare, and reverses the situation such that the gear now is the member that can be easily starved if the jet nozzle is not placed properly. Here, as can be seen, the depth of  $d_g$  improves as  $S_i$  is increased above  $S_i$ =0.6667, and when optimized by one of the three methods available in IMPOUT2 will still provide an equal impingement depth for both the pinion and gear at dimensionless depth of about  $\delta$   $\cong$  0.1, where  $\delta$  = d/whole depth. In addition, the program option 3 usually has a larger value than option 2, which is also reversed relative to standard mesh conditions.

Fig. 10 shows the results of setting  $S_i$ =0 and  $\beta_i$ =0 and adjusting the pinion and gear addendums to realize the

Fig. 7 – Effect of gear ratio on impingement depth,  $\beta_i=0$ ,  $\Delta P=138$  psi, n=5000 rpm,  $N_p=28$ 

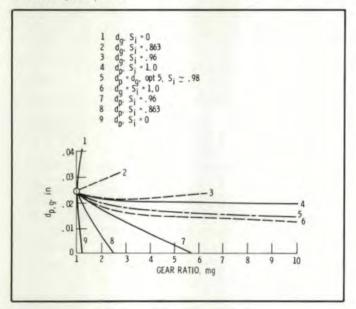
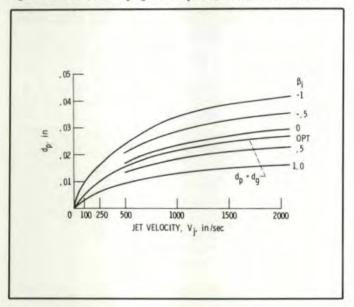


Fig. 8 – Effect of  $\beta_i$  on impingement depth,  $S_i = 1.0$ , 21/35 combination



balance of depths of impingement desired. Here as is seen in Fig. 10, as  $\Delta a_p$  is reduced,  $\Delta C$  is reduced, and at  $\Delta a_p = 0.0662$  an equal (or optimum) impingement depth is reached on the pinion and gear. The connected circles at A depict the depths when  $\Delta a_p = 0.10$  and  $\Delta a_g = -0.0375$ . Obviously we must optimize using  $S_i$  and  $\beta_i$  as discussed in Fig. 9. The connected circles at C depict the depths when  $\Delta a_p = 0.08375$  for the pinion and -0.0375 for the gear are closer together than in Case A. In addition, when  $\Delta a_p$  is further reduced to  $\Delta a_p = 0.06875$ , as in Case B, the depths get much closer together, and at  $\Delta a_p = 0.0662 = -\Delta a_g$  when optimized on  $\Delta a_i$ , the depths are equal.

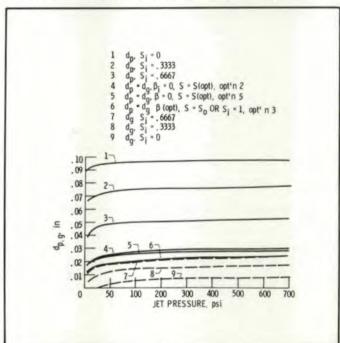
Fig. 11 shows the effect of  $\beta_i$  on  $S_{i(min)}$ . The 21/35 tooth combination was used as the example here. This figure shows that if  $\beta$  is slanted backward in the negative direction,  $S_i$  can be made smaller before the pinion is starved. Also if the offset  $S_i$  is set at 1.0, it is nearly impossible to starve the pinion at any reasonable inclination angle  $\beta$  or  $\beta_i$  ratio, even when the pressure is modest.

Fig. 12 shows the effect of the dimensionless offset  $S_i$  on the minimum velocity or its cause, jet nozzle pressure at the nozzle exit ( $V_{j(\min)}$ =velocity needed to reach pinion O.D.). As is seen in the figure,  $S_{i(\min)}$  establishes the asymptote where  $V_{j(\min)}$  approaches infinity. This makes it clear that we cannot set  $S_i$  below  $S_{i(\min)}$  and expect to obtain primary impingement on the pinion top land or profile at any jet pressure.

#### Discussion

Figs. 11 and 12 show the most important results of this study, in that they point out the importance of careful placement of the nozzle in both position and pointing direction, especially at higher gear ratios considered in Fig. 7. Also for out-of-mesh nozzle orientation, increasing the oil jet pressure

Fig. 9 – Effect of offset  $S_i$  on impingement depth: 12/43 comb.,  $\beta_i=0$ ,  $\Delta a_g=-0.0375$ ,  $\Delta c=0.0625$ ,  $\Delta a_p=0.1$ , DP=8,  $\varphi=20$  deg



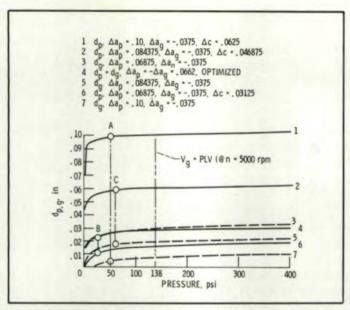


Fig. 10 – Effect of addendum modification and spread centers:  $S_i$  = 0,  $\beta_i$  = 0, for 12/43 tooth combination

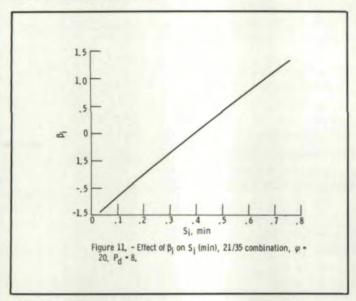


Fig. 11 – Effect of  $\beta_i$  on  $S_{i(min)}$ , 21/35 combination,  $\varphi = 20$ ,  $P_d = 8$ 

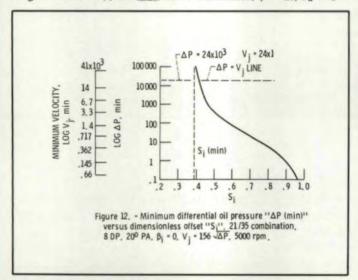
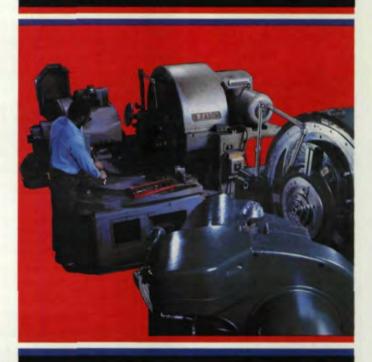


Fig. 12 – Minimum differential oil pressure  $\Delta P_{\rm (min)}$  versus dimensionless offset  $S_i$ , 21/35 combination, 8 DP, 20 deg PA,  $\beta_J=0$ ,  $V_j=156$   $\sqrt{\Delta P}$ , 5000 rpm.

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#### **NOMENCLATURE**

```
C_r = C + \Delta a_p + a_g = \text{operating center distance (in.)}
                          L_n = impingement distance for pinion (see Fig. 3) (in.)
                           L_g = impingement distance for gear (see Fig. 5) (in.)
                      N_p, N_g = number of teeth in pinion and gear, respectively
                           R = standard pitch radius of gear
           R_a = R_r \cos \beta = \text{perpendicular distance from gear center (in.)}
                          R_b = base radius of gear (in.)
                          R_i = (R_{\text{or}}^2 - R_5^2)^{1/2} offset normal (in.)
                         R_{or} = R_0 + \Delta a_g operating outside radius of gear (in.)
      R_r = C_r N_g / (N_g + N_p) = running or operating pitch radius of gear (in.)
                          R_s = R_r + S radius to gear offset from gear center (in.)
             R_x = R_x + S - x = \text{gear radius to coincidence of O.D.'s (in.)}
                           S =  offset of jet stream on gear O.D. (see Figs. 1 and 2) (in.)
                   S_i = S/S_0 = \text{dimensionless offset}
      S_1(\min) = S(\min)/S_0 = \min \max \text{ theoretical dimensionless offset when } d_n(\max) = 0 \text{ and }
                                 V_i = \infty simultaneously (see Figs. 11 and 12)
                          S_0 = offset of jet to crotch (or coincidence) of O.D.'s (see Fig. 1) (in.)
                          S_p = offset of jet stream on pinion O.D. (see Fig. 4) (in.)
                          t_{vv} = total time of rotation(s)
                           x = \text{arbitrary offset distance from } S \text{ (see Fig. 2) (in.)}
                           \beta = arbitrary inclination angle of jet stream (rad)
+\beta_i = (\beta - \beta_p)/(\beta_{max} - \beta_p) = \text{normalized positive inclination angle}
-\beta_i = (\beta - \beta_p)/(\beta_p - \beta_{min}) = normalized negative inclination angle
                        \beta_{\text{max}} = maximum inclination angle permitted
                        \beta_{\min} = minimum inclination angle permitted
                          \beta_p = inclination angle when jet passes through pitch point (rad)
                         \beta_{pp} = inclination angle when jet passes through crotch of O.D.'s and pitch
                                 point (S = S_0) (rad)
                  \delta = dP_n/2 = dimensionless impingement depth, pinion or gear
  \Delta a_g = \Delta N_g/(2P_n \cos \psi) = \text{gear addendum modification (in.)}
  \Delta a_p = \Delta N_p / (2P_n \cos \psi) = \text{pinion addendum modification (in.)}
                        \Delta N_g = tooth addendum modification for \Delta a_g
                        \Delta N_n = \text{tooth addendum modification for } \Delta a_n
                         \Delta P = differential jet pressure at nozzle exit (psi)
                          \omega_n = angular velocity of pinion (rad/s)
                          \Phi_n = normal pressure angle (rad)
                          V_i = oil jet velocity from nozzle exit (in./s)
```

and velocity to high levels may not always improve the primary impingement depth appreciably as shown in Figs. 9, 10, and 12, and in practical fact, may sometimes cause flooding in the gear case housing. Further, if the inclination angle  $\beta$  is adjusted to an extreme, per Fig. 8, to improve the impingement depth on the pinion, then the gear depth is dimished usually unacceptably.

Even when the position is fixed in the historically standard orientation position which will not allow primary impingement on the pinion for even modest gear ratios (at any jet pressure), the pinion and gear addendums can be adjusted per Fig. 10 to provide adequate impingement on both mesh members. This has been done in the past to control incipient scuffing or scoring, often without the designer realizing he was also controlling the impingement and cooling phenomena favorably.

