Influence of Geometrical Parameters on the Gear Scuffing Criterion—Part I

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Abstract

To avoid scuffing* of high speed and highly loaded gears, a criterion different from the bending strength criterion (Galilei - Lewis) and the contact pressure criterion (Hertz) is needed. The Draft International Standard, DIS 6336, Part 4, now in print, still presents two scuffing criteria. The existence of two proposals impeded the progress of writing that standard for many years. As with Columbus' egg**, a simple solution to a difficult problem was found: the only factors which differed in both formulae were purely geometric, and the comparison was reduced to a simple mathematical comparison. The maximum contact temperature in the flash temperature criterion according to Blok was approximated by the integral temperature of the other criterion. All test results expressed in integral temperature are fully applicable to the flash temperature criterion, and in an unintentional way, these results confirmed the validity of the gear-scuffing criterion according to Blok, which is still the most practical.

In Part 2 it will be shown that all geometric influences may be concentrated in one factor dependent on only four mutually independent parameters. This simple fact will be used to examine the influence of different shapes and values of the load sharing factor.

Introduction

The load capacity rating of gears had its beginning in the 18th century at Leiden University when Prof. Pieter van Musschenbroek systematically tested the wooden teeth of windmill gears, applying the bending strength formula published by Galilei one century earlier. In the next centuries several scientists improved or extended the formula, and recently a Draft International Standard could be presented.[1]

In the 19th century, metal gears suffered surface pitting which could not be predicted by the bending strength formula. Of necessity, material constants of the bending strength formula were considered empirically dependent on geometrical parameters. Attempts to find a relationship between "wear" and specific sliding were not successful. Nearly fifty years after its first publication, the theory of Hertz was used for gear rating. Gradually, experience and test results could be transformed into several influence factors completing the contact stress formula. Even in the final stage of preparing the Draft International Standard,[1] the discussions about these influence factors did not stop.

The gear technology of the twentieth century allowed increasing loads and velocities at decreasing dimensions with new materials. Again, phenomena occurred which could not be predicted by existing formulae. The scuffing of gears was studied intensively and several criteria were proposed. Two categories of criteria may be distinguished:

1. The flash temperature criterion according to Blok, based on a realistic thermodynamic theory and confirmed by tests.
2. Empirical expressions yielding one representative value, mainly based on tests and, to a lesser extent, on theoretical considerations.

The flash temperature criterion was published by Blok in 1937.[2][3][4] At the same time several tests were run. The lubricant was recognized as the third gear material and important progress was made in the development of additives to the mineral oils. An immediate application of the new theory would have been possible. However, a long delay was caused by World War II. When the Netherlands had to surrender to occupying forces in May 1940, Blok had to destroy all test results on the instruction of his employer, so after the war these could not be published in full.

Most expressions of the second category had a limited field of application and were not widely accepted. The best empirical criterion was Almen's factor, PVT. Afterwards, a relationship between PVT and the flash temperature could be shown:[5] the square root of PVT approximates the maximum value of the flash temperature, keeping in mind that the jump from "force/time" to "temperature" is accounted for by thermodynamic constants. For a rough dimensional

**During a long, dull discussion, Columbus challenged his opponents to balance an egg on one of its ends. No one could do this until Columbus planted the egg firmly on the table, causing it to stand on its crushed shell, thereby proving that even the most perplexing problem may have a simple solution.

*Scuffing and scoring are synonyms for the same phenomenon. Since scoring may also have another meaning, the ISO Technical Committee 60 decided to apply the word scuffing in the ISO standards.

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Nomenclature

(Symbols, terms and units chosen in accordance with the international standard)

