Gear Design

National Broach and Machine Division of
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A gear can be defined as a toothed wheel which, when meshed with another toothed wheel with similar configuration, will transmit rotation from one shaft to another. Depending upon the type and accuracy of motion desired, the gears and the profiles of the gear teeth can be of almost any form.

Gears come in all shapes and sizes from square to circular, elliptical to conical and from as small as a pinhead to as large as a house. They are used to provide positive transmission of both motion and power. Most generally, gear teeth are equally spaced around the periphery of the gear.

The original gear teeth were wooden pegs driven into the periphery of wooden wheels and driven by other wooden wheels of similar construction. As man's progress in the use of gears, and the form of the gear teeth changed to suit the application. The contacting sides or profiles of the teeth changed in shape until eventually they became parts of regular curves which were easily defined.

To obtain correct tooth action, (constant instantaneous relative motion between two engaging gears), *the common normal of the curves of the two teeth in mesh must pass through the common point, or point of contact, of the pitch circles of the two wheels,* Fig. 1-1. The common normal to a pair of tooth curves is the line along which the normal pressure between the teeth is exerted. It is not necessarily a straight line. Profiles of gear teeth may be any type or types of curves, provided that they satisfy the law of contact just defined. However, manufacturing considerations limit the profiles to simple curves belonging to the circle group, or those which can be readily generated or form cut, as with gear cutters on standard milling machines.

Because of inherent good properties and easy reproducibility, the family of cycloid curves was adopted early (1674) and used extensively for gear tooth profiles. The common normal of cycloidal gears is a curve, Fig. 1-2, which is not of a fixed direction, but varies from a maximum inclination with respect to the common tangent at the pitch point to coincidence with the direction of this tangent. Cycloidal gears roll with the direction of this tangent. Cycloidal gears roll with conjugate tooth action providing constant power with uninterrupted rotary motion. One disadvantage of this type of gear is that the center distance between mates must be held to fairly close tolerances, otherwise mating gears will not perform satisfactorily.

The involute curve was first recommended for gear tooth profiles in the year 1694 but was not commonly used until 150 years later. The curve is generated by the end of a taut line as it is unwound from the circumference of a circle, Fig. 1-3. The circle from which the line is unwound is commonly

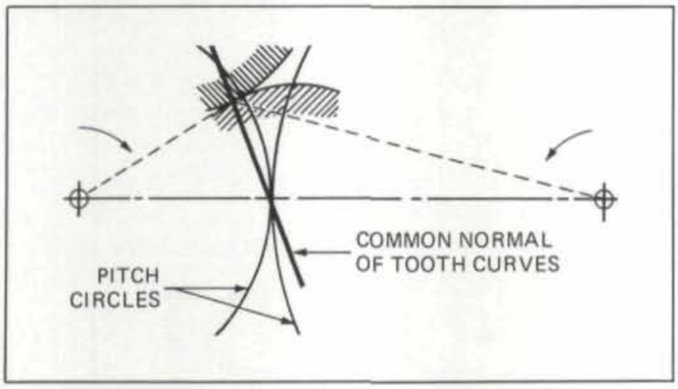

FIG. 1-1—For constant instantaneous relative motion between two engaging gears, the common normal of the curves of the two teeth in mesh must pass through the common point, or point of contact, of the pitch circles of the two gears.

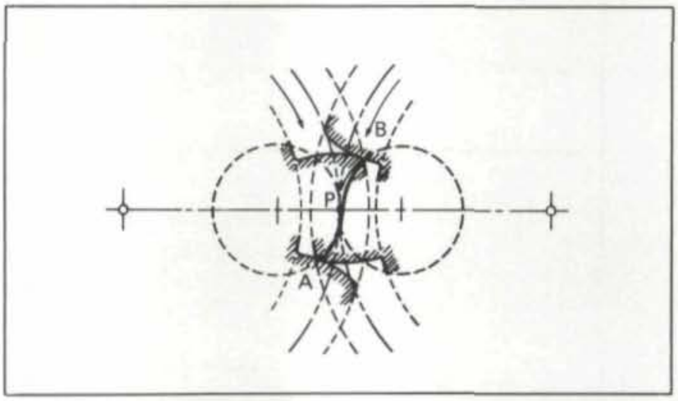

Fig. 1-2—The common normal of cycloidal gears is a curve which varies from a maximum inclination with respect to the common tangent at the pitch point to coincidence with the direction of this tangent. For cycloidal gears rotating as shown here, the arc *BP* is the*Arc of Approach*, and the arc *PA*, the *Arc of Recess*.

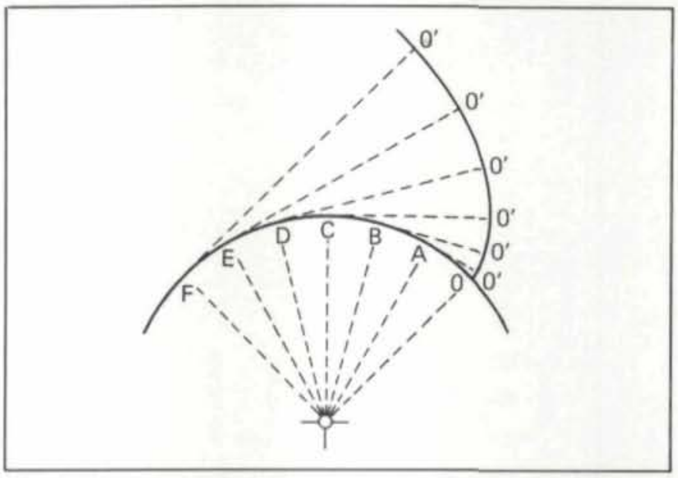

Fig. 1-3—The involute tooth form used for virtually all gearing today is generated by the end of a taut line as it is unwound from the circumference of a circle. The circle from which the line is unwound is the *Base Circle*.

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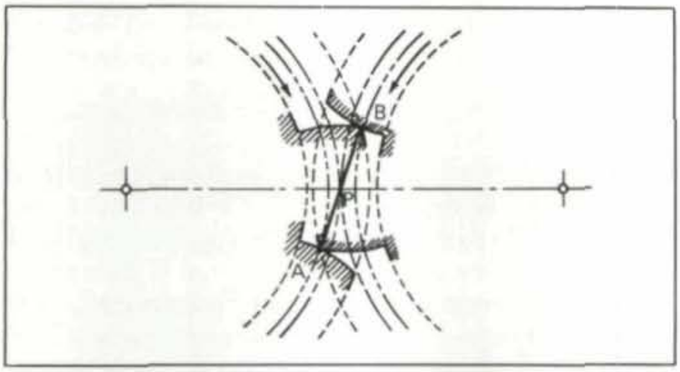

Fig. 1-4—The common normal of involute gear teeth is a straight line *AB*.

known as the "base circle". The common normal of involute gear teeth is a straight line (AB in Fig. 1-4). Gears of this type satisfy all the requirements for smooth, accurate and continuous motion. Gears with involute tooth profile are very flexible in both geometric modification and center distance variation.

There have been many other types of gear tooth forms, some related to the involute curve. One particular type of recent interest is the "circular arc" gear (where the profile is an arc from the circumference of a circle). First proposed in this country by Ernest Wildhaber in the 1920's, the circular arc gear was recently introduced by the Russians as the "Novikov" tooth form. These profiles are not conjugate. Gears with this tooth form depend upon helical overlapping of the teeth in order to roll continuously. This can and does create face width size and end thrust problems.

At the present time, except for clock and watch gears, the involute curve is almost exclusively used for gear tooth profiles. Therefore, except for an occasional comment, the following discussion will cover some of the basic elements and modifications used in the design of involute tooth form gears.

Ratio

The primary purpose of gears is to transmit motion and at the same time, multiply either torque or speed. Torque is a function of the horsepower and speed of the power source. It is an indication of the power transmitted through a driving shaft and from it the gear tooth loads are calculated. The loads applied to gear trains can vary from practically nothing to several tons or more. Gears, properly designed and meshed together in mating pairs, can multiply the torque and reduce the higher rotational speed of a power producing source to the slower speeds needed to enable the existing power to move the load. Where application requires speed rather than torque, the process if reversed to increase the speed of the power source.

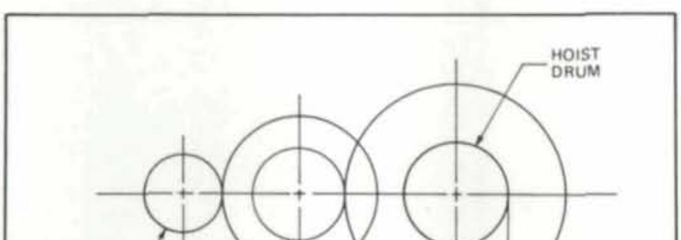

Fig. 1-5—Ratios of the gear teeth in this hypothetical hoist drive would depend upon weight (*W*) to be lifted and torque (*T*) available from the motor.

Rotational speeds of the shafts involved in power transmission are inversely proportional to the numbers of teeth (**not** the pitch diameters) in the gears mounted on the shafts. With the relative speed of one member of a pair of gears known, the speed of the mating gear is easily obtained by the equation:

$$n_G = \frac{n_P N_P}{N_G}$$

Where N_p and N_G = Number of teeth in pinion and gear.
n_p and n_G = Revolutions per minute (rpm) of pinion and gear respectively.

The ratio of speed to torque is of the utmost importance in the design of gear teeth to transmit and use the power. A typical case would involve the design of the gearing for a hoist to raise a certain weight (W) at a uniform speed, when making use of a motor with a given horsepower (hp) running at a given speed (rpm) and driving through a pinion with number of teeth N_p, Fig. 1-5.

Obviously, the ratio of the gear teeth and the number of gears needed depend entirely upon the application and the power source.

Velocity

Circumferential velocity is an important factor present in all running gears. Its value is obtained by multiplying the circumference of a given circle by the rpm of the shaft. In reference to the pitch circle it is generally referred to as "pitch line velocity" and expressed as "inches per minute" or "feet per minute".

Circumferential velocities in a complex gear train have a direct effect on the loads to be carried by each pair of gears. As the load *W*, in Fig. 1-5, is shown tangent to the perephery of the final cylinder, so the loads on gear teeth are applied tangent to the pitch diameters and normal to the gear tooth profile. Since the rpm's of mating gears are inversely proportional to the numbers of teeth, it can be shown that the pitch line velocities of the two gears are equal and the loads carried by their respective teeth will also be equal.

Elements of Gear Teeth

A very excellent reference for the names, description and definition of the various elements in gears is the American Gear Manufacturers Association (AGMA) Standard entitled "Gear Nomenclature".

Pitch

Pitch is generally defined as the distance between equally spaced points or surfaces along a given line or curve. On a cylindrical gear it is the arc length between similar points on

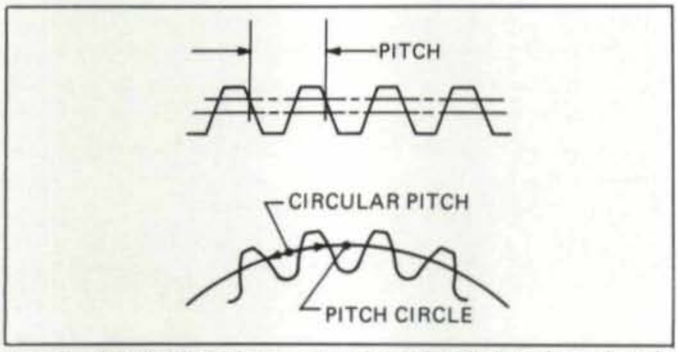

Fig. 1-6—*Circular Pitch* of gear teeth is the arc length along the pitch circle between identical points on successive teeth.

successive teeth and is known as **circular pitch** (p). See Fig. 1-6. Therefore, by definition, circular pitch of gear teeth is a function of circumference and numbers of teeth, varying with diameter and evolving into straight line elements as shown in Figs. 1-7 and 1-8. In Fig. 1-7 the teeth are shown as **helical,** or at an angle to the axis of the gear cylinder. If the teeth were parallel to the axes they would be straight or **spur** teeth as they are more commonly called. With **spur** teeth, Fig. 1-7, the **normal circular pitch** and the **transverse circular pitch** would be equal and the **axial pitch** (a straight line element) would be infinite.

