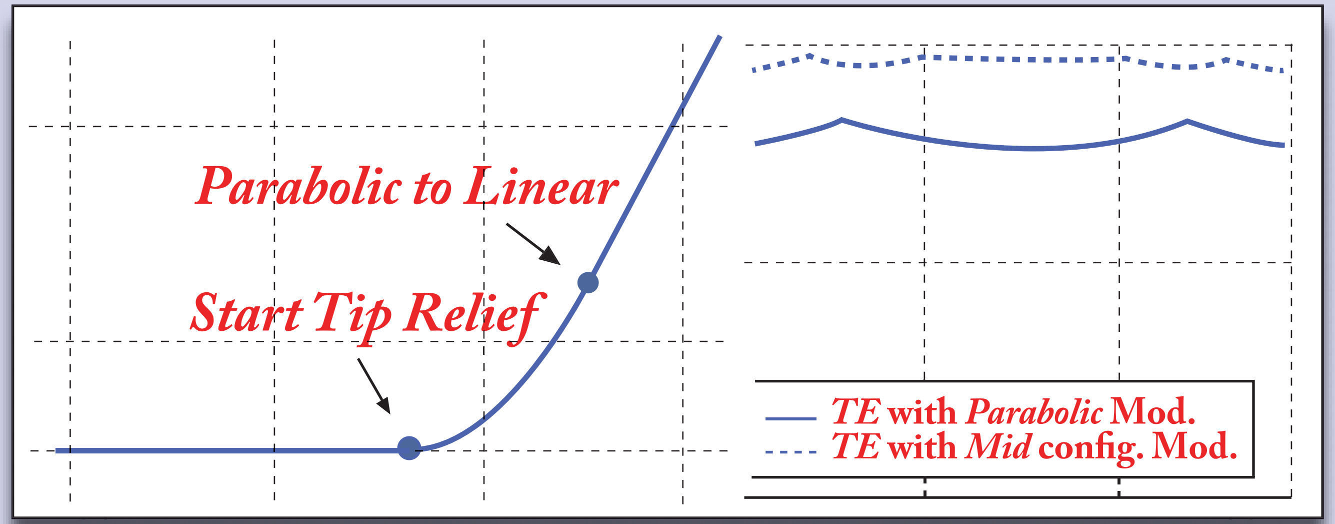
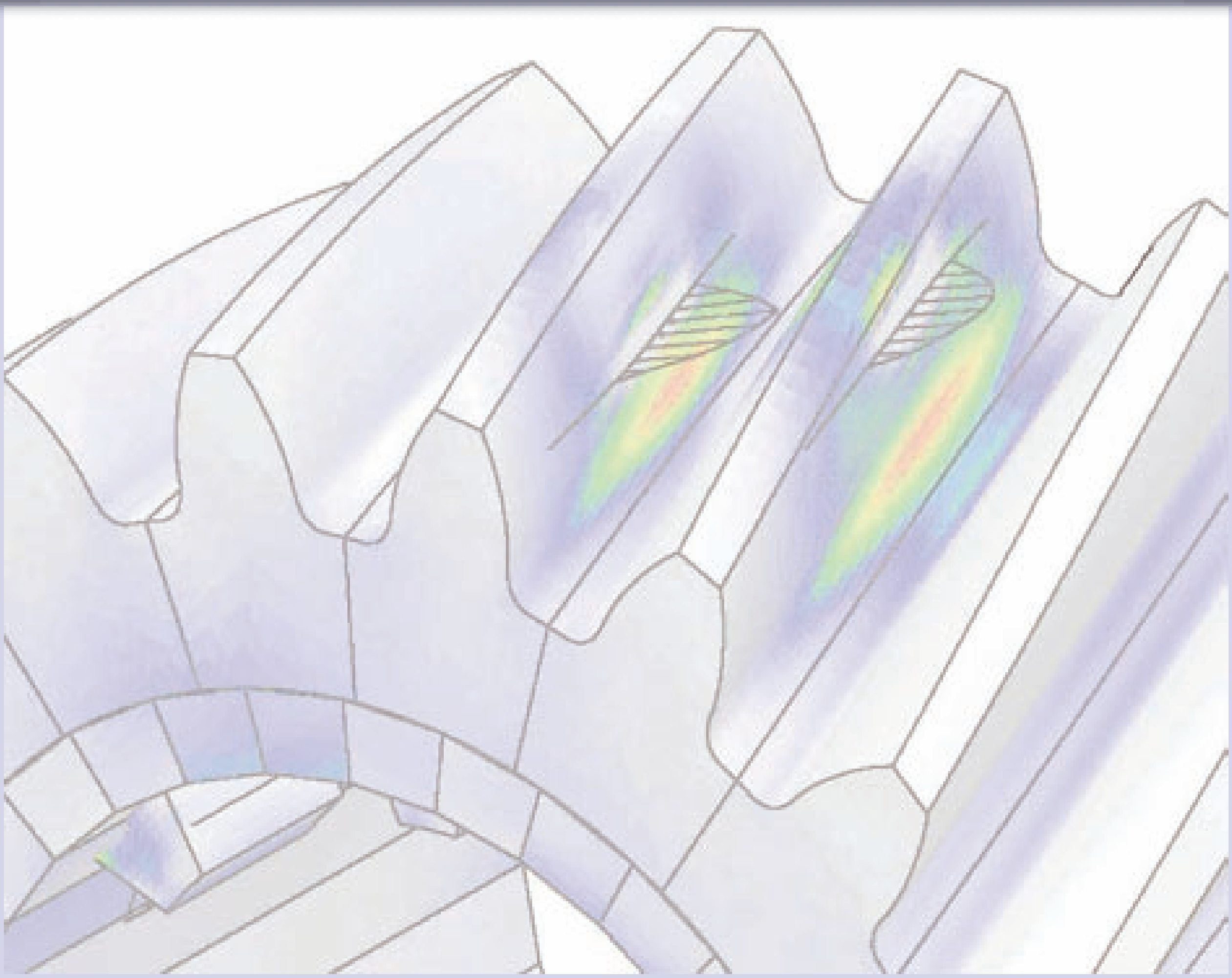
Proposal for Tip Relief Modification to Reduce Noise and Sensitivity to Meshing Conditions in Spur Gears

