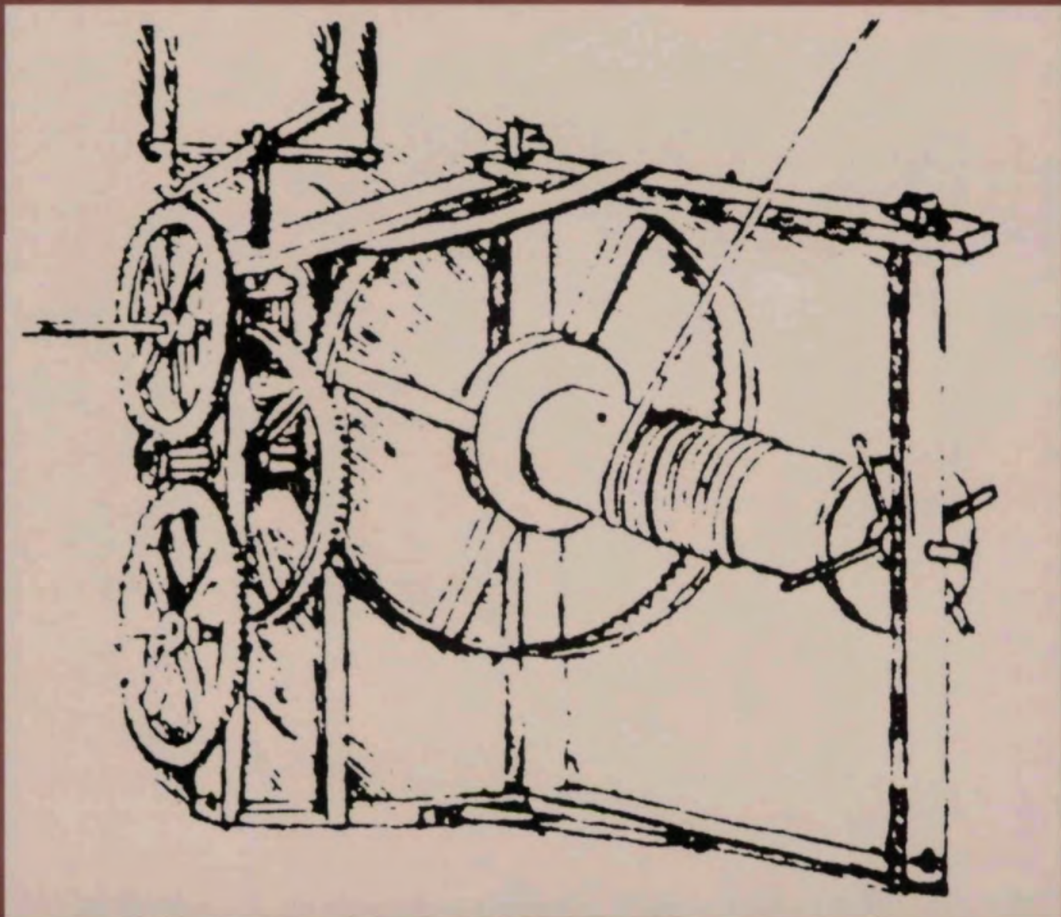
GEAR TECHNOLOGY

The Journal of Gear Manufacturing

MARCH/APRIL 1989

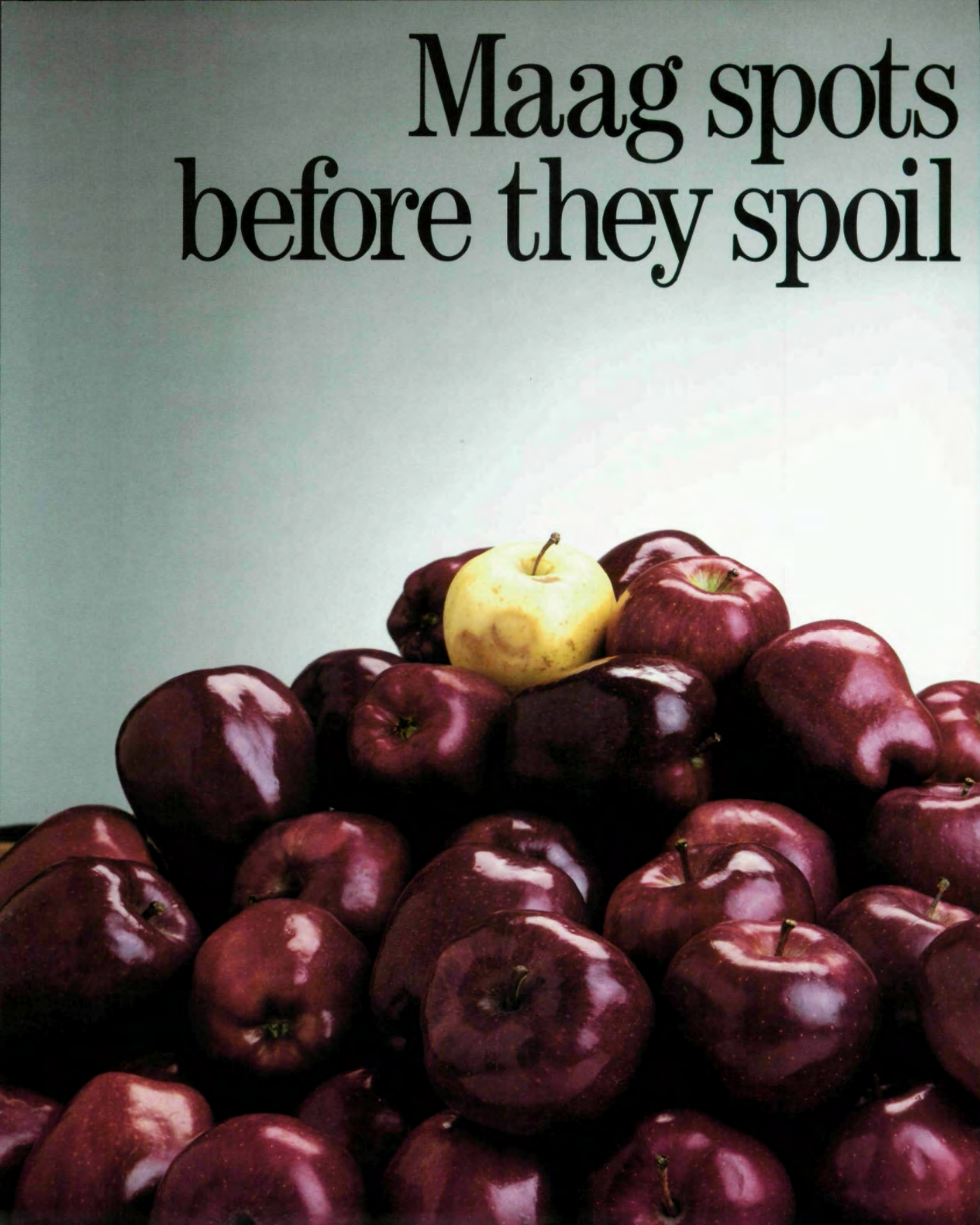


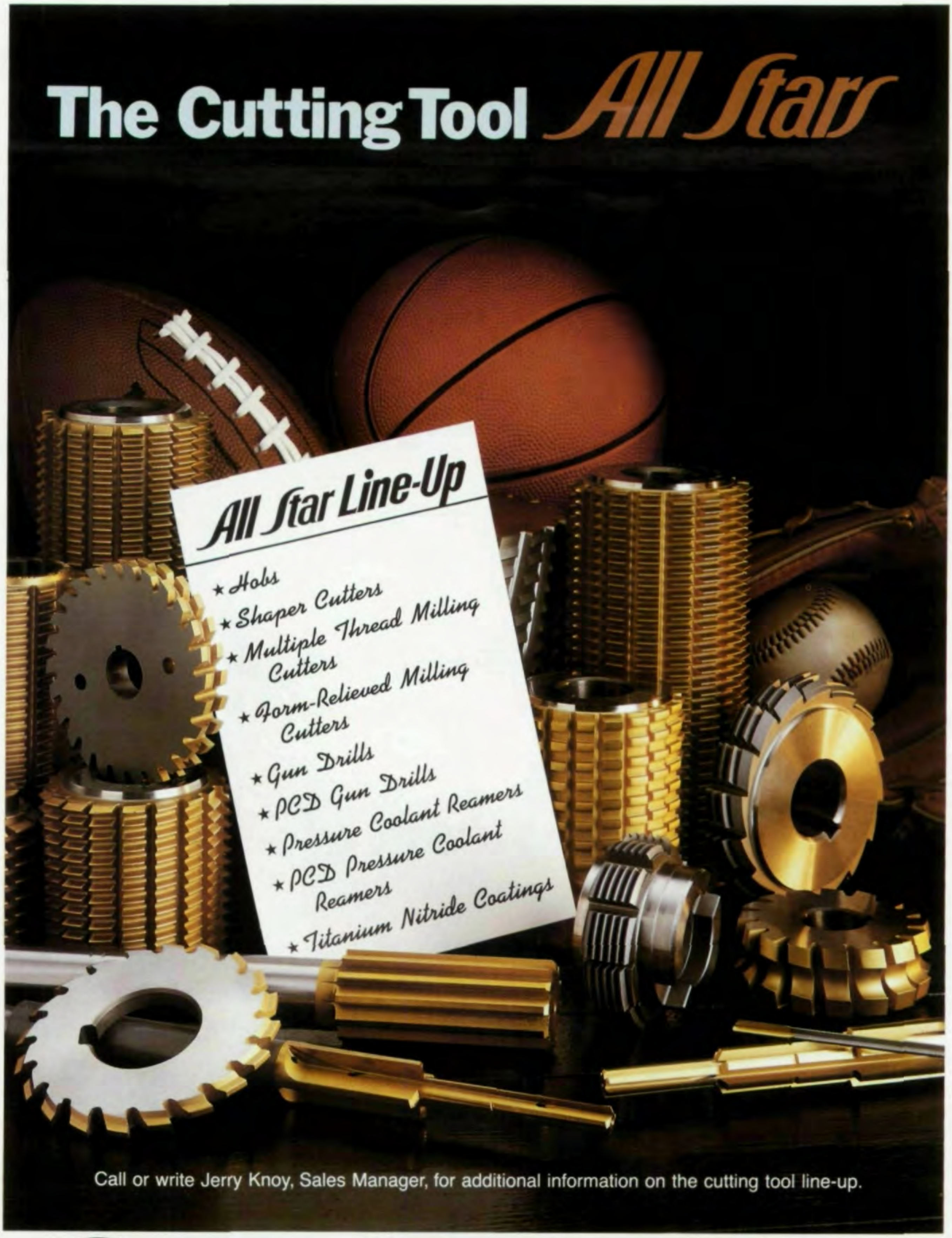
Effect of Load on Pitting Fatigue Life For Low-Contact-Ratio Spur Gears

Deburring With Power Brushes

The Wafer Shaper Cutter

Design of Internal Helical Gears

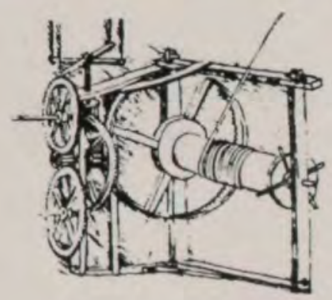

Maag spots
before they spoil


The Cutting Tool All Stars
All Star Line-Up
★ Hobs
★ Shaper Cutters
★ Multiple Thread Milling Cutters
★ Form-Relieved Milling Cutters
★ Gun Drills
★ PCD Gun Drills
★ Pressure Coolant Reamers
★ PCD Pressure Coolant Reamers
★ Titanium Nitride Coatings
Call or write Jerry Knoy, Sales Manager, for additional information on the cutting tool line-up.

GEAR TECHNOLOGY

The Journal of Gear Manufacturing

RANDALL PUBLISHING
1401 Lunt Avenue
P.O. Box 1426
Elk Grove, IL 60007
(312) 437-6604



The Advanced Technology
of
Leonardo da Vinci
1452-1519

COVER

Leonardo was a practical-minded engineer. In a time when most cooking was done over an open fire, he set his mind to making life a little easier in the kitchen. He experimented with a variety of ways to liberate the cook from the tedious task of tending a roast, including a device driven by the heat of the fire. Our cover shows a sketch of a mechanical roasting spit. It is powered by a descending weight attached to a pulley-and-gear drive.

CONTENTS — PAGE NO.

FEATURES

PREDICTED EFFECT OF DYNAMIC LOAD ON PITTING FATIGUE LIFE FOR LOW-CONTACT-RATIO SPUR GEARS — 6
David G. Lewicki, NASA Lewis Research Center, Cleveland, OH

THE WAFER SHAPER CUTTER — 26
Edward Haug, Pfauter-Maag Cutting Tools, Loves Park, IL

DEPARTMENTS

EDITORIAL — 5

FROM THE INDUSTRY. . .
DEBURRING AND FINISHING GEARS WITH POWER BRUSHES — 30
George A. Pishek, Osborn Manufacturing Co., Cleveland, OH

BACK TO BASICS . . .
DESIGN OF INTERNAL HELICAL GEARS — 34
Earle Buckingham
Eliot K. Buckingham, Buckingham Associates, Springfield, VT

CLASSIFIED — 46

TECHNICAL CALENDAR — 47

March/April, 1989 — Vol. 6, No. 2

GEAR TECHNOLOGY, The Journal of Gear Manufacturing (ISSN 0743-6858) is published bimonthly by Randall Publishing Co., Inc., 1425 Lunt Avenue, P. O. Box 1426, Elk Grove Village, IL 60007. GEAR TECHNOLOGY, The Journal of Gear Manufacturing. Subscription rates are: $40.00 in the United States, $50.00 in Canada, $55.00 in all other foreign countries. Second-Class postage paid at Arlington Heights, IL and at additional mailing office.

Postmaster: Send address changes to GEAR TECHNOLOGY, The Journal of Gear Manufacturing, 1425 Lunt Avenue, P. O. Box 1426, Elk Grove Village, IL 60007.

MANUSCRIPTS: We are requesting technical papers with an educational emphasis for anyone having anything to do with the design, manufacture, testing or processing of gears. Subjects sought are solutions to specific problems, explanations of new technology, techniques, designs, processes, and alternative manufacturing methods. These can range from the "How to . . ." of gear cutting (BACK TO BASICS) to the most advanced technology. All manuscripts submitted will be carefully considered. However, the Publisher assumes no responsibility for the safety or return of manuscripts. Manuscripts must be accompanied by a self-addressed, self-stamped envelope, and be sent to GEAR TECHNOLOGY, The Journal of Gear Manufacturing, P.O. Box 1426, Elk Grove, IL 60009, (312) 437-6604.

EDITORIAL

A STEP IN THE RIGHT DIRECTION

At the time I'm writing this editorial, the new year is barely two weeks old. The air and the papers are still full of those inevitable end-of-the-year estimates of how far we've come in one area or another and how far we have to go. Analyses of the future, both grim and humorous, abound. There are even more of these laundry lists of PROBLEMS TO BE SOLVED IMMEDIATELY than usual, since a new president will be inaugurated in a week or so. Everyone has advice for George Bush on what to do first and how to do it. Some of the advice is sound, and I hope he's listening; however, reading all these position papers can be a depressing exercise.

If one believed everything the pundits say, it would seem as though the sky really were falling. The world is full of intractable problems that have no easy solutions. A host of difficult issues need attention RIGHT NOW, and it's easy to despair. After all, what can any one of us, no matter how well-informed and well-intentioned, do about the national debt, world hunger, terrorism, acid rain or the shambles that is our public education system? But to just throw up our hands and say, "What's the use?" is to insure that whatever the problem is will only get worse and worse. The solutions to even the toughest problems frequently begin with simple steps by a few people, and if all we can do at the moment is to point out the steps and applaud, then that's what we should do.

A problem that is of immediate concern to the gear industry is the decline in the number of *trained* machinists and engineers out there to do the work we will need done in the next two decades. The number of students studying science and engineering is declining. Young men and women are not exactly beating down the doors of schools to be taught gearing. At the same time, the tough economic times of the last few years have forced gear manufacturers to cut back sharply or eliminate entirely their apprenticeship training programs. According to *Industry Week*, the average age of trained machine tool operators is now 58, and the projected supply over the next decades is only one fourth of the projected demand. Are these figures another example of the sky falling?

Not everyone sees them that way. Caterpillar Incorporated and AGMA have announced a program that, while it will not solve our engineering shortage, is a good step in the right direction. And it's the kind of step that can be taken by any gear manufacturer with the will to do it.

AGMA has announced that Caterpillar is donating a Barber Coleman hobber to both Mississippi State University and to the University of Alberta. The machines are from Caterpillar's surplus inventory and ordinarily would have been scrapped or sold on the used machinery market. Instead they will be used as an important part of the undergraduate and graduate mechanical engineering cirricula at these schools. They will provide engineering students at these schools with invaluable hands-on experience.

Mississippi State and Alberta were chosen from a larger field of universities, community colleges and vocational schools that applied for the grant of the machinery. AGMA and Caterpillar have been gratified by the response to the program so far and hope to expand it to include other companies and other academic institutions.

The idea is beautiful in its simplicity. Companies donate good, working used machinery that they otherwise would dispose of in the course of modernization. Schools that wish such machines plan undergraduate and graduate programs using them to train future engineers. They apply for one of the machines through AGMA, who administers the grant. The only restrictions are that the machines not be used for manufacture or for research. The idea is to make the machines available for training the largest number of students possible. Schools also have to arrange and pay for shipping and installation of the machinery, but AGMA will provide technical advice, and lack of funds to cover these expenses will not necessarily disqualify a school for one of these grants.

Several factors have led to a serious decline in the number of well-trained people to populate our shop floors in the future, but the time is past for hand-wringing, finger-pointing and "ain't-it-terrible" whining about the future. The fact is without skilled employees, no amount of business acumen or "lean and mean" bottom line thinking will keep U.S. companies competitive.

Caterpillar and AGMA have come up with a dynamite first step toward a solution.

It's the kind of idea that any company with the will to do so can adopt. It's the kind of idea that any educational institution with the willingness to spend the time it takes to plan for such a grant can take advantage of. It's easy to talk in generalities about how business must work in partnership with schools to insure the trained labor force it needs for the future. Here's a chance to take some very specific steps to see that it happens.

A phone call to AGMA to find out how your company can participate in this program is the only first step required. Perhaps a second phone call to your local college, university or vocational school to inform its engineering faculty that such a program is available is a good second step.

Maybe the sky isn't falling after all. There are things we can do as an industry to work for a better future for all of us. All it takes is the will to think ahead and the vision to recognize a good idea when we see one.

I say congratulations to both AGMA and Caterpillar for bringing a good idea to reality. To the rest of the gear industry I suggest that this is a case where imitation is not only the sincerest form of flattery, but also a way to work for the betterment of the industry as a whole.

Michael Goldstein

Michael Goldstein, Editor/Publisher

Photo courtesy of Harris Metals

Predicted Effect of Dynamic Load on Pitting Fatigue Life for Low-Contact-Ratio Spur Gears

David G. Lewicki,
U.S. Army Aviation Research &
Technology Activity – AVSCOM
NASA Lewis Research Center
Cleveland, Ohio

Summary:

How dynamic load affects the pitting fatigue life of external spur gears was predicted by using NASA computer program TELSGE. TELSGE was modified to include an improved gear tooth stiffness model, a stiffness-dynamic load iteration scheme and a pitting-fatigue-life prediction analysis for a gear mesh. The analysis used the NASA gear life model developed by Coy, methods of probability and statistics and gear tooth dynamic loads to predict life. In general, gear life predictions based on dynamic loads differed significantly from those based on static loads, with the predictions being strongly influenced by the maximum dynamic load during contact.

With the modified TELSGE, parametric studies were performed that modeled low-contact-ratio involute spur gears over a range of gear speeds, numbers of teeth, gear sizes, diametral pitches, pressure angles and gear ratios. Dynamic loads and pitting fatigue lives were calculated. Gear mesh operating speed strongly affected predicted dynamic load and life. Meshes operating at a resonant speed or at one-half the resonant speed had significantly shorter lives. Dynamic life factors for gear surface pitting fatigue were developed on the basis of the parametric studies. The effects of number of teeth, gear size, diametral pitch, pressure angle and gear ratio on predicted life were related to the contact ratio. In general, meshes with higher contact ratios had higher dynamic life factors than meshes with lower contact ratios. A design chart was developed for use in the absence of a computer and program TELSGE. An example illustrates the use of the design chart.

INTRODUCTION

Gears may fail from scoring, tooth fracture due to bending fatigue or surface pitting fatigue. Scoring failure is usually lubrication related and can be prevented by proper lubrication and proper operating temperatures. Tooth fractures are usually caused by poor materials, improper design or overloading and can be prevented by designing for bending stresses below the material's maximum allowable stress. The American Gear Manufacturers Association (AGMA) has a standard practice for predicting gear surface pitting fatigue.(1) The method assumes that infinite life results when the maximum surface contact stresses are less than the material's endurance limit. Surface contact stress calculations may include a dynamic factor to account for gear dynamic loading. The AGMA recommends a dynamic factor of 1 for gear teeth of high accuracy, but states that actual dynamic loads, computed or measured, can be used.(1)

Gear research authorities do not completely agree on surface pitting fatigue. Some state that gear materials do not have surface endurance limits,(2-3) as is true for rolling-element bearings. In 1975, Coy developed an improved model for the surface fatigue life of spur and helical gears, using an approach

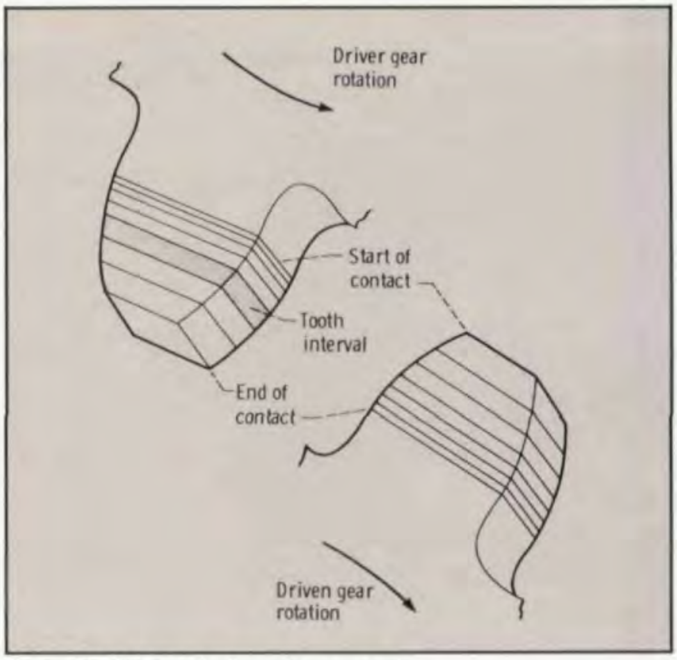


Fig. 1 – Tooth intervals of a meshing gear pair.