One of the most important pitch classifications in an involute gear is the one termed **base pitch**, in Fig. 1-8. Primarily, it is the circular pitch on the perimeter of the base circle, but by definition of the involute curve the arc distance becomes the linear normal distance between corresponding sides of adjacent teeth when raised to position as part of the **taut line.** In spur gears there is only one base pitch to consider. On the other hand, in helical gears, base pitch is definable in the section normal to the helix angle (**normal base pitch**), parallel to the gear axis (**axial base pitch**) and perpendicular to the gear axis (**transverse base pitch**), Fig. 1-9. Since the gear teeth are equally spaced, it becomes apparent that in order to roll together properly, two gears must have the same base pitch. More specifically, two mating involute gears must have the same **normal base pitch.**

Originally gears were classified and calculated beginning with circular pitch. With the number of teeth (N) and the

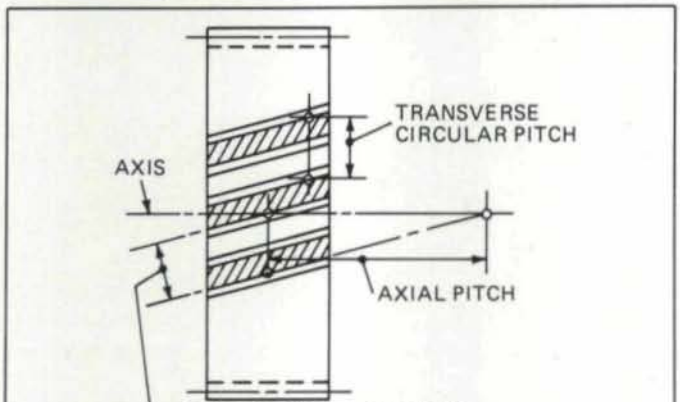

Fig. 1-7—For helical gear teeth, pitch may be measured along a line normal to the gear teeth (*Normal Circular Pitch*), in a direction perpendicular to the axis of rotation (*Transverse Circular Pitch*), and in a direction parallel to the axis of rotation (*Axial Pitch*).

circular pitch (p) given, the circumference of the circle and consequently the **pitch diameter** (D) can be calculated from

$$D = \frac{N \times p}{\pi}$$

For simplification, developers of gear design techniques created a separate term for the value of π divided by circular pitch (π/p). This is **diametral pitch** (P) Fig. 1-10 which is the ratio of teeth to the pitch diameter in inches. It is a number, it cannot be seen or measured. However, the system developed since the inception of diametral pitch is used almost exclusively wherever the decimal system of measuring is used.

Diametral pitch regulates the proportions or size of the gear teeth. The number of gear teeth and the diametral pitch regulate the size of the gear. Therefore, for a known load to be transmitted, the pitch is chosen which in turn determines the number of teeth to suit the desired ratio and size of gear. The number of teeth divided by the diametral pitch produces the diameter of the gear pitch circle, Fig. 1-9. The part of the tooth above the pitch circle is called the **addendum** and the lower part **dedendum,** Fig. 1-11. Two addendums added to the pitch diameter equal the outside diameter of the gear.

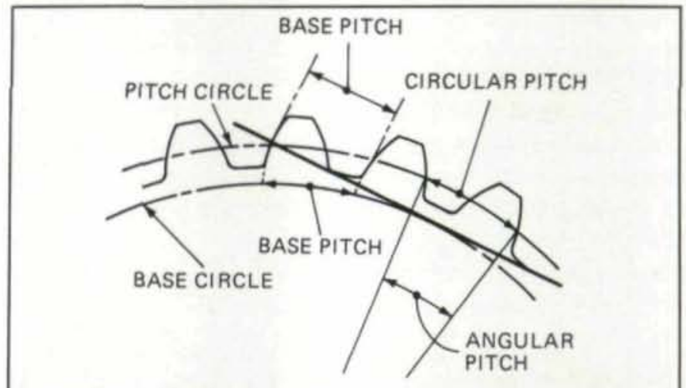

Fig. 1-8—*Base Pitch and Angular Pitch* as defined by this drawing are important gear terms. In order to roll together properly, involute gears must have the same *Normal Base Pitch*.

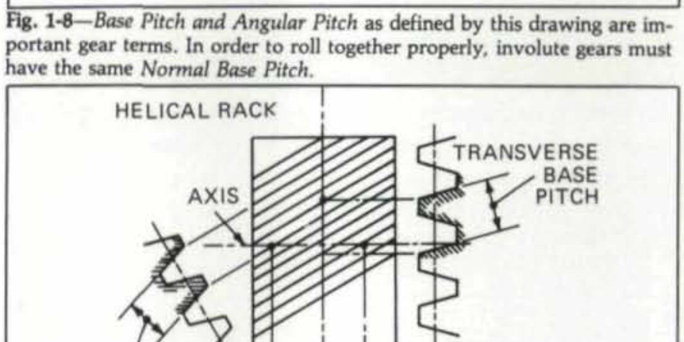

Fig. 1-9—This drawing defines *Transverse Base Pitch, Normal Base Pitch* and *Axial Base Pitch* for a helical rack.

Pressure angles

Pressure angles in involute gears are generally designated by the greek letter phi (ϕ), with subscripts to denote the various sections and diameters of the gear tooth, Fig. 1-12.

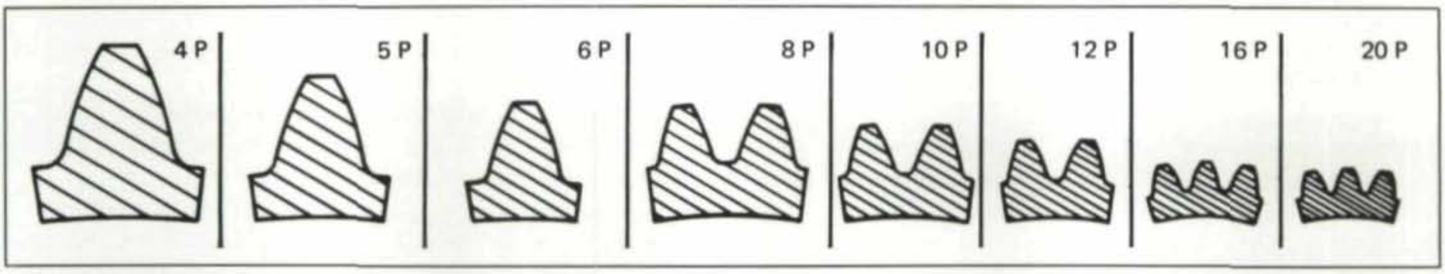

Fig. 1-10—Gear teeth of different diametral pitch, full size, 20-Deg. pressure angle.

An involute curve is evolved from origin point A on a base circle. The point P on a taut line containing point B describes the curve. The taut line is tangent to the base circle at point B, and normal to the involute curve at P. This line segment BP is known as the **radius of curvature** of the involute curve at point P and is equal in length the arc AB. The angle ϵ subtended by the arc AB is the roll angle of the involute to the point P. The angle between OP (radius r) and OB (base radius r_b) *is the pressure angle* ϕ at point P. Angle θ between the origin OA and radius OP is the polar angle of point P. (The polar angle θ and the radius r are the **polar coordinates** of point P on the involute curve). When given in radians, angle θ is known as the **involute function** of the pressure angle θ and is used extensively in gear calculations.

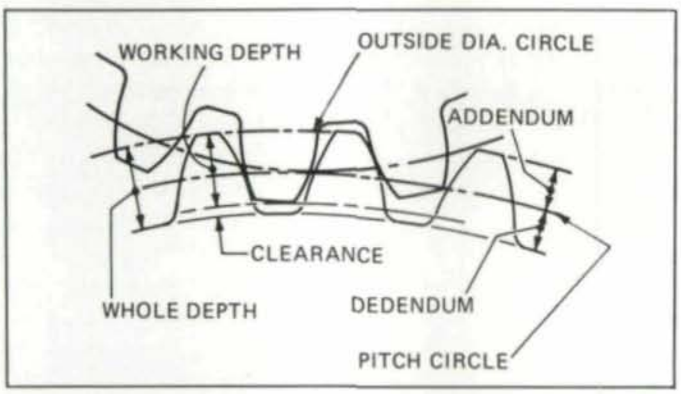

Fig. 1-11—The portion of a gear tooth above the pitch circle is called the *Addendum;* the portion of the tooth below the pitch circle is called the *Dedendum.*

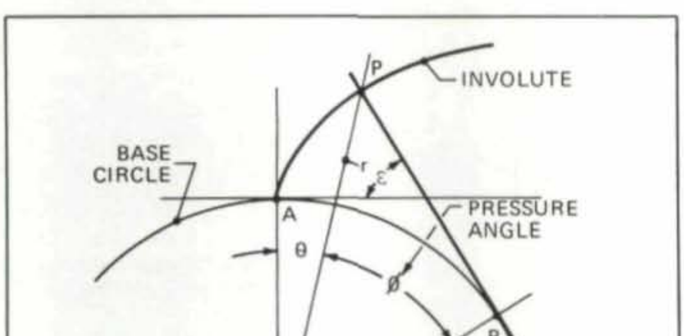

Fig. 1-12—This drawing defines *Roll Angle* (ϵ), *Pressure Angle* (ϕ) and *Polar Angle* (Θ).

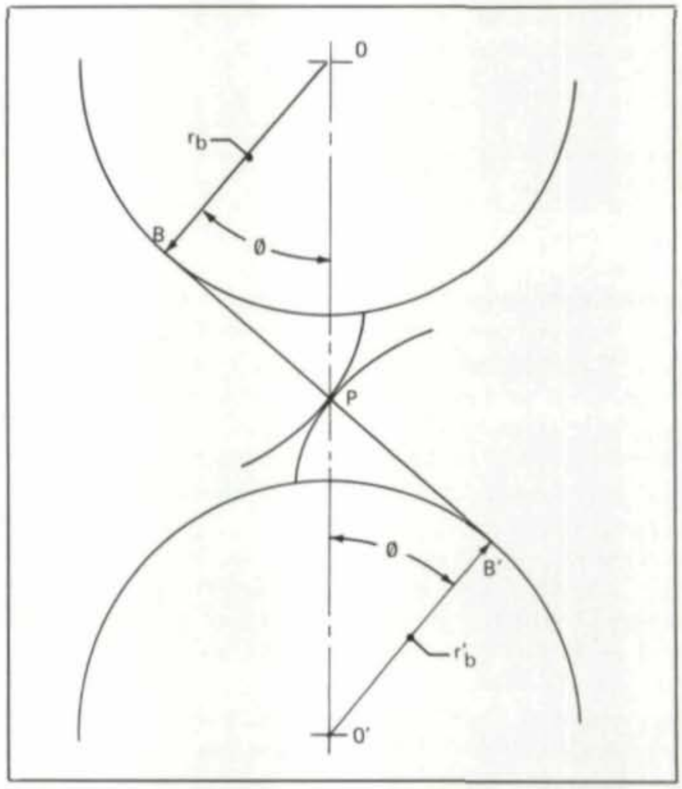

Fig. 1-13—When two involute teeth are brought into contact and made tangent at a point P, pressure angle ϕ is equal to both.

When two involute curves are brought together as profiles of gear teeth and are made tangent at a point P, the pressure angle θ is equal on both members, Fig. 1-13. The line BB' is the common normal passing through the point of contact P and is tangent to both base circles. All contact and tooth action will take place along the common normal. If one member is rotated, the involute curves will slide together and drive the other member in the opposite direction.