Ciro Santus, Fabio Presicce, Marco Beghini





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Management Summary

In this article, a new tip relief profile modification for spur gears is presented. The topography proposed here is a classical linear profile modification with a parabolic fillet (linear-parabolic modification). The parabolic fillet extension is treated as a parameter and its effect investigated.

The proposed topography combines advantages from both linear and parabolic tip relief. The evolution from linear to parabolic is discussed in terms of peak-to-peak transmission error. Anomalous contact conditions due to particular tip relief modifications are reported, and maps are obtained in order to find optimum tip relief by taking into account contact condition boundaries.

Introduction

In a gear set, transmission error (TE) is defined as the difference between the effective and the ideal position of the output shaft with reference to the input shaft. The ideal position represents a condition of perfect meshing, without geometrical errors or distortions. TE can be expressed either by an angular displacement or, more conveniently, as a linear displacement measured along the line of action tangent to the base circle. TE is considered to be the primary cause of whining noise (Ref. 1). Indeed, whining noise is produced by changes of tooth load:

- Amplitude,
- Position along the profile,
- Direction.

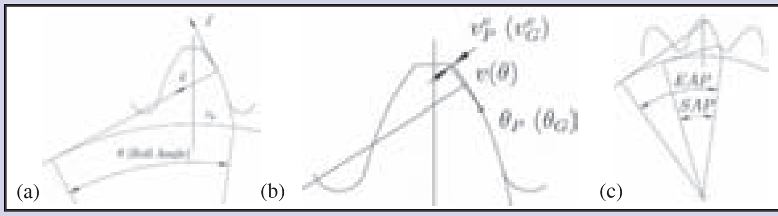

Figure 1(a)—Definition of the roll angle θ. Figure 1(b)—Definitions of: profile modification $v = v(\theta)$ as function of the roll angle, the total thickness at the end of the flank either for pinion v^e_P or gear v^e_G and the start roll angle for either pinion θ_P or gear θ_G. (c) Definition of start of active profile (SAP), end of active profile (EAP) angles.

These changes are consequences of tooth deflection, local contact deformation and body deformation, which are the origin of TE.