#### Summary of Results

An analysis was developed for the lubrication jet flow in the out-of-mesh condition. The analysis provides for the in-

```
V_{imin} = jet velocity when d_p = 0 for given \omega_p (in./s)
         a_{gr} = R_{or} - R_r = \text{operating gear addendum (in.)}
          a_{nr} = r_{or} - r_r = operating pinion addendum (in.)
                    d_o = depth of impingement on gear (in.)
                    d_n = \text{depth of impingement on pinion (in.)}
                d_{\nu(\text{max})} = \text{maximum theoretical depth on pinion } V_{\sigma} = \infty
inv\phi_{og} = tan\phi_{og} - \phi_{og} = involute function of \phi_{og} (typical) (rad.)
                    m_o = \text{gear reduction ratio}
                    p_n = normal diametral pitch
                      r = standard pitch radius of pinion
         r_a = r_r \cos \beta = \text{perpendicular distance for pinion center (in.)}
         r_b = r \cos \phi_t = \text{base radius of pinion (in.)}
         r_{or} = r_0 + \Delta a_p = operating outside radius of pinion (in.)
 r_r = C_r N_p / (N_p + N_e) = \text{running or operating pitch radius of pinion (in.)}
           r_{sp} = r_r - S_p = \text{offset radius from pinion center (in.)}
         r_x = r_r - S + x = pinion radius to coincidence of O.D.'s (in.)
                      t = generalized time of jet flight and gear rotation(s)
                     t_f = total time of flight to jet impingement(s)
                   \phi_{og} = pressure angle at O.D. of gear (rad)
                   \phi_{v2} = pressure angle at lowest impingement point on pinion (rad)
                   \phi_{op} = pressure angle at O.D. of pinion (rad)
                    \phi_t = tangential standard pressure angle (rad)
                    \phi_{tr} = tangential operating pressure angle (rad)
                     \psi = helix angle (deg)
                    \theta_{gl} = initial position of gear tooth at beginning of pinion impingement cycle
                           (t=0) (rad)
                    \theta_{e2} = initial position of gear tooth at beginning of gear impingement cycle
                    \theta_{g3} = final position of gear at max. impingement depth (t_f = t_{gg}) (rad)
                    \theta_{v1} = initial angular position of pinion tooth at beginning of pinion imp-
                           ingement cycle (t=0) (rad)
                    \theta_{n2} = final position of pinion tooth at maximum impingement depth (t_e = t_m)
                    \theta_{n3} = final position of pinion tooth when just missed by jet stream (t=0)
                    \theta_{n4} = initial position of pinion tooth at beginning of gear impingement cycle
                           (t=0) (rad)
```

clusion of modified center distances and modified addendums. The equations are developed for the limit values of variables necessary to remove the severe limitations or constraints necessary to facilitate computer analysis. A computer program was developed using these limit formulas to prevent negative impingement (missing) on the pinion. The following results were obtained:

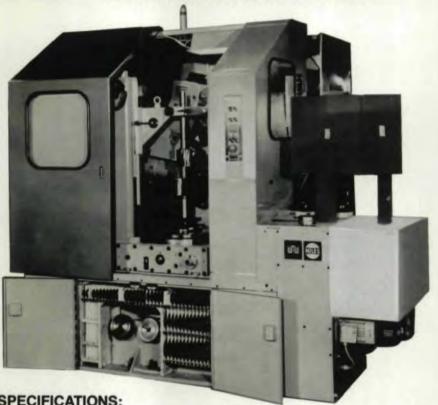
1. The industrial standard nozzle orientation usually found where the offset S=0 and inclination angle  $\beta=0$  will often cause the pinion to be deprived of primary impingement, which can be important cause of incipient gearing failure in high-speed drives.

- 2. For ratios larger than 1:1, the oil jet will only impinge on the gear teeth unless a minimum calculated jet velocity is provided to lubricate the pinion teeth.
- 3. When a minimum oil jet velocity is provided, the oil jet offset must be equal to or greater than a minimum calculated offset to assure impingement on the pinion.
- 4. As the oil jet velocity is increased above the calculated minimum value, the impingement depth will increase, but



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at a decreasing rate. The maximum impingement depth will generally not exceed 10 percent of the tooth profile depth.

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This article was previously presented by the ASME-DED Power Transmission and Gearing Committee at the October, 1984, Design Engineering Technical Conference. Paper No. 84-DET-96.

## GUEST EDITORIAL . . . (continued from page 9)

position. Our machinery and equipment people have all-out efforts underway to improve productivity. And, I know that most of you have been doing the same things.

Like most of you, I am convinced that our industries can come back, provided a reasonable atmosphere for development is maintained, and, as Lee lacocca says, a level playing field is provided. A good indicator of national industrial performance is the recent gain in U.S. manufacturing productivity, certainly a key factor in determining competitive position.

I want to remind you of the importance of constantly working hard on improving productivity. Our long head start has disappeared; there is nothing in reserve. We have to find new ways or rework previous successful methods to extract the maximum gain.

To give you some ideas for consideration, John W. Kendrick of Georgetown University, a long-time authority on productivity, has determined that since World War II advances in knowledge (essentially R and D), capital investment, and an educated workforce, contributed around 75 percent on all productivity gains. He says that R and D offers the highest payoff, contributing around 35 percent, with the other two around 20 percent each. These figures are for all manufacturing industries combined. Individual industries obviously will show different results.

Another interesting finding from Kendrick's studies was that the secondary users of basic research benefited up to ten times more than the original developers. The gains from these secondary uses of R and D are substantial. The Navy itself told Congress that they will save at least one billion dollars on the three aircraft carriers now under construction in our yard, mainly because we'll be delivering each of these ships in 17 to 18 months less time than previous carriers. Just last month, the Defense Department notified Congress that they were taking \$430 million from the money already saved on two of the three unfinished carriers, and were applying it to other programs.

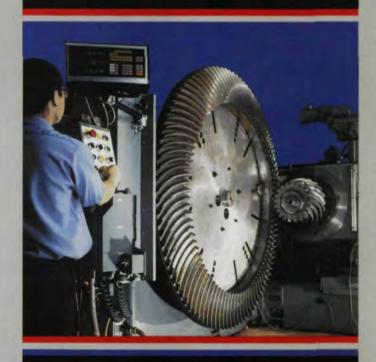
Kendrick left the remaining 25 percent of the contributors to productivity in one package. For example, you may find that mergers and joint ventures among industry members, or with foreign partners, may be necessary to reach economies of a scale to permit automated production.

Your industry as a whole can encourage research into new materials, new applications, and higher standards of quality. University research facilities can be tapped to provide the technical expertise and

(continued on page 22)

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#### GUEST EDITORIAL . . .

(continued from page 21)

facilities perhaps not generally available in your individual companies.

Individually, or as an industry project, visits can be made to foreign foundries to observe their methods. In ship building, major Japanese and Korean shipbuilders have been willing to do this type of exchange. Your visits could possibly lead to business with Far East companies with enough foresight to know that they had better put some American content in their exports to North America or face more drastic remedies.

#### **Unfair Trade Practices**

Our delemma: There are two remaining problems that we cannot solve by ourselves: (1) illegal imports and (2) the unreasonably high value of the U.S. dollar. Our Canadian friends do not have the latter problem. Saying it another way—bottomless foreign national treasuries subsidizing illegal trade and an unrealistic 40 percent exchange rate difference will overwhelm anything we can obtain through our internal cost reductions. In fact, for certain products, if we eliminated all labor, we'd still not be able to meet certain foreign selling valuesas their selling values frequently have no relationship to costs!

As to the import issue, after realizing the depth of the problem last year, President Reagan then concluded, "Unfair trade practices are the preponderant source of the injury found in the United States industry."

And Congress, last October, enacted the steel import stabilization act. This far-reaching trade statute is based upon an important Congressional finding: "The ability of our steel industry to be internationally competitive has been impeded by subsidized and dumped foreign steel."

But this problem is not confined to steel. Industry after industry is being impacted, from machinery to textiles, from computers to semi-conductors, from castings to ships.

Proof of this fact is indicated by the record volume of 203 unfair trade cases filed in 1984 by U.S. industry. Never before have the appeals from injured firms and unemployed workers for restraints on illegal and unfair import competition been so numerous and so broadly based.

Now these problems didn't develop overnight. For more than a decade, the United States and Canada have been slowly losing their capability to compete in world markets, especially in the manufacturing sectors. This has led to a significant change in our whole economic system, as basic manufacturing industries shrank or faded away and were replaced by fast food and trading companies.

#### **Problem Solving**

What can be done? Well, problem solving begins with recognizing the existence of a problem.

We now hear a multitude of voices from Maine to California demanding governmental action on our budget deficit; that's right, our budget deficit. However, almost no public debate has focused on the U.S.A. expected 1985 trade deficit of \$156 billion. And, frankly, I consider this to be the more severe long term problem.

One way or another, we must-and we willsolve the budget deficit. It'll be painful, but it can be done. On the other hand, the continual, astronomical trade deficit is having a negative long-term impact on our country's industrial base. Once an industry is gone, it cannot be reconstituted overnight-if ever.

Therefore, another suggestion: "Let's stimulate more public debate on our trade deficit and its impact on manufacturing. We must remind our fellow citizens and our public officials, especially your Congressmen that our trade problems are not confined to steel, autos, apparel, machine tools, and consumer electronics. Every sector of manufacturing is susceptible to its own version of the steel industry's dismal decade. Let's challenge the all too familiar responses such as: support free trade at all cost; they'll retaliate against our exports; or we need unlimited imports to keep American industry on its toes and inflation down.

Another recommendation: while North American markets should be open to overseas competition, let's also insist that this overseas competition comply with our fair trade laws and regulations. And, it should not be the responsibility of individual companies to be the policemen on the trade law beat. Our federal governments must be persuaded to meet their statutory responsibilities to ensure that American industry can compete on a fair basis against overseas competition.

And a third recommendation: let's force our trading partners to open up their borders to unrestricted equitable-and legal- and fair trade, to us and to the other trading partners of the world.

Japan currently enjoys a \$37 billion trade advantage over the United States, and their trade imbalance increases each year. Yet, the Japanese continue to refuse to allow American goods-from telecommunications equipment to baseball bats—to have fair access to its markets. They'll take in potatoes, but not potato chips, because that has "value added." They'll take in wood, but not furniture. In short, they're providing employment for their people—a great idea, especially if your trading partners lie back and let you do it.

#### Production Is Base of the Economy

Remember, if smokestack America slides slowly down the chute, the economic base of this country

and Canada (and the country's defense capability, by the way), slides right along with it. For example, do you know who uses the most silicon chips? It's General Motors, Ford, and Chrysler for computers in every car. If more and more cars come in by ship, they not only bring the steel and the glass and the castings and the tires with them; they bring the chips with them, too. And go ask the people in Claymont, Delaware, Lackawanna, New York, or Johnstown, Pennslvania, what happens to the drive-in hamburger business and the real estate business and the hardware business, when steel mills cut back or close down.

Or visualize us without a viable DuPont . . . Bethlehem . . . Alcoa . . . Ford . . . Caterpillar, What replaces these industrial keystones in an area's economy? Banks? McDonald's? Travel agencies? To sell to whom?

Don't get me wrong. I have nothing against these businesses. But we need balance between building and producing something—and financing it, selling it, and servicing it. If you don't have a healthy producing base, overtime, you lose the other three. Yank production out, and the entire economic base starts to collapse.

Another good reason to not depend on service industries is that the Japanese are now taking heavy aim at the banking and insurance areas.

#### **Dollar Unrealistic**

Let me turn to the second and final concern I want to discuss—the strong U.S. dollar. To talk about the problems a strong U.S. dollar creates gives rise to this response: But, our nation's interest rates are down. Inflation is down. The economy is strong. So what's the problem?

My response: I'm simply trying to convince public officials, business leaders, and opinion makers that the time to fix the leaky roof is when the sun is shining. And believe me, it's damn difficult to get America's attention on tomorrow's potential problems.

So let me repeat: Meaningful actions must be initiated to change the overvaluation of the dollar and its impact on all of us. Too often, we are told America is better off with a strong dollar and cheap imports, a formula which equates to reduced inflation.

In fact, what we're doing is putting a "heavy tax" on our exports and subsidizing imports. As a result, we're pricing ourselves out of world markets. And, as you notice from recent media reports, this problem is now even adversely impacting on our agricultural exports. American consumers may be pleased with the bargain prices, but that won't last very long if their jobs keep moving overseas.

Governmental actions—prompted and supported by all of us-must focus in on this dollar valuation problem. The old formula for managing your business is no longer valid. Trade—like the

economy—has become completely global.

Endangered industries in North America can profit from government help in some form—for example, the Section 201 relief your industry is seeking from the International Trade Commission of the United States. It is usually a slow and torturous procedure. Nevertheless, the timing appears to be right—the U.S. Congress has begun to pay attention to the trade balance problem. There's movement on Capital Hill. We don't know yet exactly where it is heading—but at least there are signs of life.

#### Individual and National Resolve

Where do we go from here? Americans must take on the challenge of competitiveness as the economic agenda for the next decadel

We must insist that our industrial base be considered as important to our country as our defense capability and our domestic support programs—it must be, as it supports the other two.