The pressure angle through the point of contact of a pair of involute curves is governed by regulating the distances between the centers of their respective base circles. A gear does not really have a pressure angle until its involute curved profile is brought into contact with a mating curve as defined in Fig. 1-13. At that time the pressure angle θ becomes the **operating** or **rolling** pressure angle between the mating gears. For a given center distance, C, and base circle diameters, the rolling pressure angle is determined by the expression.

$$\cos \phi = \frac{r_b + r_b'}{C}$$

Similar to the pitch element, the pressure angles of a spur gear are only in a plane normal to the gear axis. In helical gears, pressure angles are defined in three planes. The **transverse** pressure angle is normal to the gear axis or parallel to the gear face. **Normal** pressure angle is in the plane or section which is normal or perpendicular to the helix. In the plane of the gear axis the pressure angle is termed **axial.** This plane is used mostly in reference to involute helicoids with very high helix angles such as worms or threads.

As at point *P* the pressure angle at any radius greater than the given base radius may be defined as

$$\cos \phi = \frac{r_b}{r}$$

The actual rolling or operating pressure angle of a pair of gears is chosen by the designer as the most practical for his application. Several things should be considered, among which is the strength of the resulting tooth and its ability to withstand the specified load. Another important item is the rate of profile sliding, as mentioned earlier. However, the majority of involute gears are in a **standard** use class which can be made using methods and tooth proportions which are well proven. Generally, involute gears roll at pressure angles ranging from 14½° to 30°. Standard spur gears for general use are usually made with 20° pressure angle. The **normal** pressure angle of standard helical gears ranges from 14½° to 18½° and sometimes 20°. The higher pressure angles (25°-30°) are generally used in gear pumps. In standard gears these pressure angles are generally (but not always) the operating angle between mates. Usually the given pressure angle is the same as derived from the normal base pitch and selected normal diametral pitch, or

$$\cos \phi_n = \frac{P_{bn} P_n}{\pi}$$

Diametral Pitch, Numbers of Teeth and Pitch Circles

The number of factors which control or are controlled by diametral pitch would probably confound the inexperienced gear designer. Among these are: strength required of the gear teeth, the number of teeth to provide a given ratio, and size of the pitch circles to satisfy center distance or space requirements. It becomes obvious that pitch, number of teeth and pitch diameters are dependent upon and regulate each other.

Load to be transmitted by gear teeth will most certainly dictate tooth thickness which is regulated by diametral pitch. Choice of a pitch to handle a given load is one of the more difficult tasks for the gear designer. The inexperienced will probably design more than one set of gears for a given load before finalizing the design with the proper power rating. Actually, there is no method of choosing a pitch in advance which will carry a given load.

Once the torque is established, tables are available to aid in selection of a diametral pitch. After the basic gear design is completed, there are standard equations to "rate" the gears with the maximum load carrying capacity, to be compared against the orginal torque. If they are then under- or over-designed, corrections must be made. Sometimes a complete new design is required. Tooth load is not the only criterion in choosing the diametral pitch and consequently the number of teeth. Very often the tooth load is not a critical factor at all and is replaced by problems of correct speed, ratio and center distance.

Quite often when the load is small, or not even an important factor, consideration should be given to using fine pitches such as 12 to 20, or even finer. When applicable, the fine pitch gear offers longer service, greater capacity, and better production control. This is because there are more teeth for a given pitch diameter with finer pitch. This means there will be more teeth in mesh at any given instant and less load on each individual tooth. Since the fine pitch gear has a relatively shorter active profile, there is less sliding between profiles than with coarser pitch gears. This reduces the possibility of fatigue failure.

The diametral pitch referred to is usually the pitch of the tool producing the gear teeth and is known as the generating pitch. The chosen pressure angle is considered at the diametral pitch, or more specifically, at the pitch diameter, *D*.

$$D = \frac{N}{P}$$

By definition, the pitch diameter is proportional to the diametral pitch and the number of teeth, and the base diameter is the product of the pitch diameter and cosine of the pressure angle, Fig. 1-14.

$$2r_b = D_b = D \cos \phi$$

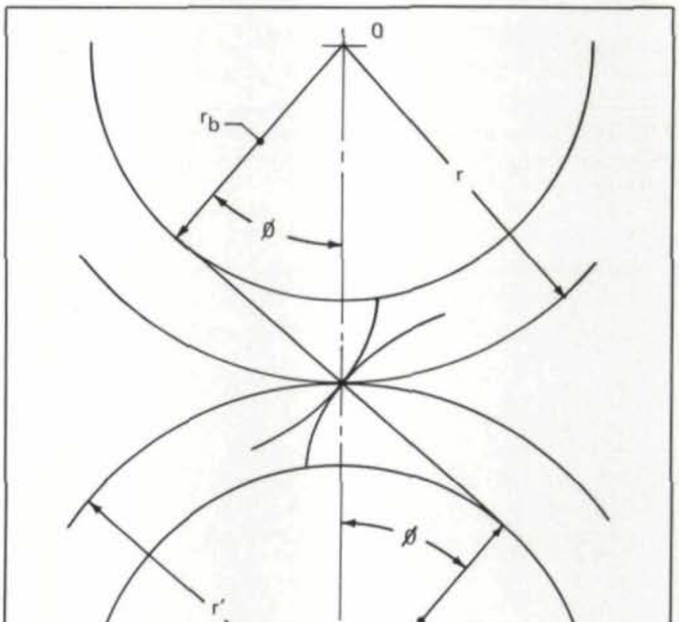

Fig. 1-14—*Base Diameter* (D_b) is twice the base radius (r_b) or the product of the pitch diameter (D) and the cosine of the pressure angle (ϕ).

In most cases a ratio controlling the amount of increase or decrease of rotational speeds is determined before the design of a gear set. The center distance $(r + r')$ is usually defined approximately by space limitations or may be given as a previously set dimension. Nominal rolling pitch circle diameters can be approximated through use of the given ratio and center distance. For example, if the desired ratio is 3 to 1, one pitch diameter will be three times larger than the other. The numbers of teeth to be used in the gears are products of the pitch diameter and the chosen diametral pitch.

$$N = PH$$

With the number of teeth defined, the next step is the design of the gear tooth and its mate.

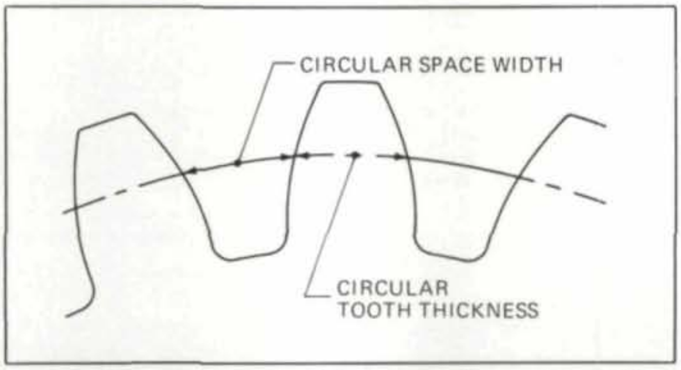

Fig. 1-15—Circular space width and circular tooth thickness defined. When dealing with these measurements, the diameter at which they are measured must be specified.

The Teeth

The elements which describe and regulate the gear tooth proportions are more or less standardized under the diametral pitch system. Most of the circular dimensions are shown in Fig. 1-8 with the radial dimensions shown in Fig. 1-11. In Fig. 1-15, the tooth and space thicknesses are illustrated. These dimensions may be given or calculated at any defined diameter.

Gear teeth having involute profiles are very versatile and adaptable to the many variations that may be required. A large percentage of involute gears have **standard** teeth.

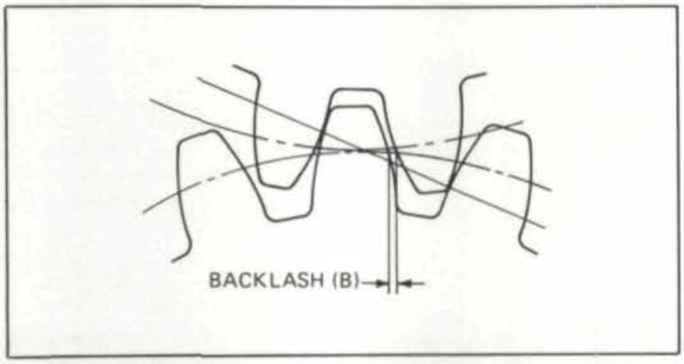

Fig. 1-16—Practical gear teeth usually are thinned to provide a certain amount of *Backlash* (*B*) to prevent tooth interference under the worst conditions of manufacturing tolerances and expansion due to temperature increase.

Standard toothed gears generally run in mating pairs on standard center distance, usually with the teeth *thinned* for backlash, Fig. 1-16. This backlash is essentially an angle, measurable in various ways and very necessary in any pair of mating gears. It must be sufficient to permit the gears to *turn freely under the worst conditions of manufacturing* tolerances and temperature variations. The amount of backlash used must be regulated and controlled within practical limits. It is possible to have too much backlash. This could be detrimental to the operation of the gear train. Basically, backlash is a factor in determining the final thickness of the gear tooth. In standard gears, the tooth thickness of one gear of a pair is determined by subtracting one half of the total desired backlash from one half of the circular pitch:

$$t = \frac{p}{2} - \frac{B}{2}$$

There are a few applications which call for no backlash. In these cases, the other dimensional elements must be held to extremely close tolerances. The required backlash depends upon where, when and how the gear set will be used. For example, in cases of tooth deflection due to heavy loading, or when extreme temperature variations are present, the amount of backlash required must be determined from experience. For general application, tables recommending backlash limits with reference to diametral pitch are available in AGMA standards and many other texts.

The whole depth of tooth, Fig. 1-11, must provide sufficient clearance for the tip of the mating tooth to swing through and make proper flank contact. Corners at the bottom, or root, of the tooth space are rounded rather than sharp. Depending on the method of manufacture, the rounding can be a true radius or a trochoid-type curve tangent to both flank and root of the tooth space. This rounding is usually expressed as **root corner radius** or more specifically as **root corner fillet.**

The expression $D + f$ is sometimes used when referring to the whole depth of gear teeth. The term has an early beginning and, for a considerable length of time, was the expression for whole depth of tooth. It was originally derived from the sum of working depth, D, and the desired root fillet, f, Fig. 1-17. Since the mating teeth make contact somewhat above the working depth circle, this method of obtaining the whole depth assured clearance between the mating tooth

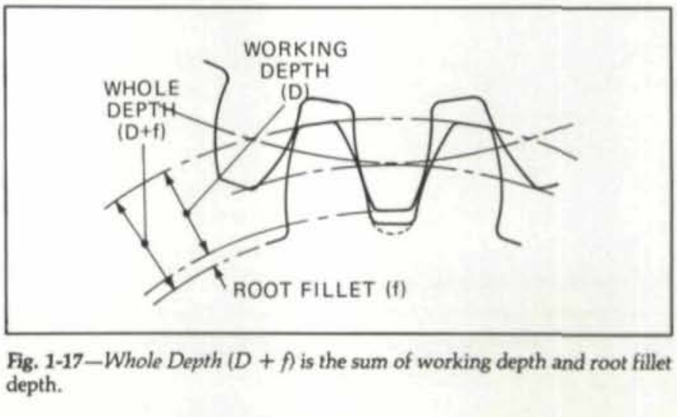

Fig. 1-17—*Whole Depth* $(D + f)$ is the sum of working depth and root fillet depth.

tip and the root fillet. The working depth is flexible and changeable within certain limits to be discussed later. With these two determinants, it was a simple matter to obtain a practical whole depth of tooth which allowed mating gears to rotate freely. The term $D + f$ is specified on most gear cutting tools to define the depth of tooth which will be produced by the tool when the tooth thickness of the gear is one-half the circular pitch.

Despite use of the term, $D + f$, whole depth remains addendum plus dedundum because the root fillet is part of the dedendum. However, with new innovations in manufacture and finishing of gear teeth, the establishment of whole depth has become more complicated in some instances.