a	center distance (mm)
A	point of path of contact at tip of wheel
b	facewidth (mm)
B	lower point of transverse single contact
C	pitch point
C_2	weight factor (value 1,5)
D	upper point of transverse single contact
E	point of path of contact at tip of pinion
F_t	tangential force at reference circle (N)
GAM	parameter on the line of action
GAMA	parameter on the line of action at point A
GAMAB	parameter on the line of action between A and B
GAMB	parameter on the line of action at point B
GAMD	parameter on the line of action at point D
GAME	parameter on the line of action at point E
GAMED	parameter on the line of action between E and D
S_B	safety factor, Equation (18)
TAA1	tangents of transverse tip pressure angle of pinion
TAA2	tangents of transverse tip pressure angle of wheel
TAT	tangents of transverse working pressure angle
trapez	number corresponding with figure number of
v	pitch line velocity (m/s)
W_{Bt}	specific tooth load[1]
x_1	addendum modification coefficient of pinion
x_2	addendum modification coefficient of wheel
X_B	geometry factor, Equation (14)
X_{BE}	geometry factor at point E
X_{Ca}	tip relief factor [1]
XGAM	load sharing factor
X_M	thermal contact coefficient, Equation (12) $(K.N^{-3/4}.s^{1/2}.m^{-1/2}.mm)$
X_Q	approach factor [1]
X_{top}	form factor, Equation (11), (9) $(K.N^{-3/4}.s^{1/2}.m^{-1/2}.mm)$
X_ϵ	contact ratio factor, [1]
X_Γ	load sharing factor, Figs. 1 to 4
z_1	number of teeth of pinion
α_t	transverse working pressure angle
α_y	pressure angle of arbitrary point
β	helix angle
Γ	linear parameter on line of action, Equation (14)
Θ_B	contact temperature, Equation (1) (°C)
Θ_{Bmax}	maximum contact temperature, Equation (9) (°C)
Θ_{fl}	flash temperature, Equation (2) (°C)
Θ_{flaint}	approximated mean value of the flash temperature, Equation (4) (°C)
Θ_{flmax}	maximum flash temperature, Equation (10) (°C)
Θ_{int}	integral temperature, Equation (3) (°C)
Θ_M	bulk temperature, Equation (16), (17) (°C)
Θ_{oil}	oil temperature (°C)
Θ_S	scoring temperature[1] (°C)
μ_{mC}	mean coefficient of friction at pitch point[1]
μ_{my}	mean local coefficient of friction[1]
π	product of factors in comparison, Equation (7)

analysis of some criteria, see Table 1, which demonstrates a certain progress towards the flash temperature criterion.

Table 1. Dimensional Comparison of Criteria

Hofer 1926: power/pitch surface	p^2v
Almen 1935: PV	p.v
Almen 1943: PVT	p.v.T.
Blok 1937: flash temperature	$(p^{1.5}v.T)^{0.5}$

p = contact stress, v = velocity, T = length

Some empirical expressions were based on an accumulation of energy along the path of contact. These expressions were rejected, one after the other, but the idea of accumulation[6] turned up finally in the concept "integral temperature" presented in 1972 to the ISO Technical Committee 60 as an alternative to the flash temperature. The existence of two proposals impeded the progress of evolving an international standard. The possibility of rejecting either of them did not appeal to the committee, nor could a well-balanced combination be found for many years. Previous comparisons between the two methods were often based on the application of several influence factors in one method and neglecting those factors in the other, to the detriment of the latter.[7]

However, the discussions began again when a clear comparison was found,[8] and in 1984 the deadlock could be broken, just in time to present the preliminary results in a Draft International Standard, together with the drafts on pitting and tooth breakage.[1]

Comparison of the Two Methods

The Draft International Standard[1] still presents the two methods with a short comparison in the appendix. The first method is the flash temperature criterion according to Blok, supplemented with a few influence factors. The second method is the integral temperature criterion, including the same or comparable influence factors. The comparison takes account of those factors which are different in both formulae.

The flash temperature criterion concerns the contact temperature Θ_B which is a temperature function along the

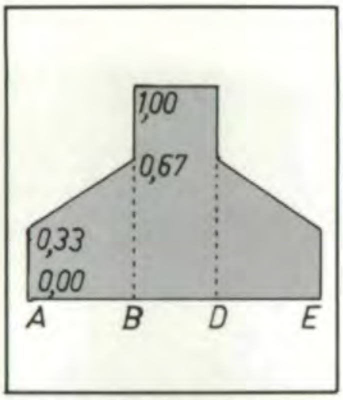

Fig. 1 – Traditional load sharing factor for a gear pair with unmodified tooth profiles.