Several authors (Refs. 2–6) studied the correlation between TE and tip relief profile modification since it is a strong tool to modify TE by looking at fixed gear parameters. Niemann proposed long and short modifications (Ref. 4). The difference in denomination is based on the starting point of tip relief along the profile. According to experimental results, gears with long modifications show reduced TE excursion, indicated as peak-to-peak transmission error (PPTE), and therefore little noise level at the design torque.

At lower torque, this optimum condition was not verified, and an intermediate or short modification is suggested.

In tooth modification design, tip relief is defined as the thickness v of the material removed along the tooth flank with reference to the nominal involute profile. Profile modification is usually defined versus the roll angle coordinate (θ), shown in Figure 1 (a, b), and measured in the direction of the inner normal, shown in Figure 1 (c).

The type of function $v = v(\theta)$ is usually indicated as profile modification topography. Meshing sensitivity to topography is explored throughout this article.

A different shape for the tip relief profile modification is proposed with the aim of reducing noise since TE can be significantly reduced if an optimized profile modification is produced (Refs. 1, 5, 3). Litvin (Ref. 7) agrees with this approach even though tooth contact analysis is considered only, instead of loaded tooth contact analysis (LTCA). This article shows how the new profile tip relief modification proposed here can influence TE meshing response, according to LTCA hypothesis.

Linear and parabolic topographies are the only shapes that have been deeply studied (Refs. 8–11). Considering recent grinding developments, it is possible to consider more complex

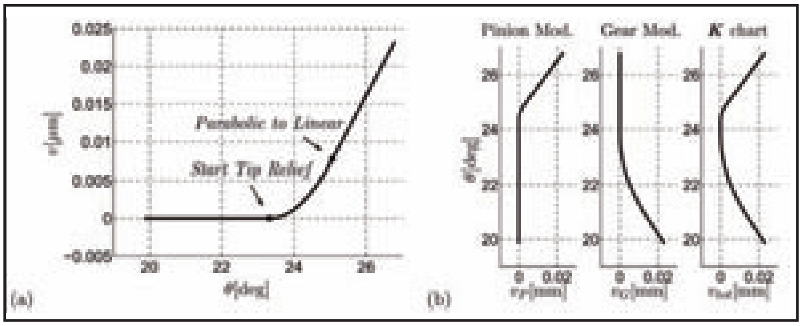

Figure 2(a)—Tip relief profile modification definition. Figure 2(b)—Example of pinion-gear tip relief modification and K-chart combination.

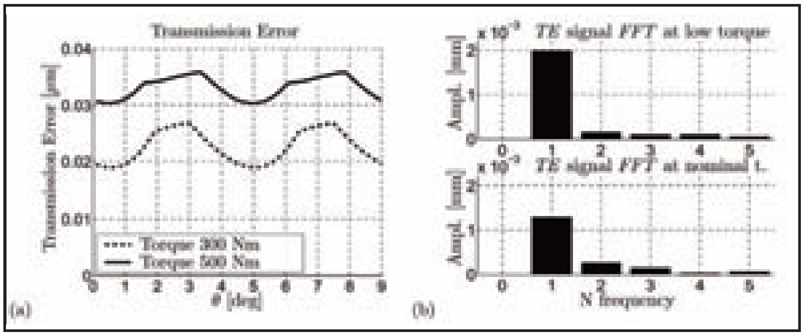

Figure 3(a)—TE at low torque and nominal torque applied. Figure 3(b)—Fast Fourier Transform (FFT) of the TE signal, both cases.

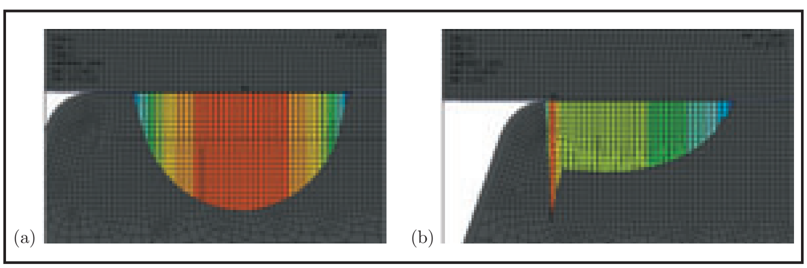

Figure 4(a)—No corner contact detected. Figure 4(b)—Corner contact detected.

shapes, in particular for spur gears in which profile modification is strategic (Ref. 12).