Rack Teeth and the Basic Rack

An involute gear with an infinite pitch diameter will have straight profiles angled at the chosen pressure angle, Fig. 1-18. Such teeth are known as **rack teeth** and when several are put together in an elongated toothed member it is known as a **gear rack.** The rack tooth has the same properties as a gear tooth with the involute curve for its profile. Gear racks are driven by cylindrical involute gears and are used extensively throughout general industry.

The basic rack tooth is a special case of the involute rack tooth form. Generally, it defines the basis for a system or family of involute gears. The general tooth proportions of the basic rack show the standard design conditions of the system defined. Any involute gear of the basic rack system should be designed to roll freely with the basic rack, regardless of modification, so that it can also roll freely with any other gear of the same system, Fig. 1-19.

By definition, a tangent to any curve is perpendicular to the radius of curvature at the point of contact. Therefore, as shown in Fig. 1-20, the involute curve from a base circle is generated by tangents and radii of curvature. Accordingly, if the profile of the basic rack tooth is considered tangent to an involute curve from the base circle, it becomes the generator of an involute gear with a given number of teeth, Fig. 1-20. A cutting tool, with teeth in the form of the basic rack and used to produce gear teeth from a solid blank, is a hob, Fig. 1-21.

The generating action of a hob, as described previously, is unique in that as long as the base pitch (p_b) is maintained (see Fig. 1-19), any number of pitch and pressure angle combinations will produce the same gear tooth. However, the large majority of hobs are **standard generating** with respect to the gears they produce. By this is meant that the gear **generating** pitch diameter (sometimes called **theoretical**) is obtained by dividing the number of gear teeth by the hob diametral pitch, and the pressure angle of the hob becomes the generating pressure angle of the gear.

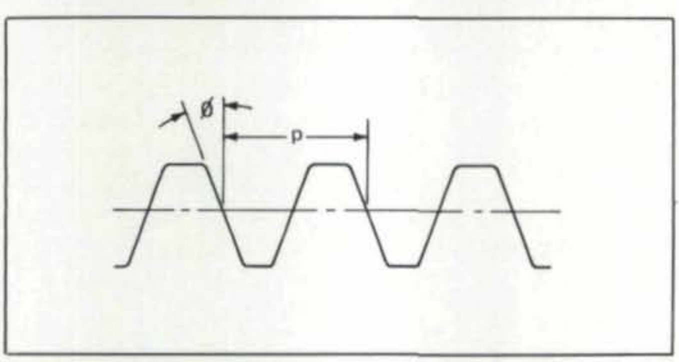

Fig. 1-18—An involute tooth form gear of infinite pitch diameter is called a *Rack*. The teeth have straight sides whose angle equals the chosen pressure angle.

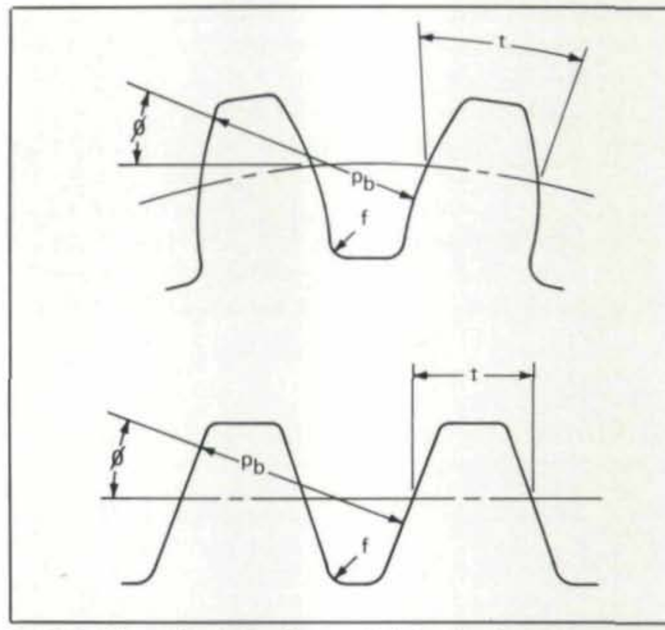

Fig. 1-19—Since a circular involute gear is designed to the same parameters as the basic rack, it will roll freely with the basic rack and with any other circular gear using the same design system.

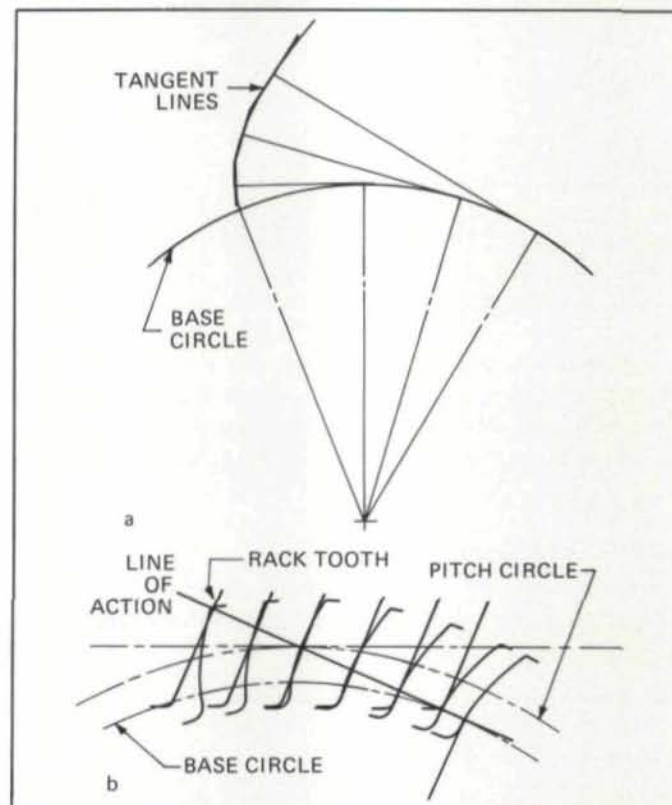

Fig. 1-20—An involute curve may be generated by a series of tangents, *a*. Therefore, if the profile of the basic rack tooth is considered to be tangent to an involute curve from a base circle, the rack becomes the generator of an involute gear with a given number of teeth, *b*.

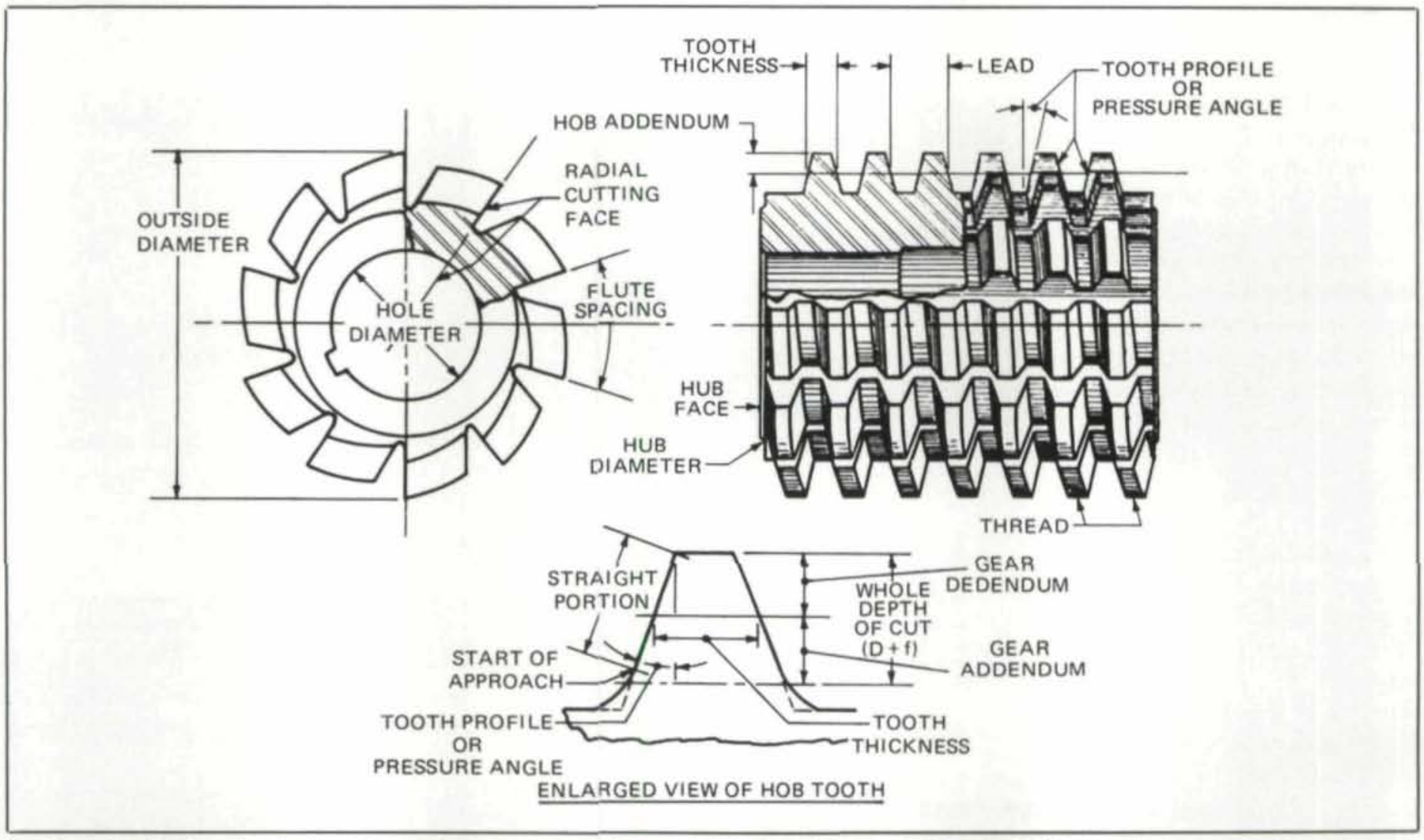

Fig. 1-21—A tool with teeth in basic rack form, which cuts gear teeth from a solid blank, is a hob. Details of a typical hob for production of involute tooth form gears are shown here.

Roots and Fillets

The gear root corner fillet is part of a trochoidal curve, generated by the corner of the hob tooth which is generally rounded. It is not practical to have a sharp corner on a hob tooth. First, the root fillet produced will be the very minimum and second, the corner will wear away—producing an inconsistent and unknown root fillet. In many cases, the root fillet produced by a standard tip corner radius of approximately 1/20 of normal circular pitch would suffice. Very often the load to be carried by the gear teeth will dictate the type and size of root fillet. Therefore, the gear designer should be aware of his control over the profile and root conditions to be produced in gears of his design.

One of the more common methods of root fillet control is the use of **short-lead** (sometimes called **short-pitch**) hobs. The term, **short-pitch**, describes and defines both tool and process. Linear lead and pitch of a hob (or rack) are one and the same as circular pitch of a cylindrical gear. Therefore, the reference **short-lead** indicates a smaller pitch. In order to maintain the base pitch, see Fig. 1-19, it is necessary to reduce the pressure angle accordingly. The difference in rolling diameters between standard generating and short lead is shown in Fig. 1-22.

The advantage of short-lead hobs is in the shape of the root corner fillets produced in the gear tooth space. A very important factor in gear tooth design is control of the root fillet tangent point in relation to the lower end of the active profile on the tooth flank. Since the tooth form is not altered, the short-lead hob allows the designer to relate root fillet magnitude with the whole depth and beam strength of the tooth. It is obvious that the closer to the center of the hob tip radius one moves the **generating** pressure angle, the closer the hob tip will come to reproducing itself, and the root fillet tangency approaches its lowest point of contact. However, there are limiting factors.

The hobbing tool designers contend that the lowest practical generating pressure angle for a hob is approximately 12 deg. for purposes of both manufacture and use. As the pressure angle of the hob is reduced, the tool has a tendency to undercut the gear tooth flank. This undercut can become excessive on smaller numbers of teeth and is a problem for the gear designer. He must decide the importance of the root fillet condition to his design.

It is possible to use hobs rolling at pressure angles greater than the theoretical pressure angle of the gear. Such hobs are known as **long-lead** and their effect on the root fillet shape is opposite that of the short-lead type. However, since conditions requiring the use of long-lead hobs are unusual, they are seldom used.

