BACK TO BASICS...

Helical Gear Mathematics Formulas and Examples

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The following excerpt is from the *Revised Manual of Gear Design, Section III,* covering helical and spiral gears. This section on helical gear mathematics shows the detailed solutions to many general helical gearing problems. In each case, a definite example has been worked out to illustrate the solution. All equations are arranged in their most effective form for use on a computer or calculating machine.

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Given the pitch radius and lead of a helical gear, to determine the helix angle:

When,
R = Pitch Radius of Gear
L = Lead of Tooth
ψ = Helix Angle

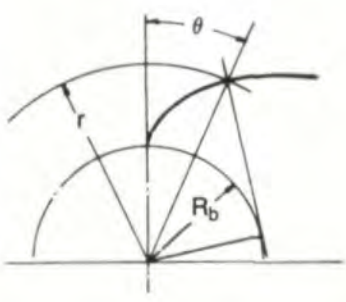
Then,

$$\text{TAN } \psi = \frac{2 \pi R}{L}$$

Example: $R = 3.000$ $L = 21.000$

$$\text{TAN } \psi = \frac{2 \times 3.1416 \times 3.000}{21.000} = .89760 \qquad \psi = 41.911°$$

The involute of a circle is the curve that is described by the end of a line which is unwound from the circumference of a circle as shown in Fig. 1.



When,
R_b = Base Radius
θ = Vectorial Angle
r = Length of Radius Vector

Then,

$$\theta = \frac{\sqrt{r^2 - R_b^2}}{R_b} - \text{ARC TAN } \frac{\sqrt{r^2 - R_b^2}}{R_b}$$

Given the arc tooth thickness and pressure angle in the plane of rotation of a helical gear at a given radius, to determine its tooth thickness at any other radius:

When,

r_1 = Given Radius

ϕ_1 = Pressure Angle at r_1

T_1 = ARC Tooth Thickness at r_1

r_2 = Radius Where Tooth Thickness Is To Be Determined

ϕ_2 = Pressure Angle at r_2

T_2 = ARC Tooth Thickness at r_2

Then,

$$\text{COS}\ \phi_2 = \frac{r_1\ \text{COS}\ \phi_1}{r_2}$$

$$T_2 = 2\ r_2\left(\frac{T_1}{2r_1} + \text{INV}\ \phi_1 - \text{INV}\ \phi_2\right)$$

Example: $r_1 = 2.500$ $T_1 = .2618$ $r_2 = 2.600$

$\phi_1 = 14.500°$ COS $\phi_1 = .96815$ INV $\phi_1 = .00554$

$$\text{COS}\ \phi_2 = \frac{2.500 \times .96815}{2.600} = .93091$$

$\phi_2 = 21.425°$ INV $\phi_2 = .01845$

$$T_2 = 2 \times 2.600\left(\frac{.2618}{5.000} + .00554 - .01845\right) = .2051$$

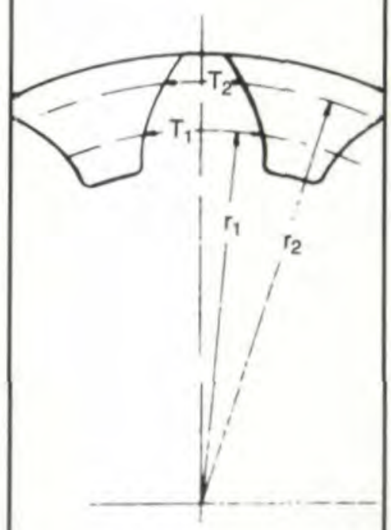


Fig. 2

Given the helix angle, normal diametral pitch and numbers of teeth, to determine the center distance:

When,

ψ = Helix Angle

N_1 = Number of Teeth in Pinion

N_2 = Number of Teeth in Gear

C = Center Distance

P_n = Normal Diametral Pitch

Then,

$$C = \frac{N_1 + N_2}{2\ P_n\ \text{COS}\ \psi}$$

Example: $\psi = 30°$ $P_n = 8$ $N_1 = 24$ $N_2 = 48$ COS $\psi = .86603$

$$C = \frac{24 + 48}{2 \times 8 \times .86603} = 5.1961$$

Given the arc tooth thickness in the plane of rotation at a given radius, to find the normal chordal thickness and the normal chordal addendum:

When, T = ARC Tooth Thickness at R in Plane of Rotation

T_n = Normal Chordal Thickness at R

Q_n = Normal Chordal Addendum

R_o = Outside Radius

R = Pitch Radius

ψ = Helix Angle at R

Then,

$$\text{ARC } B = \frac{T \cos^2 \psi}{2R}$$

$$T_n = \frac{2 R \sin B}{\cos \psi}$$

$$Q_n = R_o - \cos B$$

Example: $T = .2267 \qquad R_o = 1.8570 \qquad R = 1.7320$

$\psi = 30° \qquad \cos \psi = .86603 \qquad \cos^2 \psi = .75000$

$$\text{ARC } B = \frac{.2267 \times .7500}{2 \times 1.7320} = .04908 \qquad B = 2.812°$$

$\sin B = .04906 \qquad \cos B = .99880$

$$T_n = \frac{2 \times 1.7320 \times .04906}{.86603} = .1962$$

$$Q_n = 1.8570 - (1.7320 \times .99880) = .1271$$

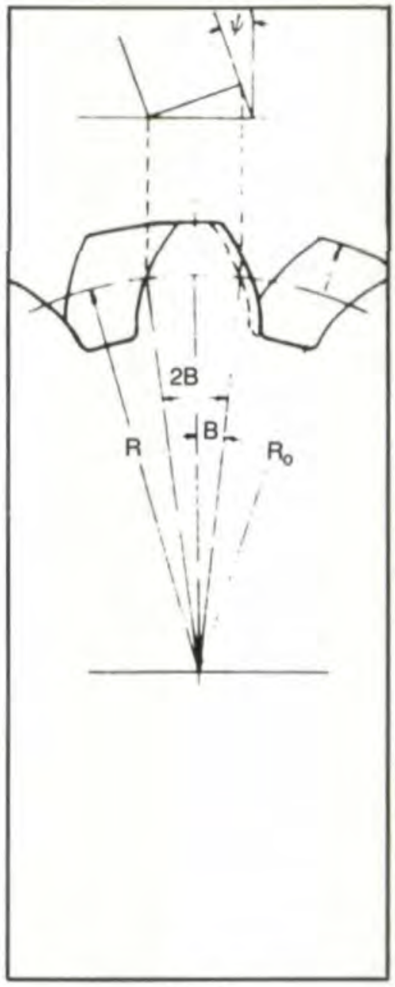


Fig. 3

Given the circular pitch and pressure angle in the plane of rotation and the helix angle of a helical gear, to determine the normal circular pitch and the normal pressure angle:

When, ψ = Helix Angle

ϕ = Pressure Angle in Plane of Rotation

p = Circular Pitch in Plane of Rotation

ϕ_n = Normal Pressure Angle

p_n = Normal Circular Pitch

Then,

$$p = p \cos \psi \qquad \tan \phi_n = \tan \phi \cos \psi$$

Example: $p = .3927 \qquad \psi = 23° \qquad \phi = 20° \qquad \cos \psi = .92050 \qquad \tan \phi = .36397$

$p_n = .3927 \times .92050 = .36148 \qquad \tan \phi_n = .36397 \times .92050 = .33503$

$\phi_n = 18.522°$

Given the arc tooth thickness and pressure angle in the plane of rotation at a given radius, to determine the radius where the tooth becomes pointed:

When, r_1 = Given Radius

r_2 = Radius where Tooth Becomes Pointed

T_1 = ARC Tooth Thickness at r_1

ϕ_1 = Pressure Angle at r_1

ϕ_2 = Pressure Angle at r_2

Then,

$$\text{INV } \phi_2 = \frac{T_1}{2\,r_1} + \text{INV } \phi_1$$

$$r_2 = \frac{r_1 \text{ COS } \phi_1}{\text{COS } \phi_2}$$

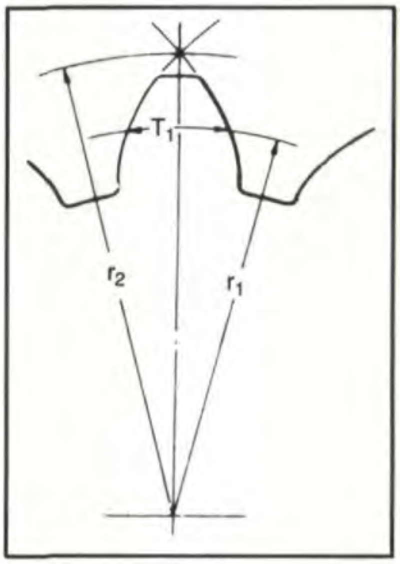


Fig. 4

Example: $r_1 = 2.500$ $T_1 = .2618$ $\phi_1 = 14.500°$

$\text{INV } \phi_1 = .00554$

$$\text{INV } \phi_2 = \frac{.2618}{2 \times 2.500} + .00554 = .05790 \text{ Radians}$$

$\phi_2 = 30.693°$ $\text{COS } \phi_2 = .85991$ $\text{COS } \phi_1 = .96815$

$$r_2 = \frac{2.500 \times .96815}{.85991} = 2.8147$$

Given the normal circular pitch, the normal pressure angle and the helix angle of a helical gear, to determine the circular pitch and the pressure angle in the plane of rotation:

When, ψ = Helix Angle

ϕ_n = Normal Pressure Angle

p_n = Normal Circular Pitch

ϕ = Pressure Angle in Plane of Rotation

p = Circular Pitch in Plane of Rotation

Then,

$$p = \frac{p_n}{\text{COS } \psi} \qquad \text{TAN } \phi = \frac{\text{TAN } \phi_n}{\text{COS } \psi}$$

Example: $\psi = 25°$ $\phi_n = 20°$ $\text{COS } \psi = .90631$ $\text{TAN } \phi_n = .36397$ $p_n = .5236$

$$p = \frac{.5236}{.90631} = .57772 \qquad \text{TAN } \phi = \frac{.36397}{.90631} = .40159 \qquad \phi = 21.880°$$

Given the tooth proportions in the plane of rotation of a pair of helical gears (parallel shafts), to determine the center distance at which they will mesh tightly:

When,

r_1 = Given Radius of 1st Gear

r_2 = Given Radius of 2nd Gear

N_1 = Number of Teeth in 1st Gear

N_2 = Number of Teeth in 2nd Gear

ϕ_1 = Pressure Angle at r_1 and r_2

ϕ_2 = Pressure Angle at Meshing Position

T_1 = ARC Tooth Thickness at r_1

T_2 = ARC Tooth Thickness at r_2

C_1 = Center Distance for Pressure Angle ϕ_1

C_2 = Center Distance for Pressure Angle ϕ_2

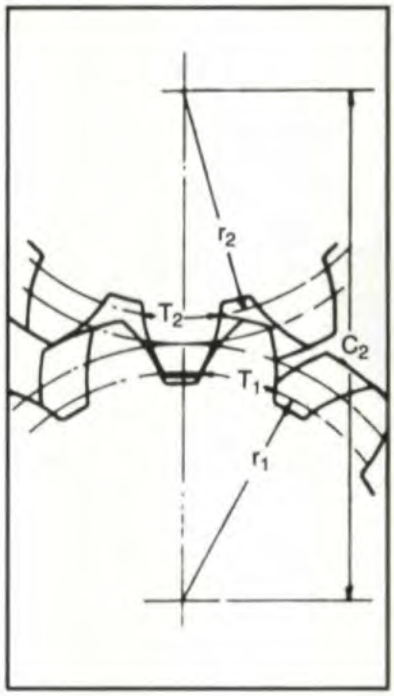


Fig. 5

Then,

$$INV\ \phi_2 = \frac{N_1 (T_1 + T_2) - 2\pi r_1}{2\, r_1 (N_1 + N_2)} + INV\ \phi_1$$