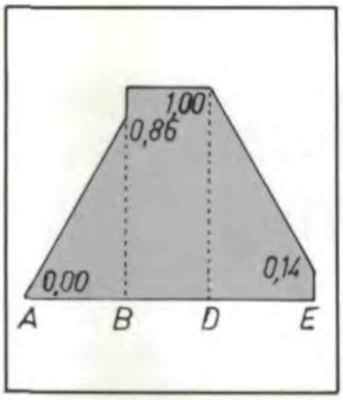

Fig. 2 – Traditional load sharing factor for a gear pair with modified tooth profile, designed for high load capacity if the pinion is driver.

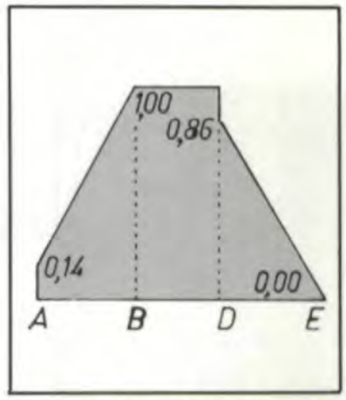

Fig. 3 – Traditional load sharing factor for a gear pair with modified tooth profile, designed for high load capacity if pinion is follower.

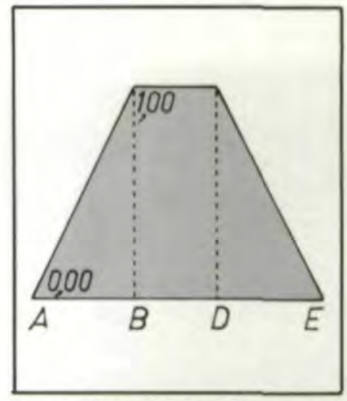

Fig. 4 – Traditional load sharing factor for a gear pair with modified tooth profile designed for smooth meshing.

path of contact, defined as the sum of a constant bulk temperature Θ_M and a varying flash temperature Θ_{fl}.

$$\left.\begin{aligned} \Theta_B &= \Theta_M + \Theta_{fl} && (1) \\ \Theta_{fl} &= \mu_{my}\, X_M\, X_B\, X_\Gamma \frac{W_{Bt}^{3/4}\, V^{1/2}}{a^{1/4}} && (2) \end{aligned}\right\}$$

in which μ_{my}, X_B and X_Γ are dependent on the point of the path of contact considered.

The integral temperature criterion concerns a single value Θ_{int}, defined as the sum of the bulk temperature Θ_M and the mean value of the flash temperature along the path of contact Θ_{flaint}, multiplied by a weight factor C_2.

$$\Theta int = \Theta_M + C_2\, \Theta_{flaint} \qquad (3)$$

$$C_2\, \Theta_{flaint} = \mu_{mC}\, X_M\, X_{BE} \frac{C_2\, X_\epsilon}{X_Q\, X_{Ca}} \frac{W_{Bt}^{3/4}\, V^{1/2}}{a^{1/4}} \qquad (4)$$

in which μ_{mc} is taken for the pitch point, X_{BE} is taken for the point, E, of the path of contact, and X_Q, X_{Ca} and X_ϵ are empirical geometric expressions.

The bulk temperature Θ_M is the same in both criteria and can be left out of consideration. Hence, the comparison concentrates on the quantities defined in Equations (2) and (4). If the quantity defined by (4) is approximately the same as the maximum value of the flash temperature (2) along the path of contact, then the two criteria will be equivalent.

$$\left.\begin{aligned} C_2\, \Theta_{flaint} &\cong \Theta_{fl} && \text{at one point} \\ C_2\, \Theta_{flaint} &\geq \Theta_{fl} && \text{elsewhere} \end{aligned}\right\} \qquad (5)$$

The thermal flash factor, X_M, the specific tooth load, W_{Bt}, the pitch line, velocity, V, and the center distance, a, cancel out after substitution of (2) and (4) in (5). Rearrangement of the factors yields

$$\left.\frac{\mu_{mC}\, X_{BE}}{\mu_{my}\, X_B} \frac{C_2 X_\epsilon}{X_Q\, X_{Ca}} \begin{aligned} &\approx X_\Gamma \text{ at one point} \\ &\geq X_\Gamma \text{ elsewhere} \end{aligned}\right\} \qquad (6)$$

Now, the comparison between the flash temperature criterion and the integral criterion is reduced to a comparison of the product of empirical factors (mainly, part of the integral temperature formula) and the load sharing factor (to be applied in the flash temperature formula).