We must convince our policymakers and lawmakers to focus on the budget deficit, the trade deficit, capital investment, the dollar value, and export promotion—all in a coordinated manner. Piecemeal legislation would be typical, but most unfortunate.

We must get across the following points:

- Foreign governments' involvement in world trade matters is no longer limited to "smokestack" industries.
- The current distortions plaguing world trade and exchange rates will not simply go away with the passage of time.
- Without an equitable world trading environment, no amount of North American knowhow, improved productivity, or reduced labor costs will arrest our slide into deeper trade deficits.
- 4. From hi-tech to agriculture, from the service industries to basic manufacturing, all of us must work for industrial policies which fit today's world.

In short, North America's competitive position is rapidly deteriorating, and we'd better do something about it.

In the final analysis, it comes down to a question of individual and national resolve, a question of whether our countries really want to get tough and compete in world trade—now and in the future.

Before we can regain control of our own economic destiny, we as one continent of people with common interests must develop a combined will to compete. We must reject any thought that we can make it through this century and into the next without getting back into the mainstream of global competition.

We must stop the great giveaway.

Mr. E. J. Campbell, President Newport News Shipbuilding Co.



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## Gear Noise and the Sideband Phenomenon

A. K. Dale GKN Technology, Ltd. Wolverhampton, England

#### Abstract

It is now well understood that gear noise is caused by the dynamics of tooth meshing, and that this can be characterized by transmission error. The frequency spectrum of gear noise is characterized by sidebands, which are not well understood either qualitatively or quantitatively. Sidebands are a crucial factor in the quality of gear noise and are entirely due to manufacturing errors in the gears. The sideband phenomemon is explained in terms of amplitude and frequency modulation of the tooth mesh component caused by faults in the gears. The theory of complex modulation is fully developed to support this explanation. Previous mysteries such as the disappearing fundamental and uneven sidebands are explained. Sidebands are related to errors in the gears, and methods are suggested to development a new generation of dynamic gear testing machinery.

#### Introduction

Gear noise can be a source of intense annoyance. It is often the primary source of annoyance even when it is not the loudest noise component. This is because of the way it is perceived. Gear noise is a collection of pure tones which the human ear can detect even when they are 10dB lower than the overall noise level. (1) Another reason for our sensitivity to transmission noise is that we associate it with impending mechanical failure.

Because of this annoyance and anxiety and ever-increasing levels of noise refinement, gear manufacturers will experience continued pressure to make quieter gears.

Although gear design and manufacturing techniques continue to advance, our understanding of the relationships between gear errors and noise is incomplete. Without this knowledge, the refinement of an existing gear pattern or the design of a new gear form is uncertain. The effects of manufacturing errors on noise generation are difficult to assess.

This study discusses the characteristics of gear noise and shows qualitatively how the frequency spectrum is generated. The spectrum is shown to be related to errors in the gears,

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and methods are suggested for making fault diagnosis directly from the transmission error spectrum or some other dynamic measurement.

#### The Nature of Gear Noise

Gear noise is generated by the transfer of load from tooth to tooth as the gears mesh. This causes a series of pressure pulses which are radiated as vibrational and acoustic energy through the transmission casing. The frequency of the noise is given by the product of gear rotational speed and the number of gear teeth. This explanation is adequate in the investigation of many gear noise problems. Fig. 1 shows a spectral map for the typical internal noise of a passenger bus. The noise is analyzed into frequency spectra for several propshaft rotational speeds. The order lines marked show the predicted noise frequencies from meshes in the gearbox and axle. These components can be easily compared and the effects of structural resonances assessed.

The simple theory fails for many reasons. Frequency components appear which cannot be related to any known tooth-meshing rate. Such a component is present just below the axle gear mesh order in Fig. 1. To study these cases, we need much finer resolution. In order-locked analysis, the data is sampled at fixed intervals of rotation of a shaft or gear instead of fixed intervals of time. The frequency axis becomes cycles per revolution or orders.

Fig. 2 shows a typical gear noise spectrum from a passenger car in which the tooth-meshing (or fundamental) frequency

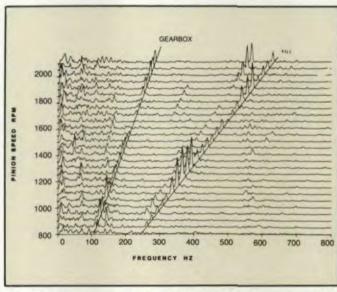


Fig. 1-Noise spectral map. Typical internal noise on passenger bus.

is present at 13 orders of pinion rotation, along with many other spectral peaks. These spectral peaks, called sidebands, are separated in frequency by multiples of the rotational speeds of the gears in mesh. They affect the timbre and our subjective perception of the noise, although the human ear cannot resolve their individual frequencies.<sup>(1)</sup>

The presence of sidebands in the gear noise spectrum has long been known<sup>(2)</sup> and several curious properties have been noted.

Sidebands can be traced at frequency spacings well away from the fundamental. Their amplitudes are asymmetrical about the fundamental, and it is not uncommon to see sideband amplitudes which are greater than the fundamental. Sometimes the fundamental will even disappear completely.

These characteristics have confused many gear noise investigations with unexplained noise peaks. Simple theory suggests that a series of pressure pulses is generated by the meshing of the gear teeth, which will be exactly periodic for a perfect pair of gears. If, however, one gear is mounted offcenter, two effects of the eccentricity can be readily appreciated: the amplitude of the pressure pulses will vary cyclically, and, as the depth of mesh increases and decreases, the speed of the output gear will vary about the mean speed. These two mechanisms are forms of modulation. They are called amplitude and frequency modulation respectively, and they are both responsible for sidebands.

Radio and television transmission exploits modulation by encoding information directly onto a carrier wave: words and pictures become sidebands. In the same way, information about the shape of gears is encoded into their sidebands. Modulation is caused by pitch error, heat treatment distortion, eccentricity, out-of-roundness and all other gear errors. It could, therefore, become possible to diagnose manufacturing errors from a spectral analysis of the gear noise or transmission error alone.

The modulation process was first suggested by Kohler, Pratt and Thompson<sup>(2)</sup> as the mechanism which controls the gear noise frequency spectrum. Thompson later used frequency modulation to predict sideband amplitudes from cumulative pitch errors.<sup>(3)</sup> He concluded that frequency modulation does not operate alone and that a complete explanation would also require amplitude and pulse modulation. Pulse modulation would account for the case of a damaged tooth. Only the general case, undamaged gears, will be considered here, although the theory developed in the appendix also could be expanded to include damaged gears.

#### Amplitude Modulation

If a sine wave is amplitude-modulated by another sine wave, the frequency spectrum will include three components: the unaffected component of the modulated sine wave (the fundamental) and a sideband spaced on each side by the frequency of the modulating wave. The symmetrical sidebands have an amplitude which is half of the product of the amplitudes of the two sine waves. For a pair of gears, we can see that one error per rev, two errors per rev, etc. in the gears will produce sidebands at the appropriate spacings from the fundamental. The AM process will, therefore, produce sidebands at the frequencies found experimentally, but will

explain neither the usual asymmetry nor the occasional disappearance of the fundamental.

#### Frequency Modulation

If a sine wave is frequency-modulated by another sine wave, then a multiple sideband structure will arise. The spectrum includes the fundamental plus sidebands spaced at all the positive and negative integer multiples of the modulating wave. If the two waves have the same phase angle, then all the upper sidebands will be in phase, as will be all the even-numbered lower ones. The odd-numbered lower sidebands will be in anti-phase. The theory developed in the appendix includes phase angles and shows in the general case that the sideband phase relationship is more complex. The amplitude of the fundamental and sidebands are controlled by Bessel functions, some of which are shown in Fig. 3. When the Bessel functions pass through zero, the fundamental or a sideband will disappear. This is illustrated by the example shown in

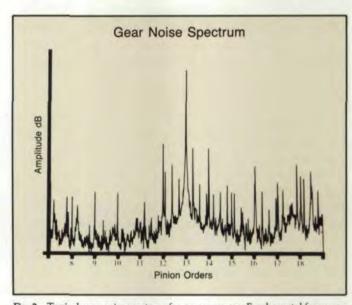


Fig. 2-Typical gear noise spectrum for passenger car. Fundamental frequency at 13 orders of pinion rotation.

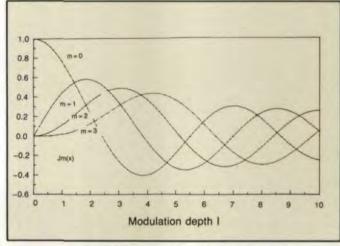


Fig. 3 - Bessel functions of the first kind.

Fig. 4. The spectrum of a 510Hz wave is shown while being frequency modulated by a 29.4Hz wave. The modulation depth varies from 0.2 to over 9. The cyclical variation in the amplitude of the fundamental and the sidebands can be seen clearly.

#### Complex Modulation

The full expression for a complex modulated waveform is given in Equation 17 of the appendix. This shows that an asymmetric sideband spectrum results from a tooth mesh fundamental modulated by two other waveforms. Each frequency component can be considered as the sum of the frequencymodulated component of the fundamental plus the amplitudemodulated sidebands from its neighbors. In addition to these main sidebands, there are secondary frequency components not seen in either AM or FM alone. Equation 17 of the appendix shows that sidebands are possible at all frequencies equal to the fundamental plus or minus all pinion multiples plus or minus all crownwheel multiples. These additional sidebands are the complex intermodulation components.

#### Discussion

From the preceding treatments of modulation, it appears that only a complex form of modulation can cause the typical asymmetric gear noise structure. However, the process by which the gear excitation becomes noise is governed by the very complex dynamics of the shafting, bearings and gear casing. It can readily be argued that either amplitude or frequency modulation can give the usual sideband structure especially in regions of high structural modal density. To explore the arguments further, it is necessary to look at a measure of the gear excitation function unaffected by dynamic response. Such a measure is transmission error, the nonuniform component of gear motion. The transmission error of a 13/43 tooth combination hypoid pair was measured and the order-locked spectrum computed. The spectrum is presented in Figs. 5A and 5B. Fig. 5A shows the low frequency components of eccentricity and distortion. Fig. 5B is

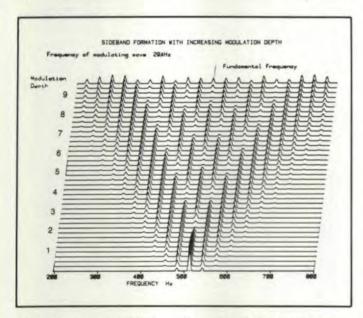


Fig. 4-Disappearance of fundamental or sideband when Bessel functions pass through zero.

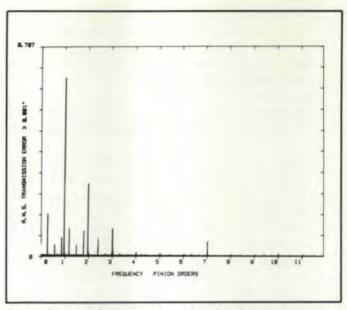


Fig. 5a - Spectrum of gear transmission error. Low frequency components of eccentricity and distortion.

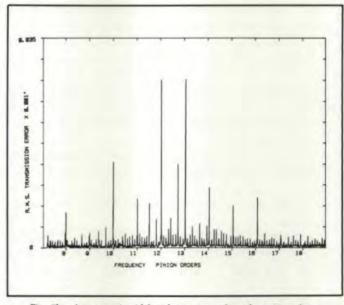


Fig. 5b - Asymmetric sideband structure of tooth at 13 orders.

centered on the tooth mesh at 13 orders and shows an asymmetric sideband structure. The predicted frequencies of fundamental plus and minus integer multiples of pinion and crownwheel frequencies agree with the sideband positions. There are also many other secondary sidebands predicted by neither AM nor FM alone. These are the intermodulation products predicted by complex modulation. For example, close to the fundamental are crownwheel minus pinion (12.3023), twice crownwheel minus pinion (12.6046), and three times crownwheel minus pinion (12.9070). All the secondary sidebands agree exactly with some combination of positive or negative multiples of the crownwheel and the pinion frequencies.