Nomenclature

B	material constant (2.23×10^8 N/m$^{1.979}$, 35,000 lb/in.$^{1.979}$, ref. 21)
C_{eq}	equivalent damping per unit face width, N sec/m^2 (lb sec/in.^{2})
C_v	dynamic life factor
c	contact ratio
d	distance of inscribed parabola, m (in.)
E	modulus of elasticity, Pa (psi)
e	Weibull exponent
f	gear tooth face width, m (in.)
h_0	tooth thickness at root radius, m (in.)
I_0	beam cross-sectional moment of inertia, m^4 (in.^{4})
i	contact position index
J	number of intervals; or polar mass moment of inertia per unit face width ($1/2mr_b^2$ for disk), kg m (lb sec^2)
j	interval index
K	combined stiffness per unit face width, Pa (psi)
K_{eq}	equivalent stiffness per unit face width, Pa (psi)
$\overline{K}_{eq}$	mean equivalent stiffness per unit face width, Pa (psi)
k	gear tooth stiffness per unit face width, Pa (psi)
L	life for 90% probabilty of survival, Mrev
l	involute length, m (in.)
M	effective mass per unit face width, J/r_b^2, kg/m (lb sec^2/in.^{2})
M_{eq}	equivalent mass per unit face width, kg/m (lb sec^2/in.^{2})
m	mass per unit face width, kg/m (lb sec^2/in.^{2})
m_o	module, mm/tooth
N	number of teeth
P	diametral pitch, teeth/in.
P_d	dynamic load per unit face width, N/m (lb/in.)
P_s	static load per unit face width, N/m (lb/in.)
p_b	base pitch, $2\pi r_b/N$, m (in.)
Q	normal load, N (lb)
$\overline{Q}$	average interval load, N (lb)
Q_t	tangential load, $Q \cos \varphi$, N (lb)
R	radius of curvature, m (in.)
r_b	base radius, $r_p \cos \varphi$, m (in.)
r_o	outside radius, m (in.)
r_p	pitch radius, $Nm_o/2 = N/2P$, m (in.)
r_r	root radius, m (in.)
s	displacement, m (in.)
T	torque, N m (lb in.)
t	tooth thickness at pitch radius, m (in.)
X	relative displacement, m (in.)
x	contact position, m (in.)
Δx	interval length, m (in.)
Y	Lewis form factor
Z	contact length, m (in.)
z_1	contact length from pitch point to start of contact, m (in.)
z_2	contact length from pitch point to end of contact, m (in.)
α	pressure angle at root radius, deg
γ	density, kg/m^3 (lb/in.^{3})
δ	beam deflection, m (in.)
ζ	damping ratio
η	life for 90% probability of survival, millions of stress cycles
θ	angular displacement, rad
Σp	curvature sum, m^{-1} (in.$^{-1}$)
$\overline{\Sigma p}$	average curvature sum, m^{-1} (in. $^{-1}$)
φ	pressure angle, deg
ω	speed, rpm
ω_n	resonant speed, rpm

Subscripts:

d	dynamic life
I	gear tooth stiffness-dynamic load iteration index
i	contact position index
j	interval index
m	mesh
max	maximum during contact position
s	static life
t	tooth
1	driver gear
2	driven gear

Superscripts:

(I)	first pair of teeth in contact
(II)	second pair of teeth in contact

similar to that for rolling-element bearings.[2-6] This work did not, however, include the effect of dynamic load.

Early contributions to gear dynamic loading were made by Buckingham, Tuplin, Richardson and Attia.[7-10] More recently computer-based analytical programs have been developed to determine gear tooth dynamic loads.[11-16] The dynamic loads of these programs depend on such factors as inertia and stiffness of rotating members, tooth spacing and profile errors, size and speed. The loads are determined by solving the equations of motion of a given gear mesh system.

The objective of the present study was to combine the dynamic load calculation procedure of Wang and Cheng[14] with the NASA gear life model of Coy[2-6] to determine how

AUTHOR:

DAVID G. LEWICKI *is employed by the United States Army Aviation Research and Technology Activity's Propulsion Directorate at the NASA Lewis Research Center, Cleveland, Ohio. He has been doing work at NASA on helicopter and turboprop transmissions and transmission components since 1982, involved in both analytical and experimental research activities. He holds a B.S. degree in Mechanical Engineering at Cleveland State University and a M.S. degree in Mechanical Engineering at the Unviersity of Toledo. He is a member of the American Society of Mechanical Engineers and is current chairman of the ASME Publicity Committee for the Design Engineering Division.*

dynamic load affects the pitting fatigue life of external spur gears. NASA computer program TELSGE, modified to include Cornell's gear tooth stiffness model,[17] a stiffness-dynamic load iteration scheme and a pitting fatigue life analysis, was used to predict gear dynamic loads and life. Parametric studies using modified TELSGE were performed for low-contact-ratio involute gears with no tooth spacing or profile errors. Gear dynamic loads and tooth stiffnesses were calculated as a function of contact position and speed. On the basis of the parametric studies dynamic life factors for gear surface pitting fatigue were developed as a function of speed and contact ratio.

ANALYSIS

Gear Life Model

Current theory. The life model proposed by Lundberg and Palmgren[18-20] is the commonly accepted theory for predicting the pitting fatigue life of rolling-element bearings. Because the fatigue failure mechanism is similar for both gears and rolling-element bearings made from high-strength steel, the Lundberg-Palmgren model for bearings has been adapted to predict gear life.[2-6] Reference 6 gives the life for a 90% probability of survival η of a single tooth on a driver or driven gear of a mesh as

$$\eta = B^{4.3} f^{3.9} \Sigma\rho^{-5} \ell^{-0.4} Q^{-4.3} \qquad (1)$$

where B is a material constant based on experimental data; f is the tooth face width; Σp is the curvature sum at the start of single-tooth contact; l is the involute surface length during single-tooth contact; and Q is the static tooth load, normal to the contact. A complete list of symbols is given in the Nomenclature.

The life of the complete driver gear (all teeth) L_1 in terms of driver gear rotations is

$$L_1 = N_1^{-1/e} \eta \qquad (2)$$

where N_1 is the number of teeth on the driver gear and e, the Weibull exponent, is a measure of scatter in fatigue life. Experimental research on AISI 9310 steel spur gears has shown gear fatigue to follow the Weibull failure distribution with $e \approx 2.5$.[3]

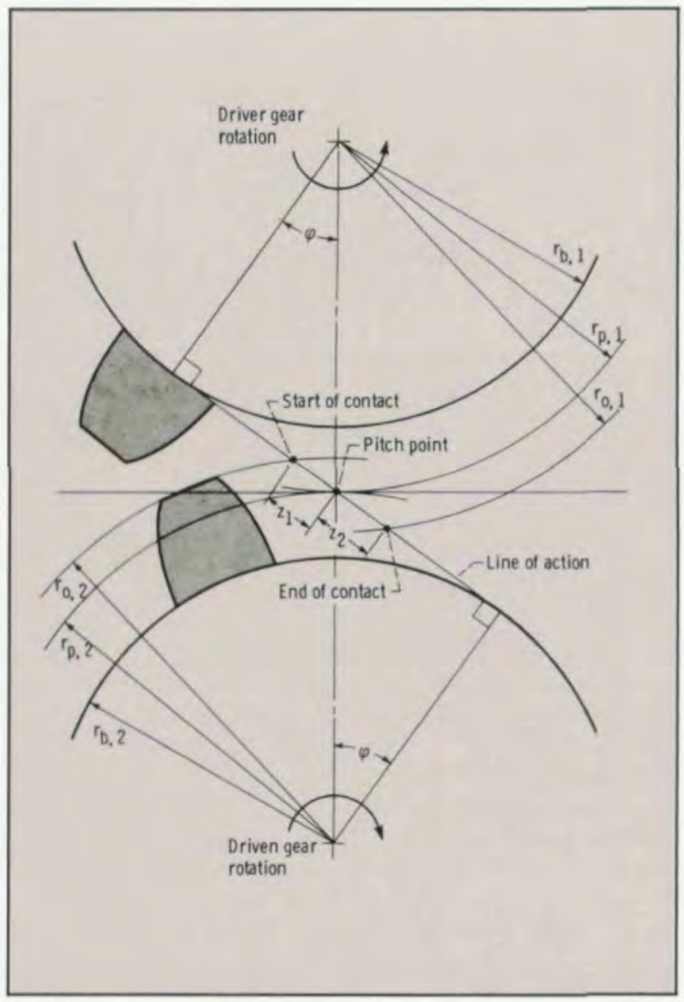


Fig. 2 – Basic geometry of a pair of external spur gears in mesh.

The life of the complete driven gear L_2 in terms of driver gear rotations is

$$L_2 = \left(\frac{N_2}{N_1}\right) N_2^{-1/e} \eta \tag{3}$$

where N_2 is the number of teeth on the driven gear. The mesh life (both driver and driven gears) L_m in terms of driver gear rotations is given by

$$L_m = \left(L_1^{-e} + L_2^{-e}\right)^{-1/e} \tag{4}$$

Expanded theory. To adapt the current gear life model for predictions based on gear tooth dynamic loads, the tooth was divided into intervals (Fig. 1). The use of intervals allowed the current gear life model to account for load and curvature sums varying with contact position. The complete gear tooth life was determined from the interval lives and methods of probability and statistics. The details are as follows.

When a pair of external spur gears is in mesh (Fig. 2), the line tangent to the base circles of both the driver and driven gears is called the line of action. The gears begin contact when the outside radius of the driven gear intersects the line of action. As the gears rotate, the contact point occurs on the line of action. The contact ends when the outside radius of the driver gear intersects the line of action. The point at which the pitch circles of the driver and driven gears intersect is called the pitch point.

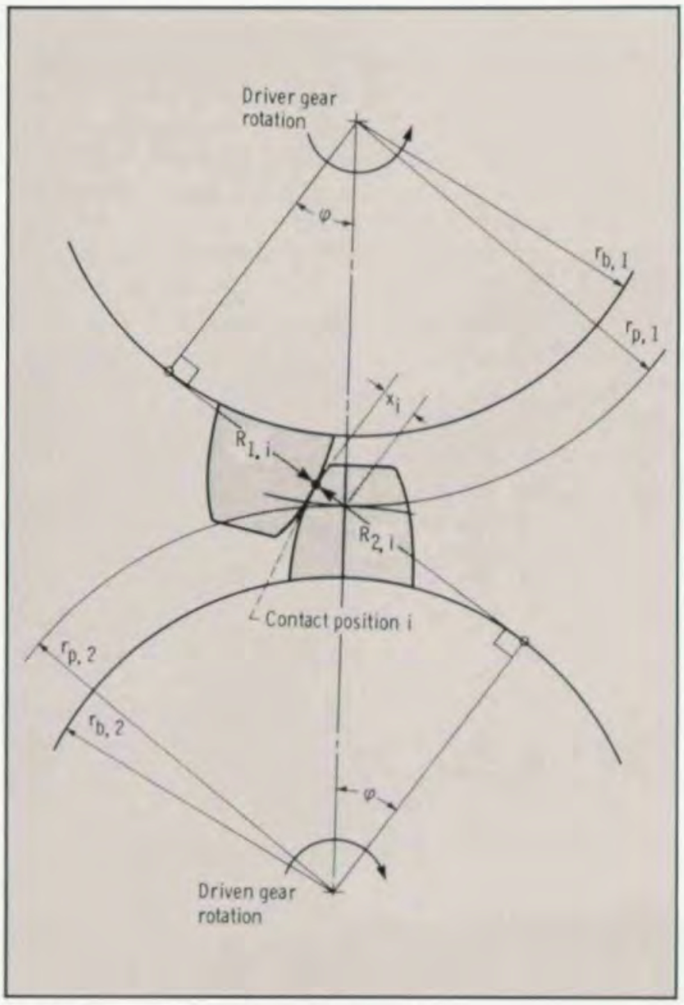


Fig. 3 – Curvatures of involute teeth in contact.

The distance along the line of action from the pitch point to the start of contact is

$$z_1 = \sqrt{r_{o,2}^2 - r_{b,2}^2} - r_{p,2} \sin \varphi \tag{5}$$

The distance along the line of action from the pitch point to the end of contact is

$$z_2 = \sqrt{r_{o,1}^2 - r_{b,1}^2} - r_{p,1} \sin \varphi \tag{6}$$

The contact length Z is defined as

$$Z = z_1 + z_2 \tag{7}$$

Dividing the contact length into equal-size intervals of length Δx gives

$$\Delta x = \frac{Z}{J} \tag{8}$$

where J is the total number of intervals on a tooth, and

$$x_i = -z_1 + (i-1)\Delta x \qquad \text{for } i = 1 \text{ to } J+1 \tag{9}$$

where x is the contact position along the line of action. The value of x is negative when contact is before the pitch point, zero when at the pitch point, and positive when after the pitch point.

The life of each interval for a 90% probability of survival is given from Equation 1 by

$$\eta_j = B^{4.3} f^{3.9} \overline{\Sigma\rho_j}^{-5} \ell_j^{-0.4} \overline{Q}_j^{-4.3} \qquad \text{for } j = 1 \text{ to } J \tag{10}$$

where B and f do not change from interval to interval. Both curvature sum and involute length, however, change with contact position.

At the i^{th} contact position the radii of curvature of the driver and driven gears (Fig. 3) are

$$R_{1,i} = r_{p,1} \sin\varphi + x_i \tag{11}$$

$$R_{2,i} = r_{p,2} \sin\varphi - x_i \tag{12}$$

The curvature sum at the i^{th} contact position is

$$\Sigma\rho_i = \frac{1}{R_{1,i}} + \frac{1}{R_{2,i}} \tag{13}$$

For the j^{th} interval the average curvature sum used in the life model is

$$\overline{\Sigma\rho_j} = \frac{\Sigma\rho_i + \Sigma\rho_{i+1}}{2} \qquad \text{for } i = j \tag{14}$$

The curvature sum varied slightly with contact position for the example gear mesh data from Table I for 100 intervals on each tooth (Fig. 4a). (The contact position was made dimensionless by dividing by the base pitch p_b.) The plot shows the curvature sum to be symmetric about the pitch point ($x=0$). This was true only because the driver and driven gears of the example were the same size.

The involute surface lengths of the driver and driven gears for the j^{th} interval (for small Δx) are

$$\ell_{1,j} = \left(\frac{\Delta x}{r_{b,1}}\right) x_i + \Delta x \tan\varphi \qquad \text{for } i = j \tag{15}$$

$$\ell_{2,j} = -\left(\frac{\Delta x}{r_{b,2}}\right) x_i + \Delta x \tan\varphi \qquad \text{for } i = j \tag{16}$$

The involute length is a linear function of contact position (Fig. 4b). Equations 15 and 16 imply that rotating a gear mesh through equal angles produces unequal involute lengths and, thus, different-size tooth intervals (as shown in Fig. 1).

By using intervals the life model considers load that can vary with contact position. For the j^{th} interval the average load used in the life model is

$$\overline{Q}_j = \frac{Q_i + Q_{i+1}}{2} \qquad \text{for } i = j \tag{17}$$

The static load variation with contact position depends on the number of teeth in contact (Fig. 4c). As a pair of teeth begin contact, the preceding pair of mating teeth are also in contact. This

Table I. Baseline Data For Both Driver And Driven Gear

Number of teeth	36
Diametral pitch	8
Outside radius, cm (in.)	6.033 (2.375)
Base pitch, cm (in.)	0.937 (0.369)
Face width, cm (in.)	0.635 (0.250)
Pressure angle, deg	20
Root radius, cm (in.)	5.318 (2.094)
Fillet radius, cm (in.)	0.102 (0.040)
Chordal tooth thickness, cm (in.)	0.485 (0.191)
Normal load, N (lb)	1718 (386)
Speed, rpm	4000
Material	Steel

double-tooth-pair contact occurs for Intervals 1 to 41, and it is assumed that half the applied load is transferred per contact. Near the pitch point single-tooth-pair contact occurs (Intervals 42-59), and all the load is transferred by it. Toward the end of contact, double-tooth-pair contact again occurs (Intervals 60-100) as the following pair of mating teeth begin contact. As before, it is assumed that half the load is transferred per contact.

The life of a complete gear tooth η_t is determined from the interval lives and methods of probability and statistics where

$$\eta_t = \left(\sum_{j=1}^{J} \eta_j^{-e} \right)^{-1/e} \tag{18}$$

The complete tooth life was always shorter than the lives of the shortest-lived intervals (Fig. 4d). Also, intervals with larger applied loads had much more influence on gear tooth life than intervals with smaller loads.

The tooth lives for a driver and driven gear in mesh are determined by the expanded life theory and Equation 18. They are equal if the driver and driven gears are the same size. They are slightly different if the driver and driven gears are different sizes because of curvature sums and involute lengths. The complete gear lives and mesh life are determined, as before, by using Equations 2 to 4 and substituting η_t for η.

The total number of tooth intervals was varied from 30 to over 400 to check convergence on life. Static loads were used. All cases predicted the same tooth life. Gear size, diametral pitch, pressure angle and gear ratio were also varied to compare mesh lives predicted by the current and expanded theories. Static loads were used. The expanded theory predicted mesh lives a little longer than, but within 10% of, those predicted by the current theory for meshes with equal-size gears. This difference was caused by the expanded theory's curvature sum variation with contact position. Thus for meshes with equal-size gears, the curvature sum variation has a small effect on life. For meshes with unequal-size gears, however, there were greater differences in the mesh lives predicted by the two theories.

Gear Tooth Dynamic Loads

Gear tooth dynamic load model. The contact load of meshing gear teeth varies as the contact point moves along the line of action. This is known as dynamic load. It is mainly caused by single- and double-tooth-pair contact transitions, tooth stiffness variation along the contact, and tooth profile deviations from true involutes (tooth profile errors). NASA computer program TELSGE[14-16] was used to determine gear tooth dynamic loads. The program models meshing gears as a pair of rigid disks connected by a spring (Fig. 5). The spring stiffness corresponds to gear teeth stiffnesses.

The dynamic load model uses the equations of motion governing the angular displacements of the driver and driven gears. By converting the angular movements of the disks to linear displacements along the line of action, and by algebraic manipulation, the equations of motion are represented by a single differential equation, where

$$M_{eq}\ddot{X} + C_{eq}\dot{X} + K_{eq}X = P_s \tag{19}$$

The dependent variable X, called the relative displacement, is the compression of the spring along the line of action,

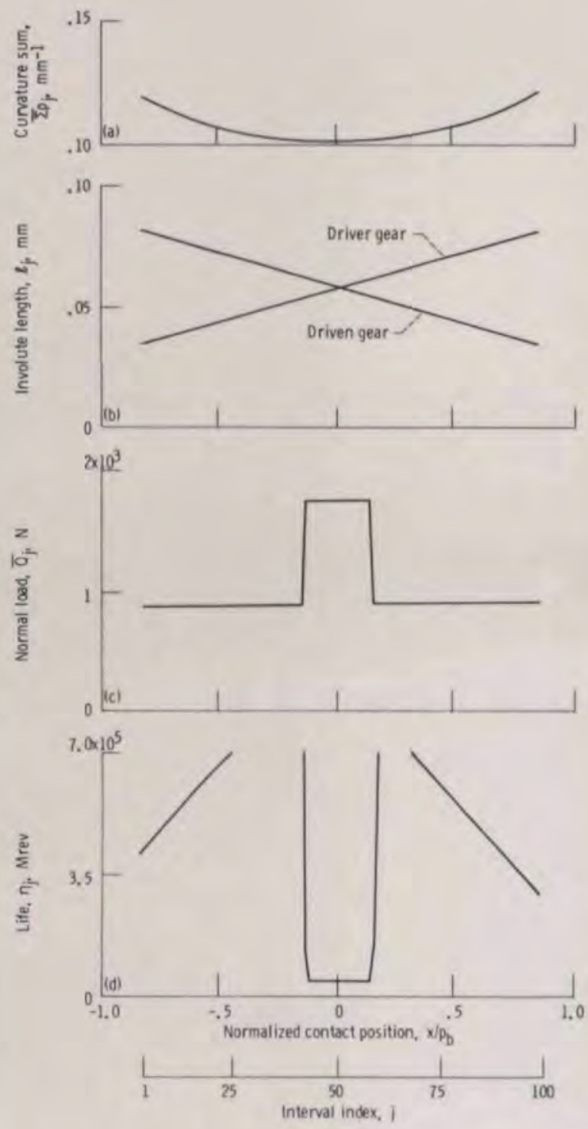
$$X = s_1 - s_2 \tag{20}$$

where

$$s_1 = r_{b,1}\theta_1 \quad \text{and} \quad s_2 = r_{b,2}\theta_2 \tag{21}$$

The equivalent mass per unit face width is

$$M_{eq} = \frac{M_1 M_2}{M_1 + M_2} \tag{22}$$



(a) Average curvature sum of interval.
(b) Length of involute surface of interval.
(c) Average normal load on interval.
(d) Life of interval for driver gear. Tooth life, 13 200 Mrev; gear life, 3100 Mrev; mesh life, 2400 Mrev.

Fig. 4 — Effect of contact position on gear life parameters for gear data from Table I.