As higher performance is demanded of gears, the shape of the entire gear root space, not just the fillet alone, has become more and more important. Relative position of the gear root fillet with respect to the form diameter is a critical factor in the load carrying capacity of the gear and is also most important to the rolling conditions with its mate and with the finishing shaving cutter.*

*The comments here and in the next four paragraphs apply to other types of gear finishing tools as well as to shaving cutters.

As mentioned previously, the hobbed or shapercut root fillet is a generated curve in the trochoid family. Although often very close, it is not part of a true circle. Shaper cutters produce higher fillets than hobs for the same depth of tooth. Consequently, it is usually necessary to cut shaper-cut gears deeper than hobbed gears to maintain a good relationship between profile form and fillet diameters.

Root fillets should never extend above the form diameter. The shaving cutter must finish the gear profile for the desired length without contacting the gear root fillet, Fig. 1-23. Therefore, gear designers must regulate form diameter, root fillet shape, and root diameter of the tooth for ease of manufacture and satisfactory performance while allowing for the required load.

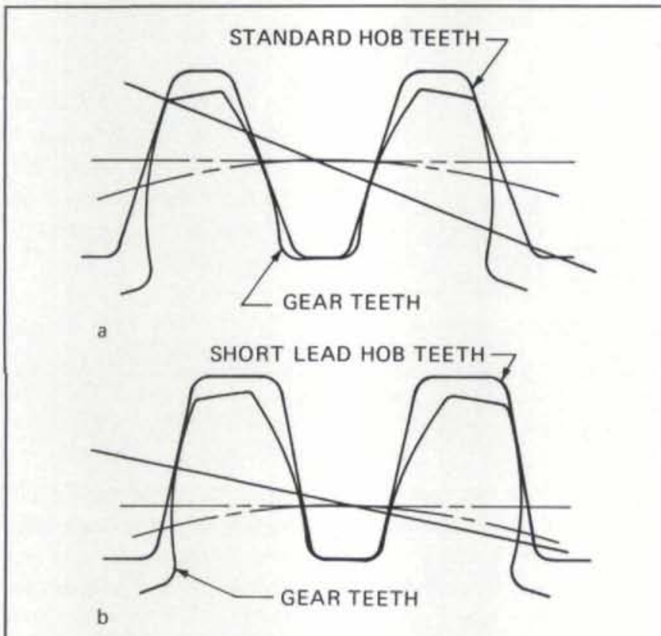

Fig. 1-22—The difference in rolling diameters for *Standard Generating, a,* and *Short Lead* hobs, *b.*

SUGGESTED CLEARANCE "A" BETWEEN START OF ACTIVE PROFILE AND TRUE INVOLUTE FORM DIA.	
Normal Diametral Pitch	Recommended Clearance (In.)
2	0.020
4	0.020
6	0.020
8	0.015
10	0.015
12	0.015
14	0.010
16	0.010
20	0.005
24	0.005
Note: Clearance depends to some extent on the size of the mating gear.	

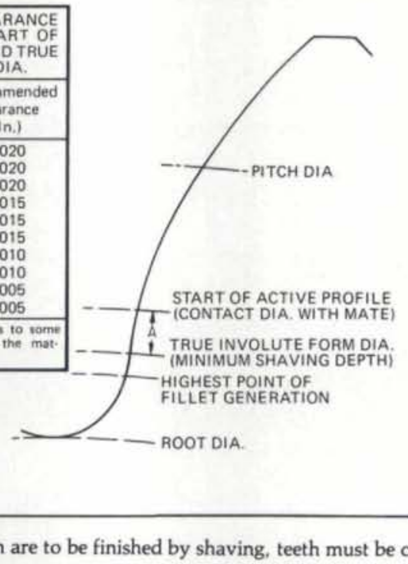

Fig. 1-23—When gear teeth are to be finished by shaving, teeth must be cut to allow for this as shown.

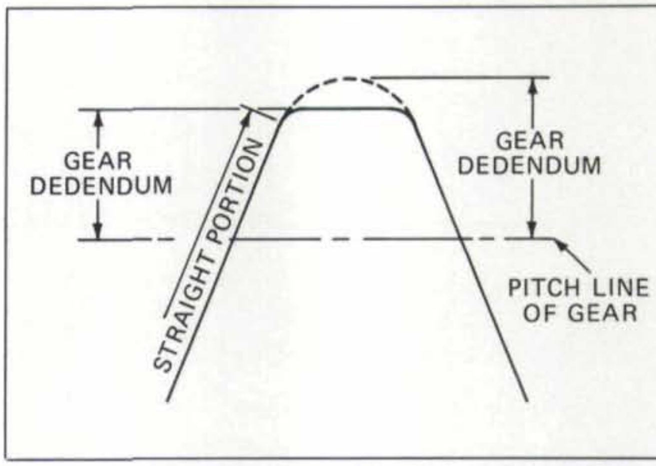

Fig. 1-24—Tooth form of preshaving hob. If full rounded root fillets are desired, additional whole depth becomes necessary.

The form diameter, as defined, is the lowest point on the gear tooth where the desired profile is to start. Often referred to as TIF (true involute form), it can also be represented in degrees roll or inches along the line of action. It is an important control point on the gear profile. Some establish the form diameter at the point of actual mating gear contact and others choose an arbitrary distance below the actual start of active profile. From a practical standpoint, the extension of the active profile should be determined by the extreme tolerances of center distance, mating gear outside diameter, size and runout.

In determining the actual whole depth and root fillet shape, the designer must consider the load which the tooth is to bear. These whole depths are based on the use of preshaving hobs and shaper cutters with a standard tip radius of 1/20 of the normal circular pitch.

If full rounded root fillets are desired, additional whole depth becomes necessary. It is advisable to maintain the same length of profile portions on the preshaving tool and determine the size of tip radius and extra depth from that point as indicated in Fig. 1-24. In this manner, the same amount of profile between form and fillet diameters is available for shaving cutter contact. When the designer must establish the fillet diameter at the maximum possible point because of load carrying demands, depending on the diametral pitch, a radial amount of 0.020/0.040 in. below the form diameter can be used safely.

During the shaving operation a small amount of material is removed from the gear tooth thickness. If the profile in the vicinity of the fillet diameter is not relieved in any manner, a step in the tooth flank will result, Fig. 1-25. This step, caused by the shaving cutter digging in, is detrimental to shaving action. It not only causes excessive wear of the shaving cutter teeth, but also affects the accuracy of the shaved profile. Thus, some amount of undercut must be provided to minimize the shaving cutter contact with the gear tooth flank.

The basic problem is to move the fillet and a short portion of tooth profile out of the path of the shaving cutter tip. In some cases with small numbers of teeth, a natural undercutting of the tooth flank occurs. Sometimes this will provide

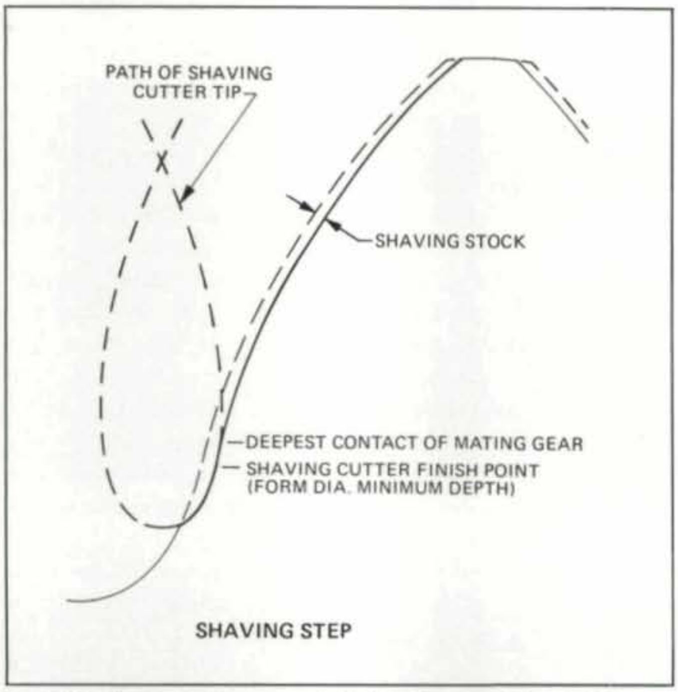

Fig. 1-25—A step in the gear tooth flank will result if the profile in the vicinity of the fillet diameter is not relieved in any way.

sufficient clearance. However, since natural undercut only occurs in specific cases, the necessary clearance must usually be provided by other means.

By putting a high point or protuberance on the flank of the preshaving tool tooth at the tip, a controlled undercut can be generated into the lower gear profile. The magnitude and shape of the undercut portion can be regulated by altering the amount of protuberance, tool tip radius, the total length of the protuberance section and/or the rolling pressure angle of the tool. Without control, profile undercut can be detrimental to the basic tooth form of the gear. If it is allowed to run up too high on the profile, it could cut away profile needed to maintain involute contact ratio with its mate. On the other hand, it might be so low that it cannot be reached by the shaving cutter and, therefore, serve no useful purpose.

Natural undercut, usually occurring in small pinions, can be controlled to some extent by changing the tool tip radius or by designing the pinion oversize from standard proportions. However, with the protuberance type tool full control of a profile undercut can be maintained.

Theoretically, the protuberance type tool should be designed for a specific gear and in accordance with the number of teeth. This becomes impractical when one desires to use the least number of tools for a range of gears with varying numbers of teeth.

Generally, the amount of undercut should be from 0.0005 to 0.0010 in. greater than the shaving stock being removed from each flank of the gear tooth. If the gear tooth flank is to be crown shaved, the depth of undercut should be increased by the amount of crown specified for each side of the tooth. The position of the undercut should be such that its upper margin meets the involute profile surface at a point below its form diameter. Sometimes, it is not possible to construct a preshave tool tooth from which will keep all generated undercut below this profile control point. In such cases, it is permissible to allow the tool to undercut the preshaved profile slightly above the form diameter, providing at least 0.0005 in. of stock is left for removal by the shaving cutter, see Fig. 1-26. However, any amount of profile removed by undercutting will reduce the involute overlap with the shaving cutter. For best control of gear tooth profile form, tooth contact ratio with the shaving cutter should never be less than 1.0 (preferably 1.2). Since the tip radius and high point of the preshave tool determine the fillet diameter and the height of undercut, it is sometimes possible to use slightly larger root fillets.

