# ENGINEERING CONSTANTS . . .

## RULES AND FORMULA *For Worm Gearing*

**(Based on Lewis Formula)**

| TO FIND | RULE | FORMULA |
|---|---|---|
| Linear pitch | Divide the lead by the number of threads. (see "thread" below) | $P = \frac{L}{N}$ |
| Addendum of worm tooth | Multiply the linear pitch by 0.3183. | $S = 0.3183P$ |
| Pitch diameter of worm | Subtract twice the addendum from the outside diameter. | $d = 0 - 2S$ |
| Pitch Diameter of Worm-Wheel | Multiply the number of teeth in the wheel by the Linear Pitch of the Worm, and divide the product by 3.1416. | $D = \frac{NP}{3.1416}$ |
| Center Distance between worm and gear | Add together the Pitch Diameter of the Worm and the Pitch Diameter of the Worm-Wheel, and divide the sum by 2. | $C = \frac{D + d}{2}$ |
| Whole depth of worm tooth | Multiply the Linear Pitch by 0.6866. | $W = 0.6866P$ |
| Bottom Diameter of worm | Subtract twice the whole depth of tooth from the outside Diameter. | $b = o - 2W$ |

| TO FIND | RULE | FORMULA |
|---|---|---|
| Helix Angle of worm | Multiply the Pitch Diameter of the Worm by 3.1416 and divide the product by the lead; the quotient is the cotangent of the Helix Angles of the Worm. | $\cot B = \frac{3.1416d}{L}$ |
| Width of thread tool at end | Multiply the Linear Pitch by 0.31. | $T = 0.31P$ |
| Minimum length of worm for complete action | Subtract four times the Addendum of the Worm thread from the outside Diameter of the wheel, square the remainder, and subtract the result from the square of the outside Diameter of the wheel. The square root of the result is the minimum length of Worm advisable. | $x = \sqrt{O^2 - (O - 4S)^2}$ |
| Outside Diameter of worm | Add together the Pitch Diameter and twice the Addendum. | $o = d + 2S$ |
| Pitch Diameter of worm | Subtract the Pitch Diameter of the Worm-wheel from twice the center distance. | $d = 2C - D$ |

$P$ = Circular Pitch of Wheel and Linear Pitch of Worm;

$L$ = Lead of Worm;

$n$ = Number of Threads in Worm;

$S$ = Addendum, or Height of Worm Tooth Above Pitch Line;

$d$ = Pitch Diameter of Worm;

$D$ = Pitch Diameter of Worm-Wheel;

$o$ = Outside Diameter of Worm;

$O$ = Outside Diameter of Worm-Wheel;

$b$ = Bottom or Root Diameter of Worm;

$N$ = Number of Teeth in Worm Wheel;

$W$ = Whole Depth of Worm Tooth;

$T$ = Width of Thread Tool at End;

$E$ = Helix Angle of Worm and Gashing Angle of Wheel;

$C$ = Distance Between Centers;

$x$ = Threaded Length of Worm;

"Thread" = It is Understood that by the Number of Threads is Meant, Not Number of Threads per Inch, But the Number of Threads in the Whole Worm — One if It is Single Threaded, Four, if It is Quadruple Threaded, etc.

## CONSIDERATIONS REGARDING STRENGTH OF WORM GEARING

The chief purpose of worm gearing is to reduce velocity. Hence, when designing worm drives it is essential that the diameter of the worm be kept as small as possible. Obviously if the diameter of worm is too large the worm gear may overheat and start undue wear.

"Industrial" Worm Gears are finished off so there are no sharp edges or corners on the teeth. This helps to eliminate friction and consequent heating. It also keeps from weakening the strength of the teeth.

*Refer to table on page 198 for determining the strength* of teeth. Table at right can be used for safe working unit stresses. The values are for single, double, triple and quadruple threads. In all cases the strength of the worm wheel is considered rather than the strength of the worm. It is safe practice to figure a worm gear as a spur gear insofar as strength of teeth is concerned.