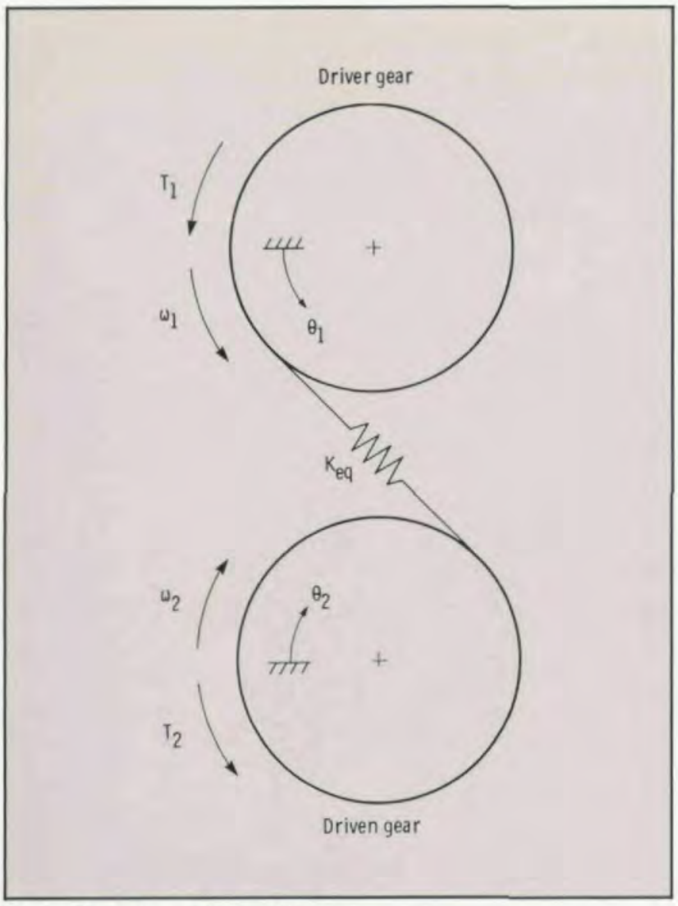
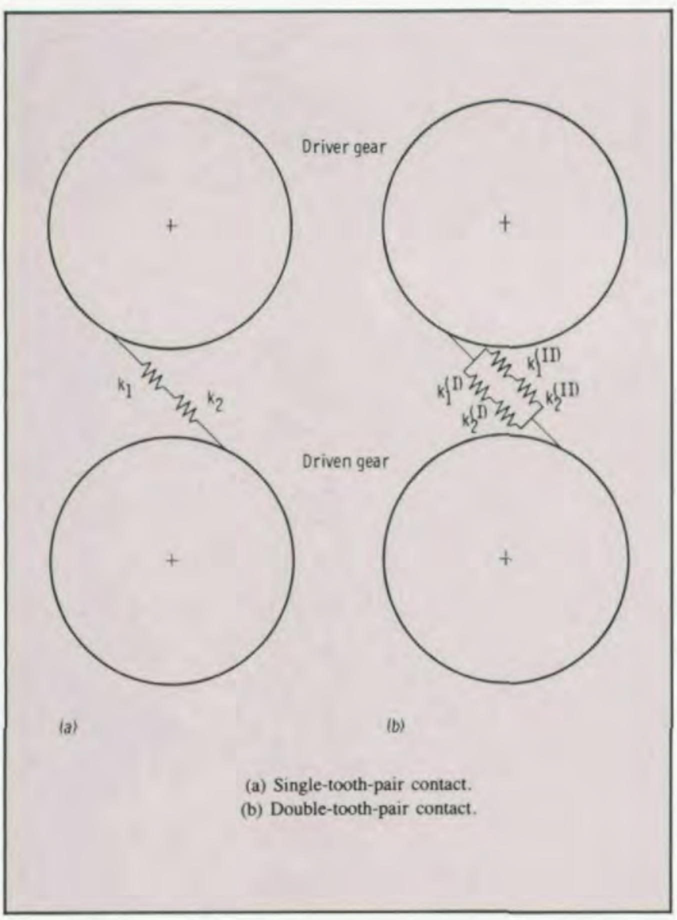


Fig. 5 – Dynamic model of meshing gears.



(a) Single-tooth-pair contact.
(b) Double-tooth-pair contact.

Fig. 6 – Gear tooth stiffness models.

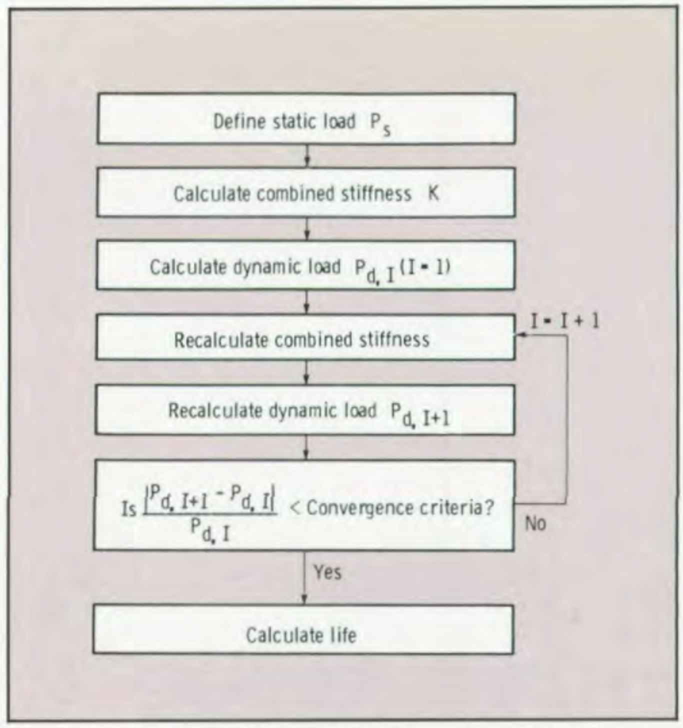


Fig. 7 – Flowchart of gear tooth combined stiffness-dynamic load interaction scheme in computer program TELSGE.

The equivalent damping coefficient per unit face width C_{eq} includes the effect of viscous damping,

$$C_{eq} = 2\zeta\sqrt{K_{eq}M_{eq}} \tag{23}$$

where ζ is the damping ratio, K_{eq} is the equivalent stiffness per unit face width (discussed in the section *Equivalent gear tooth stiffness*), and P_s is the static load per unit face width.

The relative displacement is determined as a function of contact position by using a Runge-Kutta numerical method and solving Equation 19. The dynamic load on a gear tooth is determined as a function of contact position by

$$P_d = KX \tag{24}$$

where P_d is the dynamic load per unit face width and K is the combined stiffness per unit face width (discussed in the section *Equivalent gear tooth stiffness*). Note that when X is negative, the teeth separate and the dynamic load is zero. Although tooth profile errors can be accounted for in Equations 19 and 24, they were beyond the scope of this study.

Gear tooth stiffness. Computer program TELSGE was modified to incorporate the gear tooth stiffness model of Cornell,[17] regarded as the present state of the art. The stiffness model consists of tooth bending as a cantilever beam, fillet and foundation flexibilities and local Hertzian compression, all as functions of contact position. In Cornell's model the deflections due to bending and fillet and foundation flexibilities are expressed as linear functions of load, but the deflections due to Hertzian effects are not linear with load. This makes the stiffness of a gear tooth dependent on dynamic load, and Equation 19 nonlinear.

Equivalent gear tooth stiffness. The stiffnesses of the driver and driven teeth of a mesh, k_1 and k_2 respectively, are found by

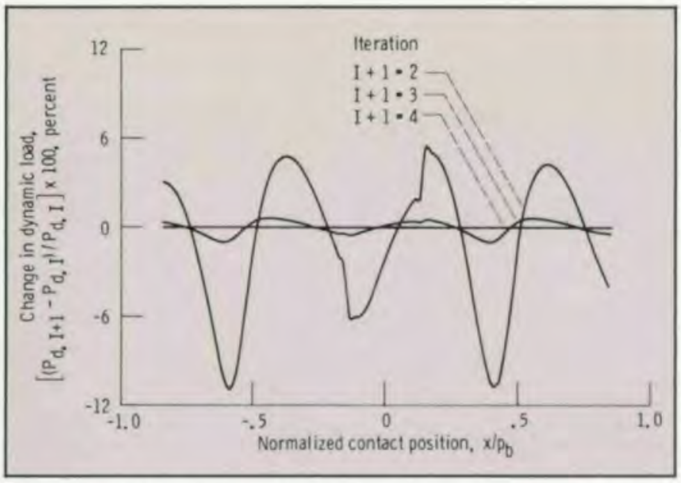


Fig. 8 – Effect of gear tooth combined stiffness-dynamic load interation scheme on dynamic load for gear data from Table I.

the methods of Cornell.[17] The combined stiffness K for a pair of teeth in contact is

$$K = \frac{k_1 k_2}{k_1 + k_2} \tag{25}$$

For single pair of teeth in contact (Fig. 6a) the equivalent stiffness is

$$K_{eq} = K \tag{26}$$

For two pairs of teeth in contact (Fig. 6b) the equivalent stiffness is

$$K_{eq} = K^{(I)} + K^{(II)} \tag{27}$$

where

$$K^{(I)} = \frac{k_1^{(I)} k_2^{(I)}}{k_1^{(I)} + k_2^{(I)}} \tag{28}$$

$$K^{(II)} = \frac{k_1^{(II)} k_2^{(II)}}{k_1^{(II)} + k_2^{(II)}} \tag{29}$$

The superscript (I) refers to the first pair of teeth in contact and (II) refers to the second pair of teeth in contact. The equivalent stiffness of Equation 19 varies from double-tooth-pair contact at the start of mesh to single-tooth-pair contact and back to double.

Iteration of gear tooth stiffness and dynamic load. Because of the Hertzian compression, gear tooth stiffness is not independent of dynamic load. TELSGE was therefore modified to iterate for dynamic load (Fig. 7). First the static load is defined. As in the example (Fig. 4c) all the load is transferred per contact during single-tooth-pair contact, and half the load is transferred per contact during double-tooth-pair contact. Next the combined stiffness is determined along the contact position by using the static load in the Hertzian deflection computation. Then the dynamic load is determined along the contact position. Next combined stiffness is recalculated by using the calculated dynamic load in the Hertzian deflection computation. Then dynamic load is

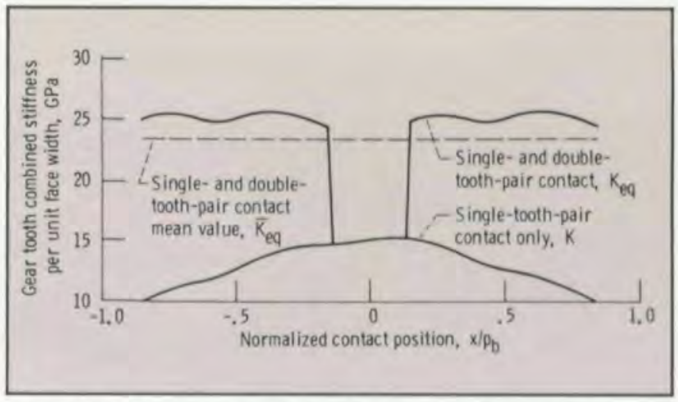


Fig. 9 – Effect of contact position on gear tooth combined stiffness for gear data from Table I.

recalculated by using the latest stiffness values. The stiffness and load calculations continue until the change in dynamic load with each iteration becomes smaller than a preset amount.

With modified TELSGE and the example data (Table I) the dynamic load required only four iterations to converge to within 0.1% (Fig. 8). So few iterations were required since the Hertizan deflection was usually only 10 to 20% of the total gear tooth deflection. The variation in equivalent stiffness due to double- and single-tooth-pair contact transitions is a major excitation in the dynamic load model (Fig. 9). The dynamic load varied appreciably from the static when the operating conditions of the example were used (Fig. 10). The maximum dynamic load during contact was about 30% greater than the static load.

Gear life using dynamic loads. The expanded gear life model, which accounts for variations of load and curvature sums with respect to contact positon, was incorporated in modified TELSGE. The dynamic loads were used in the life model, where

$$Q_i = P_{d,i}f \qquad \text{for } i = 1 \text{ to } 101 \tag{30}$$

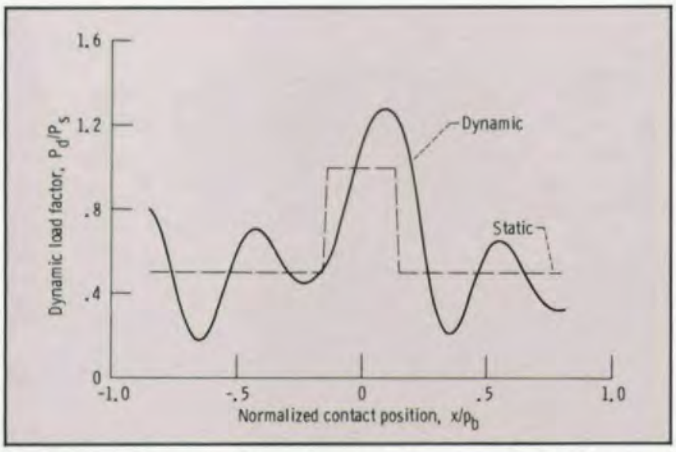


Fig. 10 – Effect of contact position on gear tooth dynamic load for gear data from Table I.

(TELSGE divides the contact length into 100 intervals.) For the data from Table I the mesh life based on dynamic loads was then 50% shorter than that based on static loads. The cause was the increase in maximum load during contact when dynamic loads were considered (Fig. 10).

RESULTS AND DISCUSSION

NASA computer program TELSGE, modified to include an improved gear tooth stiffness model, a tooth stiffness-dynamic load iteration scheme, and a pitting fatigue life prediction method was used to perform parametric studies. Dynamic loads and gear mesh life predictions were performed over a range of gear speeds, numbers of teeth, gear sizes, diametral pitches, pressure angles and gear ratios.

Effect of Speed on Dynamic Load and Life

Modified TELSGE was run using the mesh data in Table I for speeds ranging from 600 to 12000 rpm. At very low speeds the dynamic load as a function of contact position (Fig. 11) resembled the static load. However, spikes occurred at double-to single-tooth-pair contact transitions, and at single to double. As the speed increased, the dynamic load as a function of contact position differed appreciably from the static.

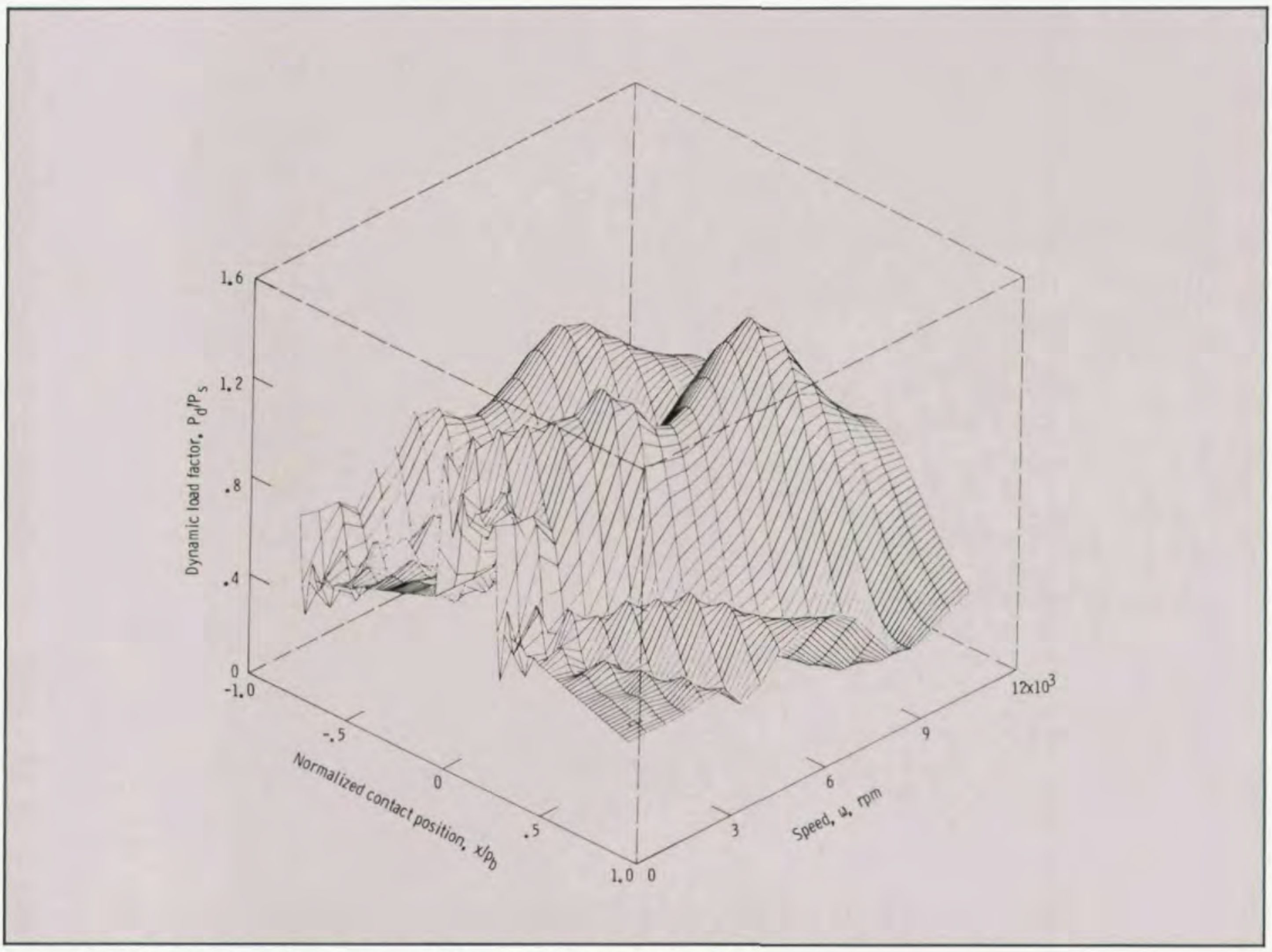


Fig. 11 – Effect of contact positon and speed on gear tooth dynamic load for gear data from Table I.

The dynamic load reached a maximum at a resonant speed ω_n of about 8500 rpm. At speeds below resonance the excitation frequency from the change in equivalent stiffness was lower than the resonant frequency, and the dynamic load was basically an oscillatory load superimposed on the static load. This produced peak dynamic loads greater than the static load. At speeds above resonance the dynamic load had a smoother response, with peaks lower than the static. This was caused by the greater inertia forces at higher speeds. The resonant speed can be approximated by

$$\omega_n = \frac{\sqrt{\bar{K}_{eq}/M_{eq}}\,\cos\varphi}{N}\left(\frac{60}{2\pi}\right) \tag{31}$$

Here, although the mean equivalent stiffness K_{eq} varies with load and speed due to Hertzian effects, its influence on ω_n is not significant.

For the data in Fig. 11 the maximum dynamic load during contact was greatest at the resonant speed (Fig. 12). It was also greater than the static load at speeds below resonance, with a secondary peak at about $\omega/\omega_n = 0.5$. At speeds above resonance the maximum dynamic load during contact decreased and was less than the static load above $\omega/\omega_n \approx 1.2$.

The gear mesh life as a function of speed for mesh data in Table I is shown in Fig. 13. The dynamic life factor is defined as

$$C_v = \frac{L_d}{L_s} \tag{32}$$

where L_d is the gear mesh life based on the expanded life theory and dynamic loads, and L_s is the gear mesh life based on the expanded life theory and static loads (as illustrated in Fig. 4). Comparing Figs. 12 and 13 shows that the gear mesh life decreased when the maximum dynamic load during contact increased. This was true even though the analysis considered the load along the complete contact length. The mesh life as a function of speed was lowest at resonance.

Effect of Mass, Stiffness and Damping on Gear Life

The mass, stiffness and damping of a gear mesh system significantly affected dynamic load and life. Modified TELSGE was run using the mesh data in Table I while varying the

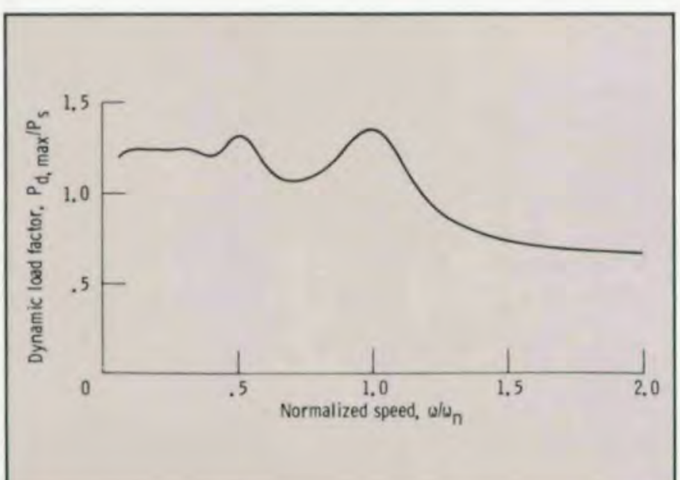


Fig. 12 – Effect of speed on maximum gear tooth dynamic load during contact for gear data from Table I.

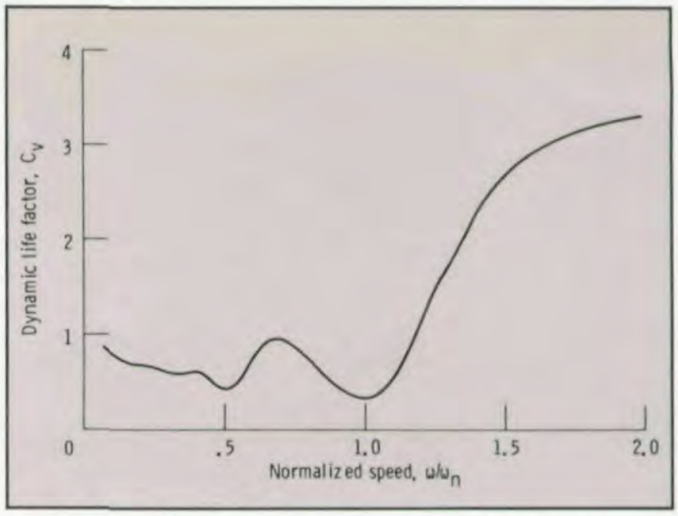


Fig. 13 – Effect of speed on gear mesh life for gear data from Table I.

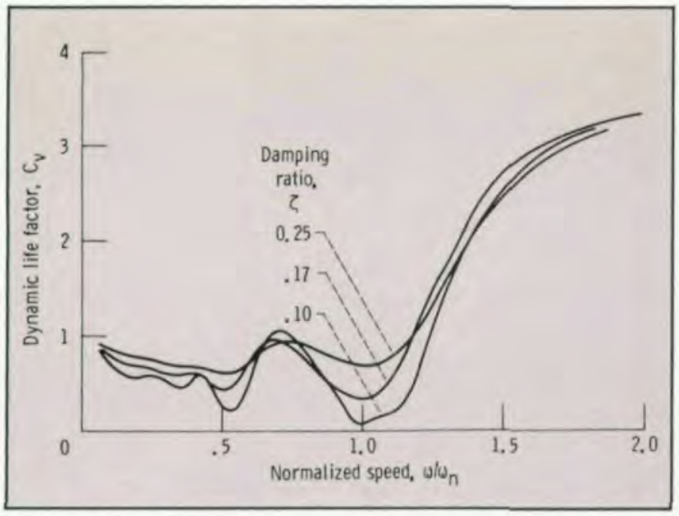


Fig. 14 – Effect of damping ratio on gear mesh life for gear data from Table I.

equivalent mass M_{eq} and keeping all other parameters the same. The life-speed results were identical when plotted on dimensionless coordinates (as in Fig. 13). Modified TELSGE was also run while varying the equivalent stiffness K_{eq} and keeping all other parameters the same. Again, the life-speed results were identical when plotted on dimensionless coordinates (as in Fig. 13). Thus, the value of the equivalent mass or the equivalent stiffness had no effect on the life-speed results when plotted on dimensionless coordinates. However, as expected from Equation 31, different values of the equivalent mass or the equivalent stiffness produced different values for the resonant speed. The equivalent mass and equivalent stiffness must accurately portray the gear mesh being modeled for the calculated resonant speed to be accurate.

The damping force in the dynamic load model depends on the gear system's viscous friction and is usually an unknown. Damping ratios ζ between 0.1 (in Equation 23) and 0.2 were used in Reference 11 to correlate analytical and experimental dynamic load gear tests. Here damping ratios of 0.10, 0.17 and 0.25 were used (Fig. 14). Decreasing the damping ratio increased the dynamic load and, thus, shortened the mesh life at speeds near the resonant speed and one-half the resonant speed ($\omega/\omega_n = 1.0$ and 0.5, respectively). A damping ratio of 0.17 was used in the original version of TELSGE and was used in this study for all other figures.