This evidence confirms that the tooth mesh component of transmission error is both amplitude- and frequencymodulated by low frequency faults in the gears. From this it may be possible to demodulate the transmission error and

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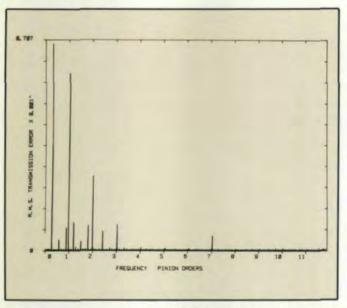
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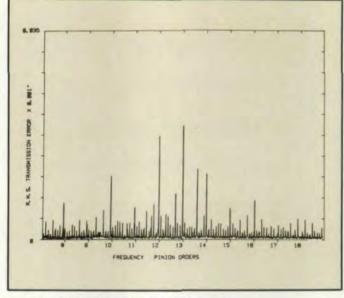


Fig. 6a and b-Spectra of transmission error caused by misaligned pinion and resulting in 0.002" runout.

obtain the modulation coefficients directly. These coefficients could then be related to manufacturing errors. This can be illustrated by introducing a deliberate error into a gear pair. This was done by misaligning the pinion from the previous example to introduce a 0.002 inch runout. The spectra of the new transmission error are shown in Figs. 6A and 6B. Fig. 6A shows a corresponding increase in the first pinion order. No other change is seen in the low frequency part of the spectrum. The high frequency part of the spectrum in Fig. 6B shows, perhaps surprisingly, that the amplitudes of the two dominant peaks have been reduced. It is interesting to speculate from this that if it were possible to control gear errors exactly, gears could be manufactured in which no particular sidebands were dominant. These gears would still generate noise, but this might be more comfortable for the human ear than a pure tone.

Because of its complexity, the equation of complex modulation is not amenable to solution. So far, attempts to solve digitally for the modulation coefficients have failed. This is because iterative techniques will not converge unless some reasonable first estimates of modulation coefficients are available. It may, however, prove possible to develop a hybrid analog/digital technique. Analog techniques can demodulate individual AM and FM signals. FM demodulators cannot fully discriminate the FM component in a signal which is also amplitude- modulated, and similar problems affect AM demodulators. They could, however, be used to provide starting estimates for a digital solution. The frequencies at which the technique would work would be much greater than those generated by a single flank tester and a frequency translation would also be required. A more practical solution, however, would be to measure the gear vibration on a very stiff rolling gear tester. The dynamic characteristics of the tester would have to be such that there were no significant resonances in the frequency range measured. If this requirement is met, present technology could lead to the development of a fast loaded rolling check of gears which indicates individual gear faults directly.

#### Conclusion

The asymmetric nature of the gear noise spectrum is caused by both amplitude and frequency modulation of gear mesh excitation. The modulation is caused by low frequency manufacturing and assembly errors in the gears.

Considerably more work is needed to demodulate the transmission error or gear excitation. If successful, a dynamic gear testing technique which would rapidly diagnose individual gear faults could be developed.

#### Appendix

	Nomenclature
A	= Constant of Amplitude
ω	= Angular Frequency
t	= Time
Ø	= Phase Angle
M(t)	= A Modulated Waveform
Δω	= Frequency Variation
θ	= Instantaneous Angle
I	= Modulation Index
$J_n(x)$	= The Bessel Function of x of the First Kind
m	= Integer Constant
n	= Integer Constant

#### Amplitude Modulation (AM)

The waveform from two perfect gears can be considered as a sinusoid represented by

$$M(t) = A_c \cos(\omega_c t + \emptyset_c)$$

For the sake of complete generality, phase angles will be included everywhere.

Now suppose the amplitude of this waveform is modulated

$$A_1 \cos(\omega_1 t + \emptyset_1)$$

Then the modulated waveform is given by

$$M(t) = A_c \left[ 1 + A_1 \cos(\omega_1 t + \emptyset_1) \right] \cos(\omega_c t + \emptyset_c) \tag{1}$$

which expands to

$$M(t) = A_c \cos (\omega_c t + \emptyset_c) + \frac{1}{2} A_c A_1 \cos [(\omega_c + \omega_1)]t + \emptyset_1 + \emptyset_c] + \frac{1}{2} A_c A_1 \cos [(\omega_c - \omega_1)t + \emptyset_c - \emptyset_1]$$
(2)

so that an amplitude-modulated wave is equivalent to the sum of three components: the unaffeted fundamental and an upper and lower sideband.

We now have a partial explanation for sidebands, although this modulation gives only one upper and lower sideband.

#### Frequency Modulation (FM)

Consider again the same fundamental

$$M(t) = A_c \cos(\omega_c t + \emptyset_c)$$

frequency-modulated by

$$\Delta\omega_{c}\cos(\omega_{2}t + \emptyset_{2})$$

where  $\Delta\omega_c$  is the maximum variation of the fundamental frequency.

The instantaneous frequency of the fundamental is given by

$$\omega_{i} = \omega_{c} + \Delta \omega_{c} \cos (\omega_{2} t + \emptyset_{2}) \tag{3}$$

and the instantaneous angle is given by

$$\theta = \int_{0}^{t} \omega_{i} dt = \int_{0}^{t} (\omega_{c} + \Delta \omega_{c} \cos(\omega_{2}t + \emptyset_{2})) dt$$

$$= \omega_{c}t + \emptyset_{c} + \frac{\Delta \omega_{c}}{\omega_{2}} \sin(\omega_{2}t + \emptyset_{2})$$
(4)

The modulated wave is given by

$$M(t) = A_{c}\cos(\omega_{c}t + \varphi_{c} + \frac{\Delta\omega_{c}}{\omega_{2}}\sin(\omega_{2}t + \emptyset_{2}))$$
 (5)

Let  $\frac{\Delta\omega_c}{\omega_2}$  = I, the modulation index or modulation depth.

Expanding (5)

$$M(t) = A_c \cos (\omega_c t + \emptyset_c) \cos (I \sin(\omega_2 t + \emptyset_2))$$

$$-A_c \sin (\omega_c t + \emptyset_c) \sin (I \sin (\omega_2 t + \emptyset_2))$$
(6)

Now it can be shown that

$$\cos (x \sin y) = J_o(x) + 2\sum_{m=1}^{\infty} J_{2m}(x)\cos 2my$$
 (7)

$$\sin (x \sin y) = 2\sum_{m=1}^{\infty} J_{2m-1}(x)\sin (2m-1)y$$
 (8)

where Jn (x) is the Bessel function of x of the first kind of order n.

Using (7) and (8) in (6)

$$M(t) = A_c \cos (\omega_c t + \emptyset_c) (J_o(I) + 2\Sigma_{m=1}^{\infty} J_{2m}(I) \cos 2m(\omega_2 t + \emptyset_2))$$

$$-A_c \sin (\omega_c t + \emptyset_c) 2\Sigma_{2m-1}^{\infty} J_{2m-1}(I) \sin (2m-1) (\omega_2 t + \emptyset_2)$$
(9)
Now expanding (9)

$$M (t) = A_{c}J_{o} (I) \cos (\omega_{c}t + \emptyset_{c})$$

$$+A_{c}\Sigma_{m=1}^{\infty} J_{2m}(I) \cos(\omega_{c}t + \emptyset_{c} + 2m(\omega_{2}t + \emptyset_{2}))$$

$$+ \cos(\omega_{c}t + \emptyset_{c} - 2m(\omega_{2}t + \emptyset_{2}))$$

$$+A_{c}\Sigma_{2m-1}^{\infty} J_{2m-1} \cos(\omega_{c}t + \emptyset_{c} + (2m-1)(\omega_{2}t + \emptyset_{2}))$$

$$- \cos(\omega_{c}t + \emptyset_{c} - (2m-1)(\omega_{2}t + \emptyset_{2})) \qquad (10)$$

making use of the property

$$J_{-m}(x) = (-1)^{m} J_{m}(x)$$
(11)



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CIRCLE A-2 ON READER REPLY CARD

we can reduce equation 10 to 
$$M(t) = A_c \sum_{m=-\infty}^{\infty} J_m(I) \cos (\psi_c t + \emptyset_c + m(\omega_2 t + \emptyset_2)) \quad (12)$$

It is important that we study the significance of this expression before moving on. It is a Fourier series with terms for the fundamental frequency and each frequency equal to the fundamental plus or minus *every* integer multiple of the modulating frequency. The amplitude of these sidebands is governed by the Bessel function of the modulation index I, so to complete our understanding of FM we must briefly study the Bessel function.

The particular Bessel function we are interested in is the one of the first kind, which is a particular solution to a differential equation and which is itself an infinite series,  $J_o(x)$ ,  $J_1(x)$ ,  $J_2(x)$  and  $J_3(x)$ , are plotted against x in Fig. 3. The function is periodic and resembles a decaying sinusoid.

If  $\Delta\omega_c = 0$ , that is, the frequency does not modulate, then in Equation 14 I = 0 and  $J_m(0) = 0$  form  $\neq 0$  and  $J_0(0) = 1$ .

So

$$M(t) = J_o(0) \cos (\omega_c t + \emptyset_c)$$

and we have the fundamental only.

As we increase  $\Delta \omega_c$ , I increases so the amplitude of the fundamental will decrease, and all the sidebands will have a finite amplitude which is smaller the further their frequency from the fundamental.



CIRCLE A-16 ON READER REPLY CARD

If we increase  $\Delta\omega_c$  until I=2.4 the amplitude of the fundamental will drop to zero because  $J_o$  (2.4) = 0 (Fig. 3). We can actually remove the fundamental and leave only sidebands if we modulate to this depth.

As  $\Delta\omega_c$  increases, the amplitude of each particular sideband varies from maximum to minimum values and passes through zero. This is illustrated by Fig. 4. Here the spectrum of a 510 Hz fundamental is shown under FM by a 29.4Hz modulating wave over a range of modulation depths from 0.2 to over 9. The cyclical variation in the amplitudes of the fundamental and sidebands is clearly shown.

#### Complex Modulation

We can now proceed from these simple treatments to a more realistic one where we consider the combination of the two modulation processes, AM and FM.

Consider again a fundamental wave

$$A_c \cos (\omega_c t + \emptyset_c)$$

amplitude modulated by

$$A_1 \cos(\omega_1 T + \emptyset_1) + A_2 \cos(\omega_2 t + \emptyset_2)$$

representing a combination from both of the gears and frequency modulated by

$$\Delta\omega_{c1}\cos(\omega_1t + \emptyset_3) + \Delta\omega_{c2}\cos(\omega_2t + \emptyset_4)$$

if we set 
$$I_1 = \frac{\Delta \omega_{c1}}{\omega_1}$$
  $I_2 = \frac{\Delta \omega_{c2}}{\omega_2}$ 

$$C = 1 + A_1 \cos(\omega_1 t + \emptyset_1) + A_2 \cos(\omega_2 t + \emptyset_2)$$
 (13)

$$D = \omega_c t + \emptyset_c + I_1 \sin(\omega_1 t + \emptyset_3) + I_2 \sin(\omega_2 t + \emptyset_4)$$
 (14)

(The modulated wave may be written)

$$M(t) + A_c C \cos D \tag{15}$$

Note that this represents a frequency modulated wave which is subsequently amplitude modulated.

It can be shown that the result is the same if the amplitude modulation takes place before frequency modulation.

