

# Generation of Helical Gears with New Surfaces Topology by Application of CNC Machines

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## Abstract

Analysis of helical involute gears by tooth contact analysis shows that such gears are very sensitive to angular misalignment leading to edge contact and the potential for high vibration. A new topology of tooth surfaces of helical gears that enables a favorable bearing contact and a reduced level of vibration is

described. Methods for grinding helical gears with the new topology are proposed. A TCA program simulating the meshing and contact of helical gears with the new topology has been developed. Numerical examples that illustrate the proposed ideas are discussed.

## Introduction

Computations by tooth contact analysis (TCA) have shown that involute helical gears are sensitive to errors such as the crossing of gear axes (instead of being parallel) and lead errors. These errors shift the bearing contact to the edge and cause transmission errors of an undesirable shape (Figs. 1 and 2). The transfer of meshing of gears with such transmission errors is accomplished with a jerk, producing a high level of vibration and noise.

A new topology of tooth surfaces has been proposed (Refs. 1-3) that provides for a more favorable bearing contact and transmission error motion, even with misalignment present. The generation of the proposed gear tooth surfaces was based on the application of existing equipment for generation of helical gears that provided linear relations between the rotations and displacements of the tool and the gear being generated. The modified gear tooth surfaces proposed in Refs. 1 - 3 could be generated as Formate<sup>®</sup>-cut by a tool of large dimension or generated point by point if computer-controlled. These methods of generation have some difficulties for manufacturing, but they may be overcome by the new approach.

This new approach is based on the

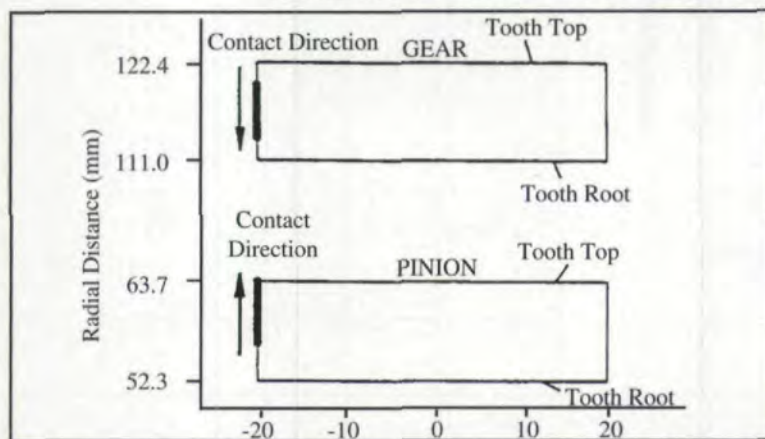


Fig. 1. Edge contact due to axis misalignment (crossing angle,  $\Delta\gamma = 5$  arc-min).

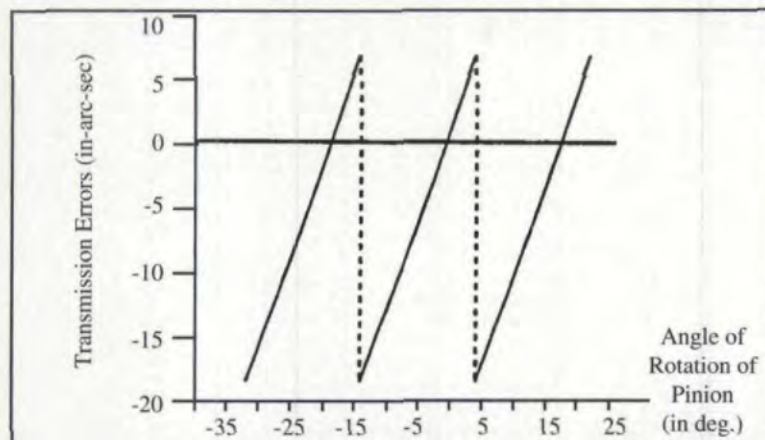


Fig. 2. Transmission errors of involute helical gears with axis misalignment (crossing angle,  $\Delta\gamma = 5$  arc-min).

application of CNC machines with five degrees of freedom that provide: (1) computer-controlled nonlinear functions that relate the motions of the tool and the gear being generated, (2) a varied plunge of the tool along the shortest center distance between the axes of the tool and the pinion and (3) a point contact of tooth surfaces that is spread over an elliptical area of controlled dimensions. This approach avoids edge contact and reduces the sensitivity of the gears to misalignment. The generation of gear tooth surfaces may be accomplished by form grinding.

The new form grinding method for helical gears provides: (1) a stabilized bearing contact, (2) better conditions of lubrication and (3) a predesigned parabolic function of transmission errors that is able to absorb an almost linear function of transmission errors caused by gear misalignment. It is expected that the new topology will eliminate edge contact and substantially reduce noise and vibrations.