Effect of Speed and Contact Ratio on Gear Life

Modified TELSGE was used to predict how speed and contact ratio affect dynamic load and gear life. Number of teeth, gear size, diametral pitch, pressure angle and gear ratio were varied. The driver gear data for the different runs are shown in Table II. The different sets had basically the same shape while displaced upward or downward when plotted on dimensionless life-speed coordinates (Fig. 15). In most sets the mesh life was shortest at the resonant speed or one-half the resonant speed and was significantly shorter than the life based on static loads at those speeds. For all sets meshes operating above resonance had

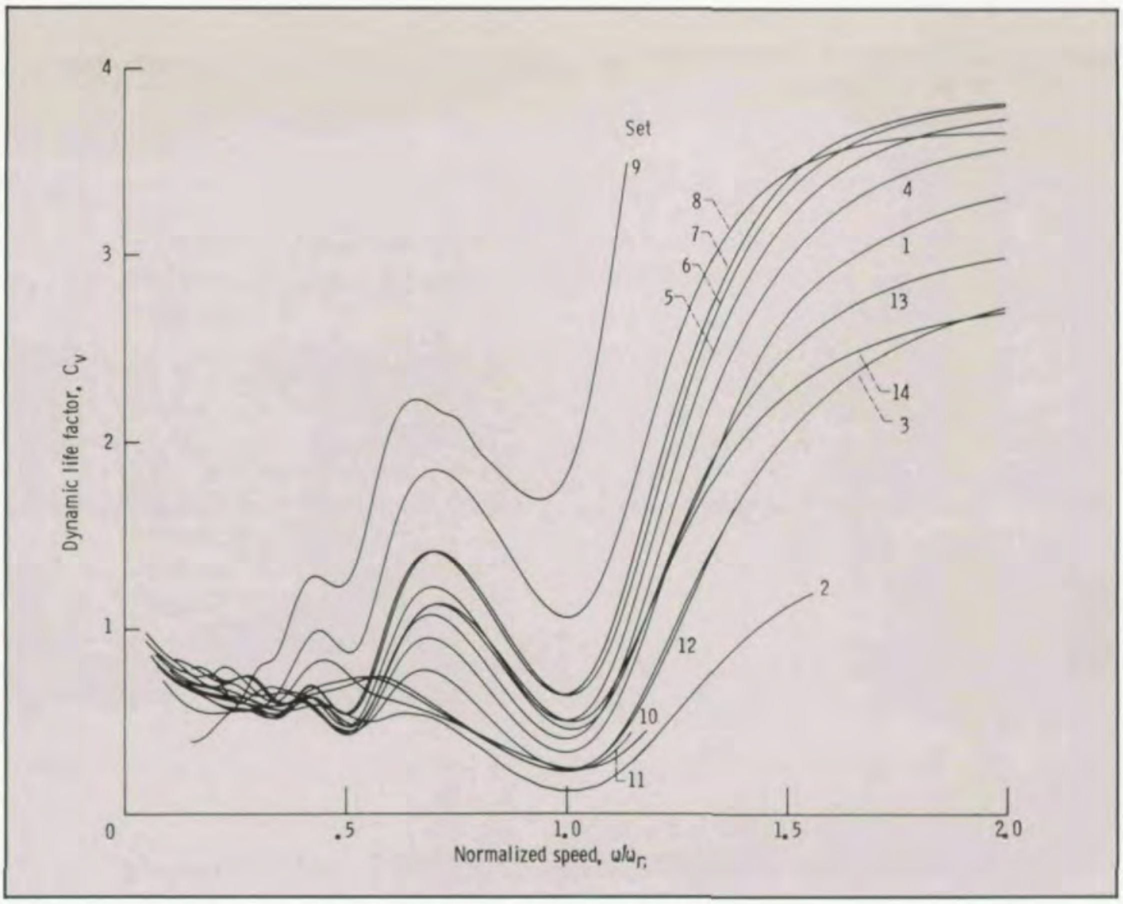


Fig. 15 – Effect of speed on gear mesh life for parametric study data from Table II.

Table II. Driver Gear Data
(Set 1 used for baseline; shaded area indicates parameter varied from baseline.)

Set	Number of teeth	Pitch radius, cm	Diametral pitch	Pressure angle, deg	Gear ratio	Contact ratio
1	36	5.715	8	20	1	1.69
2	20	3.175				1.56
3	28	4.445				1.64
4	44	6.985				1.73
5	52	8.255				1.76
6	60	9.525				1.78
7	66	6.985	12			1.80
8	99	6.985	18			1.85
9	28	4.445	8	14.5		1.92
10	20	3.175		25		1.41
11	28	4.445		25		1.46
12	36	5.715		25		1.50
13	36	5.715		20	2	1.75
14	36	5.715		20	3	1.78

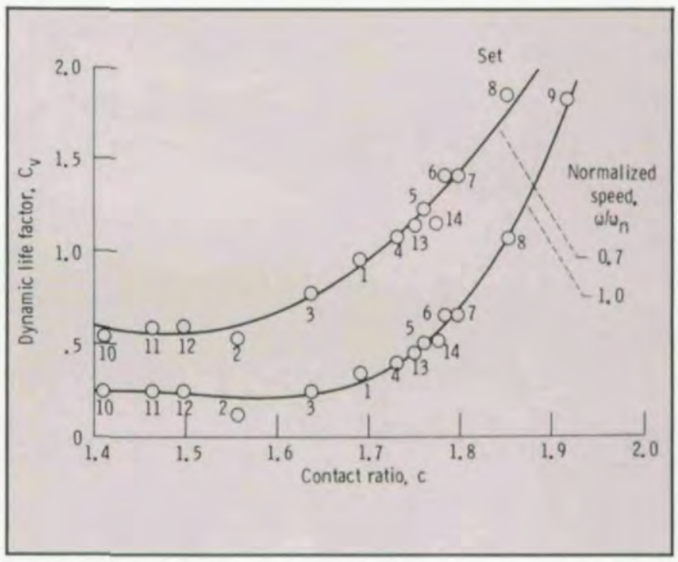


Fig. 16 – Effect of contact ratio on gear mesh life for parametric study data from Table II.

significantly longer life when compared with the static load calculations.

The contact ratio c, defined as the average number of teeth pairs in contact, is given by

$$c = \frac{Z}{p_b} \tag{33}$$

For a mesh with a contact ratio of 1.6, two pairs of teeth are in contact 60% of the time, and one pair is in contact 40% of the time. Low-contact-ratio gears have contact ratios between 1 and 2. In the parametric studies the contact ratio ranged from 1.41 to 1.92.

For the data in Fig. 15 the dynamic life factor was plotted as a function of contact ratio in Fig. 16 for speeds ω/ω_n of 0.7 and

1.0. A sixth-order polynomial curve-fit was used to generate the curves. At a constant normalized speed the dynamic life factors were about the same for meshes with contact ratios between 1.4 and 1.6, but were significantly higher for meshes with higher contact ratios.

With higher contact ratios the equivalent stiffness (Fig. 9) had a smaller duration of single-tooth-pair contact and, thus, a smoother transition of double- to single- to double-tooth-pair contact. This resulted in lower dynamic load factors and higher dynamic life factors. For the sets studied, the resonant speed varied with equivalent mass, mean equivalent stiffness, pressure angle and number of teeth.

A general design chart for the dynamic life factor of a gear mesh was developed from the parametric studies (Fig. 17). The objective was to determine the dynamic life factor as a single simple function of speed and contact ratio to be used when a computer and program TELSGE are not available. The heavy solid line represents the best fit of the results of the parametric studies. For $\omega/\omega_n \leqslant 0.5$ the dynamic life factor can be read directly from the plot by using the scale on the left. For $\omega/\omega_n > 0.5$ the dynamic life factor is the product of the value of the curve (using the scale on the right) and the contact ratio to the sixth power. The light dotted lines represent the actual results of the parametric studies and indicate the possible error when using the chart. An example problem given in Appendix A demonstrates the use of the design chart. A simplified hand calculation of gear tooth stiffness is also given in Appendix A.

SUMMARY OF RESULTS

How dynamic load affects the pitting fatigue life of external spur gears was predicted by using a modified version of the NASA computer program TELSGE to perform parametric studies. TELSGE was modified to include a surface pitting fatigue life analysis. The parametric studies modeled low-contact-ratio involute gears with no tooth spacing or profile errors. The following results were obtained:

1. Gear life predictions based on dynamic loads generally differed significantly from those based on static loads and were strongly influenced by the maximum dynamic load during contact.

2. Gear mesh operating speeds strongly affected predicted dynamic loads and, thus, gear life. In most cases studied, meshes operating at a resonant speed or one-half the resonant speed had significantly shorter lives than the life based on static loads. Meshes operating above resonance had significantly longer lives.

3. In general, meshes with higher contact ratios had higher predicted dynamic life factors than meshes with lower contact ratios.

4. Damping significantly affected predicted gear mesh life for meshes operating at or near a resonant speed or one-half the resonant speed.

5. A solution for dynamic load converged with only a few iterations of gear tooth stiffness and dynamic load because the Hertzian deflection was relatively small in comparison with the total gear tooth deflection.

Consistent Quality — Unbeatable Value

That is why Overton Gear & Tool grinds with Okamoto Gear Grinders

"The Okamoto gear grinder has stood the test with Overton Gear. The machine has proven to be dependable and accurate. Service is outstanding. No question, we made the right choice. We have one more on order — a new CNC HTG-24 High Tech Form Gear Grinder."

Kevin Walsh, Dept. Foreman, Carl Overton, Chairman, and Tony Waitekus, President.

Overton
Gear and Tool Corporation

Designed, Built and Sold by the Group That Can Solve Your Gear Grinding Problems

PMT

PEREZ MACHINE TOOL CO.

II GINGER CT. · E. AMHERST, NEW YORK 14051 · TELEPHONE 716-688-6982

Exclusive U.S.A. & Canadian Distributor of Okamoto Gear Grinders GTP-200 & GTR Series Gear Tooth Chamfering Machines

CIRCLE A-12 ON READER REPLY CARD

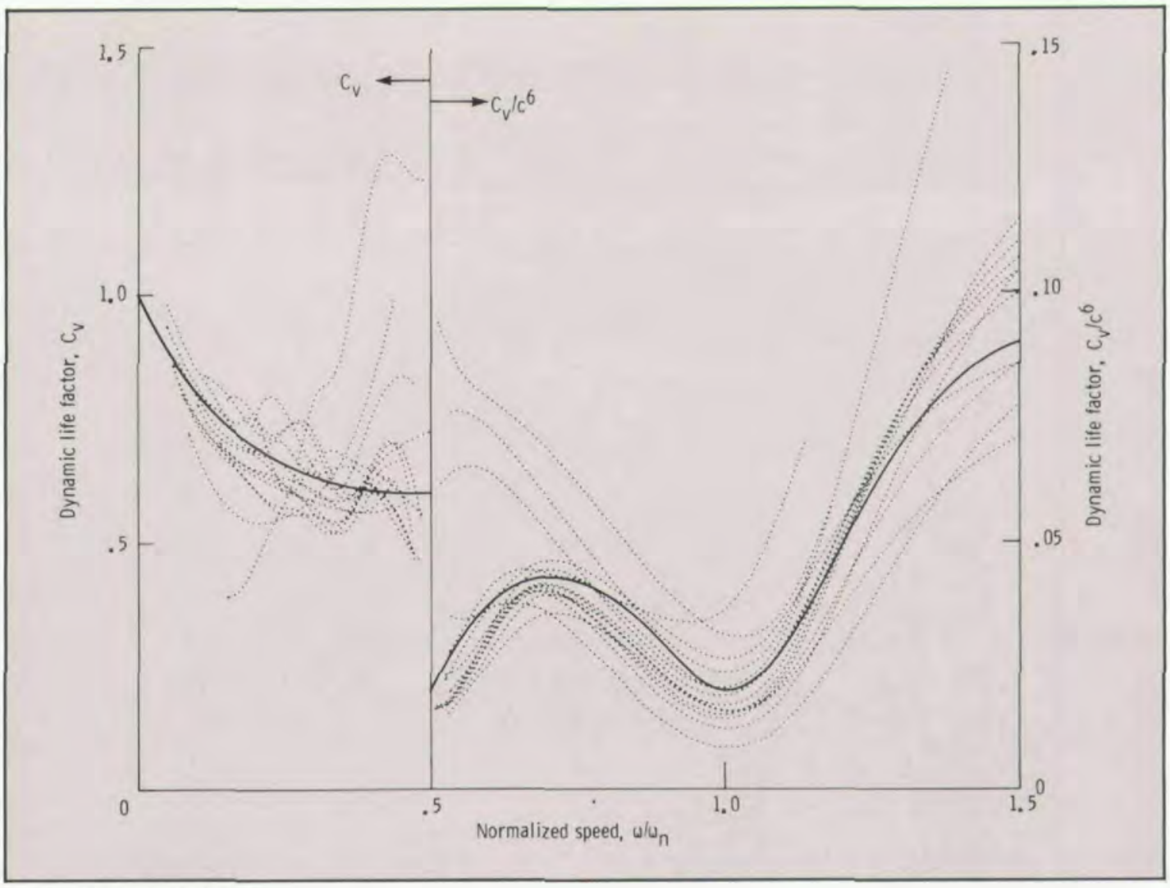


Fig. 17 – Dynamic life factor.

APPENDIX A
Example Problem

Example problem. Determine the dynamic life factor of the mesh from the data given in Table III when the driver gear is rotating at 5000 rpm.

Solution. The pitch radii are

$$r_{p,1} = \frac{1}{2}N_1 m_o = \frac{1}{2}(32 \text{ teeth})\left(4.233 \text{ mm/tooth} \times \frac{1 \text{ cm}}{10 \text{ mm}}\right)$$
$$= 6.773 \text{ cm } (2.667 \text{ in.})$$

$$r_{p,2} = \frac{1}{2}(100 \text{ teeth})\left(4.233 \text{ mm/tooth} \times \frac{1 \text{ cm}}{10 \text{ mm}}\right)$$
$$= 21.165 \text{ cm } (8.333 \text{ in.})$$

Table III. Gear Mesh Data Used In Dynamic Life Factor Example Problem

Parameter	Driver gear	Driven gear
Number of teeth	32	100
Outside radius, cm (in.)	7.196 (2.833)	21.590 (8.500)
Root radius, cm (in.)	6.246 (2.459)	20.638 (8.125)
Lewis form factor	0.433	0.521
Module, mm/tooth (Pitch, teeth/in.)		4.233 (6)
Face width, cm (in.)		6.350 (2.500)
Pressure angle, deg (rad)		25 (0.436)
Tooth thickness at pitch radius, cm (in.)		0.665 (0.262)
Modulus of elasticity, Pa (psi)		2.068×10^{11} (30×10^6)
Density, kg/m^3 (lb/in.^{3})		7833 (0.283)

The base radii are

$$r_{b,1} = r_{p,1} \cos\varphi = (6.773 \text{ cm}) \cos 25° = 6.138 \text{ cm } (2.417 \text{ in.})$$

$$r_{b,2} = (21.165 \text{ cm}) \cos 25° = 19.182 \text{ cm } (7.552 \text{ in.})$$

From Equations 5 and 6 the contact lengths from the pitch point to the start and end of contact are

$$z_1 = \sqrt{(21.590 \text{ cm})^2 - (19.182 \text{ cm})^2}$$
$$- (21.165 \text{ cm}) \sin 25° = 0.964 \text{ cm } (0.379 \text{ in.})$$

$$z_2 = \sqrt{(7.196 \text{ cm})^2 - (6.138 \text{ cm})^2}$$
$$- (6.773 \text{ cm}) \sin 25° = 0.894 \text{ cm } (0.351 \text{ in.})$$

From Equation 7 the contact length is

$$Z = (0.964 \text{ cm}) + (0.894 \text{ cm}) = 1.858 \text{ cm } (0.730 \text{ in.})$$

The base pitch is

$$p_b = \frac{2\pi r_{b,1}}{N_1} = \frac{2\pi(6.138 \text{ cm})}{32 \text{ teeth}} = 1.205 \text{ cm } (0.475 \text{ in.})$$

From Equation 33 the contact ratio is

$$c = \frac{1.858 \text{ cm}}{1.205 \text{ cm}} = 1.54$$

The masses per unit face width of the driver and driven gears can be approximated by

$$m_1 = \gamma \pi r_{p,1}^2 = (7833 \text{ kg/m}^3)\pi\left(6.773 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}\right)^2$$

$$= 112.886 \text{ kg/m } (1.637 \times 10^{-2} \text{ lb sec}^2/\text{in.}^2)$$

$$m_2 = (7833 \text{ kg/m}^3)\pi\left(21.165 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}\right)^2$$

$$= 1102.337 \text{ kg/m } (1.598 \times 10^{-1} \text{ lb sec}^2/\text{in.}^2)$$

The effective masses per unit face width are

$$M_1 = \frac{J_1}{r_{b,1}^2} = \frac{\frac{1}{2} m_1 r_{b,1}^2}{r_{b,1}^2} = \frac{1}{2} m_1 = \frac{1}{2}(112.886 \text{ kg/m})$$

$$= 56.443 \text{ kg/m } (8.185 \times 10^{-3} \text{ lb sec}^2/\text{in.}^2)$$

$$M_2 = \frac{1}{2}(1102.337 \text{ kg/m}) = 551.169 \text{ kg/m}$$

$$(7.990 \times 10^{-2} \text{ lb sec}^2/\text{in.}^2)$$

From Equation 22 the equivalent mass per unit face width is

$$M_{eq} = \frac{(56.443 \text{ kg/m})(551.169 \text{ kg/m})}{(56.443 \text{ kg/m}) + (551.169 \text{ kg/m})}$$

$$= 51.200 \text{ kg/m } (7.424 \times 10^{-3} \text{ lb sec}^2/\text{in.}^2)$$

Determining teeth stiffnesses by the methods of Cornell[17] requires the use of a computer. For this example the stiffness calculations will be simplified by modeling the gear teeth as cantilever beams of uniform strength (beams in which the section modulus varies along the beam in the same porportion as the bending moment). The pressure angles at the root radii of the driver and driven gears (Fig. 18) are

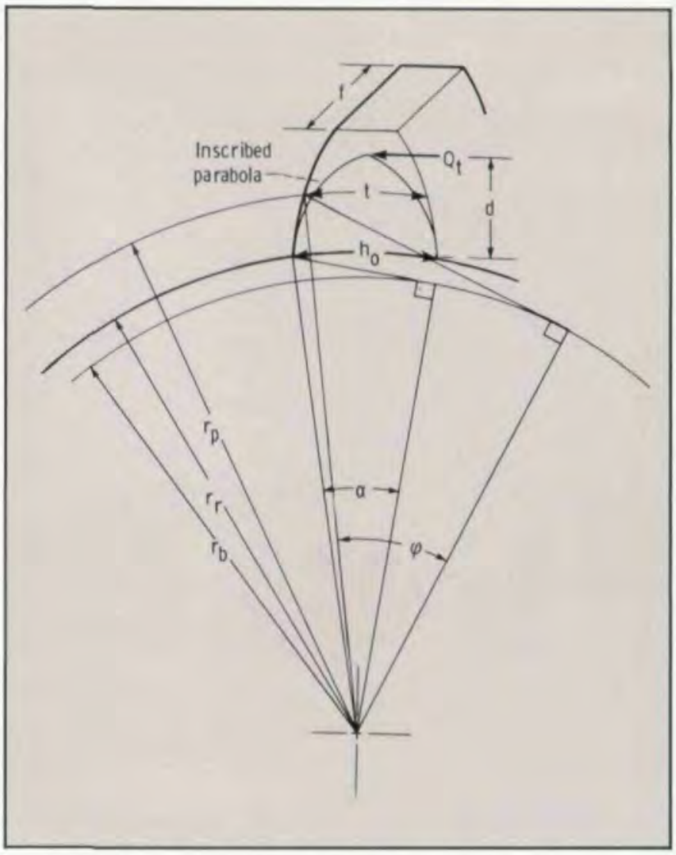


Fig. 18 – Gear tooth model for simplified stiffness calculations.

$$\alpha_1 = \cos^{-1}\left(\frac{r_{b,1}}{r_{r,1}}\right) = \cos^{-1}\left(\frac{6.138 \text{ cm}}{6.246 \text{ cm}}\right)$$

$$= 10.670° = 0.186 \text{ rad}$$

$$\alpha_2 = \cos^{-1}\left(\frac{19.182 \text{ cm}}{20.638 \text{ cm}}\right) = 21.651° = 0.378 \text{ rad}$$

From Reference 22 the tooth thickness at the pitch radius is related to the tooth thickness at the root radius by

$$t = 2r_p\left[\frac{h_0}{2r_r} + (\tan\alpha - \alpha) - (\tan\varphi - \varphi)\right]$$

Therefore the teeth thickness at the root radii are

$$h_{0,1} = 2r_{r,1}\left[\frac{t}{2r_{p,1}} - (\tan\alpha_1 - \alpha_1) + (\tan\varphi - \varphi)\right]$$

$$= 2(6.246 \text{ cm})\left[\frac{0.665 \text{ cm}}{2(6.773 \text{ cm})} - (\tan 0.186 - 0.186) + (\tan 0.436 - 0.436)\right]$$

$$= 0.960 \text{ cm } (0.378 \text{ in.})$$

$$h_{0,2} = 2(20.638 \text{ cm})\left[\frac{0.665 \text{ cm}}{2(21.165 \text{ cm})} - (\tan 0.378 - 0.378) + (\tan 0.436 - 0.436)\right]$$

$$= 1.095 \text{ cm } (0.431 \text{ in.})$$

From Reference 23 and Fig. 18 the distance of the inscribed parabola is

$$d = \frac{h_0^2}{6m_oY}$$

where Y is the Lewis form factor. Thus the distances of the inscribed parabolas are

$$d_1 = \frac{h_{0,1}^2}{6m_oY_1} = \frac{(0.960 \text{ cm})^2}{6\left(4.233 \text{ mm/tooth} \times \frac{1 \text{ cm}}{10 \text{ mm}}\right)(0.433)}$$

$$= 0.838 \text{ cm } (0.330 \text{ in.})$$

$$d_2 = \frac{(1.095 \text{ cm})^2}{6\left(4.233 \text{ mm/tooth} \times \frac{1 \text{ cm}}{10 \text{ mm}}\right)(0.521)}$$

$$= 0.906 \text{ cm } (0.357 \text{ in.})$$

The inscribed parabola in Fig. 18 is a cantilever beam of uniform strength. From Reference 24 the deflection for the beam is

$$\delta = \frac{2Q_td^3}{3EI_0} \qquad \text{where } I_0 = \frac{1}{12}fh_0^3$$

The gear tooth stiffness per unit face width is

$$k = \frac{Q_t}{\delta f} = \frac{3EI_0}{2d^3f} = \frac{Eh_0^3}{8d^3}$$

For the driver and driven gears, respectively,

$$k_1 = \frac{Eh_{0,1}^3}{8d_1^3}\ \frac{(2.068 \times 10^{11} \text{ Pa})\left(0.960 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}\right)^3}{8\left(0.838 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}\right)^3}$$

$$= 3.886 \times 10^{10} \text{ Pa } (5.636 \times 10^6 \text{ psi})$$

$$k_2 = \frac{(2.068 \times 10^{11} \text{ Pa})\left(1.095 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}\right)^3}{8\left(0.916 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}\right)^3}$$

$$= 4.564 \times 10^{10} \text{ Pa } (6.599 \times 10^6 \text{ psi})$$

From Equation 25 the combined stiffness per unit face width is

$$K = \frac{(3.886 \times 10^{10} \text{ Pa})(4.564 \times 10^{10} \text{ Pa})}{(3.886 \times 10^{10} \text{ Pa}) + (4.564 \times 10^{10} \text{ Pa})}$$

$$= 2.099 \times 10^{10} \text{ Pa } (3.040 \times 10^6 \text{ psi})$$

For this example it is assumed that the combined stiffness is constant with respect to contact position. During single-tooth-pair contact the equivalent stiffness is K. During double-tooth-pair contact the equivalent stiffness is $2K$. The mean equivalent stiffness per unit face width (Fig. 19) is given as

$$\bar{K}_{eq} = \frac{1}{Z}\int_0^Z K_{eq}\,dx$$

$$= \frac{1}{Z}\left(\int_0^{Z-p_b} K_{eq}\,dx + \int_{Z-p_b}^{p_b} K_{eq}\,dx + \int_{p_b}^{Z} K_{eq}\,dx\right)$$

$$= \frac{1}{Z}\left\{2K\left[(Z - p_b) - 0\right] + K\left[p_b - (Z - p_b)\right] + 2K\left[Z - p_b\right]\right\}$$

$$= K\left[3 - 2\left(\frac{p_b}{Z}\right)\right] = K\left(3 - \frac{2}{c}\right)$$

$$= (2.099 \times 10^{10} \text{ Pa})\left(3 - \frac{2}{1.54}\right)$$

$$= 3.571 \times 10^{10} \text{ Pa } (5.172 \times 10^6 \text{ psi})$$

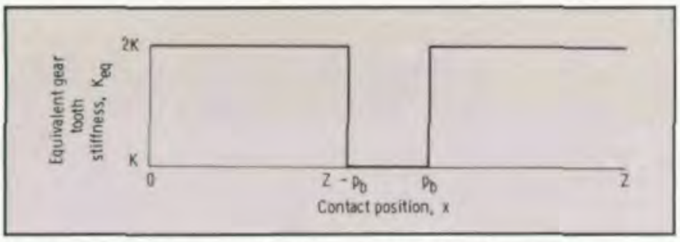


Fig. 19 – Equivalent gear tooth stiffness as a function of contact position.