Linear tip relief modification produces minimal transmission error in spur gears if properly designed at a given load (Refs. 4, 11).

Parabolic profile modification satisfies tangent continuity condition even though sharp curvature changes is produced. Linear profile modification produces an edge instead. At the modification start point, discontinuity arises. Indeed, according to solid elastic contact mechanics, a sharp edge generates singular pressure when an angular point falls inside the contact region. Therefore this modification topography is considered to be more dangerous for contact pressure, which is the primary reason of micropitting activation (Refs. 13, 14).

The new topography proposed here is able to exploit both linear and parabolic topography properties since it is a linear relief with a parabolic fillet, as shown in Figure 2.

The topography shown here has never been found by the authors in the technical literature. Transmission error produced by the modification pattern of Figure 2 is shown in Figure 3, along with a fast fourier transform analysis of TE. In Figure 3(b), the mean value of the TE signal is eliminated, first frequency component is predominant, and it is remarkable that with higher load, PPTE and first frequency amplitude can be smaller than with lower load.

As discussed, PPTE can be effectively considered as the main parameter of TE roughness (Ref. 8).

The tool exploited to find the optimum tip relief configuration is a map plotting PPTE as a function of start relief roll angles θ_P and θ_G for fixed profile modification magnitudes at the top v^e_P and v^e_G.

Methodology

The methodology proposed by the authors (Ref. 8) is applied here.

Simulations of meshing gears have been carried out by means of a hybrid method, combining the finite element technique with a semi-analytical solution (Refs. 15, 16).

The main assumptions for the analysis are the following:

- **Plain strain conditions suggested by the spur gear geometry (high ratio b/h)**. Two-dimensional plane strain analysis is adequate for this kind of tooth. Moreover, the two-dimensional version of the software requires little time both for model generation and simulations, with very precise results.
- **Static analysis**. Static TE was determined while neglecting rotational speed and inertia forces. This is the main assumption; undertaking dynamic analysis is too time consuming.
- **Friction neglected**. It is assumed that it has little effect on TE output.
- **Space error and pitch error not considered**. No statistical consideration was included in the analysis.

The quantities v^e_P (v^e_G) and θ_P (θ_G) are defined in Figure 1(b). The ranges for both of these two variables are the start of active profile roll angle (SAP) and the end of active profile roll angle (EAP) for each gear, as shown in Figure 1(c).

Research Boundaries

Corner contact boundary. Corner contact is produced when the contact region includes zones of the fillet of the tooth tip (Ref. 17). As a consequence of tooth deflection, the effective contact ratio is greater than that found according to rigid geometry. Hypothesis: The contact pressure rises locally at the tip fillet, as shown in Figure 4.

This definition of corner contact can be exploited if an FEM analysis is performed. When the corner contact is detected, the calculated pressure peak was not considered reliable since the maximum is strongly affected by the radius of the fillet, which is a very unpredictable quantity.

High curvature boundary. There is also the possibility of getting an anomalous contact condition if the tip relief is too high along the tooth profile. In this condition, high curvature occurs and then contact pressure is expected to be much higher than the Hertz model according to the nominal involute curvature.

In Figure 5, three contact pressure histories are shown in which only linear-parabolic profile modification parameters are modified.

For each case, two load conditions (nominal and low load) are shown in comparison. It is worth noting that corner contact is related to torque versus tip relief, so configuration can show corner contact if the load is high enough. This is obviously detectable only if loaded tooth contact analysis (LTCA) is considered.

PPTE Maps

This paper's main result is to obtain PPTE maps in order to find the minimum inside an acceptance domain defined according to contact pressure. Maps hereafter presented plot PPTE as a function of tip relief start roll angles (θ_P, θ_G), which are strongly PPTE-dependent parameters, for a given quantity of material removed at the top (v^e_P, v^e_G). Initial values of v^e_P and v^e_G have to be related to the nominal torque of the gear set even though they are also strongly PPTE-dependent (Refs. 8–11) .

To define completely the linear-parabolic modification (θ_P, θ_G) and (v^e_P, v^e_G) is not enough; the transition from parabolic to linear point roll angles needs to be defined (θ_{pP}, θ_{pG}) (see Figure 2(a)).