If cos D is expanded and the appropriate substitutions made, we obtain

$$\cos D = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_m(I_1)J_n(I_2) \cos(\omega_c t + \emptyset_c + m(\omega_1 t + \emptyset_3) + n(\omega_2 t + \emptyset_4))$$
(16)

This is a Fourier series with components at frequencies given by all possible combinations of

$$\omega = \omega_c \pm m \omega_1 \pm n\omega_2$$
; m & n = - \infty to \infty

Now substitute equations 13 and 14 into 15.

$$\begin{split} M(t) &= A_{c} \left[ 1 + A_{1} \cos(\omega_{1}t + \emptyset_{1}) + A_{2} \cos(\omega_{2}t + \emptyset_{2}) \right] \cos D \\ &= A_{c} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_{m}(I_{1}) J_{n}(I_{2}) \cos(\omega_{c}t \\ &+ \emptyset_{c} + m(\omega_{1}t + \emptyset_{3}) + n(\omega_{2}t + \emptyset_{4}) \\ &+ \frac{1}{2} A_{1} \cos(\omega_{c}t + \emptyset_{c} + (m+1) \omega_{1}t + m\emptyset_{3} + \emptyset_{1} \\ &+ n (\omega_{2}t + \emptyset_{4}) \\ &+ \frac{1}{2} A_{1} \cos(\omega_{c}t + \emptyset_{c} + (m-1) \omega_{1}t + m\emptyset_{3} - \emptyset_{1} \\ &+ n (\omega_{2}t + \emptyset_{4}) \\ &+ \frac{1}{2} A_{2} \cos(\omega_{c}t + \emptyset_{c} + m(\omega_{1}t + \emptyset_{3}) + (n+1) \\ &(\omega_{2}t + n\emptyset_{4} + \emptyset_{2}) \\ &+ \frac{1}{2} A_{2} \cos(\omega_{c}t + \emptyset_{c} + m(\omega_{1}t + \emptyset_{3}) + (n-1) \\ &(\omega_{2}t + n\emptyset_{4} - \emptyset_{2}) ) \end{split}$$

This is the complete expression for a complex modulated wave which at first sight appears highly complicated.

If we consider that in the case of a real gear pair the tooth ratio will have been selected to give a long hunting period, then we can treat the sidebands of each gear separately.

Thus we may look at the sidebands of the first gear:

$$A_{c} \stackrel{\text{def}}{\Sigma}_{m=1} J_{m} (I_{1}) J_{o} (I_{2}) \cos (\omega_{c} t + \emptyset_{c} + m(\omega_{1} t + \emptyset_{3}))$$

$$+ \frac{1}{2} A_{1} \cos (\omega_{c} t + \emptyset_{c} + (m + 1) (\omega_{1} t + m \emptyset_{3} + \emptyset_{1}))$$

$$+ \frac{1}{2} A_{1} \cos (\omega_{c} t + \emptyset_{c} + (m - 1) (\omega_{1} t + m \emptyset_{3} - \emptyset_{1})) (18)$$

We can now see how each primary sideband is made up from three contributory sources. The first is the sideband at that frequency directly from the frequency modulation process. The second and third contributions are due to amplitude modulation of the neighboring sidebands.

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# Improvement in Load Capacity of Crossed Helical Gears

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#### Abstract

A new method of improving the load capacity of crossed helical gear sets is introduced. The principle of the method is as follows:

(1) A line contact is introduced instead of a point contact between two teeth in mesh with each other; i.e., the tooth surface of one member of a crossed helical gear set is slightly finish cut by a tool of a form virtually identical with the other.

(2) In order to optimize the parameters, which control the load capacity of the gear set, a higher angle like 30° is used for the pressure angle.

A crossed helical gear set is experimentally designed and finished on the basis of the principle. Performances of the set and a corresponding ordinary one are examined. The load carrying capacity of the improved set is several times that of the ordinary one.

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#### Coordinate Systems (Fig. 1)

 $0_1$ - $x_1y_1z_1$  – Stationary with respect to the earth,  $z_1$  axis coinciding with the axis of the unmodified gear.

 $0_1$ - $x_1'y_1'z_1'$  - Stationary with respect to the unmodified gear,  $z_1'$  axis coinciding with  $z_1$  axis.

0<sub>2</sub>-x<sub>2</sub>y<sub>2</sub>z<sub>2</sub> - Stationary with respect to the earth, z<sub>2</sub> axis coinciding with the axis of the unmodified gear.

 $0_2$ - $x_2'y_2'z_2'$  - Stationary with respect to the modified gear,  $z_2'$  axis coinciding with  $z_2$  axis.

#### Introduction

Crossed helical gear sets are used to transmit power and motion between non-intersecting and non-parallel axes. Both of the gears that mesh with each other are involute helical gears, and a point contact is made between them. They can stand a small change in the center distance and the shaft angle without any impairment in the accuracy of transmitting motion. Also, shifting axially either member of the set makes

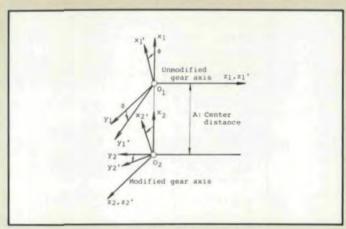


Fig. 1-Coordinate systems

no difference in meshing action, so they are the easiest of all gears to use.

The load carrying capacity of crossed helical gear sets is quite small, and their teeth surfaces tend to be easily worn out. These limitations result from the fact that they have a point contact and a higher sliding velocity. A low pressure angle and deep teeth are preferred for a higher contact ratio and a larger load capacity.

Crossed helical gear sets such as those driving an oil pump in an automobile engine are called cam shaft gears. One of the helical gears, integral with a cam shaft of the engine, drives the other on the oil pump shaft. Generally, the ratio of the gear set is equal to or nearly equal to one. Recently, there has been a tendency to increase the power of engines, making the cam shaft gears transmit a larger load. This report, explicitly concerning cam shaft gears, introduces a new method of kinematic consideration to improve the load capacity of skew gears.

#### Principle

The principle of the new method is as follows:

(1) The tooth surface of one of the gears of a mating crossed helical gear set is slightly finish cut by a tool of a form virtually identical with that of the other. Namely, one tooth surface is modified so that it becomes an envelope of the other. This makes an original point contact change into a line contact, which is a characteristic of worm gear sets.

The entire tooth surface does not need to be finish cut. The line contact and a higher load capacity are realized even if a part of the surface is modified. In this report, the gear set thus finished is called the modified gear set, and a conventional non-modified one is named the ordinary gear set. The modified member of the modified gear set is called the modified gear and the other, the unmodified gear.

(2) The pressure angle of a modified set is determined to optimize the parameters which control the load capacity. These are the relative radii of curvature, the relative sliding direction and the extent of the contact area.

The center distance, number of teeth and shaft angle of a modified set are made the same as those of an ordinary one. The pitch circle diameters, helix angles and modules of the modified and ordinary sets are not necessarily the same. Taking assembly or installation into account, the outside diameters of the modified set are made similar to those of the ordinary one.

(3) The gear tooth to be modified is rough cut by a conventional hobbing machine with an involute gear hob. Modifying cut of the tooth is the same as hobbing of a worm wheel by a worm hob, but it is difficult to make the rotation ratio of the hob and work table identical in an ordinary hobbing machine, so a special setup for modifying-cut is provided. The unmodified gear is a helical gear which is not easily cut by a hobbing machine. Although both of the mating gears can be modified, only the driven gear mounted on the oil pump shaft is modifed in this report.

#### Kinematic Analysis of Modified Gears

#### 1. Center Distance A

The center distance A and number of teeth z1 and z2 of

the modified set are identical with those of the ordinary set.

$$A = \frac{m_n}{2} \left( \frac{z_1}{\cos \beta} + \frac{z_2}{\cos (90^\circ - \beta)} \right) \tag{1}$$

2. Unmodified Tooth Surface (Involute Helicoid)

The unmodified tooth has an involute helicoid surface. Using parameters shown in Fig. 2, the surface is expressed as follows:

$$\bar{r}_{1'} = \begin{pmatrix} x_{1'} \\ y_{1'} \\ z_{1'} \end{pmatrix} = \begin{pmatrix} r_{b} (\cos \theta + u \sin \theta) \\ r_{b} (\sin \theta - u \cos \theta) \\ -q.v \end{pmatrix}$$
(2)

where,  $q=r_b/\tan \beta$  is called the reduced pitch and  $\theta = u + v + n$ .

3. Surface Normal to Unmodified Tooth

The unit surface normal to the unmodified tooth surface is

$$\overline{n}_{1}' = \begin{pmatrix} n_{x}' \\ n_{y}' \\ n_{z}' \end{pmatrix} = \begin{pmatrix} \cos \beta_{b} \sin \theta \\ -\cos \beta_{b}^{b} \cos \theta \\ -\sin \beta_{b} \end{pmatrix}$$
(3)

4. Contact Condition of Modified Gear Set

At a point of contact, the following vector equation is satisfied.

$$\overline{n}_{1} \cdot \overline{w} = 0$$
 (4)

where  $\overline{n}_1$  is the normal to the tooth surface and  $\overline{w}$  is the relative sliding velocity expressed in 01-x1y1z1 system. The vector  $\overline{\mathbf{w}}$  is derived by using Equation (2).

From Equation (4), the contact condition of the modified gear set is expressed as follows:

$$\cos H + (u-\nu \cot^2 \beta_b) \sin H + A/r_b - \cot \beta_b = 0$$
 (5) where  $H = u + \nu + \eta + \phi$ .

#### Design of Multiple Fly Cutter

1. Multiple Fly Cutter

The following difficulties appear when modifying gears by a hobbing machine:

- (1) The gear ratio one requires the work table to rotate at very high speed.
- (2) A multiple-threaded hob of a high lead angle, expensive and almost impossible to finish precisely, is needed.

Therefore, a special multiple fly cutter similar to a pinion cut-

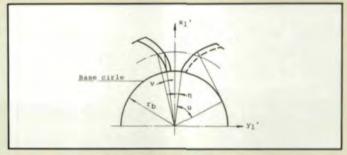


Fig. 2-Parameters for involute

ter and an experimental setup to modify the gears are provided.

#### 2. Cutting Edges

The ideal edge of the cutter is identical with the intersection between the cutting face and the unmodified gear surface. The ideal side flank, which has the profile conforming to the form of the ideal edge, is usually difficult to finish precisely, so the actual side flank is finished to be an involute surface approximating the ideal flank. Then the actual cutting edge is the intersection between the actual side flank thus finished and the face.

#### 3. Cutting Face

As shown in Fig. 3, the normal  $\overline{n}_f$  to the face is

$$\overline{n}_{f}' = \begin{pmatrix} -\sin \gamma \\ \cos \gamma \cdot \sin \beta \\ -\cos \gamma \cdot \cos \beta \end{pmatrix}$$
 (6)

At a point  $\overline{r}_f$  on the face, the following vector equation is satisfied:

$$\overline{\mathbf{n}}_{\mathbf{f}'} \cdot \overline{\mathbf{n}}_{\mathbf{f}}' = -\mathbf{r}_{\mathbf{a}} \cdot \sin \gamma \tag{7}$$

#### 4. Involute Side Flank

or

OF

The flank is finished to be an involute helicoid in contact with the ideal side flank at the pitch cylinder. Then the flank is expressed by Equation (2) in which following reduced pitch,  $q_{\rm c}$ , is substituted for q:

$$q_c = r_b/\tan (\beta - \Delta \beta_t)$$
 (P<sub>t</sub> in Fig. 4) (8)

$$q_c = r_b/\tan (\beta - \Delta \beta_1)$$
 (P<sub>1</sub> in Fig. 4)

where  $\Delta\beta_t$  and  $\Delta\beta_1$  are the clearance angles of the flanks.  $\eta$  in Fig. 2, which defines the position of the involute helicoid, is

$$\eta = a/r_b + in\nu\alpha_t \qquad \text{(for } P_t\text{)}$$

$$\eta = a/r_b + in\nu\alpha_t \qquad \text{(for } P_t\text{)}$$

where  $\alpha_t$  is the transverse pressure angle of the involute side flank. Then the flank may be written

$$\bar{\mathbf{r}}_{c'} = \begin{pmatrix} \mathbf{x}_{c'} \\ \mathbf{y}_{c'} \\ \mathbf{z}_{c'} \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{b} (\cos \theta + \mathbf{u} \cdot \sin \theta) \\ \mathbf{r}_{b} (\sin \theta - \mathbf{u} \cdot \cos \theta) \\ \mathbf{b} - \mathbf{q}_{c} \cdot \mathbf{v} \end{pmatrix}$$
(10)

where  $\theta = u + \nu + \eta$ .