From Equation 31 the resonant speed in terms of driver gear rotations is

$$\omega_n = \frac{\sqrt{\frac{\bar{K}_{eq}}{M_{eq}}}\cos\varphi}{N_1}\left(\frac{60}{2\pi}\right)$$

$$= \frac{\sqrt{\frac{3.571 \times 10^{10}\ \text{Pa}}{51.200\ \text{kg/m}}}\cos 25^\circ}{32\ \text{teeth}}\left(\frac{60}{2\pi}\right) = 7143\ \text{rpm}$$

At a driver operating speed of 5000 rpm

$$\frac{\omega}{\omega_n} = \frac{5000\ \text{rpm}}{7143\ \text{rpm}} = 0.70$$

From Fig. 17 for $\omega/\omega_n = 0.70$

$$\frac{C_v}{c^6} = 0.04$$

and the dynamic life factor is

$$C_v = 0.04\ c^6 = 0.04\ (1.54)^6 = 0.53$$

Thus about a 50% decrease in life compared with that using static loads is predicted for this example. Note that the simplified stiffness model used in the example may produce erroneous values for the resonant speed. The mean value of the equivalent stiffness per unit face width for this example was computed by using Cornell's method[17] and TELSGE, as 2.741×10^{10} Pa (3.975×10^6 psi). This produced a resonant speed of 6260 rpm and a dynamic life factor of 0.56.

References:

1. Surface Durability (Pitting) of Spur Gear Teeth. The American Gear Manufacturers Association (AGMA) Standard 210.02, Jan. 1965.
2. COY, J.J. TOWNSEND, D.P. and ZARETSKY, E.V. "Analysis of Dynamic Capacity of Low-Contact-Ratio Spur Gears Using Lundberg-Palmgren Theory." NASA TN D-8029, 1975.
3. TOWNSEND, D.P. COY, J.J. and ZARETSKY, E.V. "Experimental and Analytical Load-Life Relation for AISI 9310 Steel Spur Gears." *J. Mech. Des.*, Vol. 100, No. 1, Jan. 1978, pp. 54-60.
4. COY, J.J. and ZARETSKY, E.V. "Life Analysis of Helical Gear Sets Using Lundberg-Palmgren Theory." NASA TN D-8045, 1975.
5. COY, J.J. TOWNSEND, D.P. and ZARETSKY, E.V. "Dynamic Capacity and Surface Fatigue Life for Spur and Helical Gears." *J. Lubr. Technol.*, Vol. 98, No. 2, Apr. 1976, pp. 267-276.
6. COY, J.J. TOWNSEND, D.P. and ZARETSKY, E.V. "An Update on the Life Analysis of Spur Gears." *Advanced Power Transmission Technology*, NASA CP-2210, G.K. Fischer, ed., 1983, pp. 421-434.
7. BUCKINGHAM, E. *Analytical Mechanics of Gears*. McGraw-Hill, 1949.
8. TUPLIN, W.A. "Dynamic Loads on Gear Teeth." *Proceedings, International Confernece on Gearing*, Institution of Mechanical Engineers, London, 1958, pp. 24-30.
9. RICHARDSON, HERBERT HEATH. *Static and Dynamic Load, Stresses and Deflection Cycles in Spur Gear Systems*. Ph.D. Thesis, Massachusetts Institute of Technology, 1958.
10. ATTIA, A.Y. "Dynamic Loading on Spur Gear Teeth." *J. Eng. Ind.*, Vol. 81, No. 1, Feb. 1959, pp. 1-9.
11. ICHIMARU, K. and HIRANO, F. "Dynamic Behavior of Heavy-Loaded Spur Gears." *J. Eng. Ind.*, Vol. 96, No. 2, May 1974, pp. 373-381.
12. CORNELL, R.W. and WESTERVELT, W.W. "Dynamic Tooth Loads and Stressing for High Contact Ratio Spur Gears." *J. Mech. Des.*, Vol. 100, No. 1, Jan 1978, pp. 69-76.
13. KASUBA, R. and EVANS, J.W. "An Extended Model for Determining Dynamic Loads in Spur Gears." *J. Mech. Des.*, Vol. 103, No. 2 Apr. 1981, pp. 398-409.
14. WANG, K.L. and CHENG, H.S. "Thermal Elastohydrodynamic Lubrication of Spur Gears." NASA CR-3241, 1980.
15. WANG, K.L. and CHENG, H.S. "A Numerical Solution to the Dynamic Load, Film Thickness, and Surface Temperatures in Spur Gears Part I, Analysis." *J. Mech. Des.*, Vol. 103, No. 1, Jan. 1981, pp. 177-187.
16. WANG, K.L. and CHENG, H.S. "A Numerical Solution to the Dynamic Load, Film Thickness, and Surface Temperatures in Spur Gears. Part II, Results." *J. Mech. Des.*, Vol. 103, No. 1, Jan. 1981, pp. 188-194.
17. CORNELL, R.W. "Compliance and Stress Sensitivity of Spur Gear Teeth." *J. Mech. Des.*, Vol. 103, No. 2, Apr. 1981, pp. 447-459.
18. LUNDBERG, G. and PALMGREN, A. "Dynamic Capacity of Rolling Bearings," *Acta Polytechnica, Mechancial Engineering Series*, Vol. 1, No. 3, 1947.
19. LUNDBERG, G. and PALMGREN, A. "Dynamic Capacity of Rolling Bearings," *J. Appl. Mech.*, Vol. 16, No. 2, June 1949, pp. 165-172.
20. LUNDBERG, G. and PALMGREN, A. "Dynamic Capacity of Roller Bearings." *Acta Polytechnica, Mechanical Engineering Series*, Vol. 2, No. 4, 1952.
21. LEWICKI, D.G., et al. "Fatigue Life Analysis of a Turborprop Reduction Gearbox." *Journal of Mechanisms, Transmissions, and Automation in Design*. Vol. 108, No. 2, June 1986, pp. 255-262.
22. KHIRALLA, T.W. "On the Geometry of External Involute Spur Gears." *GEARS*, North Hollywood, 1976.
23. DEUTSCHMAN, A.D. MICHELS, W.J. and WILSON, C.E. *Machine Design – Theory and Practice*. Macmillan, 1975.
24. TIMOSHENKO, S. *Strength of Materials*. 3rd ed., Van Nostrand, 1955.

Acknowledgement: *Originally appeared as NASA Technical Paper 2610, AVSCOM Technical Report 86-C-21, June, 1986.*

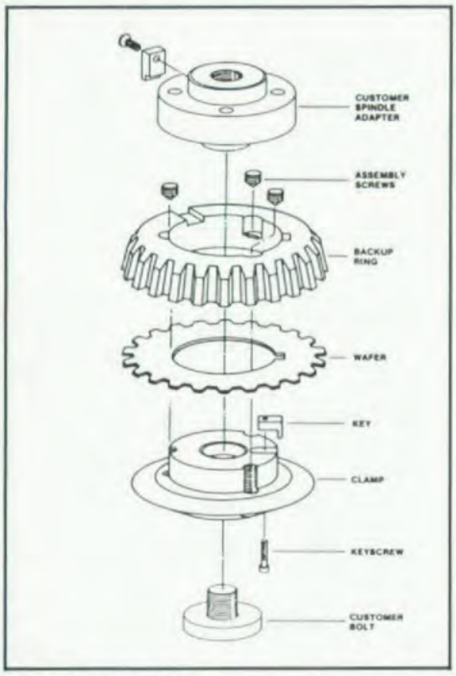


Fig. 1 – Exploded view of wafer cutter assembly.

The Wafer Shaper Cutter

Edward Haug
Pfauter-Maag Cutting Tools
Loves Park, IL

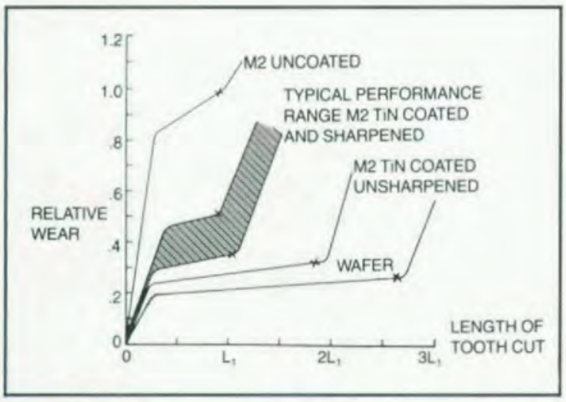


Fig. 2 – Comparison of wear characteristics of uncoated, coated and sharpened, and completely coated shaper cutters to a wafer shaper cutter.

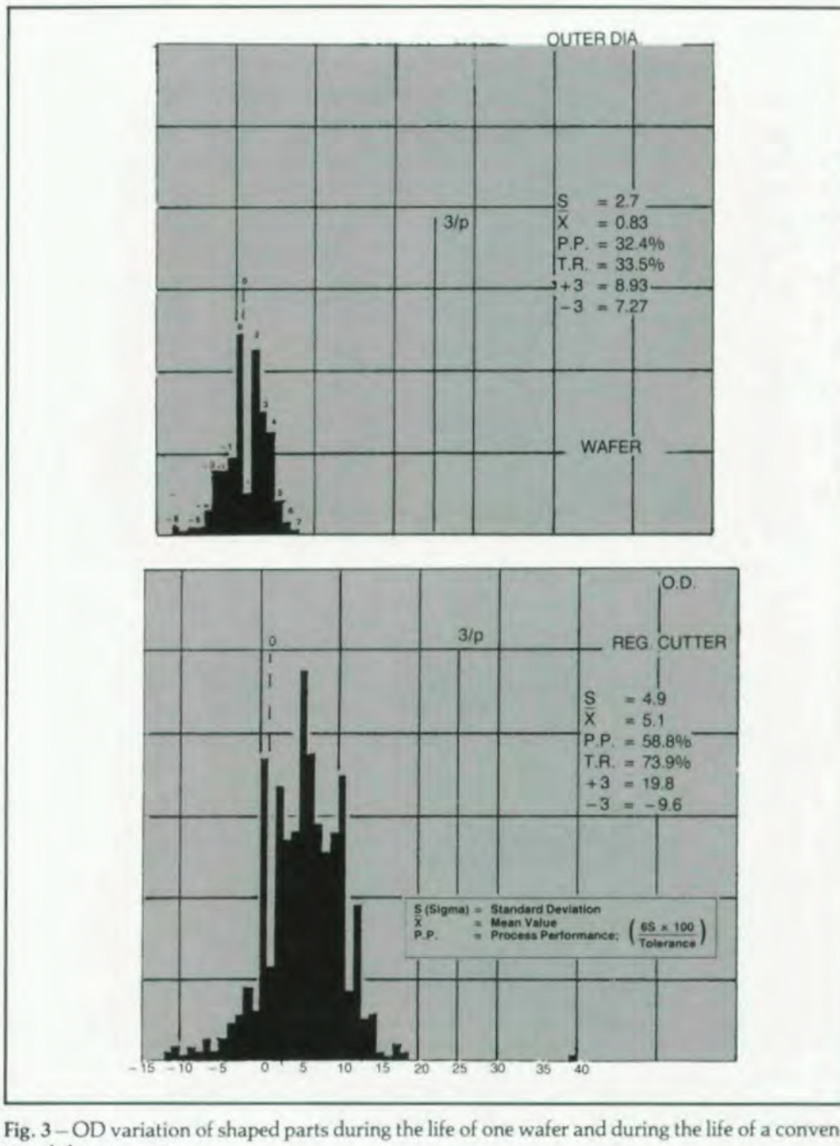


Fig. 3 – OD variation of shaped parts during the life of one wafer and during the life of a conventional shaper cutter.

In 1985 a new tooling concept for high volume gear production was introduced to the gear manufacturing industry. Since then this tool, the wafer shaper cutter, has proven itself in scores of applications as a cost-effective, consistent producer of superior quality parts. This report examines the first high-production installation at the plant of a major automotive supplier, where a line of twenty shapers is producing timing chain sprockets.

What is a Wafer Shaper Cutter?

It is a shaper cutting tool assembly, (Fig. 1) which directly replaces a conventional shaper cutter. One of its main features is that it does not have to be resharpened when worn. Rather, its cutting face is reconstituted to its original accuracy by replacement of a gear-like cutting wafer; consequently, tool mainte-

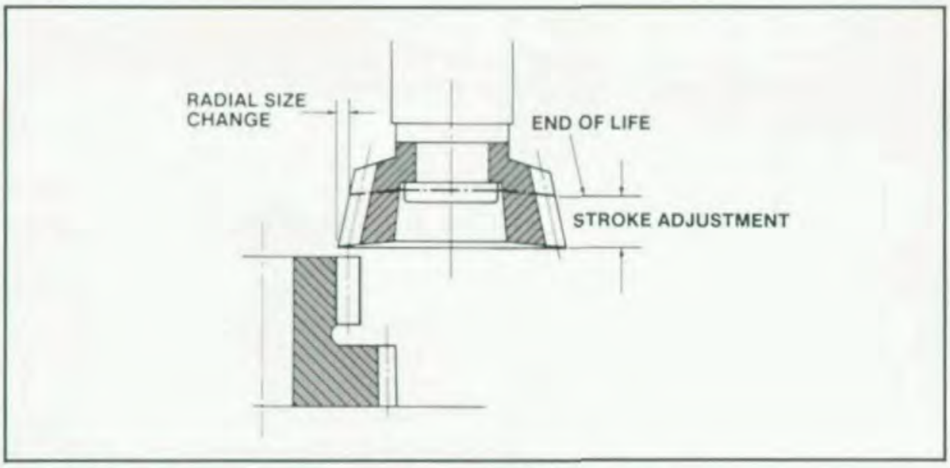


Fig. 4 – Adjustment requirements in the use of conventional shaper cutters.

Fig. 6 – Easy assembly and disassembly using three screws.

Fig. 5 – Reblading area in clean environment.

nance costs are reduced and sharpening errors are eliminated. In other words, the quality of the tooth cutting edge remains the ongoing responsibility of the tool supplier.

While the geometry of conventional shaper cutters has always been a compromise between tool sharpening life and cutting clearances, no such restrictions apply to the design of cutting wafers. This latitude permits the development of the optimum wafer tooth geometry with the most favorable cutting clearance angles tailored to every specific application.

Another reason for the superior wear characteristics of wafers is the titanium nitride (TiN) coating, which is applied to all the cutting surfaces and which has to meet stringent adherence and thickness specifications.

It is this combination of premium coating, optimum tooth geometry and cutting edge preparation, which enables a wafer shaper cutter to outperform even a new, completely coated conventional shaper cutter. It is common knowledge, of course, that a further tool life decrease occurs once the conventional cutter has been sharpened and the TiN coating has been removed from the cutting face. (Fig. 2)

The wafer material is M2 high speed steel, hardened to 64-66 Rc. Tests have shown that using premium high speed steels is of little benefit, since the superior wafer performance is the result of the TiN coating applied to all the cutting surfaces. The typical cratering wear, prevalent on the cutting face of sharpened shaper cutters, is either eliminated or greatly inhibited on wafer cutters through the high hardness and the low coefficient of friction of the TiN coating.

Part Quality

The original wafer shaper cutter development was based on the assumption that the elimination of sharpening and the increased tool life were going to be the two major benefits the new tool had to offer. The results after extensive testing lead to the conclusion, however, that consistent and improved part quality and the elimination of scrap are the most important factors for wafer cutter justification.

Statistical process control (SPC) studies have shown that wafer tools consistently produce parts with smaller deviations within the statistical population than do conventional tools. (Fig. 3)

There are many reasons. One is the tight size tolerance and accuracy to which all wafers are manufactured. There are no tool diameter and tooth profile variations, as is the case within the sharpening life of conventional cutters, and there are no sharpening errors.

A second reason is the elimination of all machine adjustments, which are usually required after a conventional cutter has been sharpened, and which traditionally have been a major source for part variations and scrap. (Fig. 4) In this first production application the fact has been substantiated that once a job is set up, wafer tools produce the correct part size on an ongoing basis. There is no longer any downtime for part qualification, usually required after the change of a conventional cutter.

Economics

Before this first production line was committed to wafer shaper cutters, extensive cost, SPC and method studies were conducted during a pilot program. Over a four month period it was documented that the wafer shaper cutter would

- reduce tool costs by 39%
- reduce scrap by 70%
- reduce manpower in the shaping department by 60%
- save 4 man-hours per day by eliminating the sharpening operation.

One of the major underlying factors for these savings is the increased machine uptime brought about by fewer tool changes. While a conventional cutter yields 250 sprockets per sharpening, a wafer cutter at the same speed and feed rates on the average produces 1200 parts before it has to be replaced.

Producing Superior Quality and Meeting Production Requirements Through a Systematic Tool Maintenance Procedure.

Soon after the pilot program was initiated at this production facility, it became evident that the reblading operation of the wafer tools was incompatible with the hostile environment of the shop floor. After all, the purpose of remounting a new wafer is to reconstitute the tool to its original accuracy, which is difficult to guarantee in an area where contamination with chips and cutting fluids is a real possibility. For this reason, a reblading station was set up in the area of the inspection lab. Here wafer cutter assemblies ready for reblading were throughly washed, disassembled and then rebladed with a new wafer. (Figs. 5 & 6) Each assembly was then checked to verify proper mounting and seating of the wafer. A ball check was routinely performed to document wafer runout before the cutter assembly was returned to the production floor; thus, assuring the operator that all tools would produce quality parts from the first to the last piece without any machine adjustments.

Range of Application

To date the wafer shaper cutter is being successfully used in a wide range of applications up to 5DP, both for internal and external gears and splines. Non-involute forms benefit particularly from the consistently accurate tool geometry of the wafer, which is always compromised with conventional shaper cutters, since they are designed to accommodate the maximum usable cutter life. Helical applications with helix angles of 20° and above are being developed and are under test in selected production applications. Economics studies are also being conducted to establish the most efficient use of wafer tools. They are being applied more aggressively with higher feeds and speeds, thus realizing lower machining cycles and lower machine costs. Based on present production results, we predict that the wafer shaper cutter will evolve as the gear cutting tool of the future, providing both superior quality and economy in high volume production. ■

AUTHOR:

EDWARD HAUG *is the Director of Engineering of Pfauter-Maag Cutting Tools, formerly the Specialty Tools Division of Barber-Colman Company. He has over thirty years of engineering experience in the field of gearing, gear tools and gear manufacturing. His present responsibilities are the development of new methods and production systems for the manufacture of cutting tools. He holds a degree in Mechanical Engineering from HTL Winterthur, Switzerland, and is a registered Professional Engineer and a member of AGMA and ASME.*

FROM THE INDUSTRY...

Deburring & Finishing Gears with Power Brushes

George A. Pishek
Osborn Manufacturing Co.,
Cleveland, Ohio

Why Brushes?

In this age of hi-tech, robots, automatic machines, machining cells, etc., is there a niche somewhere for power brushes? Let me answer by asking another question. What tool does the gear manufacturer have in his arsenal that allows him to deburr green gears, hardened gears, hobbed gears, ground gears and shaved gears? What tool allows him to deburr powder metal gears – green and sintered – brass gears, bronze gears, stainless gears made of exotic materials such as inconel, waspaloy, or hastaloy, and fiber and plastic gears? How about spur gears, helical gears, sprockets, both internal and external splines, clutch teeth and pump gears? What tool allows the finishing of pump gear journals to a one to three micro-inch finish without changing concentricity or size? What tool has the flexibility to reach into a confined area and actually round off a tooth edge or root area without creating another burr or two? What tool is so adaptable that it can be used on power tools, bench motors, semi-automatic machines, fully automatic machines and robots? What robot-programmable tool is so compliant that it will not ruin even an improperly fixtured gear? The answer to all of these questions is power brushes.