The proposed form grinding requires the application of a computer numerical controlled (CNC) machine with five degrees of freedom, but only four require control by computer. Each tooth space is generated separately and indexing is required.

#### **Bearing Contact and Transmission Errors of Misaligned Involute Helical Gears**

The authors have developed a TCA program for conventional involute helical gears that permits the investigation of the impact of misalignment. Figs. 1 and 2 show that when the crossing angle  $\Delta\gamma = -5$  arc-min, the contact is shifted to the edge, and the transmission errors have the shape shown in Fig. 2. Similar results are caused by the lead error  $\Delta\beta_1 = -5$  arc-min.

The edge contact reduces the load capacity of the gears. The transmission errors of the type shown in Fig. 2 will inevitably cause premature failure, along with increased vibration and noise.

#### **New Method for Grinding, Modified Topology**

**Pinion Form-Grinding.** The form-grinding process for the pinion with the new topology is based on the following ideas:

1. Consider initially that both tooth sides of the pinion are conventional screw involute

surfaces. Using the approach developed in Ref. 4, it is possible to determine the surfaces of a disk-shaped grinding tool that will generate the conventional screw involute surfaces. The tool performs the screw motion with respect to the pinion being generated.

2. The grinding wheel surface is modified in the axial section. The deviation of the modified tool surface from the conventional one is represented at the mean contact point by a parabolic function, which can be controlled to adapt to different applications. Both pinion tooth sides can be ground simultaneously. The surface of the pinion grinding wheel is a surface of revolution.

3. The modified grinding wheel must perform two motions with respect to the pinion: The conventional screw motion and an additional but varied translational motion along the shortest distance between the axes of the grinding wheel and the pinion. This translational motion, being deeper at the edges and less in the middle of the tooth width, prevents plunging of the grinding wheel into the space.

4. Using the methods developed in Ref. 4, it is possible to determine analytically the equations of the pinion generated as described above. These equations are necessary for the TCA that has to be applied for simulation of meshing and contact of helical gears with modified topology.

**Gear Grinding.** Consider that a conventional involute helical gear is in mesh with the pinion whose tooth surface is modified as described above. Such a gear train, if not misaligned, will transform rotation with negligible transmission errors. The bearing contact of gear tooth surfaces is localized, since the gear tooth surfaces are in point contact at every instant because of the pinion tooth surface described above.

The goal is to keep the surface point contact, but to provide a predesigned parabolic function of transmission errors. Such a function is able to absorb a linear discontinuous function of transmission errors caused by angular errors of misalignment. The goal above can be achieved by proper modification of the gear tooth surface based on the following considerations:

1. Consider that an imaginary rack-cutter is

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simultaneously in mesh with the pinion and gear provided with conventional screw involute tooth surfaces. The pinion and the gear perform rotational motions, and the rack performs translational motions  $s_t$  described as follows:

$$s_t = r_1\phi_1 = r_2\phi_2 \quad (1)$$

$$\phi_2 = \phi_1 \frac{N_1}{N_2} \quad (2)$$

where  $r_1$  and  $r_2$  are the pinion-gear centres, and  $N_1$  and  $N_2$  are the tooth numbers.

Obviously, the transmission function  $\phi_2(\phi_1)$  is a linear one, and the gears will be sensitive to angular errors of misalignment.

2. We may consider now that while the rack performs translational motion  $s_t$ , the pinion rotates through the angle  $\phi_1 = \frac{s_t}{r_1}$ , but the gear rotates through the angle

$$\phi_2 = \frac{s_t}{r_2} + \Delta\phi_2(\phi_1) \quad (3)$$

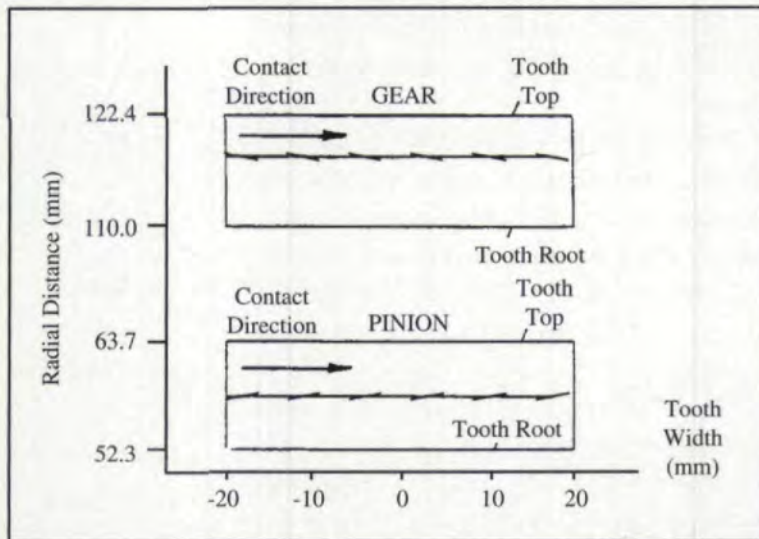


Fig. 3. Latitudinal contact path with shaft misalignment ( $\Delta\gamma = -5'$ ).

