Calculated Gear Life Values

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QUESTION

I have a query (regarding) calculated gear life values. I would like to understand for what % of gear failures the calculated life is valid? Is it 1-in-100 (1% failure, 99% reliability) or 1-in-one-thousand (0.1% failure)?

Expert response provided by Hanspeter Dinner, gear consultant and KISSsoft representative in Asia:

In gear rating per ISO, DIN or AGMA standards, permissible stresses for a required life or cycle number are calculated from the S-N curve. The S-N curve itself is determined along procedures described in the mentioned standards, using just a few values (e.g., the endurance limit and the material type) to describe the S-N curve. The basis of the S-N curve is measurements done at different gear labs, e.g. — at the Technical University of Munich — where gear strength was measured both for finite and infinite life.

The original S-N curves, as measured, are based on probability of failure of $P_a = 50\%$. Obviously, this high probability of damage is not suitable for a gear design in most cases and the S-N curves included in the mentioned standards have a far lower probability of damage or higher reliability. There, the basis is a probability of damage of $P_a = 1\%$ for a safety factor of $S = 1.00$. This means that if a safety factor of $S = 1.00$ is used, one out of 100 gears should fail by design within its design life for the rated load.

This probability of damage of $P_a = 1\%$ may be higher than acceptable or it may be lower than necessary. If it is too high, (as in a helicopter transmission, where the consequence of failure is catastrophic), a safety factor $S > 1.00$ should be introduced, thus reducing the allowable stress number. If it is lower than required (as in a gearbox of a low-cost power tool), higher allowable stress numbers may be introduced in the gear design, resulting in smaller gears at lower cost and accepting a higher probability of damage.

It is therefore of interest to convert allowable stress numbers — as listed in gear rating standards for probability of damage $P_a = 1\%$ or reliability $P_ü = 99\%$ — to other reliability levels. While the AGMA 2001 series includes a factor K_R for this conversion, DIN 3990 and ISO 6336 do not. Some guidelines are provided below on how values can be converted to different levels of reliability for ISO 6336 (and DIN 3990), based upon the scattering of strength values as given in the listed references and some statistics.

The S-N Curve for Root and Flank Strength

The concept of the S-N curve. S-N curves are measured for a probability of survival or reliability level of $P_ü = 50\%$. Correspondingly, the probability of damage is of the same value — $P_a = 50\%$. They are measured with a scatter in terms of achieved life at a constant stress in the limited life domain (where the curve has a slope p and a scatter in terms of achieved stress level for long life (where gears in test no longer fail), expresses as the standard deviation of the allowable stress number σ (Fig. 1). The scatter in terms of achieved life is far greater than the scatter in terms of achieved stress for long life. The

Abbreviations and symbols

Abbreviation	For
σ	Standard deviation
σ	Stress
σ_{Flim}	Allowable stress number, root, ISO definition
σ_{Hlim}	Allowable stress number, flank, ISO definition
AGMA	American Gear Manufacturers Association
CHD	Case hardness depth
DIN	Deutsches Institut für Normung
f_{xF}	Factor for conversion of allowable stresses for different reliabilities, root
f_{xH}	Factor for conversion of allowable stresses for different reliabilities, flank
ISO	International Organization for Standardization
K_R	Reliability factor along AGMA 2001
ME	Material quality level, highest level
ML	Material quality level, lowest level
MQ	Material quality level, normal level
p	Life exponent, slope of the S-N curve in the limited life domain
P_a	Probability of failure
$P_ü$	Probability of survival, reliability
S	Safety factor
s_{ac}	Allowable stress number, root, AGMA definition
s_{at}	Allowable stress number, flank, AGMA definition
S-N	Stress – cycle curve
Y_Z	Reliability factor along AGMA 2101
z	Standard score for normal distribution

comments below are valid for the long-life domain.