This is not to say that power brushes are the answer to every finishing and deburring problem, but they are a potential solution that should be considered when these problems arise. Power brushes will fit in with most production and budget requirements. Most of the time, cycles for gear deburring with power brushes are measured in seconds. Equipment costs begin under $1,000, and systems can be upgraded as much as budget considerations allow. The power brush is a throw-away item which means no regrinding or reworking. For most applications, even if there are angles or convex or concave forms, the power brush will conform and eventually wear into the shape or form so that no pre-dressing is required. A power brush will also stress-relieve a gear, thereby strengthening heat treated gears, because power brushes will break and round all sharp edges and fillets, reducing gear tooth breakage.

Sometimes power brushing is used to remove burrs and break edges for assembly purposes only. The brushing before assembly makes the gears easier to handle, and part-to-part fit is much improved. This brushing can help to solve many assembly problems.

Types of Brushes

Power brushes can be made with wire, natural fibers or synthetic materials. All these materials have their uses in gear deburring and finishing.

Wire Brushes: Wire brushes come in two types: crimped, where each strand of wire has crimps or bends in it, and straight wire, where the wire is used as is. Two distinct styles are involved here. The crimped wire brushes are made in solid wheel types, while the straight wire brushes are in twists or knots. The reason for these differences is the manufacturing process. Both styles are actually wires bent like hair pins. When the wires in the crimped brushes are doubled back, the crimps interlock and are secured inside the face plates. The straight wire brushes, however, will not interlock, so in order to lock the wire in the face plates, the wires must be twisted and knotted in individual clumps.

AUTHOR:

MR. GEORGE PISHEK *is Manager of Applications and Training Services for Osborn Manufacturing Co., Cleveland, OH. His responsibilities include troubleshooting in the plant and at customers' sites and training of customers and sales force in brush applications and uses. Mr. Pishek is a member of SME and the author of two other technical papers.*

The straight wire brush is most commonly used in gear deburring because of its very aggressive nature. The reason the straight wire brush is more aggressive is that being straight, all wire points are on the O.D. where they belong. The crimped wire brush, on the other hand, has areas around the periphery that are dull because the bends of the wire are exposed. This cuts down considerably on the amount of work performed. The hardness of the wire approaches hardened tool steel so that most gear materials can be deburred or edge-broken with wire brushes. Wire brushes are used where finish is not critical, where burrs are heavy and where time cycles must be short. Most of the work performed by a wire brush takes place in the initial revolution of the gear past the brush.

A wire brush works by lifting the burrs and bending them back against the parent metal of the gear. It then attacks the burr at this juncture and wears it away. The brush will be aggressive on all edges, but will do little work on the solid areas of the gear. This is one of the reasons brushes work well on automated equipment, such as robots. Spur gears are ideal candidates for wire brush treatment, as the heavy burrs are removed quickly, and an even edge break is generated around the tooth profile. The result is a rounded edge (not quite a true radius) and a stronger gear because all sharp edges are rounded. Brushing has reduced gear failures by 90% in some heavy duty transmissions.

Helical gears do present a problem for wire brushes if the gears are not shaved. A wire brush will roll some of the parent gear material into the tooth on the acute side only. The obtuse side is never a problem, so all attention should be directed to the acute side. If the gears are first hobbed, then brushed so that a large break is generated, and then shaved for the precision needed, the rollover will be gone. Since there is already a break on the edge, it will be burr-free when finished.

Wire brushes are also made with stainless steel wire in both crimped and straight styles. Stainless wire brushes should be used only where contamination is a problem or on some aluminum parts. Stainless wire is not as hard as carbon wire, so it doesn't cut as well. Stainless wire work hardens quickly, so it does not have the life of carbon wire. Because of less life and higher initial cost, stainless brushes should be used only where necessary.

Natural Fiber Brushes: Two natural fiber-brushes have application value in gear deburring and finishing. Tampico, either treated or untreated, is one of them. It is a form of sisal plant grown only in Mexico. When made into a brush with the material as is, the bristles are very flexible with little cutting power. When either liquid or solid abrasive compounds are added, it becomes a very good brush for polishing worms or other such difficult parts. This type of brush will give color or cut or both if desired. The variations come with the various compounds selected. The abrasive compounds can be applied by hand or automatically with compound applicators.

Treated tampico brushes have a solution added to their fibers which permanently keeps them stiff and tacky. This brush is always used with an abrasive compound. The tacky substance holds the compound in the brush and makes the brush either cut or polish, depending on the compound applied. The treated tampico is excellent for blending and deburring exotic materials and is, therefore, used quite heavily in the jet engine industry. Finish as well as cut is excellent, and the smooth polished finish makes for strong gear teeth. The treated tampico will round off gear teeth as long as pressure and compound is applied. Gear teeth can actually be crowned with this method.

Synthetic Brushes: Many of the tampico and compound applications have been taken over by the newest in brush materials, abrasive filaments. The base material, nylon, is impregnated with silicon carbide, aluminum oxide, pumice or other abrasive materials. This material has the compound built in, saving the trouble of applying another additive. The strands, being made of nylon, are tough and durable, giving long wear. Cut is limited in terms of size, but the silicon carbide will break the edges on the hardest of materials, even carbide inserts used in various machining operations.

The abrasive brush performs best on ground gears. They are not aggressive enough to remove heavy hobbing burrs. If some type of skiver is used to break down the heavy burrs, then the abrasive brush would break the edges on the gear teeth. The abrasive brushes work extremely well on exotic materials, so are widely accepted in the jet engine industry. The abrasive material is an excellent tool for robots, as it has good compliance, will remove burrs and leave smooth edges with no rollover.

Unlike wire brushes, which roll burrs into the acute angle on helical gears, the abrasive brush will round the edges without rolling the burr, so these brushes can be used after shaving. Abrasive brushes are also good for removing the sharp edges from powdered metal gears after sintering. There are times when powdered metal gears in the green state can be brushed also. Splines are good deburring prospects for this material, as the material is removed from the spline teeth and the assembly of the mating part is made easier. Hardened pump gears are another good application. Abrasive brushes produce the light, well-rounded edge break required for these gears.

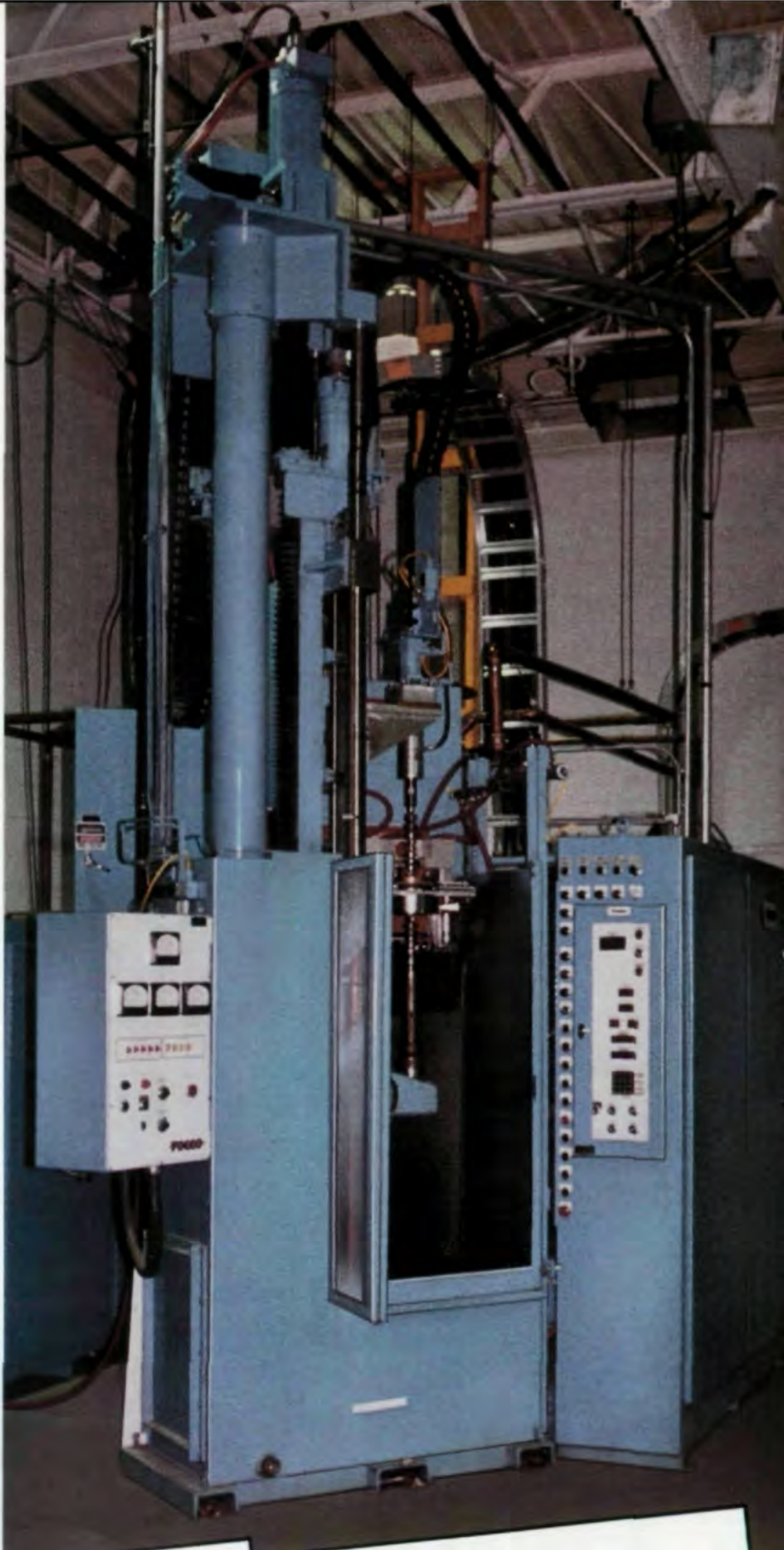
Bufcut Brushes: While talking about pump gears, the Bufcut brushes should be mentioned. The material used in them is a type of sisal that grows only in Italy. When used with the proper compound, a finish of one to three micro-inches can be obtained on hardened pump gear journals. To obtain this finish, a grind of 10-12 micro-inches is necessary. The Bufcut brush will not change the concentricity of the journal, nor will it effect size; however, it will not correct a concentricity problem either. This brush must be used with an abrasive compound, as it will do nothing when used by itself.

Cup and End Brushes: The cup brush, where it can be used, generally gives excellent brush life and is used on tough rugged jobs such as sprockets. The cup type brush will take off most heavy

FLEXA-TROL™

High Intensity, Multi-Frequency Scanners

REDUCE HEAT TREATING COSTS up to 40%



A recent TOCCO development, High Intensity Induction Hardening (HIIH)™, reduces heat treating costs substantially, in a variety of applications. HIIH has been used successfully with applications on automotive and off-highway engine and drive train components, as well as specialized gear applications. The HIIH scanning process, with present technology, can be adapted to parts up to 15″ diameter and lengths to 120″.

Parts that previously were flame hardened in a batch process, can now be induction hardened, in-line, with greater flexibility, more precision, individual quality monitoring, and at lower cost.

High Intensity Induction Hardening is indeed, a process of "The Future".

HIGH INTENSITY HARDENING PATTERNS

Cross section of camshaft lobe demonstrates uniformity of hardening. Multi-frequency, high intensity hardening of gears produces a high hardness characteristic for improved surface contact fatigue life. High residual compressive stress in the root area is also achieved for improved tooth strength.

NEW . . . TDS

TOCCO System Diagnostics are offered to document Induction System performance during dynamic, fault or standby conditions. This advanced TDS monitors digital and analog signals while recording, printing or modem transferring data to designated locations for system performance evaluation or trouble-shooting. Ask your TOCCO sales engineer for further details.

For the latest state-of-the-art heat treating technology, contact your TOCCO representative, or call: TOCCO, Inc., 30100 Stephenson Highway, Madison Hts., MI, 48071. Phone: 313-399-8601 or 1-800-468-4932 (outside Michigan).

TOCCO®

A unit of Park Ohio Industries, Inc.

CIRCLE A-16 ON READER REPLY CARD

burrs. Cup brushes are filled with wire and abrasive filament materials. An even break may be generated with only one cup brush, but using two is desirable to achieve faster time cycles. On many applications cup brushes can be more cost-effective than radial brushes. The cup brush generally requires a vertical spindle, while radial brushes require a horizontal one. Some thought should be given to this when purchasing or designing a new machine.

End brushes are desirable for some applications. This type of brush can be fixtured to form odd sizes and get into areas such as internal splines, that cannot be reached by any other type brush.

Power Brush Set-up

The setting-up of power brushes to deburr and finish gears is frequently thought to be a very simple matter, or, because of the variety of gears and brushes, quite complicated. Actually, both are misconceptions. If approached in an orderly manner, set-up is neither so simple nor as complicated as it may have at first appeared. The proper set-up is important, however, and most brushing failure is due to improper set-ups.

Almost all gear deburring involves at least two brushes being used at the same time. The brushes are spaced apart so that an even edge break is generated around the tooth profile. The spacing will vary, depending on the size of the gear being processed. The spacing is determined by visualizing the gear as the face of a clock. One brush should be placed in the five o'clock position, and one brush in the seven o'clock position. With the two brushes rotating towards the center of the gear and the gear itself rotating, an even edge break will be generated. If more break is desired on the sides of the teeth, move the brushes to the four and eight o'clock positions. On the other hand, if a larger break is desired in the root section, bring the brushes closer together. By setting the brushes in this manner, it is not necessary to reverse them. The five o'clock brush gets the left side of the tooth and half of the root area. The seven o'clock brush gets the right side of the tooth and the other half of the root area. When only one brush is used, the brush must be reversed in order to get both sides of the tooth, and the edge break will not be even.

Costs

Costs per part can vary greatly. They can run anywhere from pennies to several dollars. If costs are high, the choice of brush and improper set-up may be at fault. Efforts should be made to find the best possible combination for a particular application. The proper brush for the application must be chosen and then used to its best advantage. A brush must physically be able to come across and through the gear teeth and not be hampered in any way. If the brush material must be forced to reach these areas, poor brush life will result. Use the largest diameter brush available. While it is much easier to design "in" a small brush, a large brush will give better performance and life. Common sense and a little knowledge gained from brush manufacturers will keep costs down and give the best brush use for dollars spent.

Acknowledgement: *Reprinted courtesy of Society of Manufacturing Engineers. Presented at the 1987 Gear Processing and Manufacturing Clinic, Nov. 17-19, 1987, Southfield, MI.*

BACK TO BASICS...

Design of Internal Helical Gears

Earle Buckingham
Eliot K. Buckingham
Buckingham Associates,
Springfield, VT

In principal, the design of internal helical gear teeth is the same as that for external helical gears. Any of the basic rack forms used for external helical gears may be applied to internal helical gears. The internal gear drive, however, has several limitations; not only all those which apply to external gears, but also several others which are peculiar to internal gears. As with external gears, in order to secure effective tooth action, interferences must be avoided. The possible interferences on an internal gear drive are as follows:

1. Involute interference. To avoid this, all of the working profile of the internal tooth must be of involute form.

2. Tip interference. This exists when the tips of the pinion teeth interfere with the tips of the internal gear teeth as the teeth come into and go out of mesh. To avoid this, the size of the pinion must be a sufficient amount smaller than the size of the internal gear.

3. Fillet interference. This exists when the tips of the teeth

AUTHOR:

MR. ELIOT BUCKINGHAM *is President of Buckingham Associates, Inc., a gear consulting firm. He received his B.S. from the Massachusetts Institute of Technology and his M.S. from the University of New Mexico. During his career he has worked as an engineer and supervisor for several companies, and he was also an instructor at the University of Mexico. He has been a gear consultant since 1960. Mr. Buckingham is the author of numerous articles on gearing. A member of ASME, he is also Registered Professional Engineer in the State of Vermont.*

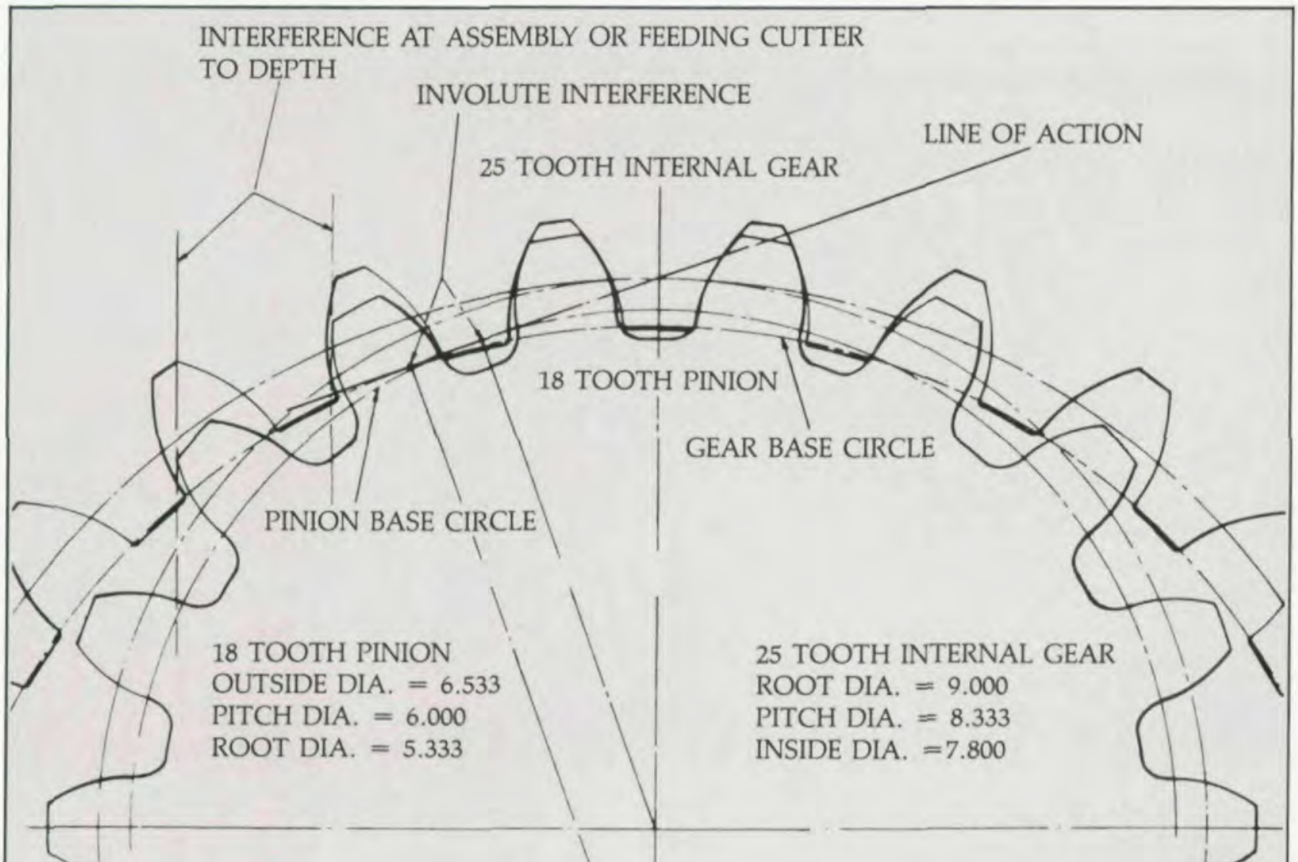


Fig. 1—3 DP-20° Stub Tooth Form — Conventional Design

of one member interfere with the fillets at the roots of the teeth of the mating member. Proper tooth proportions must be selected to avoid this interference.

Another limitation on the assembly of the drive must sometimes be considered. When the difference in the numbers of teeth between the pinion and internal gear is small, the tooth action may be correct, but it may not be possible to assemble the pair, except by sliding them together in an axial direction. In many cases, this may not be objectionable, but in other cases, it may prevent the assembly of the drive.

At the present time, the methods available for producing the internal helical gear tooth forms are limited in number. These internal gear tooth forms may be formed by casting, shaping with a formed tool or by milling with a formed milling cutter. For the more accurate internal gears, however, only one method of generating these internal gear tooth forms is generally available — the use of a pinion-shaped or Fellows' cutter. In this case, the size of the cutter imposes certain restrictions on the tooth proportions of the internal gear. If the cutter is too large, the tips of two or more teeth in the internal gear will be trimmed as the cutter is fed to depth. To avoid this, smaller, special cutters are often employed. However, if the cutter is too small, imperfect tooth forms will be developed on the internal gear.

To secure an effective internal gear drive, much more effort must be put into its design than into that of external gears. The most practical design for generated gears will be one which avoids the need for special cutters.

There are three major uses for internal helical gears; namely, internal gear drives or simple pairs, internal differential drives and internal plantary drives. The following data pertains to these three applications.

The 20° stub involute system (in the plane of rotation) is the form used on standard helical cutters and will, therefore, be used for these drives. In order to avoid involute interference, the internal radius of the internal gear must be increased over the conventional size for the smaller gears. In order to maintain effective contact, the size of the pinion must be increased over the conventional size. In order to generate full involute profiles on the internal gear teeth, the cutter must have not less than 16 teeth.