It is clear that:

$$\begin{aligned} \theta_P < \theta_{pP} < EAP_P \\ \theta_G < \theta_{pG} < EAP_G \end{aligned} \quad (1)$$

To let the transition from linear to parabolic be described in natural fashion, configurations at fixed ratios are considered in singular maps.

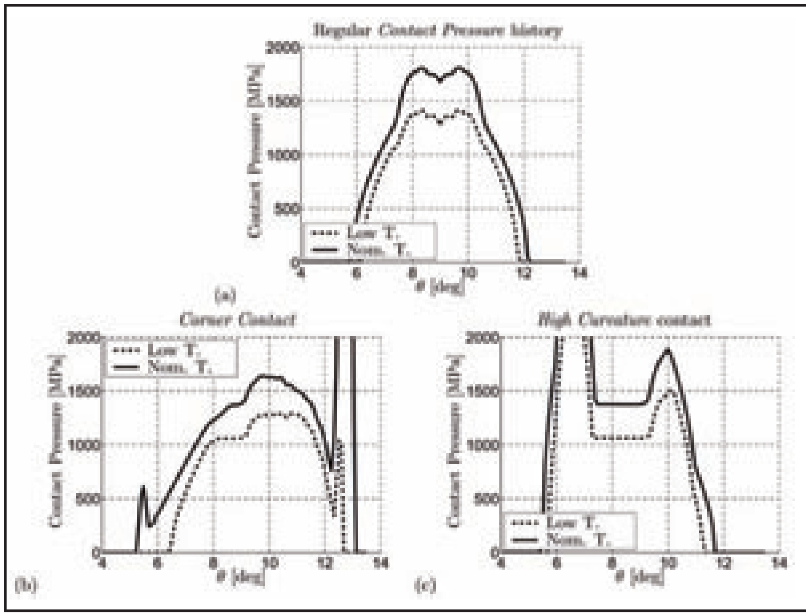

Figure 5(a)—Regular contact pressure history configuration. Figure 5(b)—Corner contact configuration. Figure 5(c)—High curvature configuration.

Table 1—LCR gear set design parameters.			
Modulus	1.75 mm	Pressure angle	22.5°
Pinion N. teeth	80	Gear N. teeth	80
Pinion external diameter	143.2 mm	Gear external diameter	143.2 mm
Pinion root diameter	135.3 mm	Gear root diameter	135.3 mm
Pinion face width	11.0 mm	Gear face width	11.0 mm
Pinion v^e_P	23.3	Gear v^e_G	23.3

$$\begin{aligned} \lambda_P = (\theta_{pP} - \theta_P)/(EAP_P - \theta_P) \\ \lambda_G = (\theta_{pG} - \theta_G)/(EAP_G - \theta_G) \end{aligned} \quad (2)$$

Limit configurations are $\lambda_{P(G)} = 0$ (linear modification) and $\lambda_{P(G)} = 1$ (parabolic modification). Maps with λ ranging from 0 up to 1 are reported hereafter.

Computational Performances

To perform parameter sensitivity analysis, a common PC platform was used with the following characteristics:

- CPU 2.6 GHz
- RAM 1 GB

Plane strain analysis was performed by ExtPair2D™ (Refs. 18, 19). Analyses were automatically performed in about 4.5 CPU hours, simulating 50 time steps for each meshing, for 225 different tip relief (θ_P, θ_G) configurations.

Results

The aforementioned methodology was applied to a low contact ratio (LCR) gear set. The main parameters of the set are reported in Table 1.

For the equal number of the set, several sym-

Table 2—Profile modification ratios and analysis of configurations.