If a large number of measurements is available, S-N curves — not only for $P_{\ddot{u}} = 50\%$ — may be determined, as may, for example, $P_{\ddot{u}} = 10\%$ and $P_{\ddot{u}} = 90\%$ as lower and upper bound. Alternatively, S-N curves for other reliability levels may be estimated from the S-N curve for $P_{\ddot{u}} = 50\%$ and a standard deviation σ based on literature (which should then be based on a large population of other tests). Factors to convert the S-N curve, or rather the allowable stress number for root and flank, from one reliability level to another, have been reported (Refs. 6–11) and are explained in the following text.

Probability of damage in ISO 6336, S-N curve. Allowable stress numbers for the flank and the root, σ_{Hlim} and σ_{Flim}, as given in ISO 6336-5 (Refs. 1–2), are valid for 1% probability of damage P_a. Probability of survival (or reliability) is therefore $P_{\ddot{u}} = 1 - P_a = 99\%$. This value is applicable for a safety factor of S = 1.00.

As per ISO 6336-5:2016 (Ref. 2), it is permissible to use σ_{Hlim} and σ_{Flim} values for other reliability levels. It is stated that "statistical analysis enables adjustment of these values in order to correspond to other probabilities of damage." It continues, "When other probabilities of damage (reliability) are desired, the values of σ_{Hlim}, σF_{lim} and σ_{FE} are adjusted by an appropriate "reliability factor." When this adjustment is made, a subscript shall be added to indicate the relevant percentage (e.g., σ_{Hlim10} for 10% probability of damage)." It is recommended to use methods described in ISO 12107 (Ref. 3) for this.

ISO 6336 does not give further guidelines on how to calculate allowable stress numbers for other reliability levels. No formulas or factors are given for a conversion of the allowable stress numbers from, for example, 1% to 10% probability of damage.

Reliability factor in AGMA 2001. Allowable stress numbers in this standard "are determined or estimated from laboratory tests and accumulated field experiences. They are based on unity overload factor, 10 million stress cycles, unidirectional loading and 99 percent reliability." This means that the probability of damage associated with the S-N curves is the same as in ISO 6336 and DIN 3990.

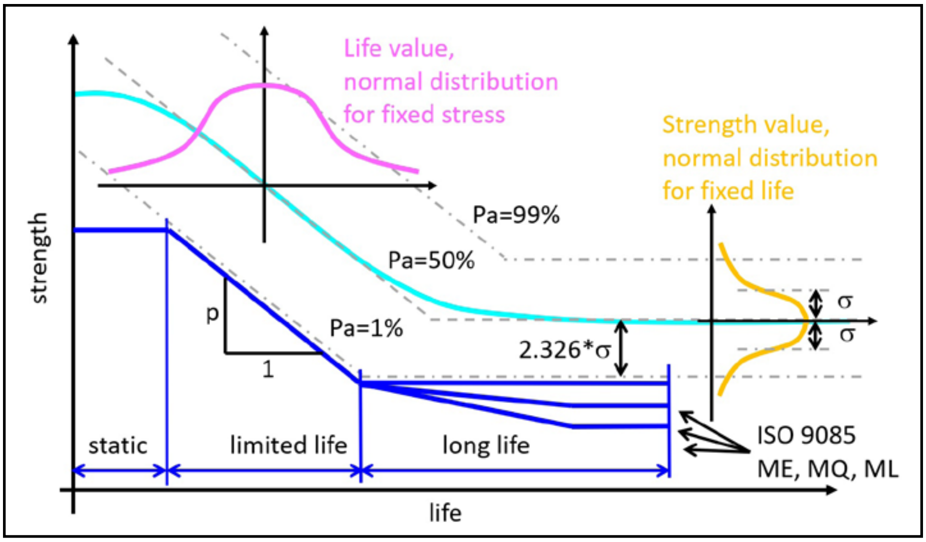

Figure 1 **Measured S-N curve for a probability of damage of $P_a = 50\%$ (cyan). S-N curves along ISO 6336 for probability of damage $P_a = 1\%$ (blue). Normal distribution of life in the limited life domain (pink). Normal distribution of endurance limit in the long life domain (yellow).**

The AGMA 2001 series (Refs. 4–5) is, however, more detailed in this regard; they include a factor that allows for gear rating for different reliability levels. A reliability factor K_R (Y_Z in AGMA 2101; Ref. 5) is introduced; it may be used to modify allowable stresses for another reliability level. The numbers are reportedly based on data developed by the U.S. Navy (Table 1).