The following tooth proportions for pinions of 16 teeth and larger will meet the foregoing conditions:

When,

R_0 = Outside Radius of Pinion
R_1 = Pitch Radius of Pinion
N_1 = Number of Teeth in Pinion
N_2 = Number of Teeth in Internal Gear
R_2 = Pitch Radius of Internal Gear
R_i = Internal Radius of Internal Gear
C = Center Distance
P = Diametral Pitch

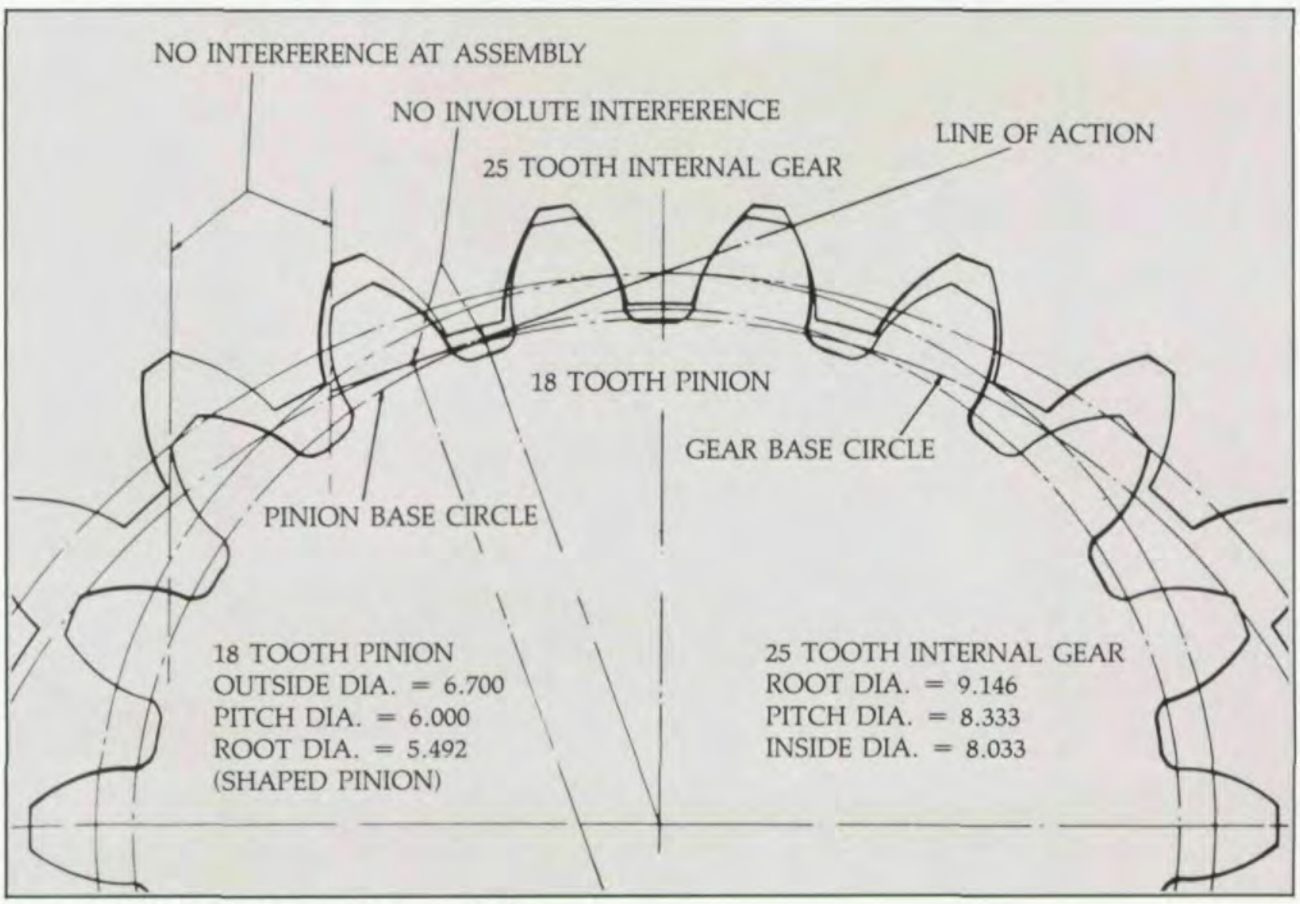


Fig. 2–3 DP-20° Stub Tooth Form – Recommended Design

Then,

$${}_1=\frac{N_1}{2P} \qquad R_2=\frac{N_2}{2P}$$

$$C=\frac{N_2-N_1}{2P}=R_2-R_1$$

$$R_0=\frac{N_1+2.100}{2P}=R_1+\frac{1.050}{P}$$

$$R_i=\frac{N_2-.900}{2P}=R_2-\frac{.450}{P}$$

T_1 = Arc Tooth Thickness of Pinion at R_1
T_2 = Arc Tooth Thickness of Internal Gear at R_2

$$T_1=\frac{1.7528}{P} \qquad T_2=\frac{1.3888}{P}$$

When the number of teeth in the pinion is less than 16, the pinion and internal gear must be enlarged still more.

To avoid trimming when feeding the cutter to depth on the internal gear, the gear must be seven or more teeth larger than the cutter.

To avoid dragging on relief while cutting, the gear may need to be up to 15 teeth larger than the cutter.

The following tables give the proportions of 1 DP small pinions from five to 16 teeth, to run at standard center distances with the internal helical gears tabulated on Tables 4-9 inclusive. For other diametral pitches, the tabulated values must be divided by the diametral pitch used. No allowance for backlash has been made in these tables. With these small pinions, the backlash should be obtained by cutting the teeth of the internal helical gear deeply enough to obtain the backlash desired.

ACKNOWLEDGEMENT: *The above material is taken from the* Revised Manual of Gear Design, Section 3, *©1980 by Eliot K. Buckingham. Reprinted by permission of the author.*

TABLE 1
PROPORTIONS OF 1 DP SMALL PINIONS (SHAPED) FOR INTERNAL HELICAL GEAR DRIVES – 20° STUB TOOTH FORM

No. of TEETH	OUTSIDE RADIUS	PITCH RADIUS	BASE RADIUS	TOOTH THICKNESS AT R_1	$\sqrt{R_0^2-R_{b1}^2}$
N_1	R_{01}	R_1	R_{b1}	T_1	
5	3.958	2.500	2.34923	2.0858	3.1863
6	4.466	3.000	2.81908	2.0555	3.4638
7	4.924	3.500	3.28892	2.0253	3.6645
8	5.383	4.000	3.75887	1.9950	3.8532
9	5.841	4.500	4.22862	1.9647	4.0293
10	6.300	5.000	4.69846	1.9344	4.1969
11	6.758	5.500	5.16831	1.9042	4.3542
12	7.216	6.000	5.63816	1.8739	4.5035
13	7.675	6.500	6.10800	1.8436	4.6473
14	8.133	7.000	6.57785	1.8133	4.7830
15	8.592	7.500	7.04769	1.7831	4.9145
16	9.050	8.000	7.51754	1.7528	5.0387

TABLE 2
ROOT RADIUS OF 1 DP SMALL PINIONS (SHAPED) FOR INTERNAL HELICAL GEAR DRIVES
20° STUB TOOTH FORM

PINION TEETH	NUMBER OF TEETH IN CUTTER					
N_1	18	21	25	28	32	35
5	2.1049	2.1126	2.1217	2.1274	2.1337	2.1380
6	2.5765	2.5825	2.5904	2.5953	2.6007	2.6045
7	3.0454	3.0516	3.0570	3.0624	3.0674	3.0707
8	3.5146	3.5196	3.5256	3.5290	3.5334	3.5361
9	3.9827	3.9872	3.9920	3.9950	3.9988	4.0012
10	4.4501	4.4538	4.4579	4.4604	4.4639	4.4653
11	4.9165	4.9197	4.9231	4.9254	4.9277	4.9296
12	5.3824	5.3849	5.3877	5.3895	5.3915	5.3929
13	5.8472	5.8493	5.8515	5.8528	5.8544	5.8559
14	6.3116	6.3132	6.3148	6.3160	6.3174	6.3193
15	6.7751	6.7764	6.7777	6.7786	6.7794	6.7803
16	7.2381	7.2387	7.2397	7.2405	7.2412	7.2416

TABLE 3
ROOT RADIUS OF 1 DP SMALL PINIONS (SHAPED) FOR INTERNAL HELICAL GEAR DRIVES
20° STUB TOOTH FORM

PINION TEETH	NUMBER OF TEETH IN CUTTER					
N_1	42	49	56	63	70	84
5	2.1460	2.1524	2.1575	2.1618	2.1653	2.1721
6	2.6116	2.6170	2.6219	2.6258	2.6286	2.6338
7	3.0767	3.0816	3.0856	3.0890	3.0916	3.0961
8	3.5415	3.5456	3.5491	3.5521	3.5544	3.5580
9	4.0059	4.0091	4.0123	4.0145	4.0166	4.0200
10	4.4693	4.4724	4.4749	4.4770	4.4786	4.4816
11	4.9328	4.9353	4.9374	4.9391	4.9405	4.9429
12	5.3956	5.3976	5.3994	5.4006	5.4019	5.4039
13	5.8579	5.8598	5.8610	5.8622	5.8631	5.8646
14	6.3198	6.3211	6.3222	6.3231	6.3239	6.3250
15	6.7815	6.7824	6.7834	6.7839	6.7846	6.7857
16	7.2425	7.2433	7.2438	7.2445	7.2449	7.2455

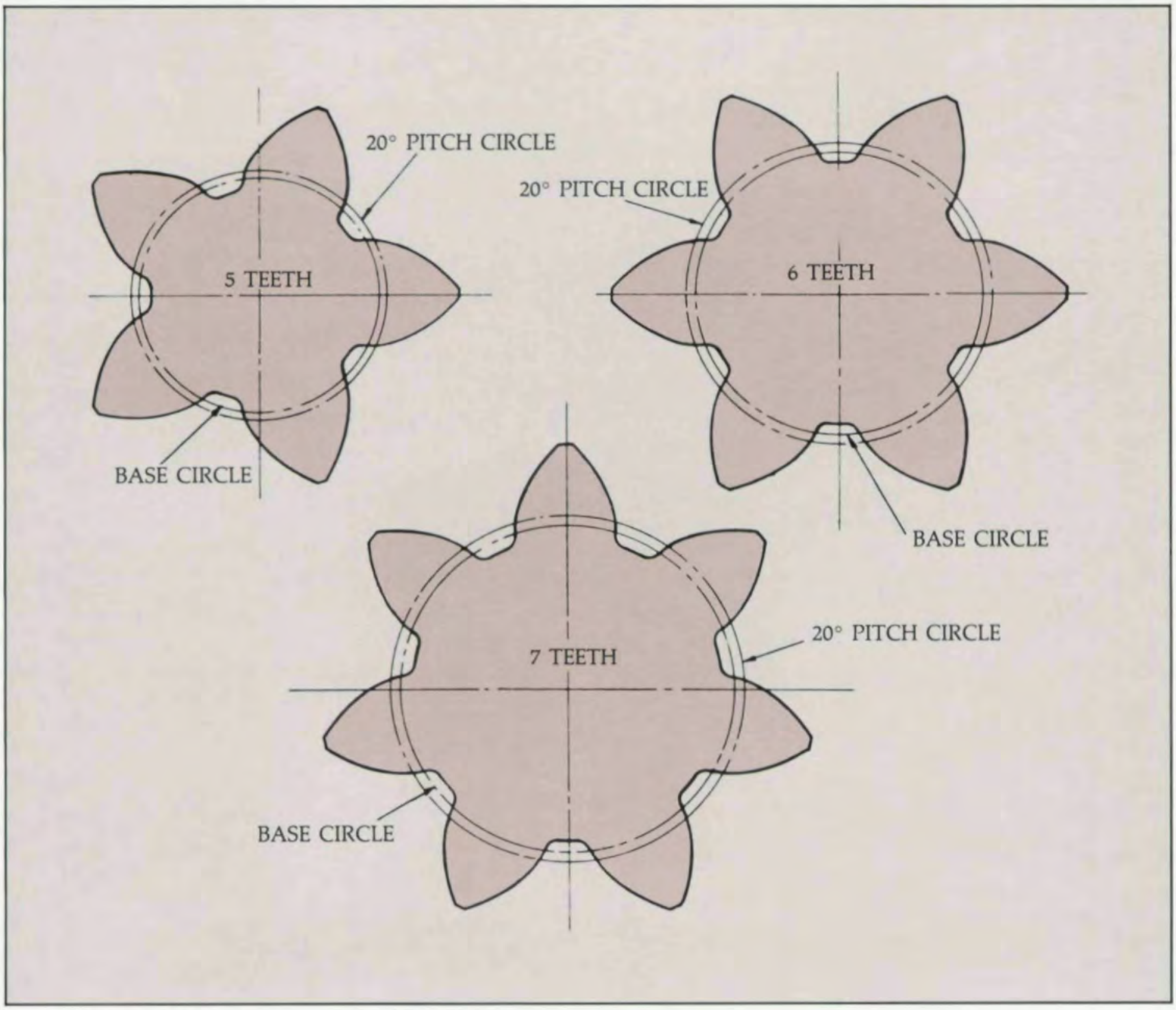


Fig. 3 – Design of Small Pinions

TABLE 4

PROPORTIONS OF 1 DP INTERNAL HELICAL GEARS – 20° STUB INVOLUTE FORM MESHING WITH SMALL PINIONS – 5 TO 16 TEETH							
NO. OF GEAR TEETH		NUMBER OF TEETH IN PINION					
		5	6	7	8	9	10
25	R_r*	14.038	14.010	13.981	13.954	13.924	13.898
	R_i**	12.391	12.360	12.329	12.298	12.267	12.236
26	R_r	14.544	14.516	14.487	14.460	14.430	14.403
	R_i	12.891	12.860	12.829	12.798	12.767	12.736
27	R_r	15.050	15.022	14.993	14.966	14.936	14.908
	R_i	13.391	13.360	13.329	13.298	13.267	13.236
28	R_r	15.556	15.528	15.499	15.471	15.441	15.412
	R_i	13.891	13.860	13.829	13.798	13.767	13.736
29	R_r	16.062	16.034	16.005	15.976	15.946	15.916
	R_i	14.391	14.361	14.329	14.298	14.267	14.236
30	R_r	16.568	16.539	16.510	16.480	16.450	16.420
	R_i	14.891	14.861	14.829	14.798	14.767	14.736
31	R_r	17.073	17.044	17.015	16.984	16.954	16.923
	R_i	15.391	15.360	15.329	15.298	15.267	15.236
32	R_r	17.578	17.549	17.519	17.488	17.457	17.426
	R_i	15.891	15.860	15.829	15.798	15.767	15.736
33	R_r	18.083	18.053	18.023	17.992	17.960	17.929
	R_i	16.391	16.360	16.329	16.298	16.267	16.236
34	R_r	18.587	18.557	18.526	18.295	18.463	18.432
	R_i	16.891	16.860	16.829	16.798	16.767	16.736
35	R_r	19.091	19.061	19.030	18.998	18.966	18.934
	R_i	17.391	17.360	17.329	17.298	17.267	17.236
36	R_r	19.595	19.564	19.533	19.501	19.469	19.436
	R_i	17.891	17.860	17.829	17.798	17.767	17.736
37	R_r	20.099	20.067	20.036	20.004	19.972	19.938
	R_i	18.391	18.360	18.329	18.298	18.267	18.236
38	R_r	20.602	20.570	20.538	20.506	20.474	20.440
	R_i	18.891	18.860	18.829	18.798	18.767	18.736
39	R_r	21.105	21.073	21.041	21.009	20.976	20.942
	R_i	19.391	19.360	19.329	19.298	19.267	19.236
40	R_r	21.608	21.575	21.543	21.511	21.478	21.444
	R_i	19.891	19.860	19.829	19.798	19.767	19.736
41	R_r	22.111	22.078	22.046	22.013	21.980	21.946
	R_i	20.391	20.360	20.329	20.298	20.267	20.236
42	R_r	22.613	22.580	22.548	22.515	22.481	22.447
	R_i	20.891	20.860	20.829	20.798	20.767	20.736
43	R_r	23.116	23.083	23.050	23.017	22.983	22.949
	R_i	21.391	21.360	21.329	21.298	21.267	21.236
44	R_r	23.618	23.585	23.552	23.518	23.484	23.450
	R_i	21.891	21.860	21.829	21.798	21.767	21.736
45	R_r	24.120	24.087	24.054	24.020	23.986	23.952
	R_i	22.391	22.360	22.329	22.298	22.267	22.236
46	R_r	24.622	24.589	24.555	24.521	24.487	24.453
	R_i	22.891	22.860	22.829	22.798	22.767	22.736
47	R_r	25.124	25.091	25.057	25.023	24.989	24.954
	R_i	23.391	23.360	23.329	23.298	23.267	23.236
48	R_r	25.626	25.592	25.558	25.524	25.490	25.455
	R_i	23.891	23.860	23.829	23.798	23.767	23.736
49	R_r	26.128	26.094	26.060	26.026	25.991	25.956
	R_i	24.391	24.360	24.329	24.298	24.267	24.236
50	R_r	26.629	26.595	26.561	26.527	26.492	26.457
	R_i	24.891	24.860	24.829	24.798	24.767	24.736

*R_r – Root radius. **R_i – Internal radius of internal gear.

TABLE 5

PROPORTIONS OF 1 DP INTERNAL HELICAL GEARS – 20° STUB INVOLUTE FORM
MESHING WITH SMALL PINIONS – 5 TO 16 TEETH

NO. OF GEAR TEETH		NUMBER OF TEETH IN PINION					
		5	6	7	8	9	10
51	R_r	27.131	27.097	27.063	27.028	26.993	26.958
	R_i	25.391	25.360	25.329	25.298	25.267	25.236
52	R_r	27.632	27.598	27.564	27.529	27.494	27.459
	R_i	25.891	25.860	25.829	25.798	25.767	25.736
53	R_r	28.134	28.100	28.065	28.030	27.995	27.960
	R_i	26.391	26.360	26.329	26.298	26.267	26.236
54	R_r	28.635	28.601	28.566	28.531	28.496	28.460
	R_i	26.891	26.860	26.829	26.798	26.767	26.736
55	R_r	29.137	29.102	29.067	29.032	28.997	28.961
	R_i	27.391	27.360	27.329	27.298	27.267	27.236
56	R_r	29.638	29.603	29.568	29.533	29.498	29.462
	R_i	27.891	27.860	27.829	27.798	27.767	27.736
57	R_r	30.140	30.104	30.069	30.034	29.999	29.963
	R_i	28.391	28.360	28.329	28.298	28.267	28.236
58	R_r	30.641	30.605	30.570	30.535	30.500	30.464
	R_i	28.891	28.860	28.829	28.798	28.767	28.736
59	R_r	31.142	31.107	31.071	31.036	31.000	30.965
	R_i	29.391	29.360	29.329	29.298	29.267	29.236
60	R_r	31.643	31.608	31.572	31.537	31.501	31.465
	R_i	29.891	29.860	29.829	29.798	29.767	29.736
61	R_r	32.144	32.109	32.073	32.038	32.002	31.966
	R_i	30.391	30.360	30.329	30.298	30.267	30.236
62	R_r	32.645	32.610	32.574	32.539	32.502	32.466
	R_i	30.891	30.860	30.829	30.798	30.767	30.736
63	R_r	33.146	33.111	33.075	33.040	33.003	32.967
	R_i	31.391	31.360	31.329	31.298	31.267	31.236
64	R_r	33.647	33.612	33.575	33.540	33.504	33.467
	R_i	31.891	31.860	31.829	31.798	31.767	31.736
65	R_r	34.148	34.113	34.076	34.041	34.004	33.968
	R_i	32.391	32.360	32.329	32.298	32.267	32.236
66	R_r	34.649	34.613	34.577	34.541	34.504	34.468
	R_i	32.891	32.860	32.829	32.798	32.767	32.736
67	R_r	35.150	35.114	35.078	35.042	35.005	34.969
	R_i	33.391	33.360	33.329	33.298	33.267	33.236
68	R_r	35.651	35.615	35.579	35.543	35.506	35.470
	R_i	33.891	33.860	33.829	33.798	33.767	33.736
69	R_r	36.152	36.116	36.080	36.044	36.007	35.970
	R_i	34.391	34.360	34.329	34.298	34.267	34.236
70	R_r	36.653	36.616	36.580	36.544	36.507	36.470
	R_i	34.891	34.860	34.829	34.798	34.767	34.736
71	R_r	37.154	37.117	37.081	37.045	37.008	36.971
	R_i	35.391	35.360	35.329	35.298	35.267	35.236
72	R_r	37.655	37.618	37.582	37.545	37.508	37.471
	R_i	35.891	35.860	35.829	35.798	35.767	35.736
73	R_r	38.155	38.119	38.082	38.046	38.009	37.971
	R_i	36.391	36.360	36.329	36.298	36.267	36.236
74	R_r	38.656	38.619	36.583	38.546	38.509	38.472
	R_i	36.891	36.860	36.829	36.798	36.767	36.736
75	R_r	39.156	39.120	39.083	39.047	39.009	38.973
	R_i	37.391	37.360	37.329	37.298	37.267	37.236

TABLE 6

PROPORTIONS OF 1 DP INTERNAL HELICAL GEARS – 20° STUB INVOLUTE FORM
MESHING WITH SMALL PINIONS – 5 TO 16 TEETH

NO. OF GEAR TEETH		NUMBER OF TEETH IN PINION					
		5	6	7	8	9	10
76	R_r	39.657	39.621	39.584	39.547	39.510	39.473
	R_i	37.891	37.860	37.829	37.798	37.767	37.736
77	R_r	40.157	40.121	40.084	40.048	40.010	39.973
	R_i	38.391	38.360	38.329	38.298	38.267	38.236
78	R_r	40.658	40.622	40.585	40.548	40.511	40.474
	R_i	38.891	38.860	38.829	38.798	38.767	38.736
79	R_r	41.158	41.122	41.085	41.049	41.011	40.974
	R_i	39.391	39.360	39.329	39.298	39.267	39.236
80	R_r	41.659	41.623	41.586	41.549	41.511	41.474
	R_i	39.891	39.860	39.829	39.798	39.767	39.736
81	R_r	42.160	42.124	42.086	42.050	42.012	41.975
	R_i	40.391	40.360	40.329	40.298	40.267	40.236
82	R_r	42.661	42.624	42.587	42.550	42.512	42.475
	R_i	40.891	40.860	40.829	40.798	40.767	40.736
83	R_r	43.161	43.125	43.087	43.051	43.013	42.975
	R_i	41.391	41.360	41.329	41.298	41.267	41.236
84	R_r	43.662	43.625	43.587	43.551	43.513	43.476
	R_i	41.891	41.860	41.829	41.798	41.767	41.736
85	R_r	44.162	44.126	44.088	44.052	44.013	43.976
	R_i	42.391	42.360	42.329	42.298	42.267	42.236
86	R_r	44.663	44.626	44.588	44.552	44.514	44.476
	R_i	42.891	42.860	42.829	42.798	42.767	42.736
87	R_r	45.163	45.127	45.089	45.052	45.014	44.977
	R_i	43.391	43.360	43.329	43.298	43.267	43.236
88	R_r	45.664	45.627	45.589	45.553	45.514	45.477
	R_i	43.891	43.860	43.829	43.798	43.767	43.736
89	R_r	46.165	46.128	46.090	46.053	46.015	45.977
	R_i	44.391	44.360	44.329	44.298	44.267	44.236
90	R_r	46.665	46.628	46.590	46.553	46.515	46.477
	R_i	44.891	44.860	44.829	44.798	44.767	44.736
91	R_r	47.166	47.129	47.091	47.054	47.016	46.978
	R_i	45.391	45.360	45.329	45.298	45.267	45.236
92	R_r	47.666	47.629	47.591	47.554	47.516	47.478
	R_i	45.891	45.860	45.829	45.798	45.767	45.736
93	R_r	48.167	48.130	48.092	48.054	48.016	47.978
	R_i	46.391	46.360	46.329	46.298	46.267	46.236
94	R_r	48.667	48.630	48.592	48.555	48.517	48.479
	R_i	46.891	46.860	46.829	46.798	46.767	46.736
95	R_r	49.168	49.130	49.092	49.055	49.017	48.979
	R_i	47.391	47.360	47.329	47.298	47.267	47.236
96	R_r	49.668	49.631	49.593	49.555	49.517	49.479
	R_i	47.891	47.860	47.829	47.798	47.767	47.736
97	R_r	50.169	50.131	50.093	50.056	50.018	49.980
	R_i	48.391	48.360	48.329	48.298	48.267	48.236
98	R_r	50.669	50.631	50.593	50.556	50.518	50.480
	R_i	48.891	48.860	48.829	48.798	48.767	48.736
99	R_r	51.170	51.132	51.094	51.056	51.018	50.980
	R_i	49.391	49.360	49.329	49.298	49.267	49.236
100	R_r	51.761	51.632	51.594	51.556	51.518	51.480
	R_i	49.891	49.860	49.829	49.798	49.767	49.736

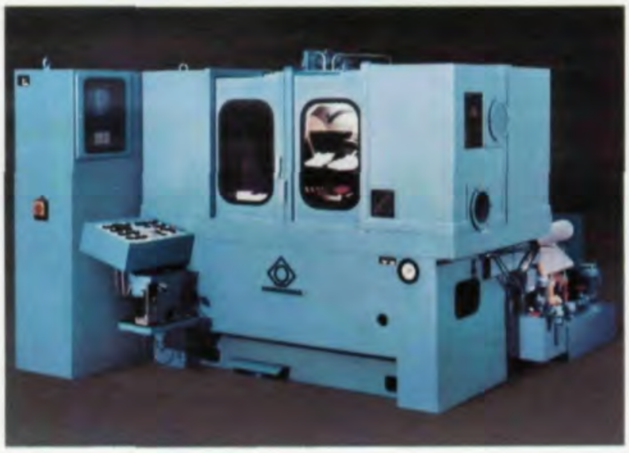
(continued on page 42)

SHARP!