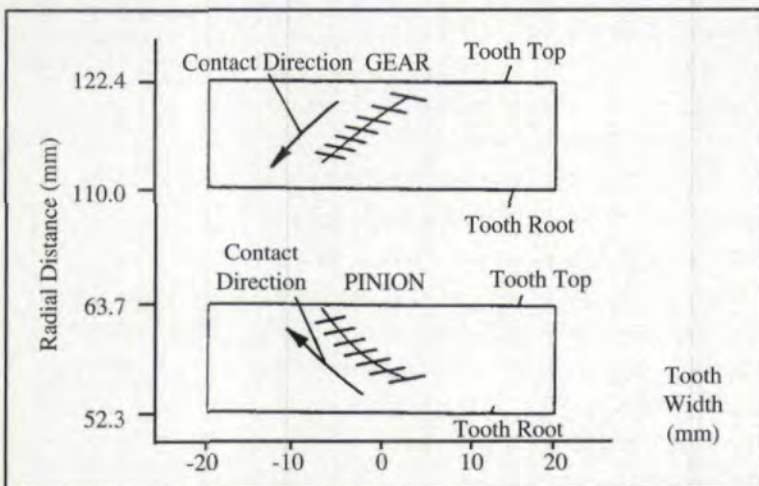


Fig. 4. Longitudinal contact path with shaft misalignment ( $\Delta\gamma = 2'$ ).

where

$$\Delta\phi_2(\phi_1) = a\phi_1^2, \quad -\frac{\pi}{N_1} \leq \phi_1 \leq \frac{\pi}{N_1} \quad (4)$$

is a parabolic function of the period of cycle of meshing determined as  $\phi_1 = \frac{2\pi}{N_1}$ .

Obviously, the transmission function of the pinion and gear generated as described above is determined as

$$\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + a\phi_1^2 \quad (5)$$

where  $a\phi_1^2$  is the predesigned parabolic function of transmission errors.

3. The nonlinear transmission function (Ref. 5) exists even in the case when the gear train is aligned. The advantage of such a function is the ability to absorb a linear but discontinuous function  $b\phi_1$  ( $0 \leq \phi_1 \leq \frac{2\pi}{N_1}$ ) that is caused by gear misalignment. This is based on the fact (Refs. 1 and 5) that the sum of functions represented as

$$\Delta\phi_2(\phi_1) = a\phi_1^2 + b\phi_1 \quad (6)$$

can be transformed into the parabolic function

$$\Delta\phi_2(\phi_1^*) = a(\phi_1^*)^2 \quad (7)$$

Parabolic functions  $(a\phi_1^2)$  and  $(a(\phi_1^*)^2)$  have the same slope. Transformation of function (6) into function (7) is equivalent to coordinate transformation when the coordinate system  $(\phi_2, \phi_1)$  is translated keeping the orientation of coordinates axes.

4. We have assumed above that the pinion tooth surface is a conventional involute screw surface  $\Sigma_1$ . In reality, the pinion tooth surface  $\Sigma_1^*$  is a modified one as mentioned above. However, a synthesized function of transmission errors of the parabolic type exists in the case of modification of the pinion tooth surface as well. This is based on the fact that surfaces  $\Sigma_1$  and  $\Sigma_1^*$  are in tangency at the mean point and only slightly deviate along the helix on  $\Sigma_1$  that passes through the mean point.

5. These methods of modification of tooth surfaces  $\Sigma_1$  and  $\Sigma_2$  enable one to localize the bearing contact of  $\Sigma_1$  and  $\Sigma_2$  and provide a predesigned parabolic type of transmission error to absorb the undesired linear function

caused by gear misalignment.

6. There are other methods for grinding the modified gear tooth surface besides the form-grinding method proposed here. The generation can also be achieved by either a grinding plane or by a grinding worm. However, a nonlinear function that relates the motions of the grinding wheel and the gear being generated is required for both alternative cases.

#### TCA for Helical Gears with New Topology

A TCA computer program to simulate the meshing and contact of the gears with the new topology has been developed.