The allowable stress number for root s_{at} and for flank s_{ac} for a desired reliability is then calculated from the s_{at} or s_{ac} value, as listed in the AGMA standard (valid for $P_a = 1\%$) divided by above reliability factor K_R. The reliability factor increases in a more or less linear fashion if reliability is increased in orders of magnitude (Fig. 2).

Allowable Stresses for Different Reliability Levels

Conversion of allowable stress numbers to reliability levels other than 50%. The data derived from measurements at $P_{\ddot{u}} = 50\%$ needs to be transformed to a probability of survival of $P_{\ddot{u}} = 99\%$, as used in ISO 6336 (or DIN and AGMA standards). The allowable stress numbers σ_{Hlim} for flank and σ_{Flim} for root for a probability of damage of 50% may be converted to a probability of damage of x% using factors f_{xH} and f_{xF} as follows (Ref. 9):

$$\sigma_{Hlim}(P_{\ddot{u}} = x\%) = \sigma_{Hlim}(P_{\ddot{u}} = 50\%)*f_{xH}$$
$$\sigma_{Flim}(P_{\ddot{u}} = x\%) = \sigma_{Flim}(P_{\ddot{u}} = 50\%)*f_{xF}$$

Values for f_{xH} and f_{xF} are listed (Tables 2 and 3) for x% = 99%. Values are taken from different sources (all of them originating from Germany) but they probably are based on the same data basis.

The standard deviation σ of the measured allowable stress number σ_{Hlim} for flank is reported (Ref. 10) at $\sigma = 2.8\%$ (for higher case hardness depth [CHD]) and $\sigma = 4.3\%$ (for lower CHD). For the allowable stress number σ_{Flim} for root the values reported are $\sigma = 3.4\%$ for shot peened gears and $\sigma = 6.0\%$ for non-shot peened gears. (*Be careful to note that the same symbol σ is used to denote the standard deviation and stress!*) This is in line with values reported (Ref. 12) for shot peened gears, where the standard deviation of the allowable stress number for the root σ_{Flim} is $\sigma = 3\%$. Note that these values are applicable for the "long life" section of the S-N curve.

An example calculation. Let us consider an example. Assume that $\sigma Flim = 500$ MPa (i.e. — a case-carburized gear with high core strength, quality grade MQ; see ISO 6336-5, Fig. 10, line "MQ, a"). This means that only 1% of the gears will have a strength lower than 500 MPa and 99% of the gears will have a strength higher than 500 MPa. Let us use the abovementioned standard deviation of $\sigma = 6\%$ (for non-shot peened gears, which is the underlying assumption in ISO 6336-5, Fig. 10).

Assuming normal distribution, we know the negative z-score for $P_a = 1\%$ ($P_{\ddot{u}} = 99\%$) is $z = -2.326$ (use a "negative z score table," e.g. — from Ref. 13 to find this value). This means that between the mean value for σ_{Flim} with $P_{\ddot{u}} = 50\%$ (as originally measured in experiments) and the value for σ_{Flim} with $P_{\ddot{u}} = 99\%$, a distance of $-2.326^{*}\sigma$ exists. (See Fig. 1 black vertical arrow between grey lines denoting S-N curve for $P_a = 50\%$ and $P_a = 1\%$. Or: $\sigma_{Flim}\ (P_{\ddot{u}} = 99\%) = \sigma_{Flim}\ (P_{\ddot{u}} = 50\%)\ -2.326^{*}6\%^{*}\sigma_{Flim}\ (P_{\ddot{u}} = 50\%)$, giving $\sigma_{Flim}\ (P_{\ddot{u}} = 99\%) = 0.86^{*}\sigma_{Flim}\ (P_{\ddot{u}} = 50\%)$.