## TABLE OF WORKING STRESSES

***For the Strength of Worm Gears***

**Used with "Lewis" Formula**

| Velocity in Feet Min. = V | Strength Factors = Y | Safe Working Unit Stress = S in Pounds per Sq. In. | |
|---|---|---|---|
| | | Cast Iron | Phosphor Bronze |
| 0 | 1.000 | 5300 | 8000 |
| 100 | .857 | 4550 | 6800 |
| 200 | .750 | 4000 | 6000 |
| 300 | .666 | 3550 | 5350 |
| 450 | .571 | 3000 | 4500 |
| 600 | .500 | 2650 | 4000 |

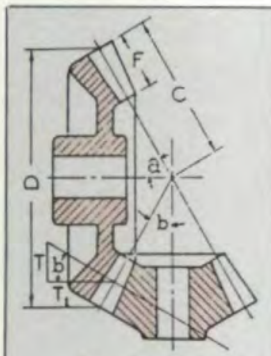
# RULES AND FORMULAS *For the Strength of Bevel Gears*
## *(Based on the Lewis Formula)*

*USE RULES AND FORMULAS (1) TO (4) IN ORDER GIVEN*

| No. | TO FIND | RULE | FORMULA |
|---|---|---|---|
| 1 | Velocity in feet per minute at the Pitch Diameter. | Multiply the product of the Pitch Diameter in inches; and the number of Revolutions per minute by 0.262. | $V = 0.262\ DR$ |
| 2 | Allowable Unit stress at given Velocity. | Multiply the allowable static stress by 600 and divide the result by the Velocity in feet per minute plus 600. | $S = Ss \times \dfrac{600}{600 + V}$ |
| 3 | Maximum safe Tangential load at Pitch Diameter. | Multiply together the allowable stress for the given velocity, the width of face, the tooth outline factor and the difference between the Pitch Cone Radius and the width of face: divide the result by the product of the Diametral Pitch and the Pitch Cone Radius. | $W = \dfrac{SFY\ (C\text{-}F)}{PC}$ |
| 4 | Maximum safe Horsepower. | Multiply the safe load at the pitch line by the velocity in feet per minute, and divide the result by 33,000. | $H.P. = \dfrac{WV}{33{,}000}$ |

D = Pitch Diameter of Gear in Inches;
R = Revolutions per Minute;
V = Velocity in Feet per Minute at Pitch Diameter;
Ss = Allowable Static Unit Stress for Material; (or the allowable stress at zero velocity);
S = Allowable Unit Stress for Material at Given Velocity;
F = Width of Face;
$N^1$ = Number of Teeth in Equivalent Gear; (see diagram in table below);
Y = Outline Factor; (see table below);
P = Diametral Pitch (if circular pitch is given, divide 3.1416 by circular pitch to obtain diametral pitch);
C = Pitch Cone Radius;
W = Maximum Safe Tangential Load in Pounds at Pitch Diameter;
H.P. = Maximum Safe Horsepower.