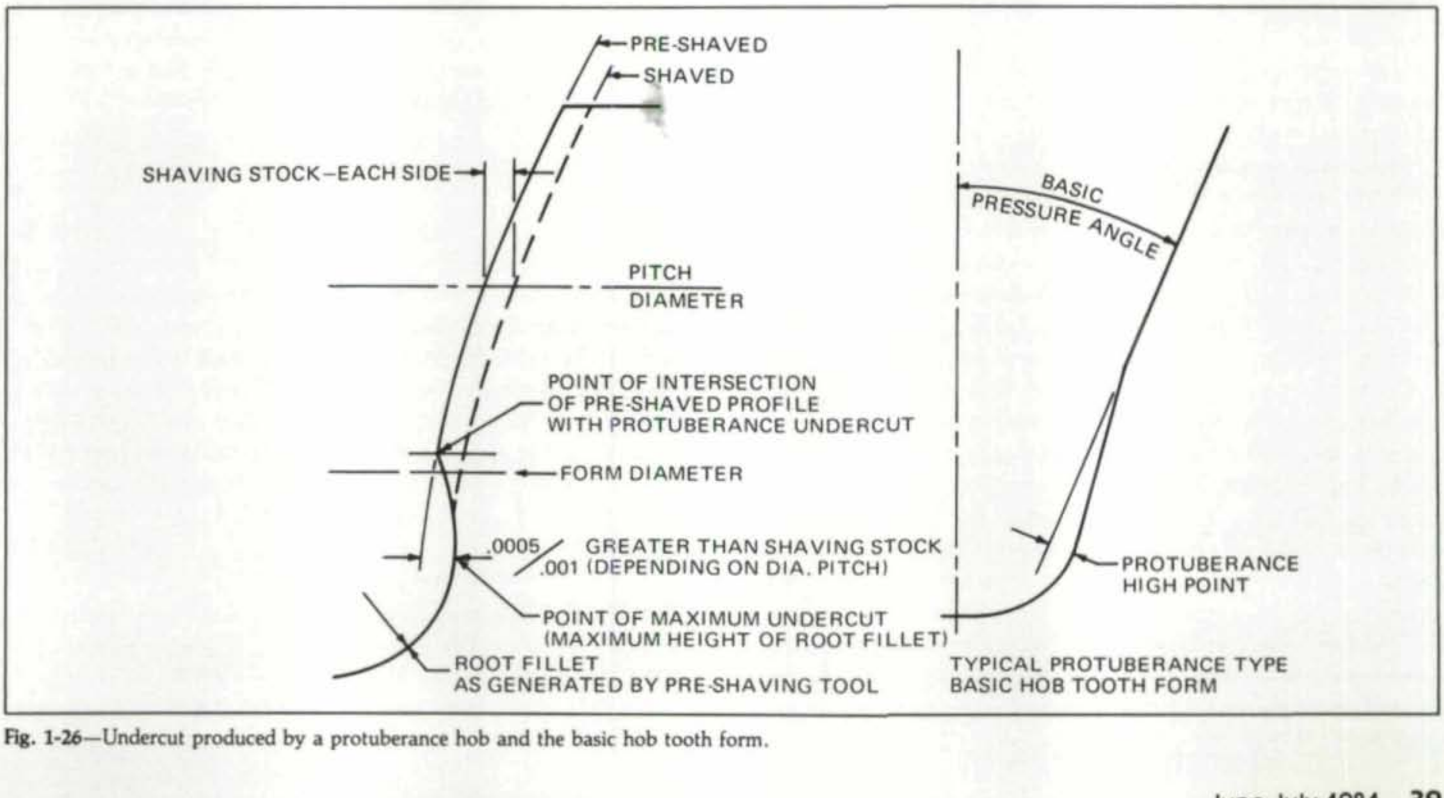

Fig. 1-26—Undercut produced by a protuberance hob and the basic hob tooth form.

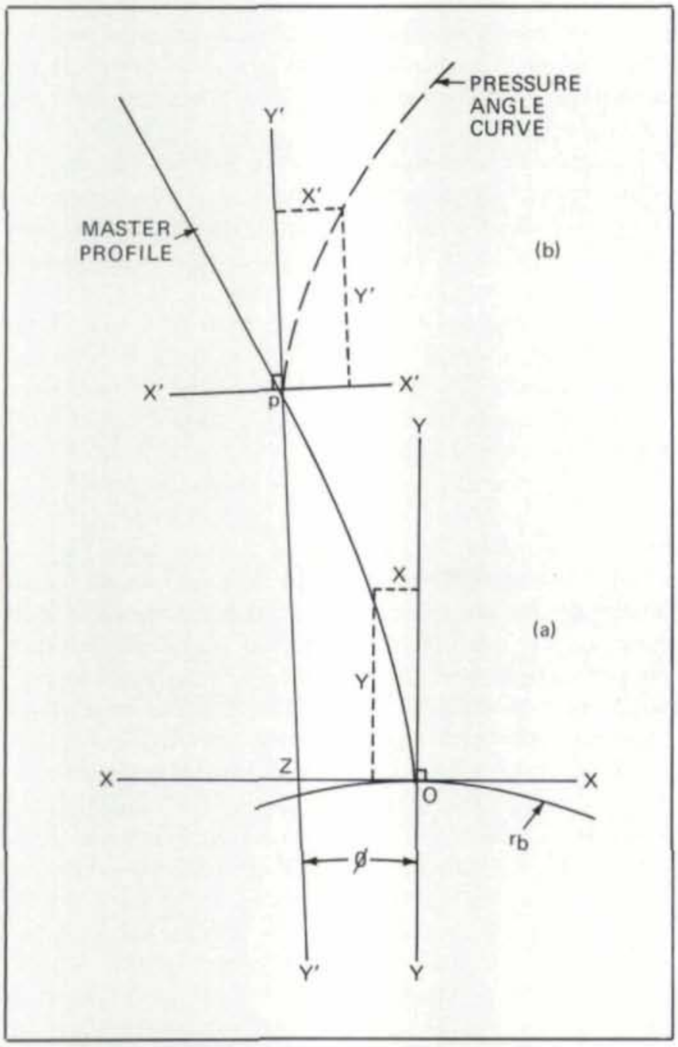

Fig. 1-27—Construction of, *a*, the master involute profile, and, *b*, the pressure angle curve.

When considering the root fillet area of the gear tooth space, it is advisable to simulate as closely as possible the conditions which will be prevalent in the gear. Some of the elements are readily calculable, such as: fillet diameter (or SRP) from a given preshave tool tip radius; the reverse calculation of a tool tip radius needed to produce a desired fillet height above a given root diameter, and the necessary increase in gear dedendum when changing from a corner radius to a full tip radius on the tool. However, proper analysis and study of the root fillet shape and capabilities can only be achieved by simulating the actual generated condition on the drawing board or in a computer.

Computer users can program the rolling action of hob or shaper cutter and receive results in approximately the same amount of time. However, by graphic or layout method, the shaper cutter presents more of a problem than the hob. Since the hob is the most prevalent of all preshaving tools, a graphic method of generating the root fillet area of gear teeth with or without the use of protuberance will be detailed.

These layouts can be made on an average size drafting board at scales of 50 to 150 times size. Scales of the layouts are limited only by normal diametral pitch and gear base diameter.

Generating Gear Root Fillets

First step is to make a master profile chart on some transparent material. The chart has a master involute profile curve which is laid out by means of rectangular co-ordinates. The base circle diameter must be large enough to produce desirable scales for the sizes of gears being used. The scale of the layout is determined by the ratio of the base circle diameters of the master profile and the work gear. Depending upon the scale desired, a single master involute profile could be used for a number of rolling layouts.

In addition to the master involute curve, the master profile chart has opposing involute curves radiating from the original involute curve. These curves represent the loci of the pitch point between the profiles of the gear teeth and the generating tools. Their number, position, and points of origin are determined by the various pressure angles of the tools to be rolled out.

The master profile chart also has a series of straight lines which are tangent to both convolutions of the involute curve. These lines represent the position of the side or profile of the generating hob or rack tooth cutter at the various degrees of pitch diameter roll.

The second step in the graphical method is to lay out the profile of the hob or rack tooth cutter on transparent material to the proper scale.

To lay out a fillet, a copy of the master profile chart is made on a white background. Then, the hob layout is laid over the master profile in various generating positions, and the fillet generated by pricking the master profile chart print. A line connecting these prick points on the print gives the true generated root fillet.

Master Profile Chart Layout

In the development of the initial master involute profile, Fig. 1-27a, it is best to choose a base radius which will provide a fair enlargement of the tooth profiles in general use. The X and Y axes are laid off on transparent material. The intersection point O is the origin of the involute curve lying on the circumference of the base circle radius, r_b, The general equations for the involute curve are:

$$Y = \cos\epsilon + \epsilon\sin\epsilon;\ X = \sin\epsilon - \epsilon\cos\epsilon$$

where ϵ is the roll angle for various points on the curve. By using a series of roll angles starting from 0 degrees, values for x and y (basic co-ordinates) may be tabulated. Tables of these rectangular co-ordinates, covering a wide range of roll angles, have been published. Subsequent co-ordinates of points on the curve are determined by the product of the chosen base radius and the values of the co-ordinates for the various roll angles:

$$Y' = r_b\,(\cos\epsilon' + \epsilon'\sin\epsilon' - 1)$$
$$X' = r_b\,(\sin\epsilon' - \epsilon'\cos\epsilon')$$

Sufficient points are calculated to form an accurate curve, which represents the involute profile of a gear tooth from the base radius.

In Fig. 1-27b, the origin of the X and Y axes for the pressure angle curve lies at point P on the master profile. To establish point P, the new Y axis is drawn through point Z on the original X axis at an angle θ from the original axis. The intersection of this new Y axis and the master profile curve is point P. Therefore,

$$\text{arc}\ \theta = \text{inv}\ \theta = \tan\theta - \text{arc}\ \theta$$
$$OZ = r_b \tan\theta$$

where θ is the pressure angle of the hob used in generating the gear tooth form. The new X axis is drawn perpendicular to the new Y axis through point P.

The radius r to the point P becomes the base radius of the pressure angle curve which is of involute form and opposed to the curve of the master profile. Radius r is determined through use of the master profile base radius and the chosen pressure angle:

$$r = \frac{r_b}{\cos\theta}$$

Substituting r for r_b in the equations for Y' and X' and using the same roll angles and basic co-ordinates as used for the master involute, the co-ordinates for the points on the pressure angle curve are calculated and plotted. Subsequent curves are established and plotted through the use of various pressure angles.

The straight lines shown in Fig. 1-28 are the control lines representing the straight side or profile of the hob tooth generating the gear tooth form. They are drawn tangent to the master involute profile and intersect all pressure angle curves. Their number depends only upon the desired accuracy of the gear fillet layout and their position relative to the original Y axis is of no consequence. Since the flank of a hob tooth is always tangent to the profile of the generated gear tooth, it is important that these straight lines are always tangent to the master involute profile.

The steps illustrated in Figs. 1-27 to 1-29 complete the involute profile chart. Actual generating layout work is done on prints of the chart, not on the original drawing. In this manner, gears of different diametral pitches and pressure angles may be generated on the same form. The scale of each layout is determined by the ratio of the base radius of the master involute form and the base radius of the gear to be generated:

$$s = \frac{r_{bl}}{r_{bG}}$$

where s is the layout scale, r_{bl} is the base radius of the master layout, and r_{bG} is the base radius of the gear. The scale is used to determine the dimensions of the hob profile which generates the gear tooth fillet and undercut. The height of the fillet and undercut is then reconverted to a radial distance above the gear base circle.

Hob Tooth Layout

The hob tooth form is laid out as shown in Fig. 1-29. This layout is made on transparent paper using the scale calculated from the equation of s. Construction lines KK, MM and NN are laid off first. The distance a is the radial dedendum of the gear tooth space from the pitch radius r to the desired or produced root radius r_r:

$$a = r - r_r$$

Point P' is the pitch point on the hob profile which coin-

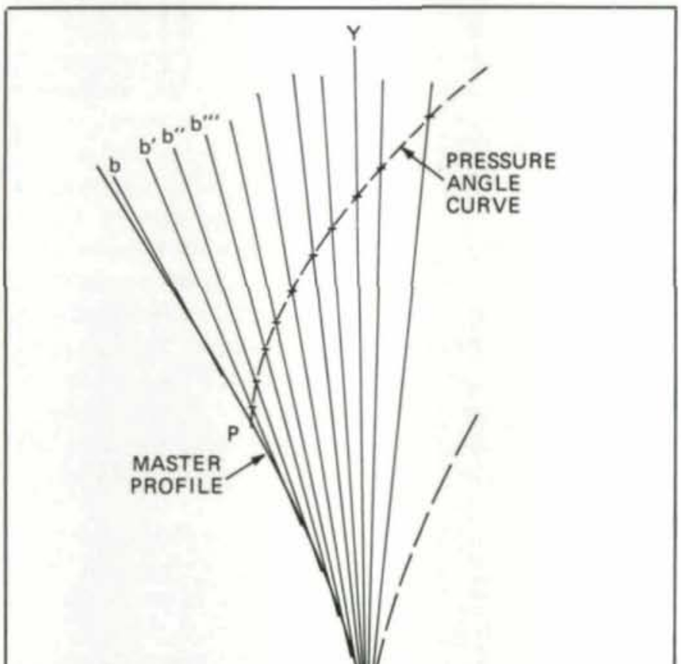

Fig. 1-28—Control lines representing positions of straight side of hob in generation of gear tooth form.