From this we find the value $\sigma_{Flim}\ (P_{\ddot{u}} = 50\%) = \sigma_{Flim}\ (P_{\ddot{u}} = 99\%)/0.86 = 500\ \text{MPa}/0.86 = 581\ \text{MPa}$. See the vertical orange, dashed line (Fig. 2) indicating the mean value for the allowable stress number, root, of 581 MPa. The standard deviation σ is 6% thereof (assuming gears are not shot peened) $\sigma = 35$ MPa (see horizontal blue arrow (Fig. 3)).

The distance between this mean value for the allowable stress number, $\sigma_{Flim}\ (P_{\ddot{u}} = 50\%) = 581$ MPa, and the value for $P_a = 1\%$ (as used in ISO 6336) is $2.326^{*}\sigma = 2.326^{*}35\ \text{MPa} = 81\ \text{MPa}$; it is indicated in cyan color (Fig. 3).

Thus the above factor $f_{xF} = 0.86$ is nothing but: $f_{xF} = 1 + z\ (P_a = 1\%)^{*}\sigma = 1 - 2.326^{*}0.06 = 0.86$ (note $z < 0$)

If we now want to determine $\sigma_{Flim}\ (P_{\ddot{u}} = 90\%)$, as sometimes used, for instance, in vehicle transmission design, we find the z-score (Ref. 13) for $P_a = 10\%$ is $z = -1.282$. From this we find $\sigma_{Flim}\ (P_{\ddot{u}} = 90\%) = \sigma_{Flim}\ (P_{\ddot{u}} = 50\%) - 1.282^{*}6\%^{*}\sigma_{Flim}\ (P_{\ddot{u}} = 50\%) = 581\ \text{MPa}^{*}(1 - 1.282^{*}0.06) = 536\ \text{MPa}$. (Fig. 3) indicating this value and the horizontal pink arrow of length $|z|^{*}\sigma = 1.282^{*}35\ \text{MPa} = 44.7\ \text{MPa}$).

Comparison and recommendations. We have calculated the allowable stress number for the root for $P_a = 10\%$ at $\sigma_{Flim}\ (P_{\ddot{u}} = 90\%) = 536$ MPa for a case carburized, non-shot peened case. This was done by using the base line value $\sigma_{Flim}\ (P_{\ddot{u}} = 99\%) = 500$ MPa along ISO 6336 and conversion factors based on the standard deviation of the measured strength values.

Now let us compare this value with the value determined along the AGMA approach. There, $K_R = 0.85$ for the conversion from $P_a = 1\%$ to $P_a = 10\%$ is applicable (Table 1). If we apply this factor to the allowable stress number (per ISO notation), $\sigma_{Flim}\ (P_{\ddot{u}} = 90\%)$ would then be $\sigma_{Flim}\ (P_{\ddot{u}} = 90\%) = \sigma_{Flim}\ (P_{\ddot{u}} = 99\%)/K_R = 500\ \text{MPa}/0.85 = 588\ \text{MPa}$. This value is higher than the value of 536 MPa reported earlier in this presentation, and so is, best-case — less conservative, and worst case — unsafe. While it cannot be determined whether the literature cited or AGMA 2101 is more trustworthy, there is indication that at least the values for K_R should be used with caution.

ISO 6336 also cautions the user of such reliability factors. While it allows their use, it also states that "such adjustments need to be considered very carefully and may require additional, specific tests or detailed documentation of the source of the information used to derive the confidence level of the failure probabilities." This means that for the average gear engineer, "tuning" that results by "playing" with different reliabilities is not encouraged.