The CNC Controlled Hob Sharpening Machines

Our advanced hob sharpener, the SNC-30 with its high productivity and field proven design is now complemented with the introduction of Klingelnberg's SNC-50. The model SNC-50 offers the same economical advantages and high accuracies to both large hob manufacturers and users, as the SNC-30 has in the past.

The operator friendly SNC series provides unparalleled speed and accuracy during wet grinding of straight or spiral fluted hobs up to 20.5" OD and flute depths to 5.1".



A host of standard features, like fully programmable grinding cycles, self-diagnostics, automatic wheel dressing and profiling for spiral flutes complement the traditionally excellent Klingelnberg design.

For gear manufacturers using a wide variety of hobs, an expandable program storage capacity (up to 120 part programs) can be provided.

The SNC-30 and -50 models are suitable for grinding with vitrified, CBN or Diamond grinding wheels for total hob sharpening flexibility.

For further details on how you can SHARPEN hobs and your bottom line, contact:

Klingelnberg Corporation, 15200 Foltz Industrial Parkway, Cleveland, OH 44136 .
Phone (216) 572-2100; FAX: (216) 572-0985.

KLINGELNBERG

CIRCLE A-18 ON READER REPLY CARD

(continued from page 40)

TABLE 7

PROPORTIONS OF 1 DP INTERNAL HELICAL GEARS – 20° STUB INVOLUTE FORM MESHING WITH SMALL PINIONS – 5 TO 16 TEETH							
NO. OF GEAR TEETH		NUMBER OF TEETH IN PINION					
		11	12	13	14	15	16
25	R_r	13.869	13.839	13.809	13.779	13.749	13.719
	R_i	12.205	12.174	12.143	12.112	12.081	12.050
26	R_r	14.373	14.343	14.313	14.282	14.251	14.220
	R_i	12.705	12.674	12.643	12.612	12.581	12.550
27	R_r	14.877	14.846	14.815	14.784	14.753	14.721
	R_i	13.205	13.174	13.143	13.112	13.081	13.050
28	R_r	15.381	15.351	15.320	15.288	15.255	15.222
	R_i	13.705	13.674	13.643	13.612	13.581	13.550
29	R_r	15.885	15.854	15.823	15.791	15.757	15.724
	R_i	14.205	14.174	14.143	14.112	14.081	14.050
30	R_r	16.389	16.357	16.325	16.293	16.259	16.225
	R_i	14.705	14.674	14.643	14.612	14.581	14.550
31	R_r	16.892	16.860	16.827	16.795	16.761	16.727
	R_i	15.205	15.174	15.143	15.112	15.081	15.050
32	R_r	17.395	17.362	17.329	17.297	17.263	17.228
	R_i	15.705	15.674	15.643	15.612	15.581	15.550
33	R_r	17.897	17.864	17.831	17.799	17.764	17.729
	R_i	16.205	16.174	16.143	16.112	16.081	16.050
34	R_r	18.399	18.366	18.333	18.300	18.265	18.230
	R_i	16.705	16.674	16.643	16.612	16.581	16.550
35	R_r	18.901	18.868	18.835	18.801	18.766	18.731
	R_i	17.205	17.174	17.143	17.112	17.081	17.050
36	R_r	19.403	19.370	19.336	19.302	19.267	19.231
	R_i	17.705	17.674	17.643	17.612	17.581	17.550
37	R_r	19.905	19.872	19.838	19.803	19.768	19.732
	R_i	18.205	18.174	18.143	18.112	18.081	18.050
38	R_r	20.407	20.373	20.339	20.304	20.269	20.233
	R_i	18.705	18.674	18.643	18.612	18.581	18.550
39	R_r	20.909	20.875	20.840	20.805	20.769	20.733
	R_i	19.205	19.174	19.143	19.112	19.081	19.050
40	R_r	21.410	21.376	21.341	21.306	21.270	21.234
	R_i	19.705	19.674	19.643	19.612	19.581	19.550
41	R_r	21.912	21.877	21.843	21.807	21.771	21.734
	R_i	20.205	20.174	20.143	20.112	20.081	20.050
42	R_r	22.414	22.378	22.343	22.307	22.271	22.235
	R_i	20.705	20.674	20.643	20.612	20.581	20.550
43	R_r	22.915	22.879	22.844	22.808	22.772	22.735
	R_i	21.205	21.174	21.143	21.112	21.081	21.050
44	R_r	23.416	23.380	23.345	23.309	23.273	23.236
	R_i	21.705	21.674	21.643	21.612	21.581	21.550
45	R_r	23.917	23.881	23.846	23.810	23.773	23.736
	R_i	22.205	22.174	22.143	22.112	22.081	22.050
46	R_r	24.418	24.382	24.347	24.310	24.273	24.237
	R_i	22.705	22.674	22.643	22.612	22.581	22.550
47	R_r	24.919	24.883	24.847	24.811	24.774	24.737
	R_i	23.205	23.174	23.143	23.112	23.081	23.050
48	R_r	25.420	25.384	25.348	25.311	25.274	25.237
	R_i	23.705	23.674	23.643	23.612	23.581	23.550
49	R_r	25.921	25.885	25.848	25.812	25.775	25.738
	R_i	24.205	24.174	24.143	24.112	24.081	24.050
50	R_r	26.421	26.386	26.349	26.313	26.276	26.238
	R_i	24.705	24.674	24.643	24.612	24.581	24.550

(continued on page 44)

(continued from page 42)

TABLE 8

PROPORTIONS OF 1 DP INTERNAL HELICAL GEARS – 20° STUB INVOLUTE FORM MESHING WITH SMALL PINIONS – 5 TO 16 TEETH							
NO. OF GEAR TEETH		NUMBER OF TEETH IN PINION					
		11	12	13	14	15	16
51	R_r	26.922	26.887	26.850	26.813	26.776	26.738
	R_i	25.205	25.174	25.143	25.112	25.081	25.050
52	R_r	27.423	27.388	27.350	27.314	27.277	27.239
	R_i	25.705	25.674	25.643	25.612	25.581	25.550
53	R_r	27.924	27.889	27.851	27.814	27.777	27.739
	R_i	26.205	26.174	26.143	26.112	26.081	26.050
54	R_r	28.424	28.389	28.351	28.315	28.277	28.239
	R_i	26.705	26.674	26.643	26.612	26.581	26.550
55	R_r	28.925	28.890	28.852	28.815	28.777	28.739
	R_i	27.205	27.174	27.143	27.112	27.081	27.050
56	R_r	29.425	29.390	29.352	29.315	29.278	29.240
	R_i	27.705	27.674	27.643	27.612	27.581	27.550
57	R_r	29.926	29.891	29.853	29.816	29.778	29.740
	R_i	28.205	28.174	28.143	28.112	28.081	28.050
58	R_r	30.427	30.391	30.353	30.316	30.278	30.240
	R_i	28.705	28.674	28.643	28.612	28.581	28.550
59	R_r	30.927	30.892	30.854	30.817	30.779	30.740
	R_i	29.205	29.174	29.143	29.112	29.081	29.050
60	R_r	31.428	31.392	31.354	31.317	31.279	31.240
	R_i	29.705	29.674	29.643	29.612	29.581	29.550
61	R_r	31.929	31.893	31.855	31.818	31.780	31.740
	R_i	30.205	30.174	30.143	30.112	30.081	30.050
62	R_r	32.429	32.393	32.355	32.318	32.280	32.241
	R_i	30.705	30.674	30.643	30.612	30.581	30.550
63	R_r	32.930	32.893	32.855	32.818	32.780	32.741
	R_i	31.205	31.174	31.143	31.112	31.081	31.050
64	R_r	33.430	33.394	33.356	33.318	33.280	33.241
	R_i	31.705	31.674	31.643	31.612	31.581	31.550
65	R_r	33.931	33.894	33.856	33.818	33.780	33.741
	R_i	32.205	32.174	32.143	32.112	32.081	32.050
66	R_r	34.431	34.394	34.356	34.319	34.281	34.241
	R_i	32.705	32.674	32.643	32.612	32.581	32.550
67	R_r	34.932	34.895	34.857	34.819	34.781	34.741
	R_i	33.205	33.174	33.143	33.112	33.081	33.050
68	R_r	35.432	34.395	35.357	35.319	35.281	35.242
	R_i	33.705	33.674	33.643	33.612	33.581	33.550
69	R_r	35.933	35.895	35.857	35.819	35.781	35.742
	R_i	34.205	34.174	34.143	34.112	34.081	34.050
70	R_r	36.433	36.395	36.357	36.319	36.281	36.242
	R_i	34.705	34.674	34.643	34.612	34.581	34.550
71	R_r	36.934	36.896	36.858	36.820	36.782	36.742
	R_i	35.205	35.174	35.143	35.112	35.081	35.050
72	R_r	37.434	37.396	37.358	37.320	37.282	37.242
	R_i	35.705	35.674	35.643	35.612	35.581	35.550
73	R_r	37.935	37.896	37.858	37.820	37.782	37.742
	R_i	36.205	36.174	36.143	36.112	36.081	36.050
74	R_r	38.435	38.397	38.358	38.320	38.282	38.243
	R_i	36.705	36.674	36.643	36.612	36.581	36.550
75	R_r	38.935	38.897	38.858	38.820	38.782	38.743
	R_i	37.205	37.174	37.143	37.112	37.081	37.050

(continued on page 48)

THE WORLD IS GRINDING A PATH TO WMW:NILES GENERATING GEAR GRINDERS

WMW:NILES
ZSTZ 08/EG-CNC

WMW:NILES Gear Generating Grinders are in demand everywhere! The world's most technologically advanced nations look to WMW:NILES for this equipment—and for good reason.

- The NILES Gear Generating Process is the fastest and most cost effective method for grinding small and medium production runs.
- NILES meets your requirements with a full line of automatic grinders with gear diameter capacities from 1/2" to 158".
- ALL NILES gear grinders have accuracies to AGMA 12/14.

Customized machines can be ordered with even higher guaranteed accuracies.

You won't find a better Gear Generating Grinder anywhere. That's why companies like yours have bought over 4600 of them, making WMW:NILES Grinders most preferred throughout the world.

PARTIAL USAGE SURVEY (as of 1988)

COUNTRY	# OF MACHINES
Federal Republic of Germany	350
Japan	287
Italy	175
France	160
Great Britain	101
North America	41
Switzerland	15

Complete information is available on request.

WMW Machinery, Inc.
570 Bradley Hill Road
Blauvelt, New York 10913
Phone (914) 358-3330
Fax (914) 358-2378
Telex 4756017

CIRCLE A-20 ON READER REPLY CARD

CLASSIFIED

Rates: Classified Display—per inch (minimum 3) **1X**-$130, **3X**-$120, **6X**-$110. Type will be set to advertiser's layout or *Gear Technology* will set type at no extra charge. **Word Count:** 35 characters per line, 7 lines per inch.

Payment: Full payment must accompany classified ads. Mail copy to *Gear Technology*, P.O. Box 1426, Elk Grove Village, IL 60009. **Agency Commission:** No agency commission on classifieds.

Material Deadline: Ads must be received by the 25th of the month, two months prior to publication. **Acceptance:** Publisher reserves the right to accept or reject classified advertisements at his discretion.

COMPUTER AIDS

GEAR

DESIGN ANALYSIS
IBM COMPATIBLE COMPUTERS
WORM • SPUR • HELICAL • INTERNAL

- Source Code Provided (Basic)
- Self Prompting
- Complete Instruction Manuals
- Non-Standard Center Distances
- Recess Designs
- Non-Standard Tooth Depths
- Contact Diagram for Worm Drives
- Tolerances per AGMA 390.03

LOAD ANALYSIS per

- Buckingham Equations
- AGMA 218.01 or 440.04 for WORMS
- Buckingham Gear Books Available

BUCKINGHAM ASSOCIATES INC.
Springfield, VT 05156

CIRCLE A-21 ON READER REPLY CARD

GEAR ESTIMATING

The COSTIMATOR® computer aided cost estimating system insures speed and consistency in the difficult task of estimating the costs of all types of gears.

Used by small shops and Fortune 500 companies throughout the country.

For complete information contact us today.

Manufacturers Technologies, Inc.
59G Interstate Dr.
West Springfield, MA 01089
(413) 733-1972

CIRCLE A-22 ON READER REPLY CARD

SERVICE

GEAR GRINDING

REISHAUER RZ 300E GEAR GRINDERS

OPERATIONAL OR MAKE COMPLETE TO AGMA CLASS 15

R & D OR PRODUCTION QUANTITIES TO 12" P.D.

SPURS, HELICALS, INTERNALS, CROWNING AND INTRICATE GEARS/SPLINE GRINDING REQUIRING SMALL DIAMETER WHEELS

COMPLETE GEAR INSPECTION INCLUDING 100 PERCENT RED LINE

RENAR INDUSTRIES, INC.
9710 DISTRIBUTION AVE.
SAN DIEGO, CA. 92121
TEL. (619) 566-7610 FAX (619) 566-6524

CIRCLE A-23 ON READER REPLY CARD

SERVICE

GEAR TESTING AND DESIGN FACILITIES

- GEAR DESIGN (NOISE — STRENGTH)
- ROTATING GEAR (TORQUE — SPEED CONTROL) TEST MACHINES.
- SINGLE TOOTH BENDING FATIGUE TESTING.
- STATISTICAL PLANNING — ANALYSIS.
- WROUGHT STEELS, SINTERED METALS, NON-METALLIC MAT'LS.
- CAD FACILITIES FOR LOW COST SET-UP.
- CUSTOM TEST MACHINE DESIGN
- EXPERIENCED PERSONNEL.

PACKER ENGINEERING
312/355-5722, ext. 214
BOX 353, NAPERVILLE, IL 60566

CIRCLE A-24 ON READER REPLY CARD

GEAR TOOTH GRINDING SERVICE

Open Capacity On Our

REISHAUER GEAR TOOTH GRINDERS

We are capable of grinding your gears to AGMA standards for salvage or as a planned operation in prototype or production quantities.

CERTIFIED GEAR INSPECTION

For immediate service phone:
716-892-0400

Or send inquiries to:
PRO-GEAR COMPANY
400 Kennedy Road
Buffalo, N.Y. 14227

CIRCLE A-25 ON READER REPLY CARD

SERVICE MAINTENANCE PARTS:

Analytical
Gear Measuring
Instruments
and Recorders

PRECISE

PRECISE INSPECTION
27380 Gratiot Avenue
Roseville, Michigan 48066
313-445-6959

CIRCLE A-26 ON READER REPLY CARD

Bring in new customers for your business by advertising in *GEAR TECHNOLOGY*, The Journal of Gear Manufacturing.
Call (312) 437-6604

There's still time . . .
order closing date for a classified ad in the May/June issue is March 10th.

TECHNICAL CALENDAR

MARCH 19-22, 1989. First International Applied Mechanical Systems Design Conference. Convention Center, Nashville, TN. Presentation of papers, tutorials and panel sessions on various design topics. For further information, contact: Dr. Cemil Bagci, Tennessee Technological University, Cookeville, TN 38505. (615) 372-3265.

MARCH 28-30, 1989. Gear Design Seminar, University of Northern Iowa. Call (319) 273-6899 for more information.

APRIL 5, 1989. Gear Technology Conference, Hartford, CT. Seminar on gear drive design, gear and spline geometry, AGMA ratings, computerized gear testing, dynamic loading and gear/spline manufacture by some of the country's leading specialists. Sponsored by Hartford Section ASME. Call Ken (203) 565-5701 or Bill (203) 528-4028 for more information.

APRIL 25-27, 1989. ASME 5th Annual Power Transmission & Gearing Conference, Chicago, IL. Presentations on emerging technologies for gears, couplings and other power transmission devices, gear geometry, noise, manufacturing and other gear-related subjects. For more information, contact Donald Borden, P.O. Box 502, Elm Grove, WI 53122.

AGMA TECHNICAL EDUCATION SEMINARS. AGMA is offering a new series of technical education seminars, each one focusing on a different aspect of gear manufacturing and taught by industry experts.

March 7/8, Rochester, NY. "Source Inspection of Loose Gears from the Customer's Standpoint."

For more information, contact: Bill Daniels, AGMA, 1500 King St., Suite 201, Alexandria, VA, 22314. (703) 684-0211.

(continued from page 44)

TABLE 9

PROPORTIONS OF 1 DP INTERNAL HELICAL GEARS – 20° STUB INVOLUTE FORM MESHING WITH SMALL PINIONS – 5 TO 16 TEETH							
NO. OF GEAR TEETH		NUMBER OF TEETH IN PINION					
		11	12	13	14	15	16
76	R_r	39.436	39.397	39.359	39.321	39.282	39.243
	R_i	37.705	37.674	37.643	37.612	37.581	37.550
77	R_r	39.936	39.898	39.859	39.821	39.782	39.743
	R_i	38.205	38.174	38.143	38.112	38.081	38.050
78	R_r	40.437	40.398	40.359	40.321	40.282	40.243
	R_i	38.705	38.674	38.643	38.612	38.581	38.550
79	R_r	40.937	40.898	40.859	40.821	40.782	40.743
	R_i	39.205	39.174	39.143	39.112	39.081	39.050
80	R_r	41.437	41.398	41.359	41.321	41.282	41.244
	R_i	39.705	39.674	39.643	39.612	39.581	39.550
81	R_r	41.938	41.899	41.860	41.822	41.783	41.744
	R_i	40.205	40.174	40.143	40.112	40.081	40.050
82	R_r	42.438	42.399	42.360	42.322	42.283	42.244
	R_i	40.705	40.674	40.643	40.612	40.581	40.550
83	R_r	42.938	42.899	42.860	42.822	42.783	42.744
	R_i	41.205	41.174	41.143	41.112	41.081	41.050
84	R_r	43.438	43.399	43.360	43.322	43.283	43.244
	R_i	41.705	41.674	41.643	41.612	41.581	41.550
85	R_r	43.938	43.899	43.860	43.822	43.783	43.744
	R_i	42.205	42.174	42.143	42.112	42.081	42.050
86	R_r	44.439	44.400	44.361	44.323	44.283	44.244
	R_i	42.705	42.674	42.643	42.612	42.581	42.550
87	R_r	44.939	44.900	44.861	44.823	44.783	44.744
	R_i	43.205	43.174	43.143	43.112	43.081	43.050
88	R_r	45.439	45.400	45.361	45.323	45.283	45.244
	R_i	43.705	43.674	43.643	43.612	43.581	43.550
89	R_r	45.939	45.900	45.861	45.823	45.783	45.744
	R_i	44.205	44.174	44.143	44.112	44.081	44.050
90	R_r	46.439	46.400	46.361	46.323	46.283	46.244
	R_i	44.705	44.674	44.643	44.612	44.581	44.550
91	R_r	46.940	46.901	46.862	46.824	46.784	46.744
	R_i	45.205	45.174	45.143	45.112	45.081	45.050
92	R_r	47.440	47.401	47.362	47.324	47.284	47.244
	R_i	45.705	45.674	45.643	45.612	45.581	45.550
93	R_r	47.940	47.901	47.862	47.824	47.784	47.744
	R_i	46.205	46.174	46.143	46.112	46.081	46.050
94	R_r	48.440	48.401	48.362	48.324	48.284	48.244
	R_i	46.705	46.674	46.643	46.612	46.581	46.550
95	R_r	48.940	48.901	48.862	48.824	48.784	48.744
	R_i	47.205	47.174	47.143	47.112	47.081	47.050
96	R_r	49.441	49.402	49.363	49.324	49.284	49.244
	R_i	47.705	47.674	47.643	47.612	47.581	47.550
97	R_r	49.941	49.902	49.863	49.824	49.784	49.744
	R_i	48.205	48.174	48.143	48.112	48.081	48.050
98	R_r	50.441	50.402	50.363	50.324	50.284	50.244
	R_i	48.705	48.674	48.643	48.612	48.581	48.550
99	R_r	50.941	50.902	50.863	50.824	50.784	50.744
	R_i	49.205	49.174	49.143	49.112	49.081	49.050
100	R_r	51.441	51.402	51.363	51.324	51.284	51.245
	R_i	49.705	49.674	49.643	49.612	49.581	49.550

FORMASTER

GRINDING WHEEL PROFILER

EASY TO INSTALL — Because of its small size and weight, the **FORMASTER** does not require major machine modifications and can be installed on nearly any grinder. Installation can usually be accomplished in less than a day.

EASY TO OPERATE — Two axis design simplifies programming and operation. You can choose between four popular controls that feature menu and G-Code programming, graphic simulation, automatic corner rounding, automatic diamond thickness compensation, and more.

IMPROVES ACCURACY

MADE IN U.S.A.

REDUCES WHEEL DRESSING TIME

Patent No. 4,559,919

ACCURATE — To within ± .0001″ of programmed dimension, with repeat accuracy to within .00006″. Extra precision roller bearing ways, pre-loaded roller screws and optical linear encoders, as well as superior design and construction, give the **FORMASTER** the ability to hold inspection gage accuracy.

PRODUCTIVE — No templates or special diamond rolls are needed, so lead times and tooling inventories are reduced. Most forms can be programmed and dressed in, ready to grind in 30 to 45 minutes. Refreshing the form between grinding passes is accomplished in seconds.

VERSATILE — Can be used with single point diamonds or with optional rotary diamond wheel attachment. Nearly any form can be dressed quickly, easily and accurately.

DURABLE — Hard seals are closely fitted and are air purged to totally exclude contamination. Sealed servo motors, automatic lubrication and totally enclosed encoders minimize down time and ensure long service life.

P.O. Box 69
Arden, NC 28704
(704) 684-1002

NORMAC

P.O. Box 207
Northville, MI 48167
(313) 349-2644

CIRCLE A-27 ON READER REPLY CARD