The Load Sharing Factor

The load sharing factor, X_G, accounts for the load sharing of succeeding pairs of meshing teeth. Dynamic effects due to vibrations of pinion and wheel are left out of consideration. The Draft International Standard[1] represents four variants of the load sharing, depending on the system of profile modification applied. See Figs. 1 to 4.

By convention, the load sharing factor is a discontinuous trapezoid function on the path of contact. The path of contact is marked on the line of action by the points, A to E.

See Fig. 5. It is composed of the approach path of transverse single contact, AB, the path of transverse double contact, BD, and the recess path of transverse single contact, DE.

The product of factors in the left member of (6), to be compared with the load sharing factor, is

$$p = \frac{m_{mC}}{m_{my}} \cdot \frac{X_{BE}}{X_B} \cdot \frac{C_2 X_e}{X_Q} \cdot \frac{1}{X_{Ca}} \tag{7}$$

The geometry factors, X_B, X_{BE}, the contact ratio factor, X_e, the approach factor, X_Q, and the tip relief factor, X_{Ca}, are defined by geometrical functions. Likewise, the quotient of the coefficients of friction, m_{mC}/m_{my} is a geometrical function. The weight factor, C_2, is a constant. Hence, the product p is a geometrical function on the path of contact.

The shape and value of p was examined numerically by computing it for a sufficiently large amount of data to cover the whole field of application. Such data may be selected adequately, keeping in mind that the geometry of a gear pair is determined by only seven independent parameters. Table 2 shows these parameters together with the fixed parameters of the standard basic tooth rack profile determining a gear pair. In this case, three of them, the center distance, the facewidth and the helix angle, can be left out of consideration. For 54 combinations of parameters, see Table 3. The product, p, was calculated separately for the cases pinion driver and follower.

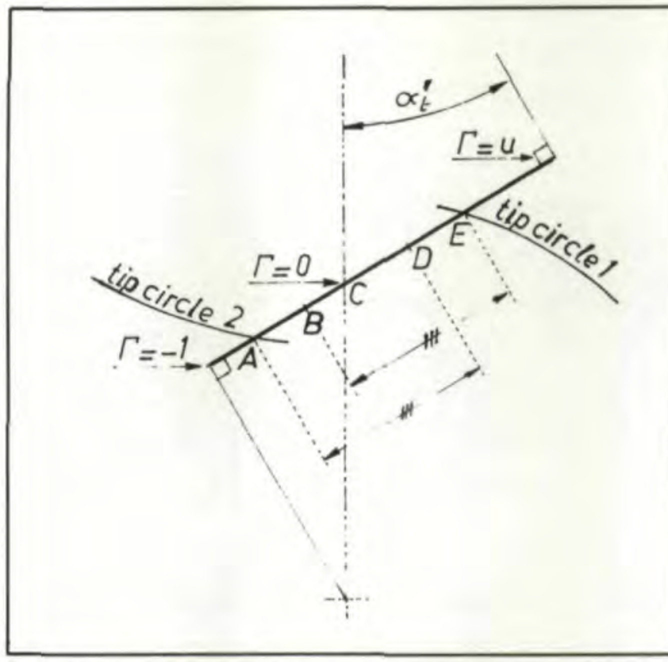

Fig. 5 – Path of contact AD and parameter Γ on the line of action.

Table 2. Independent and Fixed Parameters

a	center distance
b	facewidth
b	helix angle
u	gear ratio
z_1	number of teeth of pinion
x_1	addendum modification coefficient of pinion
x_2	addendum modification coefficient of wheel

Table 3. Combinations of Parameters

u	z_1		
1	80	140	250
2	45	80	140
4	25	45	80
8	14	25	45

x_1	x_2		
0.00		0.00	
0.20	−0.20	0.20	(u1)
0.40	0.00	0.40	(u1)

The tip relief factor, X_{Ca}, according to the Draft International Standard[1] depends on the tip relief. That tip relief may be estimated to be proportional to the specific tooth load, F_t/b, and, therefore, it is roughly proportional to the modulus or to another dimension of the gear pair. However, the tip relief factor, X_{Ca}, should be a dimensionless factor and the complicated guiding line of the estimation of X_{Ca} seems to be incomplete. To continue the examination, the modulus of a test gear pair was chosen in the calculation procedure of the factor, X_{Ca}. The load sharing factor was taken for smooth meshing. See Fig. 4.