Table 1 Reliability factor K_R along AGMA 2001

Requirements of application	Reliability factor K_R
Fewer than one failure in 10,000	1.50
Fewer than one failure in 1,000	1.25
Fewer than one failure in 100	1.00
Fewer than one failure in 10	0.85
Fewer than one failure in 2	0.70

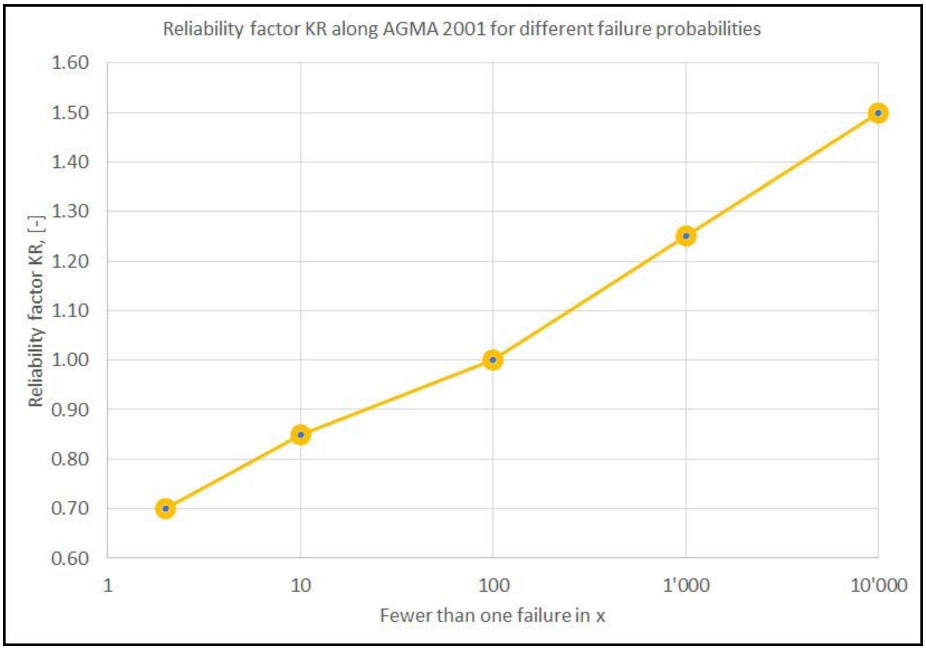

Figure 2 **Reliability factor KR along AGMA 2001; reference value is K_R=1.00 for less than one failure in 100, or a reliability of 99%.**

Table 2 Factor f_{xF} to convert allowable stress numbers, root, for $P_{\ddot{u}} = 50\%$ to other reliability levels, according to different sources

	Factor for root, f_{xF}	Reference
case hardened gears	0.86, for $P_{\ddot{u}} = 99\%$, not shot peened	[6], average value read from graph
Unhardened gears	0.90, for $P_{\ddot{u}} = 99\%$, not shot peened	[6] , average value read from graph
Case hardened gears	0.86, for $P_{\ddot{u}} = 99\%$, not shot peened	[9] , average value read from graph
Unhardened gears	0.90, for $P_{\ddot{u}} = 99\%$, not shot peened	[9] , average value read from graph
FVA guideline	0.86, for $P_{\ddot{u}} = 99\%$, not shot peened 0.92, for $P_{\ddot{u}} = 99\%$, shot peened	[8], tabulated value

Table 3 Factor f_{xH} to convert allowable stress numbers, flank, for $P_{\ddot{u}} = 50\%$ to other reliability levels, according to different sources

	Factor for flank, f_{xF}	Reference
Different materials, ground gears	0.84…0.90, for Pü = 99% 0.91…0.95, for Pü = 90% 0.05…1.08, for Pü = 10%	[7] , tabulated values