The computations have been performed for a drive with the following design parameters:

$N_1 = 20$ ,  $N_2 = 40$ ,  $P_n = 0.19685 \frac{1}{\text{mm}}$ ,  $\alpha_n = 20^\circ$ ,  $\beta_p = 30^\circ$ , and tooth face width  $F_w = 40.64$  mm.

Two types of path of contact can be provided as shown in Figs. 3 and 4. These can be obtained controlling the modification of the topology of pinion-gear tooth surfaces in the longitudinal and profile directions.

The influence of the crossing angle  $\Delta\gamma$  is shown for the above data in Fig. 5.

The results of the investigation show that the almost linear function of transmission errors caused by misalignment of conventional involute helical surfaces (shown in Fig. 5) is indeed absorbed by the parabolic type of transmission errors for the modified surfaces (see Fig. 6).

The major axis of the contact ellipse, under an assumed light load, has been determined as shown in Figs. 3 and 4. The undesirable displacement of the path of contact to the bottom and the top of the gear tooth can be controlled by the modification of the surface of the grinding wheel for the pinion generation.

#### Conclusion

The conclusions of this study are as follows:

1. A TCA program for simulation of meshing and contact of conventional involute helical gears has been developed. This program has shown that such gears are very sensitive to angular misalignment, and high vibration is inevitable.

2. A new topology of helical gear tooth surfaces has been developed. Methods for grinding tooth surfaces have been developed. The bearing contact of gears with the proposed topology is localized and the transmission

errors are reduced.

3. The TCA program for helical gears with the new topology has been developed. The influence of crossing angle on the location of the path of contact and on the transmission errors has been investigated. ■

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**Acknowledgement:** This article was originally presented at the AIAA/SAE/ASME/ASEE 29th Joint Propulsion Conference and Exhibit, June 28-30, 1993, Monterey, CA. Copyright © AIAA 1993. Reprinted with permission.

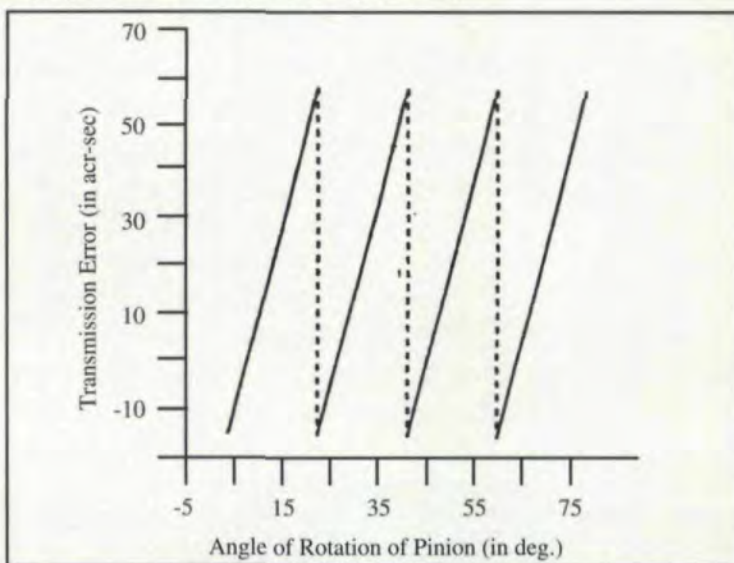


Fig. 5. Influence of misalignment on transmission errors ( $\Delta\gamma = -5'$ ).

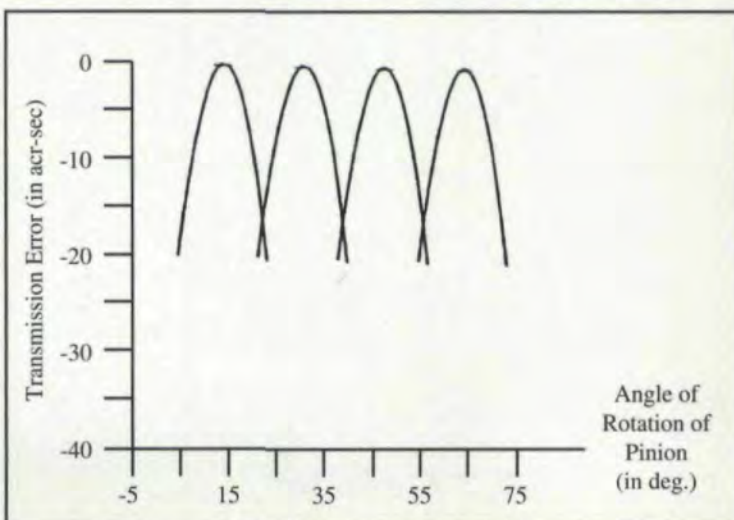


Fig. 6. Influence of misalignment on transmission errors ( $\Delta\gamma = -5'$ ).