$$C_1 = r_1 + r_2$$

$$C_2 = \frac{C_1\ COS\ \phi_1}{COS\ \phi_2}$$

Example:

$r_1 = 2.500$ $\quad T_1 = .2800$ $\quad N_1 = 30$ $\quad \phi_1 = 14.500°$

$r_2 = 4.000$ $\quad T_2 = .2750$ $\quad N_2 = 48$ $\quad C_1 = 6.500$

$$INV\ \phi_2 = \frac{30\ (.2800 + .2750) - 2\pi \times 2.500}{2 \times 2.500\ (30 + 48)} + .00545 = .007955$$

$\phi_2 = 16.315°$ $\qquad COS\ \phi_2 = .95973$

$$C_2 = \frac{6.500 \times .96815}{.95973} = 6.5570$$

Given the pitch radius and helix angle of a helical gear, to determine the lead of the tooth.

When,

R = Pitch Radius

L = Lead of Tooth

ψ = Helix Angle

Then,

$$L = \frac{2\ \pi\ R}{TAN\ \pi}$$

Example:

$R = 2.500$ $\qquad \psi = 22.50°$ $\qquad TAN\ \psi = .41421$

$$L = \frac{2 \times 3.1416 \times 2.500}{.41421} = 37.9228$$

Given the number of teeth, helix angle and proportions of the normal basic rack of a helical gear, to determine the pitch radius and the base radius:

When,
- N = Number of Teeth
- ψ = Helix Angle at R
- P_n = Normal Diametral Pitch
- R = Pitch Radius
- ϕ_n = Normal Pressure Angle
- ϕ = Pressure Angle in Plane of Rotation
- R_b = Base Radius

Then,

$$R = \frac{N}{2\ P_n \cos \psi} \qquad \tan \phi = \frac{\tan \phi_n}{\cos \psi}$$

$$R_b = R \cos \phi = \frac{N \cos \phi}{2\ P_n \cos \psi}$$

Example: $N = 30 \quad \psi = 25^\circ \quad P_n = 6 \quad \phi_n = 14\tfrac{1}{2}^\circ \quad \cos \psi = .90631 \quad \tan \phi_n = .25862$

$$R = \frac{30}{2 \times 6 \times .90631} = 2.7584$$

$$\tan \phi = \frac{.25862}{.90631} = .28535 \qquad \phi = 15.926^\circ \qquad \cos \phi = .96162$$

$$R_b = \frac{30 \times .96162}{2 \times 6 \times .90631} = 2.65256$$

Given the normal diametral pitch, numbers of teeth and center distance, to determine the lead and helix angle:

When,
- N_1 = Number of Teeth in Pinion
- N_2 = Number of Teeth in Gear
- P_n = Normal Diametral Pitch
- C = Center Distance
- ψ = Helix Angle
- L_1 = Lead of Pinion
- L_2 = Lead of Gear

Then,

$$\cos \psi = \frac{N_1 + N_2}{2\ P_n\ C} \qquad L_1 = \frac{\pi N_1}{P_n \sin \psi} \qquad L_2 = \frac{\pi N_2}{P_n \sin \psi}$$

Example: $P_n = 6 \quad N_1 = 18 \quad N_2 = 30 \quad C = 4.500$

$$\cos \psi = \frac{18 + 30}{2 \times 6 \times 4.500} = .88889 \qquad \psi = 27.266^\circ \qquad \sin \psi = .45812$$

$$L_1 = \frac{3.1416 \times 18}{6 \times .45812} = 20.5728 \qquad L_2 = \frac{3.1416 \times 30}{6 \times .45812} = 34.2880$$