#### 5. Point on Cutting Edge

At a point on the edge; i.e., the intersection between the involute side flank and the cutting face, the following vector equation is satisfied.

$$\bar{\mathbf{r}}_{c'} \cdot \bar{\mathbf{n}}_{f'} = -\mathbf{r}_{a} \cdot \sin \gamma$$
 (11)

Similarily, at a point on the ideal edge; i.e., the intersection between the ideal involute tooth surface and the face, the following is satisfied.

$$\bar{\mathbf{r}}_{1}' \cdot \bar{\mathbf{n}}_{f}' = -\mathbf{r}_{a} \cdot \sin \gamma$$
 (12)

#### 6. Profile Error of Cutting Edge

It is necessary to examine the difference between the ideal edge by Equation (12) and the actual one by Equation (11). These results differ, depending on the use of the ideal flank or the involute flank. Fig. 5 shows the profiles of these two surfaces in a plane perpendicular to their axes. In this figure,  $\Delta$  is the circumferential difference between two surfaces and  $\Delta_n$  is the normal difference.

$$\Delta_n = \Delta \cos \alpha_x \tag{13}$$

If  $\Delta_{\eta}$  is not small enough, the involute side flank must be finished so that its measured profile error coincides with the  $\Delta_{\eta}$  curve.

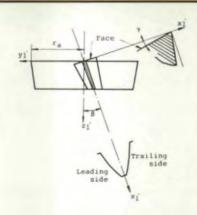


Fig. 3-Multiple fly cutter

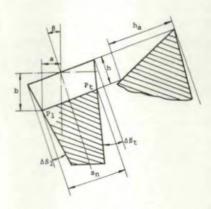


Fig. 4-Points on cutting edge P1 and Pt

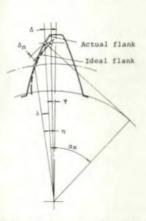


Fig. 5-Difference between side flanks

#### TABLE 1 Dimensions of Gearsets

	Ordinary gearset		Modified gearset	
	Driver	Driven	Unmodified	Modified
Center distance mm	42.917		42.917	
Normal pressure angle	14.5°		30.0°	
Normal module	2.5		2.41	
Number of teeth	12 (L.H.)		12 (L.H.)	
Helix angle	50.0°	40.0°	55.13°	34.87°
Pitch circle diameter mm	46.672	39.162	50.585	35.249

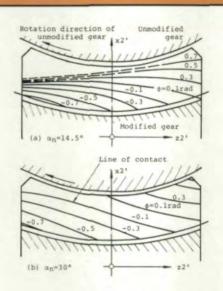


Fig. 6-Lines of contact

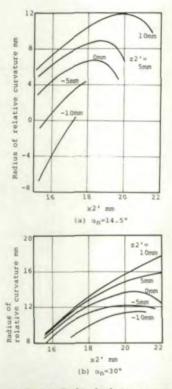


Fig. 7-Radii of relative curvature

#### Design

#### 1. Dimensions

Table 1 shows the dimensions of the ordinary and modified crossed helical gear sets.

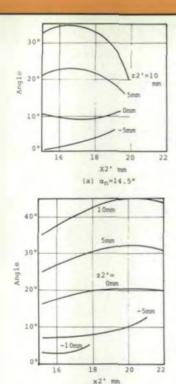
2. Lines of Contact and Radii of Relative Curvature

Fig. 6 shows the lines of contact on the modified gear surface. When the pressure angle is the same as that of the ordinary set ( $\alpha_{\eta} = 14.5^{\circ}$ , Fig. 6a), some of the lines of contact shown by the dotted lines exceed the effective length of the unmodified gear and do not actually exist. Fig. 6b shows the case of  $\alpha_{\eta} = 30^{\circ}$ . The lines of contact exist all over the surface. The contact area of this case is wider than that of  $\alpha_{\eta} = 14.5^{\circ}$ .

Fig. 7 shows the radii of relative curvature. The radii of  $\alpha_{\eta} = 30^{\circ}$  are larger than those of  $\alpha_{\eta} = 14.5^{\circ}$ .



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(b) an=30° Fig. 8-Angles between lines of contact and relative sliding direction

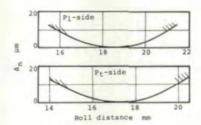


Fig. 9-Inspection curve for cutter

Fig. 8 shows the angles between a line of contact and the direction of the relative sliding velocity. The angles of  $\alpha_n = 30^{\circ}$  are also larger than those of  $\alpha_n = 14.5^{\circ}$ . From these, a high pressure angle  $\alpha_n = 30^{\circ}$  is determined.

#### 3. Profile Error of Cutting Edge

Fig. 9 shows the  $\Delta_n$  curves of the involute side flanks of the multiple fly cutter designed to agree with the designed modified gear set. The side flank clearance angles  $\Delta\beta_1$  and  $\Delta\beta_t$  are 3.5°. Errors of 13  $\mu$ m occur at the tip and root if the side flanks are finished to the correct involute; therefore, they are finished with the profile errors shown in Fig. 9.

#### Manufacture

#### 1. Hob and Multiple Fly Cutter

Fig. 10 shows a hob and a multiple fly cutter provided for a modifled gear set. The unmodified gear is finish cut by the hob. The modified one is rough cut by the hob and then finish cut by the multiple fly cutter. The number of threads and the helix angle of the cutter coincide with that of the unmodified gear.

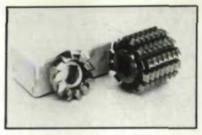
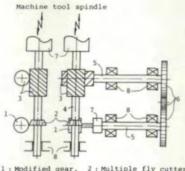


Fig. 10-Hob and multiple fly cutter



- 1 : Modified gear, 2 : Multiple fly cutter
- 3 : Master gears, 4 : Axis, 5 : Axis
- 6 : Spur gears, 7 : Chuck, 8 : Bearings

Fig. 11 - Schematic of modifying-cut setup



Fig. 12-Modifying-cut setup

#### 2. Gear Modifying Setup

Fig. 11 shows an experimental setup to modify a gear. The axis (4), on which the multiple fly cutter (2) is mounted, is supported at one end by a slide bearing. The other end of the axis is clamped by a chuck on a machine tool spindle so that the axis and the cutter are driven axially and rotationally by the spindle. A gear set (3) is the master gear of this system. It converts the axial and rotational movement of the cutter into the rotation of the axes (5) coupled to each other by a spur gear set (6). At the end of the axis (5), the modified gear (1) is chucked.

Both of the gear ratios of the gear sets (3) and (6) are one. The lead of the master gear mounted on the cutter axis is equal to that of the cutter; i.e., the unmodified gear. Therefore, when the cutter moves axially and rotationally to modify the gear, the relative movement between the cutter and the modified gear coincides with that between the unmodified gear and the modified gear, thus realizing the correct generating motion.

#### 3. Finish Cut

Fig. 12 shows the setup in operation. The modified and

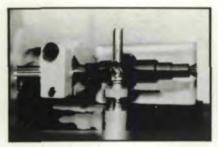


Fig. 13-Tooth bearing check

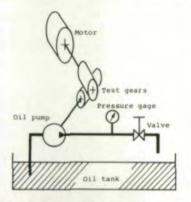


Fig. 14 - Schematic of performance test

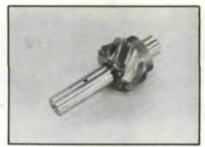


Fig. 15-Modified gear after 15 hours run

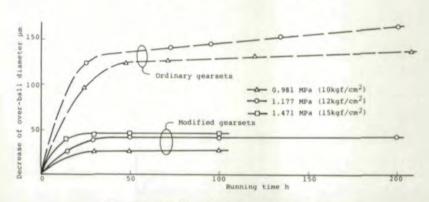


Fig. 16 - Wear of modified and ordinary gearsets

unmodified gears are made from SCM22 (Chromium Molybdenum Steel) and carburized to a hardness greater than HRC55.

The tooth bearing is checked as shown in Fig. 13. At white parts of the modified gear (upper gear), the contact between the gear teeth has occurred, and at black parts, no contact has occurred. The surface is partly whitened because only its central area is modified by finish cut.

#### Performance Test

#### 1. Test Setup

Fig. 14 shows the setup for a running test. Similar to a cam shaft gear set in an automobile engine, the pump driving performance of the modified set is tested. The load is adjusted by changing the outlet pressure of the pump. The speed of the gear rotation is fixed at 1650 rpm. The over-ball diameter of the modified gear is measured at regular intervals, and the decrease of the diameter is seen to represent the tooth wear. 2. Results

Fig. 15 shows the tooth surface of the modified gear after 15 hours run at 1.471 MPa outlet pressure. The unevenness of the surface by finish cutting is clearly observed.

Fig. 16 shows the decrease of the over-ball diameter by the running test. After a 30 hours run, the wear of the modified gear set stops even if the highest outlet pressure; i.e., the highest load, is applied. The unevenness is still apparent after 200 hours run.

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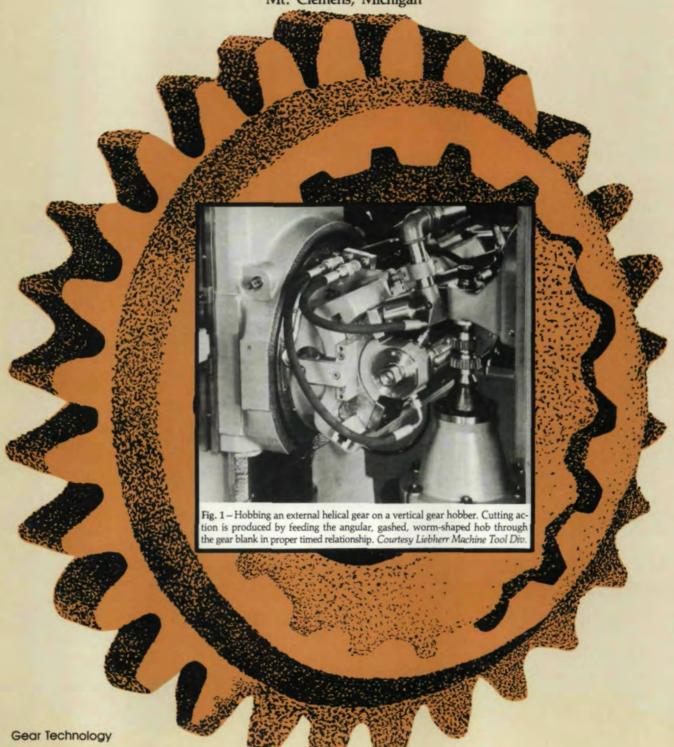
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## BACK TO BASICS...

# Gear Manufacturing Methods – Forming The Teeth

by
National Broach and Machine,
Mt. Clemens, Michigan



The forming of gear teeth has traditionally been a timeconsuming heavy stock removal operation in which close tooth size, shape, runout and spacing accuracy are required. This is true whether the teeth are finished by a second forming operation or a shaving operation.