For each combination, the product, π, had about the same shape and value. The gear ratio, u, and the addendum modification coefficient of the pinion, x_1, had the most influence. All other parameters had a low or negligible influence. The results in Figs. 6 to 8 answer surprisingly well the condition (5) for equivalence of criteria:

$$\left.\begin{array}{ll}\pi \approx X_\Gamma & \text{along a part of the curve} \\ \pi \geq X_\Gamma & \text{elsewhere}\end{array}\right\} \qquad (8)$$

This coincidence is not accidental, but is due to the extreme dependence of the integral temperature criterion on the flash temperature criterion. The rejection of one empirical criterion after the other and the steady acceptance of the flash temperature criterion on the one side and the recollection of the older concept of accumulation of energy on the other may have been a basis for the concept of the integral temperature.

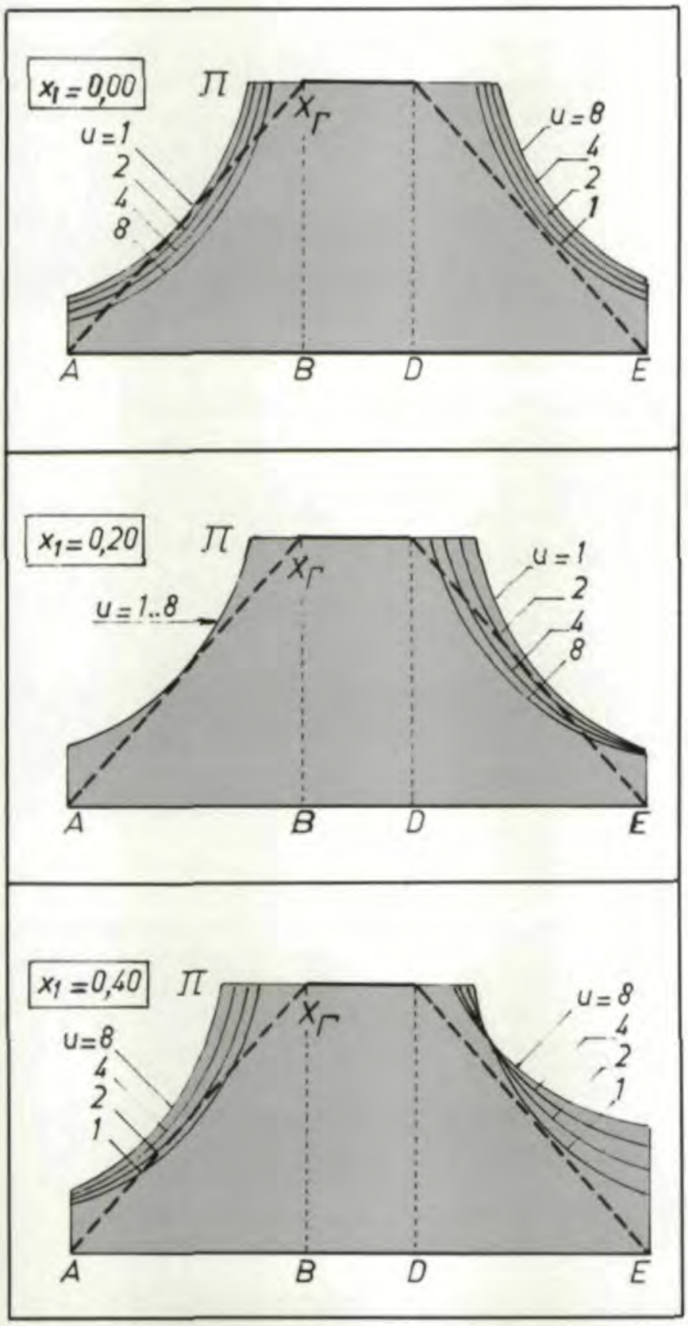

Fig. 6, 7, 8 – Product of factors, Π, to be compared with the load sharing factor for smooth meshing.