## *Factors for Calculating Strength* OF BEVEL GEARS




$$N^1 = \frac{\text{Number of Teeth}}{\text{COS.a}}$$

**Table of Outline Factors (Y) for 14½° and 20° Involute**

| $N^1$ | Outline Factor = Y: 14½° Involute (Std.) | Outline Factor = Y: 20° Involute | $N^1$ | Outline Factor = Y: 14½° Involute (Std.) | Outline Factor = Y: 20° Involute |
|---|---|---|---|---|---|
| 12 | 0.210 | 0.245 | 27 | 0.314 | 0.349 |
| 13 | 0.220 | 0.261 | 30 | 0.320 | 0.358 |
| 14 | 0.226 | 0.276 | 34 | 0.327 | 0.371 |
| 15 | 0.236 | 0.289 | 38 | 0.336 | 0.383 |
| 16 | 0.242 | 0.295 | 43 | 0.346 | 0.396 |
| 17 | 0.251 | 0.302 | 50 | 0.352 | 0.408 |
| 18 | 0.261 | 0.308 | 60 | 0.358 | 0.421 |
| 19 | 0.273 | 0.314 | 75 | 0.364 | 0.434 |
| 20 | 0.283 | 0.320 | 100 | 0.371 | 0.446 |
| 21 | 0.289 | 0.327 | 150 | 0.377 | 0.459 |
| 23 | 0.295 | 0.333 | 300 | 0.383 | 0.471 |
| 25 | 0.305 | 0.339 | Rack | 0.390 | 0.484 |

*(Also see top of page 68)*

## END THRUST ON BEVEL AND MITRE GEARS

The Method of Calculation of End Thrusts is as Follows:

A = Pressure Angle of Gear Teeth.

K = Tooth Pressure at Middle of Tooth Face.

F = Separating Force = K x Tan. A.

B = Pitch Angle of Pinion.

T = Thrust on Pinion = K x Tan. A x Sin. B.

$T_1$ = Thrust on Gear = K x Tan. A x Cos. B.

The table at right gives the factors by which the tooth pressure is multiplied to find the thrust which give practically the same values found by solving the formulae for T and $T_1$ given above.

| Gear Ratio | Pressure Angle A: 14½° Gear | 14½° Pinion | 20° Gear | 20° Pinion |
|---|---|---|---|---|
| 1 -1 | .183 | .183 | .257 | .257 |
| 1½-1 | .215 | .143 | .303 | .202 |
| 2 -1 | .232 | .116 | .325 | .163 |
| 2½-1 | .240 | .096 | .338 | .135 |
| 3 -1 | .246 | .082 | .345 | .115 |
| 3½-1 | .249 | .071 | .350 | .100 |
| 3¾-1 | .250 | .067 | .352 | .094 |
| 4 -1 | .251 | .062 | .353 | .088 |
| 4½-1 | .253 | .056 | .355 | .079 |
| 5 -1 | .254 | .051 | .357 | .072 |
| 5½-1 | .255 | .046 | .358 | .065 |

## RULES AND FORMULAS *For the Strength of Gear Teeth*
### (Based on the Lewis Formula)

*USE RULES AND FORMULAS (1) TO (4) IN THE ORDER GIVEN*

| No. | To Find | Rule | Formula |
|---|---|---|---|
| 1 | Velocity in feet per min. at the pitch diameter | Multiply the product of the diameter in inches and the number of revolutions per minute, by 0.262 | $V = 0.262\ DR$ |
| 2 | Allowable unit stress at given velocity | Multiply the allowable static stress by 600 and divide the result by the velocity in feet per min. plus 600 | $S = Ss \times \frac{600}{600 + V}$ |
| 3 | Maximum safe tangential load at pitch diameter | Multiply together the allowable stress for the given velocity, the width of face, and the tooth outline factor; divide the result by the diametral pitch | $W = \frac{SAY}{P}$ |
| 4 | Maximum safe horsepower | Multiply the safe load at the pitch line by the velocity in feet per minute, and divide the result by 33,000 | $H.P. = \frac{WV}{33{,}000}$ |

D = Pitch Diameter of Gear in Inches;
R = Revolutions per minute;
V = Velocity in Ft. per Min. at Pitch Diameter;
Ss = Allowable Static Unit Stress for Material;
S = Allowable Unit Stress for Material at Given Velocity;
A = Width of Face in Inches;
Y = Outline Factor (see table below);
P = Diametral Pitch (if circular pitch is given, divide 3.1416 by circular pitch to obtain diametral pitch);
W = Maximum Safe Tangential Load in Lbs. at Pitch Diameter;
H.P. = Maximum Safe Horsepower.