Originally gear teeth were produced with form-milling cutters on milling machines equipped with index heads. Later the popular gear hobbing process, Fig. 1, was developed to produce external gears. The shaper-cutting method was developed primarily to produce internal gears and gears on blanks that would not permit passage of a hobbing tool.

Today internal gears are being broached at high production rates. External gears are also being produced at high production rates by pot broaching methods. Other methods such as high energy rate forming and rolling of fine-pitch teeth from the solid are being applied and investigated.

#### Gear Hobbing and Shaping

One of the key problems in hobbing and shaping of gear teeth is the specification of a properly proportioned tooth form. Most of the problem occurs in the fillet area. However, when semi-topping hobs or shaper cutters are used to produce tip-protective chamfers, Fig. 2, a loss of active profile can result if the outside diameter of the blank has not been

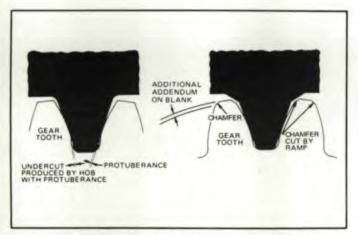


Fig. 2 - Typical hob tooth shapes. Protuberance type is at left, semi-topping type at right. Courtesy Star Cutter Co.

increased beyond the theoretical outside diameter to provide additional stock for the chamfer.

If the fillet produced by hobbing or shaping is too high, finishing tool interference and breakage can result, and the accuracy of the produced profile can be affected. If the hob or shaper cutter tooth has a full radius form on the tip, maximum wear life of the tools is provided.

Referring to Fig. 3, it can be seen that forming of the teeth with a gear-shaped shaper cutter or a rack-shaped hobbing tool differs considerably from the in-fed form tool operation. Hobbing, gear-shaping, and rotary gear shaving have tooth tip paths which produce fillets that are actually trochoidal curves generated by the tip corner of each tooth.

As a result, the point of tangency between this curve and the generated involute profiles is higher than that of the radius on the form tool. The shaper-cut fillet tangency point is slightly higher than that produced by a hob of the same working depth. Thus, the shape of a fillet on a gear drawing is correctly specified as that produced by a specific hob or shaper cutter tooth form with a specific tip radius or form.

The generating action of hobs and shaper cutters with and without protuberance to provide necessary shaving cutter tip clearance is illustrated in Fig. 4.

The amount of total undercut (shaving stock plus 0.0005 to 0.001-in.) produced by pre-shaving, protuberance-type tools, Table 1, varies with the pitch of the gear teeth. Posi-

Table 1-Recommended Shaving Stock and Total Undercut for Pre-Shave Gear Cutting Tools

Normal Diametral Pitch	Shaving Stock (In. per Side of Tooth)	Total Undercut (In. per Side of Tooth)
2 to 4	0.0015 to 0.0020	0.0025 to 0.0030
5 to 6	0.0012 to 0.0018	0.0023 to 0.0028
7 to 10	0.0010 to 0.0015	0.0015 to 0.0020
11 to 14	0.0008 to 0.0013	0.0012 to 0.0017
16 to 18	0.0005 to 0.0010	_
20 to 48	0.0003 to 0.0008	-
52 to 72	0.0001 to 0.0003	-

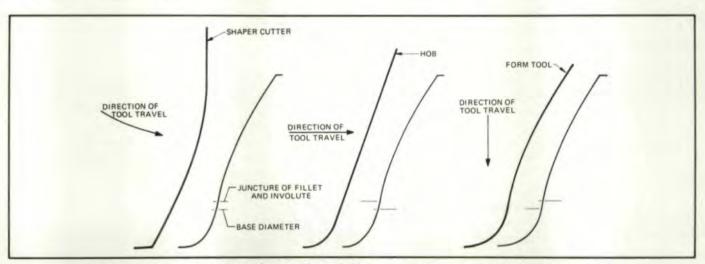


Fig. 3-Generating action of gear shaping, left, and hobbing, center; compared with index form-milling, right.

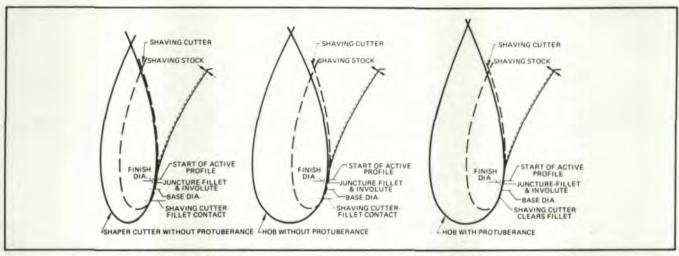


Fig. 4-Generating paths of shaper cutter, hobs and rotary-shaving cutters in fillet area on same tooth.

tion of the protuberance-produced under-cut fillet produced by a specific hob or shaper cutter varies with the number of teeth in the gear. Usually the undercut will generate too high on gears with small numbers of teeth and reduce the necessary amount of involute profile. Use of the same tool on gears with large numbers of teeth will provide an undercut too low to serve any useful purpose.

Theoretically protuberance-type hobs and shaper cutters should be designed for a gear with a specific number of teeth. However, this method is not economically feasible when a variety of gears with different tooth numbers are being processed. Often a tool with no protuberance may be used for gears with small numbers of teeth. This method makes use of the natural undercut produced by generating-type tools that extend below the base circle on gears with small tooth numbers. Fig. 5 left illustrates this condition.

On long and short-addendum gears, the amount and position of protuberance on hobs and shaper cutters must be carefully specified because of the different generating action in producing the teeth.

The fillet shapes of typical hobbed and shaped gears are shown in Figs. 6 and 8. The finish produced by these two generating forming methods is shown in Figs. 7 and 9.

The effect of the generating action of hobs and shaper cutters on the finish in the fillet area is shown in the two enlarged sketches in Fig. 5.

#### Applying the Processes

Careful consideration should be given to the tooling for hobbers and shapers. Where possible this tooling should locate on the rim or side of the gear blank, just below the root diameter of the teeth. Proper mounting of hobs, including indication for runout within 0.0005-in., and careful machine setup for tooth size are most important for good results in the subsequent shaving operation.

Optimum machine performance and economy results when only sufficient stock is left for shaving to clean up the gear and assure the removal of semi-finishing errors or their reduction to specified tolerance limits. Leaving an excessive amount of stock to be removed by shaving unduly reduces cutter life,

increases shaving time and may result in the shaving cutter hitting the fillet.

Table 1 shows the amount of stock left on each side of a tooth under average conditions for removal in the shaving operation.

It is also important that the involute profile and lead of a hobbed or shaped helical gear be held as close as possible to that of the gear as shaved if maximum shaving cutter life is to be attained. Uniform stock removal in the shaving operation equalizes cutter wear and results in more pieces shaved before the cutter has to be reground. This is not the case when the cutter has to correct too great an error in involute profile and excessive wear is concentrated on only part of the tool. This results in hollow spots on the cutter which in turn leave high spots on the shaved gear tooth profiles.

It is good practice to process a pilot group of gears to the desired lead, heat treat them and then carefully check the amount of distortion caused by the heat treatment. The resulting average of this check will serve as a guide for compensating the lead in processing the remainder of the lot.

Changes in helix angle also produce changes in involute profiles. Thus, both must be adjusted in machining gears which are to be heat-treated. The gears should be hobbed or shaper cut as closely as possible to the adjusted lead. This is particularly true if maximum shaving cutter life is desired in producing wide face gears.

Clutch gears having rounded or pointed teeth should have all chips and burrs removed from their ends before they are shaved. Otherwise, these chips can become imbedded in the serrations of the cutter teeth and cause breakage.

Blank machining, hobbing or shaping speeds and feeds should not be so excessive that they cause cold working or burnishing of gear tooth surfaces. This practice will prolong shaving cutter life and avoid excessive heat treat distortion.

The selection of the type of hobbing tool has an important economic effect on the overall cost of gear processing. At one time, because they were used on finish-hobbing operations before the development of rotary gear shaving, only single-thread, Class "A", ground-form hobs were used as preshaving tools. With the advent of shaving, less-expensive

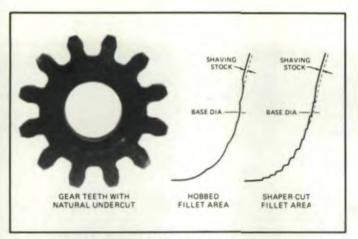


Fig. 5-A 12-tooth pinion, left, showing natural shaper-cutter undercut. Enlarged fillets, right, show type of finish generated by hobbing and shaping.

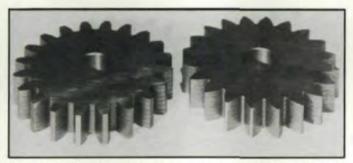


Fig. 6 - Hobbed 4-DP, 5-in. PD, 20-tooth gears with 20°-PA, left, and 30°-PA,

single-thread, Class "B", ground-form pre-shaving hobs were successfully applied.

Today even lower-cost Class "C", accurate unground-form hobs are widely applied as preshaving tools. To reduce the required number of hobbing machines for roll-finished, finepitch helical transmission gears, multiple-thread, Class "C" accurate unground-form hobs are also being utilized as preshaving tools.

Multiple-thread hobs with straight gashes are usually larger in diameter than single-thread hobs because of the requirement for a low thread-angle. In actual production of 14 and 16-NDP helical transmission pinions, high production hobbing rates are being achieved by using 3-thread, 3-in. dia., Class "C" hobs instead of 21/2-in. dia., single-thread hobs of the same class. The number of threads in multiple-thread hobs should not be prime with the number of teeth in the work gear.

Accurate unground-form, Class "C" single and multiplethread hobs can be produced by rack form-tool methods to provide extremely close tolerances for such features as protuberance, semi-topping and full-radius fillet design.

#### **Broaching Internal Gears**

Internal spur and helical gears can be most economically produced in high production by a single pass of a full-form finishing broaching tool assembly as shown in Fig. 10. A wide variety of automotive transmission internal running gears up to 6-in. pitch dia. with 6 to 20-DP teeth can be produced by

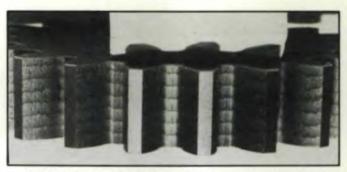


Fig. 7 - Hobbed finish of left-hand gear in Fig. 6 as produced by a 4-in. dia., 10-flute, single-thread hob rotating at 71-rpm and fed at 0.150-in. per revolution.

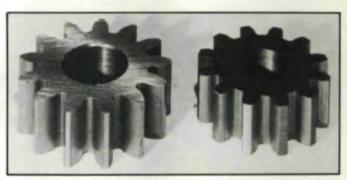


Fig. 8-Gear-Shaped 5-DP, 20°-PA, 13-tooth gear, left, produced by a 20-tooth, 41/2-in. OD shaper cutter; compared wth a 5/7-DP, 20°-PA, 12-tooth gear, right, produced by a 15-tooth, 3.325-in. OD cutter.

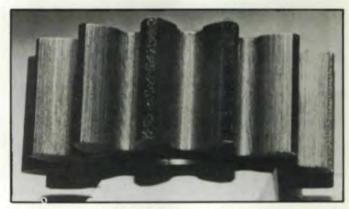


Fig. 9-Shaper-cut finish of left-hand gear in Fig. 8 as produced with the cutter making 121 strokes per minute and feeding at a rate of 0.001-in. per stroke.



Fig. 10 - Full-form finishing broach showing roughing section, finishing shell, tailpiece and broached gears.