In an unintentioned way the integral temperature criterion had to be supplemented with empirical constants, which brought the integral temperature value close to the maximum value of the contact temperature.

Comparison of Influence Factors

As mentioned before, previous comparisons were far from complete. However, the presentation of the two methods in the Draft International Standard, Part 4 and its appendix[1] created a better balance. The formulae of the two methods are identical in many respects and differ only in a few factors which have a geometrical meaning. The differing factors are partly systematic (mathematically precisely defined):

$$\mu_{mC} \text{ versus } \mu_{my}$$
$$X_{BE} \text{ and } X_\epsilon \text{ versus } X_B$$

The choice of the coefficient of friction, μ_{my}, in a varying point of the path of contact versus its choice to a fixed point, μ_{mC}, contributes unimportant differences.

The choice of the point, E, for the factor, X_{BE} in the integral temperature formula is arbitrary and only emphasizes the integral temperature as a single value, whereas the flash temperature, and in consequence the contact temperature, are functions of the path of contact.

The systematic factor, X_ϵ is due to the concept of integration along the path of contact, reminiscent of the older concept of accumulation of energy. However, this physical interpretation is dubious and not necessary. Hence, the factor,

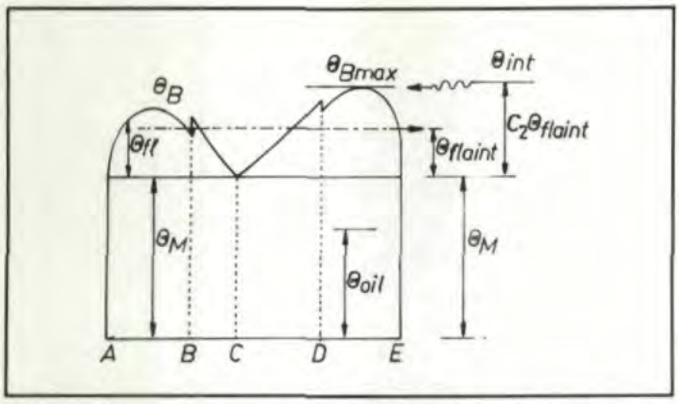

Fig. 9 – The contact temperature Θ_B varies along the path of contact. The integral temperature Θ_{int} approximates the maximum value of the contact temperature.

X_ε may be considered to be empirical.

The other factors which differ in both formulae are purely empirical.

$$C_2,\ X_Q,\ X_{Ca} \text{ versus } X_\Gamma$$

The factor C_2 is a constant (value 1.5). The factor X_Q varies from 1.00 down to 0.60, but for commonly applied gears, it seldom differs from the value 1.00.

The empirical factor, X_{Ca}, accounting for the influence of tip relief, is not decisive for a choice between the flash temperature method and the integral temperature method. The question how to improve X_{Ca} into a dimensionless factor can be left out of consideration.

No other numerical difference exists between the two methods other than those expressed in the product of factors π compared with the load sharing factor. The deviation in calculated data of the integral temperature as compared with the contact temperature data is very small, provided a proper selection of the tip relief factor or the load sharing factor in both formulae is made. The two methods correlate to such a high degree that mutual independence has to be rejected. Moreover, it is a remarkable fact that the two purely empirical factors, C_2 and X_Q contribute a value, C_2/X_Q = 1.5, up to 2.5 by which the resulting "temperature" diverges from the mean temperature and approximates the maximum value closely. See Fig. 9. Hence, the integral temperature method denies its own concept of integrating, and it confirms the significance of the maximum value of the contact temperature.

In the statistical research of scuffing phenomena a larger deviation of observed data may be expected than known from corresponding tests of tooth strength or pitting, due to more uncertain influences of a hydrodynamic, thermodynamic and chemical nature. On account of the systematic dependence of the two methods deduced above, any assertion that the integral temperature method would be statistically superior to the flash temperature method is based on a misunderstanding.

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Acknowledgement: *This article was first presented at the 2nd World Congress on Gearing, March 3-5, 1986, Paris, France.*