$\lambda_{P(G)}$	0	1/4	2/4	3/4	1
0	(Lin)				
1/4		(1-1)	(1-2)	(1-3)	
2/4		(2-1)	(2-2)	(2-3)	
3/4		(3-1)	(3-2)	(3-3)	
1					(Par)

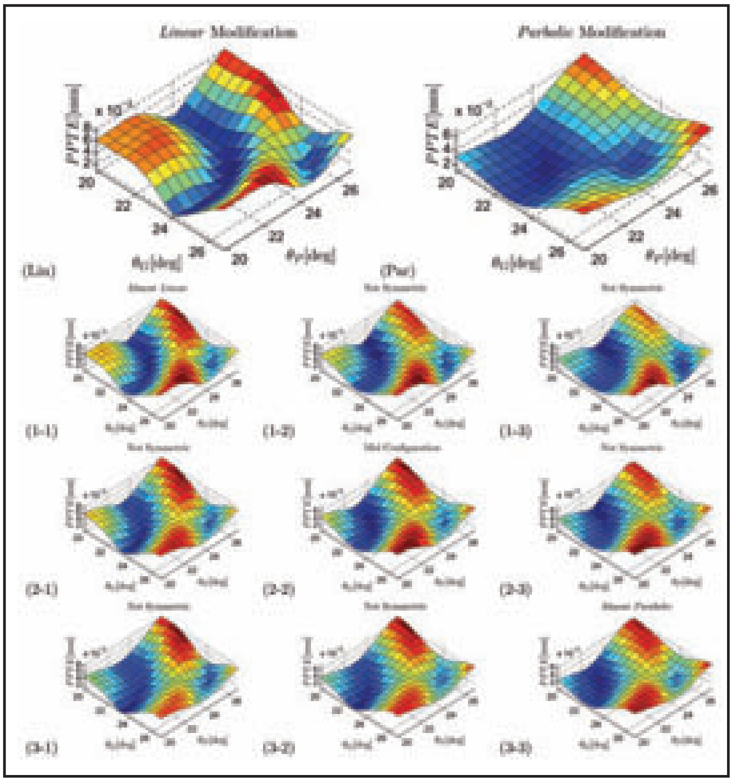

Figure 6—Nominal load (Lin) PPTE obtained with linear modification topography. (Par) PPTE obtained with parabolic modification topography. Intermediate configuration meshing conditions shown in (1-2), (1-3), (2-1), (2-3), (3-1) and (3-2).

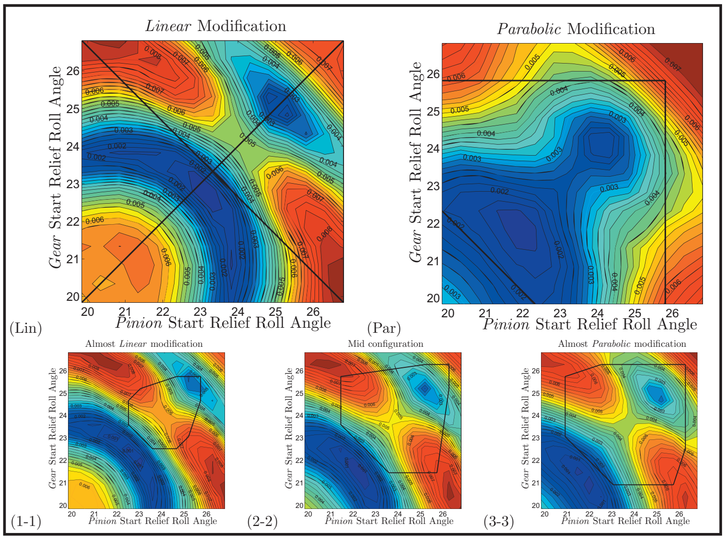

Figure 7—Nominal load (Lin) linear modification. No boundary can consistently be applied. (Par) parabolic modification, boundaries are wider. Intermediate configurations shown in (1-1), (2-2) and (3-3).

metry properties are expected. Configurations analyzed are reported in Table 2.

Two load conditions applied to the gear set were considered in this article:

- 300 Nm—considered a low load for the gear set,
- 500 Nm—nominal load for the gear set.

Nominal load

For nominal torque, the effect on PPTE of the migration from linear to parabolic topography is depicted in Figure 6.

Furthermore, boundaries were applied. Results are reported only for symmetric configurations in Figure 7.

When searching the minimum inside boundary, it is clear that parabolic modification is the one that offers a wider acceptance domain. Therefore, larger areas in which PPTE produces an absolute minimum can be considered.

Minimum values for configurations shown in Figure 7 are reported in Table 3, not considering boundaries.

Among minimum values reported in Table 3, only the one referring to parabolic modification is acceptable. Though others are lower, they fall outside the boundary. Taking boundary into account, parabolic modification leads to the best result.