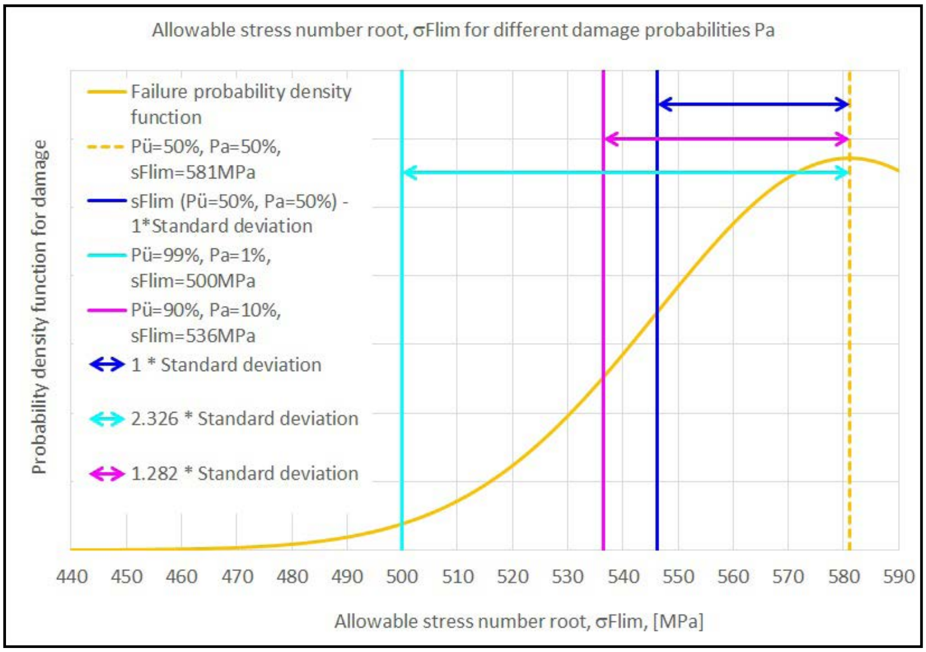

Figure 3 **Normal distribution of the allowable stress number, resulting allowable stress number for different probability of damage; standard deviation of the distribution.**

References

1. ISO 6336-5:2003. Calculation of Load Capacity of Spur and Helical Gears — Part 5: Strength and Quality of Materials.
2. ISO 6336-5:2016. Calculation of Load Capacity of Spur and Helical Gears — Part 5: Strength and Quality of Materials.
3. ISO 12107. Metallic Materials — Fatigue Testing — Statistical Planning and Analysis of Data.
4. ANSI/AGMA 2001-D04. Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth.
5. ANSI/AGMA 2101-D04, Fundamental Rating Factors and Calcualtion Methods for Involute Spur and Helical Gear Teeth, Metric Edition of ANSI/AGMA 2001-D04.
6. Weiss, T. Zahnfussfestigkeitsuntersuchungen an Gebräuchlichen Zahnradstählen, Antriebstechnik 25 (1986) Nr 12.
7. Joachim, F.J. "Streuungen der Grübchentragfähigkeit," Antriebstechnik 21 (1982) Nr. 4.
8. FVA Richtlinie 563 I. "Empfehlung zur Vereinheitlichung von Tragfähigkeitsversuchen an Vergüteten und Gehärteten Zylinderrädern," Ausgabe 2012.
9. Niemann, G. and H. Winter. *Maschinenelemente*, *Band II*, Zweite Auflage, Springer 1985.
10. Klocke, F. and C. Brecher. "Zahnrad- und Getriebetechnik," Hanser 2017.
11. Höhn, B.-R., H. Winter, K. Michaelis and K. Stahl. "Lebensdauerstatistik," FVA Vorhaben Nr. 305/II, Heft 580. 1999.
12. Sollich, Kugelstrahlen, Steigerung der Schwingfestigkeit von Verzahnungen, Braunschweig, 2011.
13. Negative Z-Score Table, *http://www.math.odu.edu/stat130/normal-tables.pdf.*

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