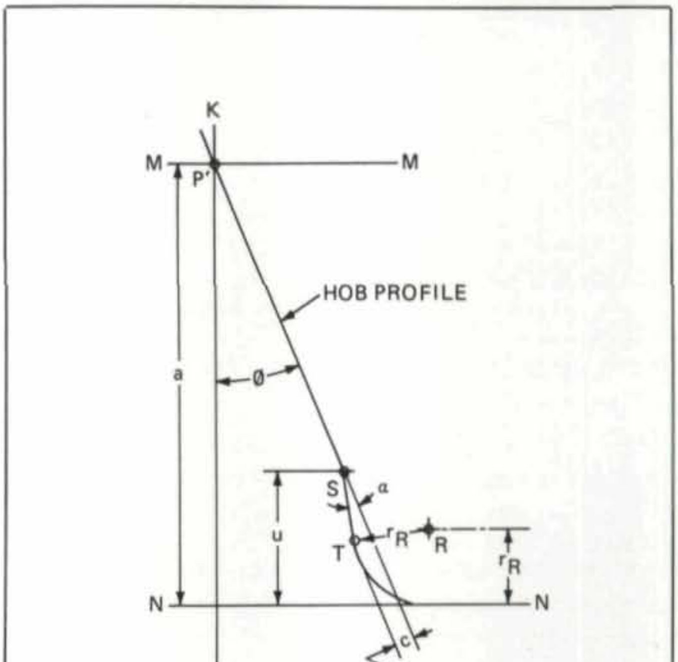

Fig. 1-29—Hob tooth profile.

cides with point P on the master profile. A line representing the main profile of the hob is laid off through P' at pressure angle ϕ to line KK and extending to the tip of the tooth on line NN. The amount of protuberance high point is measured-off parallel to the hob profile, and the desired tip radius of the hob is laid-in tangent to the distance c and the construction line NN. The approach to the protuberance is made tangent to tip radius r_R *at point* T and intersecting the hob profile at point S. The distance u to the start of approach is usually given on hob tool prints supplied by the vendors. However, 5 degrees is a good approximation of the angle a for the approach. Points P', S, T and R are encircled for future use. In order to regulate or control the amount and position of undercut produced, it may be necessary to change the tip radius, protuberane and points of intersection several times before completing a satisfactory tooth form.

When no protuberance formed under cut is desired, it is only necessary to make the tip radius tangent to the tip and profile of the hob. Then point T will lie on the hob profile and point S will cease to exist.

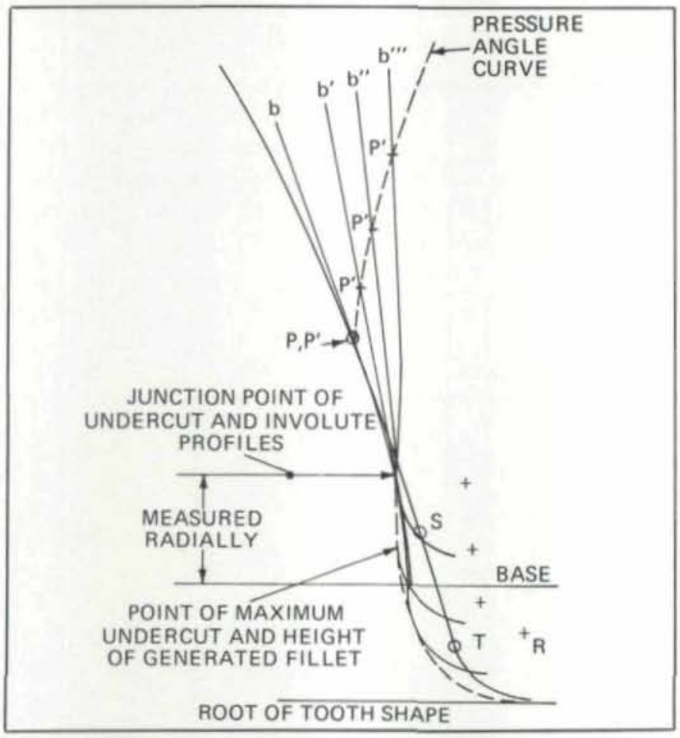

Fig. 1-30—Generation of root fillet by hob protuberance.

Using the Master Profile

For most efficient use, prints or duplications of the master profile chart should be of the type which show the lines of the layout on a white background. Then as shown in Fig. 1-30, the layout of the hob is laid over the master profile with point P' on the hob profile directly over P on the master profile and with the main profile of the hob coincident with the initial control line b which is tangent to the profile at point P. Using the point of a pricker, point P' is aligned with P as perfectly as possible and then points S, T and R on the hob form are punched through to the print below. A compass set to the scaled tip radius of the hob is centered at the transferred point R and the radius is inscribed through the transferred point T. Points S and T are joined by a straight line. In all cases, point S should fall on a control line coinciding with the main profile of the hob. Subsequent positions of the hob tip are determined by the same procedure, using the intersection points between the control lines and the pressure angel curve for locating P'. As P' proceeds along the pressure angle curve, the fillet and the undercut are formed to the root of the gear space by the hob tip radius, protuberance and approach.

Modifications of Standard Gear Tooth Forms

Quite often gears with teeth of standard proportions are either ill-suited or inadequate for the purpose intended. The versatility of the involute system makes it applicable in such cases. As long as a few basic rules are observed, the possible types of modified tooth forms are quite extensive.

Stud Tooth Gears

One of the most common types is the **Stub Tooth Form.** This tooth form differs from the standard type in tooth depth, Fig. 1-31. The shorter height makes a stronger tooth and minimizes undercut produced in small pinions. However, the length of contact between mating gears is shortened, which tends to offset the increase in tooth strength as well as raise the noise level of running gears.

The actual amount of reduction in tooth height depends upon the gear application and has definite limits. The length of the line of contact should never be less than one base pitch long. This, of course, is to maintain continuous action from tooth to tooth. Through experience, development and use, several standard stub tooth form systems have been established. Among these are: American Standard 20-Degree, Fellows Stub-Tooth and the Nuttall Stub-Tooth systems. These standard systems all have definite formulas for arriving at tooth proportions. They are very well defined in AGMA Standards, *Machinery's Handbook* and other texts.

A special case of stub tooth design is in its use as non-running involute spline teeth. Involute splines have maximum strength at the base; they can be accurately spaced and are self-centering which equalizes the bearing and stresses. *The teeth can be measured and fitted accurately.* Normally the tooth height is standardized at 50% of that based on the diametral pitch. For example, a 5 diametral pitch spline tooth would have a 10 pitch addendum and whole depth. Usually the pressure angle is 30 deg. However, this is not mandatory and quite often is changed to suit design conveniences. Involtue spline teeth may be either helical or spur.

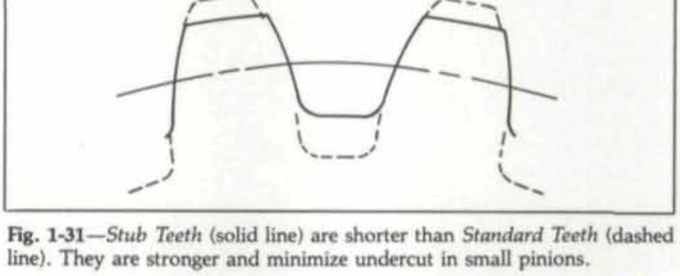
Fig. 1-31—*Stub Teeth* (solid line) are shorter than *Standard Teeth* (dashed line). They are stronger and minimize undercut in small pinions.

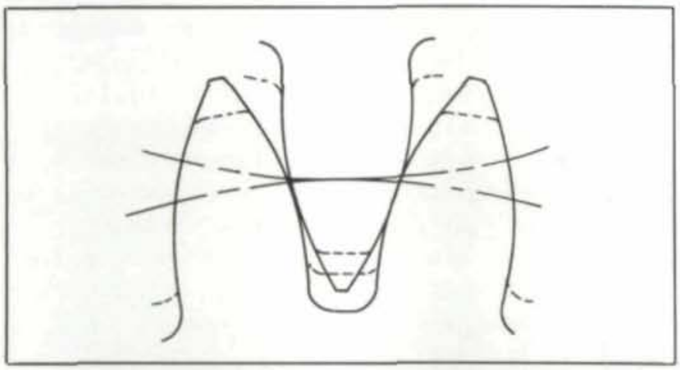

Fig. 1-32—*Extended Addendum Gears* (solid line) have longer period of contact to provide a smoother roll between adjacent mating teeth.

Extended Addendum Gears

Sometimes it is desirable to have the maximum possible length of contact between mating gears. The reasons for gear designs of this type are specialized and will vary with each application. However, when properly designed and accurately manufactured, it can be assumed that the longer period of contact will provide a smoother roll between adjacent mating teeth.

Normally, gears of this type are designed to roll on standard center distance with standard tooth thickness and backlash requirements. The exception to full standard tooth proportions is the extended addendum on both mating gears. This results in a longer radial working depth which requires lower root diameters, see Fig. 1-32. The length of the addendum is limited by the minimum allowable top land of the tooth and loss of beam strength due to the tooth length and possible undercut of the tooth flank.

Long and Short Addendum Gears

Occasionally a design will require a gear set wherein one member is considerably smaller than the other. If the tooth proportions are made standard, root fillet conditions produced in the small pinion and mating gear contacts may

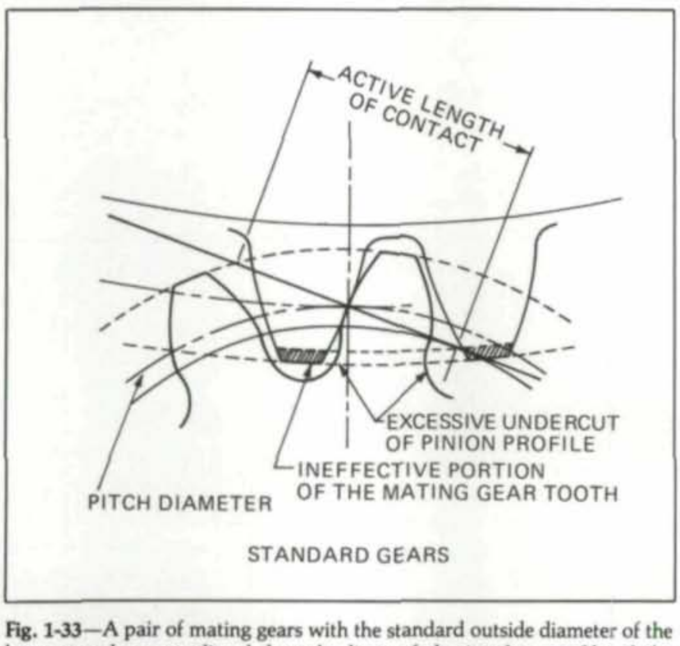

Fig. 1-33—A pair of mating gears with the standard outside diameter of the larger member extending below the limit of the involute profile of the pinion.

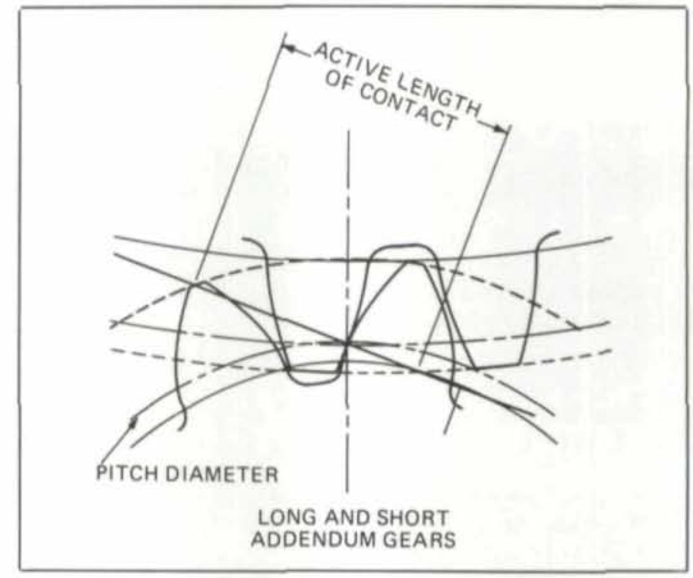

Fig. 1-34—*Long and Short Addendum Gears* minimize the problems where the pinion is very small compared to the mating gear.

result in a poorly operating set of gears. As an example, Fig. 1-33 shows a pair of mating gears with the standard outside diameter of the larger member extended beyond the limit of the involute curved profile of the pinion. Since all profile action stops at the base circle, the mating gear addendum extending beyond the line of action represents a loss of contact between mates. Further, if the root of the pinion flank had not been excessively undercut, thus weakening the tooth, the mating gear tip would have found fillet metal as interference to its trochoidal sweep. The **Long and Short Addendum Method** was devised to alleviate this type of situation.