In Figure 8, a transmission error trace is reported both for parabolic optimum (continuous line) and for mid-configuration minimum PPTE configuration. PPTE is lower for mid-configuration, but edge contact is evident. Indeed, this configuration falls outside the acceptance boundary as shown in Figure 7.

Low Load

For low torque, the migration from linear to parabolic topography is depicted in Figure 9. For this load level, no boundary related to contact pressure was applied since load is not expected to be dangerous at this level. Even though there are also necessary conditions of anomalous contact (edge contact or high curvature) at lower load, it is possible to assume that they are confined to useless configurations. Minimum values found at the different configurations are reported in Table 4.

It is worth noting that neither linear nor parabolic topography generates minimum PPTE at both loads.

Table 3—Minimum PPTE values at the nominal load.		
(LIN) 1.14 μm		
(1-1) 0.63 μm	(2-2) 0.76 μm	(3-3) 0.57 μm
(Par) 1.39 μm		

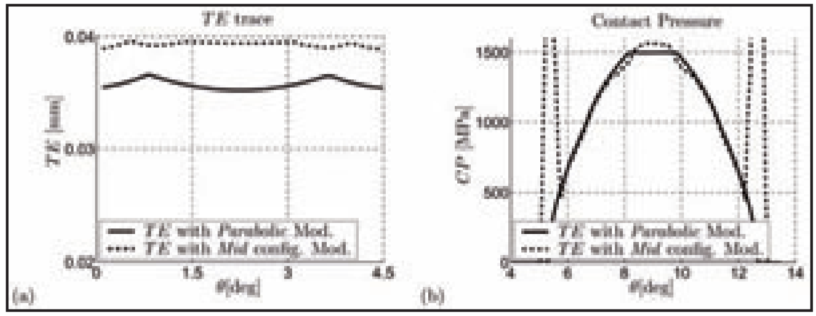

Figure 8(a)—Parabolic optimum TE trace. Figure 8(b)—Mid-configuration linear with parabolic fillet optimum TE trace.

In the configuration proposed here, minimum PPTE output is generated by a linear with parabolic fillet configuration. However, the optimum is dependent on the load. Therefore, the aim of this article is not to propose this result as a "golden" rule, but to show that the parabolic fillet can be an effective parameter for obtaining an optimum solution.

Sensitivity to Center Distance

In order to avoid a huge amount of graphical output, sensitivity to center distance offset is considered for linear modification at nominal load. Results are shown in Figure 10.

The entities of offset are around ±0.2 mm. In comparison with modulus, it is clear that perturbations induced by the meshing condition have an influence on the PPTE map.

Table 4—Minimum PPTE values at the low load.		
(Lin) 0.87 μm		
(1-1) 0.39 μm	(2-2) 0.53 μm	(3-3) 0.52 μm
(Par) 0.95 μm		