Combining steps 2, 3 and 4 from above chart.

$$H.P. = \frac{Ss \times A \times Y \times V}{55 \times P \times (600 + V)}$$

## FACTORS FOR CALCULATING *Strength of Gear Teeth*

| No. of Teeth | Outline Factor = Y: 14½° Involute and Cycloidal | Outline Factor = Y: 20° Involute | No. of Teeth | Outline Factor = Y: 14½° Involute and Cycloidal | Outline Factor = Y: 20° Involute | No. of Teeth | Outline Factor = Y: 14½° Involute and Cycloidal | Outline Factor = Y: 20° Involute |
|---|---|---|---|---|---|---|---|---|
| 12 | 0.210 | 0.245 | 20 | 0.283 | 0.320 | 43 | 0.346 | 0.396 |
| 13 | 0.220 | 0.261 | 21 | 0.289 | 0.327 | 50 | 0.352 | 0.408 |
| 14 | 0.226 | 0.276 | 23 | 0.295 | 0.333 | 60 | 0.358 | 0.421 |
| 15 | 0.236 | 0.289 | 25 | 0.305 | 0.339 | 75 | 0.364 | 0.434 |
| 16 | 0.242 | 0.295 | 27 | 0.314 | 0.349 | 100 | 0.371 | 0.446 |
| 17 | 0.251 | 0.302 | 30 | 0.320 | 0.358 | 150 | 0.377 | 0.459 |
| 18 | 0.261 | 0.308 | 34 | 0.327 | 0.371 | 300 | 0.383 | 0.471 |
| 19 | 0.273 | 0.314 | 38 | 0.336 | 0.383 | Rack | 0.390 | 0.484 |

## WORKING STRESSES *For the Strength of Gear Teeth*
### Used in the Lewis Formula

| Velocity in Feet per Minute = V | Strength Factors | Safe Working Unit Stress = S, in Pounds Per Square Inch | | | | | |
|---|---|---|---|---|---|---|---|
| | | Cast Iron | | Phosphor Bronze | | Steel | |
| | | Ordinary Workmanship | High-Grade Workmanship | Ordinary Workmanship | High-Grade Workmanship | Ordinary Workmanship | High-Grade Workmanship |
| 0 | 1.000 | 6,000 | 8,000 | 9,000 | 12,000 | 15,000 | 20,000 |
| 100 | 0.857 | 5,150 | 6,850 | 7,700 | 10,300 | 12,800 | 17,100 |
| 200 | 0.750 | 4,500 | 6,000 | 6,750 | 9,000 | 11,200 | 15,000 |
| 300 | 0.666 | 4,000 | 5,350 | 6,000 | 8,000 | 10,000 | 13,300 |
| 450 | 0.571 | 3,400 | 4,550 | 5,150 | 6,850 | 8,550 | 11,400 |
| 600 | 0.500 | 3,000 | 4,000 | 4,500 | 6,000 | 7,500 | 10,000 |
| 900 | 0.400 | 2,400 | 3,200 | 3,600 | 4,800 | 6,000 | 8,000 |
| 1,200 | 0.333 | 2,000 | 2,650 | 3,000 | 4,000 | 5,000 | 6,650 |
| 1,800 | 0.250 | 1,500 | 2,000 | 2,250 | 3,000 | 3,750 | 5,000 |
| 2,400 | 0.200 | 1,200 | 1,600 | 1,800 | 2,400 | 3,000 | 4,000 |

*(continued on page 48)*

## ROTARY GEAR HONING . . .

*(continued from page 37)*

of tool and a relatively short honing cycle be used. What is produced in the way of surface finish, then, represents a compromise. First, the honing tool must remove nicks and burrs; then it should make minor tooth corrections that will improve sound level and wear life. The improvement in surface finish, which is in reality a by-product of the honing process, is a valuable adjunct which will help promote long wear life as well as improving sound characteristics.