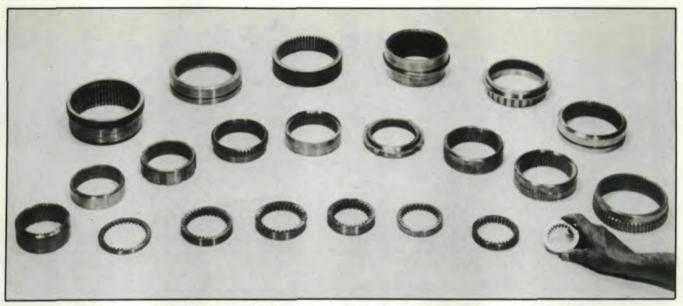


Fig. 11 - Internal-broached spur and helical transmission pump and running gears ranging from 2 to 6-in diameter.

this method. See Fig. 11.

Full-form finish broaching provides fine surface finishes, precision involute form, accurate tooth thicknesses and precision tooth spacing and lead.

Internal helical gears are usually broached on vertical broaching machines. Accurate leads are produced by the action of a precision lead bar, follower nut, and associated gearing, which rotate the broach as it is pulled through the blank. See Fig. 12

Where close control of internal gear tip contact with mating

Fig. 12—Full-form finish broaching of internal helical gears two-at-a-time on a vertical broach.

pinions is desired, the broached tooth form can be notched as shown in Fig. 13 to provide absolute control of length of roll.

In one application, two fully-automated full-form finishing broaching machines produce internal helical gears at a rate of 180 pieces per hour. The internal gear has 72, 15.5-DP, 17½°-PA teeth with a 22° 11′, 30″ right hand helix angle. The gear blank has a 6-in. OD and is about 1-9/16-in. wide. Brinell hardness of the SAE 4028 blank is from 179 to 217.

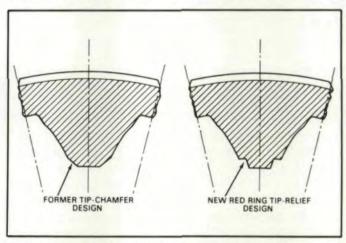


Fig. 13 – Conventional broached internal gear tip chamfer and improved tip relief, length-of-roll-control design.



Fig. 14 – Large internal spur differential running gears that are broached to precision tolerances.



Fig. 15 - A variety of external cast iron and steel clutches, cams and splines produced by push-up pot broaching.

The broaching tool is 82-in, long and has a chip load of 0.0036-in. per tooth.

Originally the gear was shaped and shaved. It took 3 minutes to shaper-cut the teeth and 11/4 minutes to shave it. Each broaching machine makes a finished gear every 40 seconds. The former method required 28 gear shaper spindles and six rotary gear shavers. Total life of individual broaching tools is about 100,000 pieces.

Internal spur differential running gears with 5/7-DP teeth up to 9.400-in. pitch diameter have been produced by nibbling-type broaching tools, Fig. 14.

#### **Broaching External Gears**

The fastest way to produce medium and high production external gears, splines and parts with specially formed teeth like those in Fig. 15, is by pot broaching. A new process called push-up pot broaching uses a machine, Fig. 16, in which the part is pushed upward through a fixed pot broaching tool of either stick-type or wafer-type design to produce external

PART UNLOADER SIZE AND FORM GAGE

Fig. 16 - A 25-ton automated push-up pot broaching machine that can produce external gears, splines and tooth forms at rates up to 450 pieces per hour.

teeth under ideal conditions that assure quick and complete chip removal from the broach teeth. Coolant is flushed into the tool area through a quick-disconnect coupling.

Fine finish and precision tooth form, size, and spacing are provided in gears and splines produced by push-up pot broaching.

The process is ideally adapted to full automation. Finished parts are ejected at the top of the pot broach where gravity force can help move them on to the next operation.

The 60-tooth, 12-DP, 141/2°-PA, SAE 5130 involute spline (second from the right in Fig. 15) has a 5-in. PD and is 0.800-in. long. The teeth are broached and the outside diameter finished with a ring-type broaching tool at a rate of 240 pieces per hour by pot broaching. Total life of the tool in this application is about 600,000 pieces.

(continued on page 48)

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Yet on the other hand (and

most puzzling) others will rant on and on and on that don't even try it's just impossible — all those Business

Lcans Programs are strictly the Chryslers the Lockheeds big corporations not for the little y or small companies



BUSINESS

Still there are those who

I need money right now and small business government loans take too darn long. It's impossible to qualify. No one ever gets one of

Or you may hear these comments

"... My accountant's junior assistant says he thinks it might be a waste of my time!" "Heck, there's too much worriesome paperwork

and red tape to wade through!"
Frankly — such rantings and ravings are just a lot of "bull" without any real basis - and only serve to clearly show that lack of knowledge misinformation and and not quite fully understanding the UNITED STATES GOVERN-MENT'S Small Business Administration's (SBA) Programs have unfortunately caused a lot of people to ignore what is without a doubt — not only the most important and generous source of financing for new business start ups and existing business expansions in this country — but

of the entire world!

Now that you've heard the "buil"
about the United States Government's SBA Loan Program - take a few more moments and read the

following facts

Only 9.6% of approved loans were actually made to minorities

"small business" actually applies to 97% of all the companies in the nation

Red tape comes about only when the loan application is sent back due to applicant not providing the requested infor-mation...or providing the wrong

The SBA is required by dollar amount in business loans each fiscal year in order to law-fully comply with strict quotas. (Almost 5 billion this year)

Yet, despite the millions who miss out - there are still literally thousands of ambitious men ar women nationwide who are properly applying — being approved — and obtaining sufficient funds to either start a new business, a franchise, or buy out or expand an existing one. Mostly, they are all just typical Americans with no fancy titles, who used essentially the same effective know-how to fill out their applications that you'll find in the Business Opportunity Seekers' Loans Manual

So don't you dare be shy about applying for and accepting these guaranteed and direct government loans. Curiously enough the government is actually very much

GUARANTEE #1

Simply — look over this most effective money raising loan e preparation assistance manual — if you decide to keep the manual — and you apply for an actually help you obtain the Business Loan you need right away — just return it for a full and prompt retund will be returned in full and prompt return of the second prompt ret \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

interested in helping you start a business that will make a lot of money it's to their advantage the more money you make the more they stand to collect in taxes. in fiscal 1986, our nation's good old generous "uncle" will either lend directly or guarantee billions of dollars in loan requests along with technical assistance and even sales procurement assistance Remember. If you don't apply for these available SBA funds somebody else certainty will.

Don't lose out — now is the best me to place your order for this comprehensive manual. It is not sold in stores. Available only by mail through this ad, directly from Financial Freedom Co., the exclusive publisher, at just a small fraction of what it would cost for the services of a private loan advisor or to attend a seminar. For example:

Initially, this amazing Guaran-teed and Direct Loans Manual was specially designed to be the basis of a Small Business Loan Seminar — where each registrant would pay an admission fee of \$450. But our company felt that since the manual's quality instructions were so exceptionally crystal-clear that anyone who could read, could successfully use its techniques without having to attend a seminar or pay for costly private loan advisory assistance services.

Therefore, for those purchasing the manual by mail, no 3 day class. no course and accommodations are required. And rather than \$450 we could slash the price all the way down to just a mere \$20 — a small portion of a typical serfinar attendance fee — providing you promptly fill in and mail coupon below with fee while this special seminar-in-print" manual offer is still available by mail at this rela-tively low price!

tively low price!

Remember, this most unique manual quickly provides you with actual sample copies of SBA Loan application and all other required forms—already properly filled in for you to easily use as reliably accurate step-by-step guides—thus offering you complete assurance that your application will be properly prepared and thereby immediately putting you on the right road to obtaining fast no red-tape loan approval.

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Of course, no one can guarantee that every request will be approved -- but clearly we are firmly con-vinced that any sound business request properly prepared—showing a reasonable chance of repayment and submitted to SBA-will be approved.

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#### IMPROVEMENT IN LOAD CAPACITY . . .

(continued from page 39)

The ordinary gear sets of the same material and hardness are also tested. The wear of the sets is several times that of the modified one, and after 200 hours run, the wear still advances. When the highest load is applied, the advance of wear is too rapid to measure the over-ball diameter.

#### Conclusion

A new method to improve load capacity of crossed helical gear sets is introduced. The method is based on kinematic consideration of skew gears. Results of a running test show that the wear of the improved gear set is far less than that of the ordinary crossed helical gear set. The method is worthy of further practical development.

#### Nomenclature

	7.10411011011101			
a,b,h	parameters for expressing the cutting edge position			
A	center distance			
m <sub>n</sub> ,	normal module			
$\overline{n}_1$ ,	a unit normal vector to $\overline{r}_1$ ,			
$\overline{n}_{\rm f}$	a unit normal vector to cutting face $\bar{r}_f$			
ra	outside radius of a cutter			
r <sub>b</sub> ,	base circle radius of an involute			
$\overline{\mathbf{r}}_{\mathbf{l}}$ ,	vector representing an involute helicoid			
rc	vector representing an involute side flank			
u,v	angular parameters for expressing an involute			
$\alpha_{\rm n}$	normal pressure angle			
β	helix angle			
γ	rake angle			
η	parameter for expressing an involute			
φ	rotation angle of an unmodified gear			
$\Delta,\Delta_n$	difference between two side flanks			

#### References

- 1. MERRITT, H.E., Gear Engineering, Pitman, 1971.
- 2. SHIMOKOHBE, A., et al., "Line of Contact and Relative Curvature of Hourglass Worm Gears", Bull.T.I.T., No. 123, 1974,

This article was previously presented at a 1984 ASME conference. Paper No. 84-DET-206.

#### GEAR MANUFACTURING METHODS . . .

(continued from page 45)

External helical gears can also be produced by pot broaching. (See Fig. 17) The 4-in. O.D., 3/4-in. wide cast iron helical gear has eighty-seven, 24-DP 22°-HA teeth.

The gears are produced on a special lead-bar-equipped vertical press by a solid HSS pot broaching tool in 15-sec. floorto-floor time. Total tool life is 1,250,000 pieces.

#### Forming Teeth in Solid Blanks

Forming of fine-pitch gear teeth from the solid with gear rolling dies before roll-finishing is a process method that

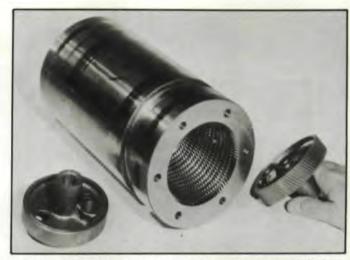


Fig. 17 - A solid HHS pot broaching tool that produces external helical cast iron running gears.

shows considerable promise. It is currently in the development stage.

High energy rate forging machines use high-pressure gas to drive a forming punch or die at speeds of up to 1,100-in. per second, Gears produced by this process are said to be 10 to 50-times stronger than those made by conventional forging and tooth-cutting methods.

To produce a blank with integrally-formed teeth, a raw billet is put in a blocker die to convert it into a preform. Then the preform is put into a finish die and is HERF-forged into a gear in a single blow. The gear is then trimmed to remove flash. Dies are 63Rc high-nickel, high-chrome, hardened steel.

Tooth grinding or rotary-shaving operations are performed after the forged blanks are machined. A typical HERF process makes thirty SAE 9310 gas turbine engine spur gears per hour. The gears have 10-DP, 25°-PA teeth on a 41/2-in. pitch diameter. HERF forging tolerances for the gears are plus or minus 0.005-in. with stock left for finish-shaving.



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The system, with workpiece truing station, storage pallets, and cutter pallet, shuttles cutters and workpieces to and

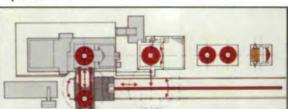
from the machine thereby effecting a complete automatic changeover. Off-machine preparation coupled with high positioning accuracy ensures effortless, precise, safe changeovers. The Liebherr Pallet Shuttle System and the LC 1002 Hobbing Machine—the solution for true automation and flexibility in large hobbing machines.

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Pallet Shuttle System Schematic

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