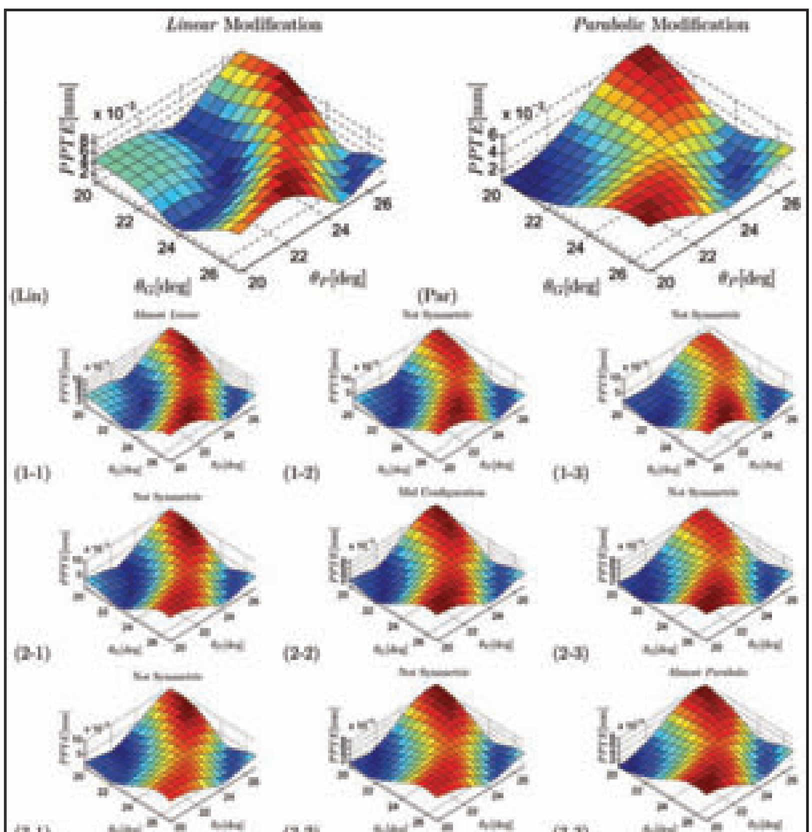

Figure 9—Low load: (Lin) PPTE obtained with linear modification topography. (Par) PPTE obtained with parabolic modification topography. Intermediate configuration meshing conditions shown in (1-2), (1-3), (2-1), (2-3), (3-1) and (3-2).

Conclusions

In this article, a new tip relief profile modification is proposed along with loaded tooth contact analysis methodology. Static peak-to-peak transmission error is considered as the main meshing output since it can be related to noise level. Extensive parametric numerical analysis results are presented. PPTE maps are reported as a function of pinion and gear start of tip relief roll angles. Migration from linear to parabolic modification is effectively described.

Contact pressure anomalies are presented and applied as limits of acceptance on PPTE maps.

Modifications that are close to parabolic can show a wider acceptance domain, since edge contact and strong curvature issues are less dangerous.

The optimum configuration is found here for a gear set at a given load. The new topography introduces a degree of freedom that can be useful in designing optimum profile modifications for any gear set.

Simulations are performed with the aim of gear contact dedicated tools produced by Advanced Numerical Solutions (ANSol)

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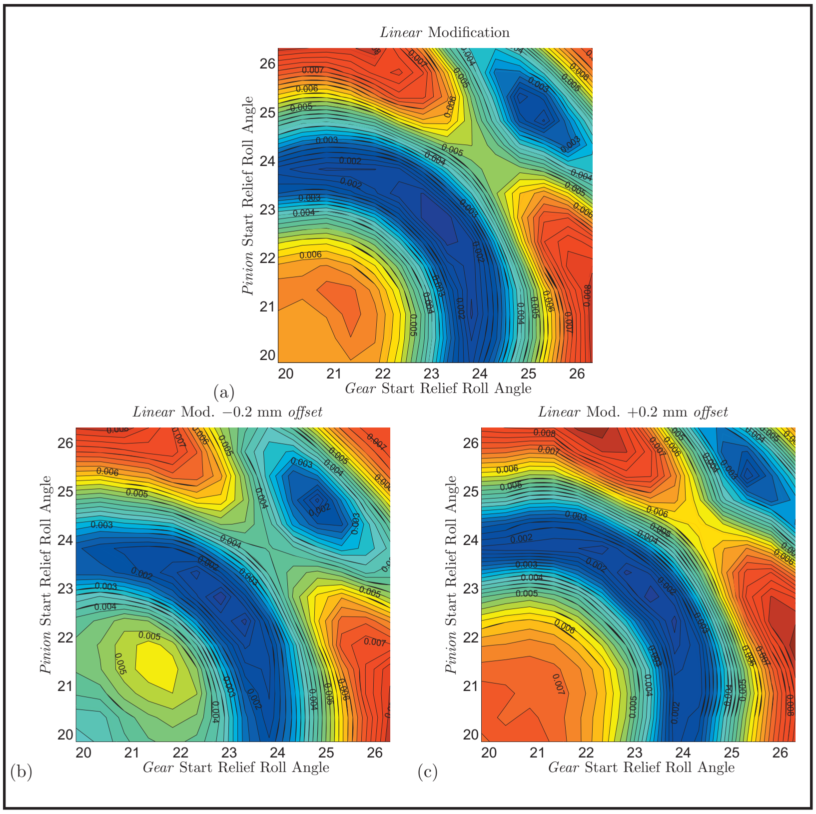

Figure 10(a)—Linear modification. Figure 10(b)—Linear modification with offset –0.2 mm. Figure 10(c)—Linear modification with offset +0.2 mm.

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