Normally, the method is applied after a gear set is already designed as *standard* and the undesirable conditions have been discovered. The pinion outside diameter can be increased (long addendum), and the gear outside diameter decreased (short addendum) by an equal amount, so that the numbers of teeth, ratio, standard working depth and center distance remain unchanged, see Fig. 1-34.

Dependent on the pressure angle and the numbers of teeth, the amount of diameter revision can be varied. Limiting factors include the minimum acceptable top land on the pinion tooth, excessive profile sliding and the beam strength requirements for the gear. These design items must be checked after the preliminary tooth proportions have been established.

The actual amount of *profile shift*, as it is known, can be determined in various ways, including an outright guess. However, there are more exact methods. The difference between this new outside diameter and the original standard design divided by two is the *profile shift*. An approximate maximum shift would be one-half of standard addendum for the given diametral pitch. In any event, changing the diameters by two times the profile shift results in an oversize (long addendum) pinion and an undersize (short addendum) gear compared with the original standard units. Consequently, it becomes necessary to revise the tooth thicknesses.

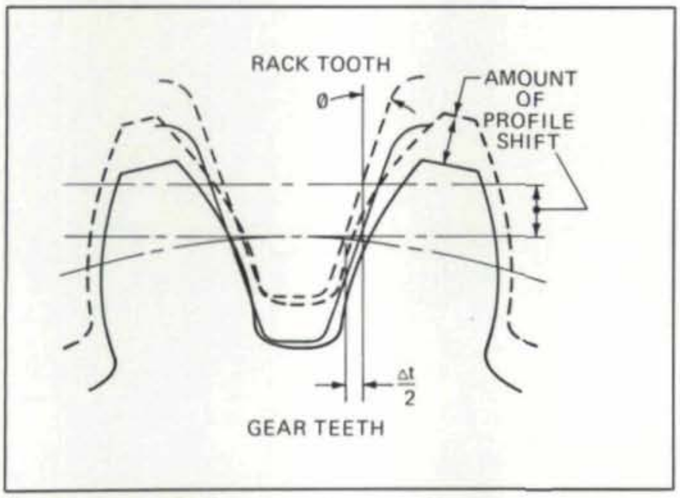

Fig. 1-35—Long addendum pinion showing profile shift.

Consider the basic rack tooth and its relationship to the gear tooth space. When the rack (hob) tooth is fed radially in towards the gear center a thinner gear tooth results. If the rack tooth is pulled out away from the center, the gear tooth becomes thicker, Fig. 1-35.

Therefore,

$$\frac{\Delta t}{2} = \text{Profile Shift} \times \text{Tangent } \phi$$

$$\text{Pinion tooth thickness} = t_p = \frac{p}{2} + (\Delta t) - \frac{B}{2}$$

$$\text{Gear tooth thickness} = t_G = \frac{p}{2} - (\Delta t) - \frac{B}{2}$$

These calculations can be used for both spur and helical gears. In helical gears, the dimensions used are normal to the helix angle at the generating pitch diameter.

The preceding equations and discussions are based on maintaining standard center distance, where the generating pitch diameters are the actual rolling pitch diameters. Therefore, the calculations for the tooth thicknesses are both simple and exact.

Non-Standard Center Distance Gears

Sometimes it is necessary to operate a pair of gears on center distances other than standard. Although the involute gear system lends itself readily to either an increased or decreased center distance, it is usually more expedient to consider a center distance greater than standard for a more efficient gear set. From the previous discussion, it is obvious that to operate on a spread center distance, the gear teeth themselves can no longer be standard and must be specially computed. The calculations involved are a little more tedious than those for long and short addendum gears, but they are not difficult.

Equations are also available for obtaining the outside diameters, size control and other dimensions necessary to complete the gear set design.

Over- and under-sized gear designs of this type are quite common. In the speed reducer and automotive transmission fields, it is much more expedient to change the design of the gear teeth for a ratio change than to incur the larger tooling cost for changing the gear housing center distance.

This ability, to modify physically the shape of the teeth and still have an efficiently operating gear set, is one of the tremendous assets of the involute gear system.

Internal Gear Teeth

Until now, the discussion has dealt exclusively with external-type gears where the teeth protrude outwardly from a cylindrical body and have convexly curved profiles. *The counterpart to the external is the internal gear where the* teeth protrude inwardly from the inside diameter of a ring, Fig. 1-36. Similar to external-type gears, internal gears can heave almost any tooth shape as long as it will roll continuously with or conform to the mating tooth profile. Internal gears are largely used in reduction gear trains and as spline teeth in non-running units.

Involute internal gear teeth have concave curved profiles. Theoretically, an external and internal gear with the same number of teeth and tooth proportions would conform exactly, except for root clearances. Internal gears, with involute curve profiles, have the same basic fundamentals as the externals. In some of the equations, it will be necessary to subtract instead of adding. However, the same forms and modifications used with external gears may be used with internal gears.

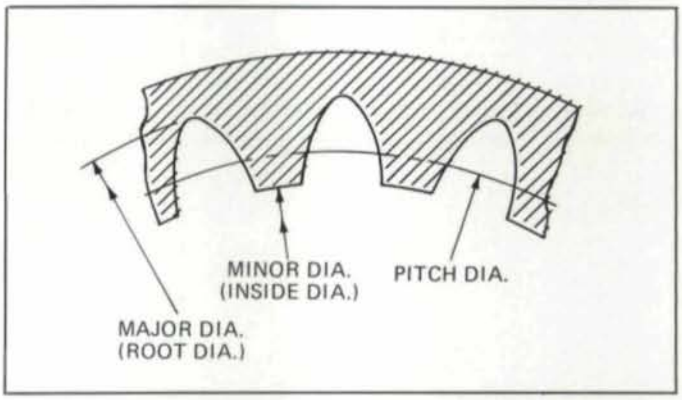

Fig. 1-36—Internal gear teeth protrude inwardly from the inside diameter of a ring.

Modification of Tooth Profile and Lead

Quite often errors in manufacture, deflections of mountings, deflections of teeth under load, and distortion of materials in heat treatment all combine to prevent the attainment of true involute contact between mating teeth. Besides contributing to objectionable noise, these undesirable meshing conditions are inefficient and lead to premature failure of the teeth. Some of these factors are illustrated in Fig. 1-37.

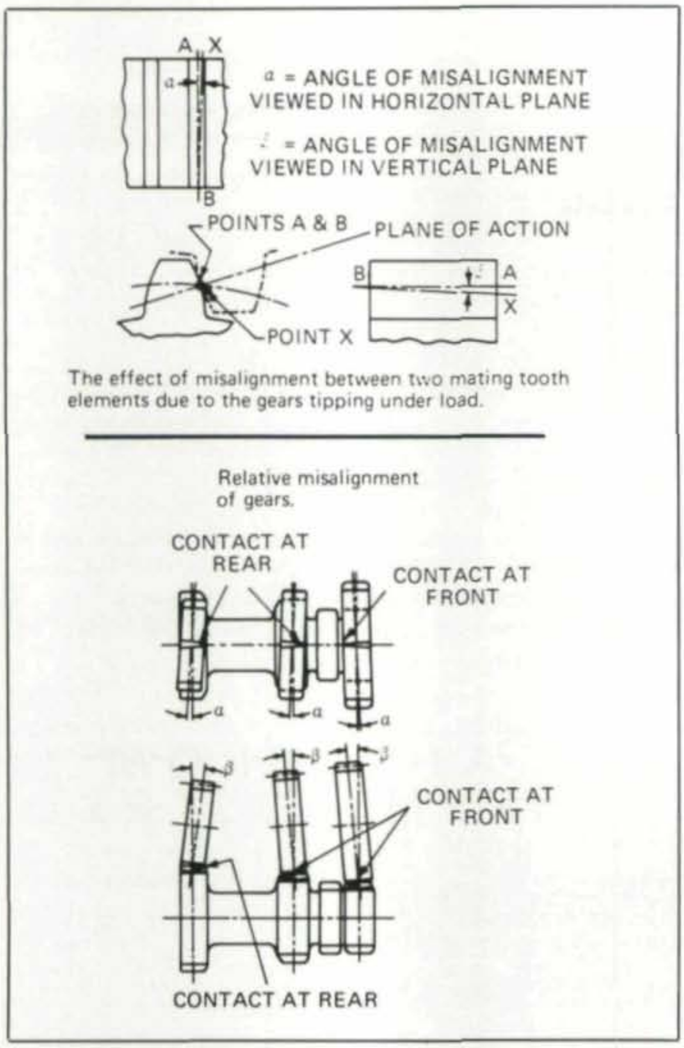

Fig. 1-37—Certain undesirable meshing conditions lead to premature failure of gear teeth. Some of these are shown here.

To alleviate and minimize the effect of these errors, the profiles and leads of the gear teeth are modified. The modifications are departures from the true theoretical form and designed to offset the undesirable contacts caused by the original errors, Fig. 1-38.

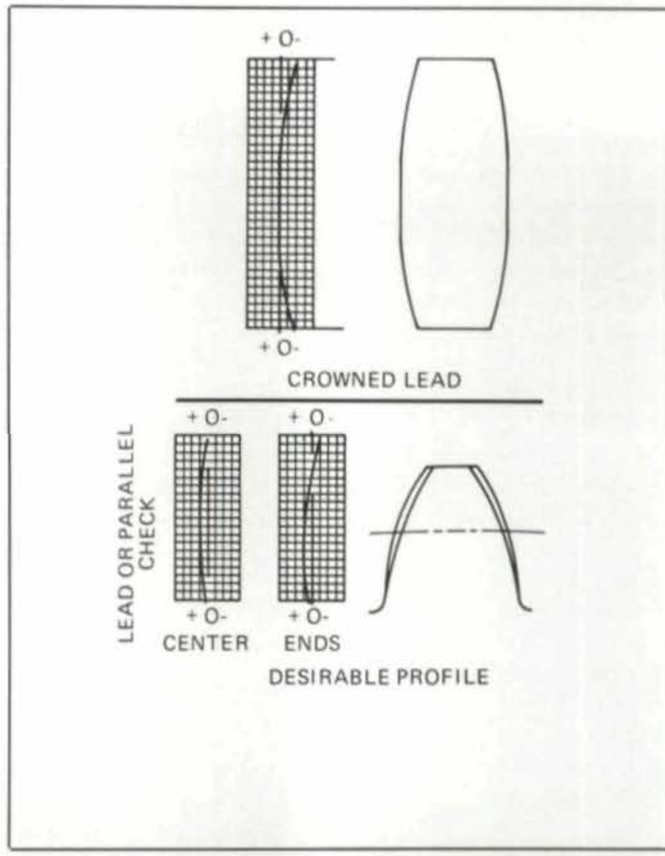

Fig. 1-38—Crowned gear teeth minimize the undesirable effects of departures from theoretical tooth form.

Normally the errors are discovered after design and manufacture of the prototype or initial lot of gears. Modifications are then developed to suit the conditions. Sometimes the designer may be experienced enough to predict, at least approximately, the distortions to take place and, from his experience, be able to order initial modifications.

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