### Honing Gound Gears

In the aerospace industry, gears are traditionally operated at high speeds under heavy loads. They are usually cut, heat treated and ground to provide tooth surfaces (usually of sophisticated modified forms) of the highest order of accuracy. However, tests with exotic surface measuring equipment have shown that ground surfaces have a jagged, wavy profile that will not support heavy loads or wear long unless costly break-in procedures are carried out.

Ground tooth surfaces usually have a surface finish in the 16 to 32 $\mu$ range. Honing with type "AA" honing tools can bring this surface finish down to the 8 to 10 $\mu$ range (Fig. 2). In one 39-tooth, 5-D.P., 20° P.A., 7.800" P.D. spur helicopter drive gear, honing of the gear teeth down to 8 $\mu$ surface finish increased wear life by 1,000% and increased load carrying capacity by 30%. Other tests by the gearing industry have shown 100% load carrying capacity increases by honing ground layers.

**Acknowledgement:**
*Presented at the SME Gear Processing and Manufacturing Clinic, November 11-13, 1986, Schaumburg, Illinois*

## ENGINEERING CONSTANTS . . .

*(continued from page 33)*

### Comparison of DIN and AGMA Qualities

A closer comparison between the various tolerance systems is beyond the scope of a simple table, for the corresponding quality fields never coincide exactly; moreover the various national standards do not make comparable adjustments to gear error tolerances to allow for the influence of the gear diameter and size of pitch: a given class in one standard can, in certain ranges of diameter and pitch, cover several classes of another standard.

A comparison between AGMA run-out tolerances and the equivalent DIN has been omitted, due to the differences in definition. AGMA Quality Classes for lead tolerances have been included in spite of minor differeces in definition.

| | Reference diameter | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Adjacent pitch error and difference between adjacent pitches | up to 15.8 in. (up to 400 mm) | DIN | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| | | AGMA | | | 15 | 15 | 13 | 12 | 12 | 11 | 10 | 9 |
| | over 15.8 in. (over 400 mm) | DIN | 2 | 2 | 2–3 | 2–3 | 3–4 | 4–5 | 4–5 | 5–6 | 6–7 | 7–8 |
| | | AGMA | 15 | 15 | 14–15 | 14–15 | 13 | 12 | 12 | 11 | 10 | 9 |
| Total profile error | | DIN | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 5 | 6 | 6 |
| | | AGMA | | | | | | 14–15 | 14–15 | 14–15 | 13–14 | 13–14 |
| Maximum accumulated pitch error | | DIN | 3 | 3 | 4 | 5 | 6 | 7 | 5 | 6 | 7 | 8 |
| Radial run-out | | DIN | 3 | 4 | 5 | 5–6 | 6 | 7 | 5 | 6 | 7 | 8 |
| Total composite error (double flank) | | DIN | | | 5 | 6 | 7 | 8 | 6 | 7 | 8 | 9 |
| | | AGMA | | | | 12 | 11–12 | 10–11 | 12 | 11–12 | 10–11 | 9 |
| Tooth-to-tooth composite error | up to 15.8 in. (up to 400 mm) | DIN | | | 5 | 5 | 6 | 7 | 6 | 7 | 8 | 9 |
| | over 15.8 in. (over 400 mm) | DIN | | | 4–5 | 4–5 | 5–6 | 6–7 | 5–6 | 6–7 | 7–8 | 8–9 |
| Total tooth alignment error | up to 15.8" (400 mm) $P \geqq 4$ ($m \leqq 6$) | DIN | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| | up to 15.8" (400 mm) $P<4$ ($m>6$) | DIN | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 5 | 5 |
| | over 15.8" (400 mm) $P \geqq 4$ ($m \leqq 6$) | DIN | | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 5 | 5 |
| | over 15.8" (400 mm) $P<4$ ($m>6$) | DIN | | 3 | 3 | 4 | 4 | 5 | 5 | 5 | 6 | 6 |