Given the tooth proportions in the plane of rotation of a helical gear, to determine the position of a mating rack of different circular pitch and pressure angle:

When,
ψ_1 = Given Helix Angle at R_1
ψ_2 = Helix Angle for Mating Rack
ψ_b = Base Helix Angle
ϕ_{n1} = Normal Pressure Angle at R_1
ϕ_{n2} = Pressure Angle of Mating Rack
ϕ_1 = Pressure Angle at R_1 in Plane of Rotation
ϕ_2 = Pressure Angle of Mating Rack in Plane of Rotation
R_1 = Given Pitch Radius
R_2 = Pitch Radius with Mating Rack
R_b = Base Radius
a = Addendum of Rack
T_1 = ARC Tooth Thickness at R_1
N = Number of Teeth
X = Distance from Center of Gear to Tip of Rack Tooth
p_{n1} = Normal Circular Pitch at R_1
p_{n2} = Normal Circular Pitch of Rack
Note: (p_{n1} COS ϕ_{n1} Must Be Equal To (p_{n2} COS ϕ_{n2})

Then,

$$\text{SIN}\ \psi_b = \text{SIN}\ \psi_1\ \text{COS}\ \phi_{n1}$$

$$\text{SIN}\ \psi_2 = \frac{\text{SIN}\ \psi_b}{\text{COS}\ \phi_{n2}} = \frac{\text{SIN}\ \psi_1\ \text{COS}\ \phi_{n1}}{\text{COS}\ \phi_{n2}}$$

$$\text{TAN}\ \phi_2 = \frac{\text{TAN}\ \phi_{n2}}{\text{COS}\ \psi_2} \qquad R_2 = \frac{R_b}{\text{COS}\ \phi_2}$$

$$X = R_2 - a + \frac{1}{2\ \text{TAN}\ \phi_2}\left[2\ R_2\left(\frac{T_1}{2\ R_1} + \text{INV}\ \phi_1 - \text{INV}\ \phi_2\right) - \frac{\pi\ R_2}{N}\right]$$

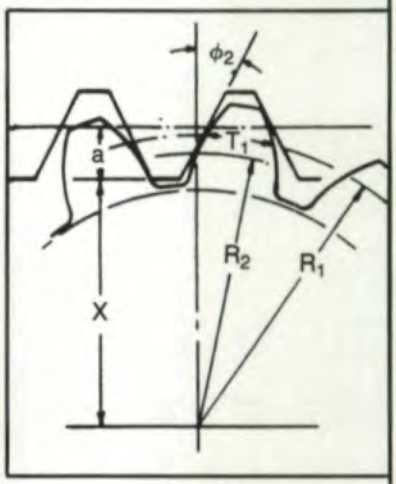


Fig. 6

Example: $\psi_1 = 25°$ $\phi_{n1} = 14\frac{1}{2}°$ $\phi_1 = 15.926°$ $R_1 = 2.7584$ $R_b = 2.65256$

$\phi_{n2} = 20°$ $a = .185$ $T_1 = .2888$ $N = 30$

SIN ψ_1 = .42262 COS ψ_1 = .90631 TAN ϕ_{n1} = .25862 TAN ϕ_{n2} = .36397

p_{n1} = .5236 p_{n2} = .53946 COS ϕ_{n1} = .96815 COS ϕ_{n2} = .93969

[p_{n1} COS ϕ_{n1} = .50692] = [p_{n2} COS ϕ_{n2} = .50692]

$$\text{SIN}\ \psi_2 = \frac{.42262 \times .96815}{.93969} = .43542 \qquad \psi_2 = 25.812° \qquad \text{COS}\ \psi_2 = .90023$$

$$\text{TAN}\ \phi_2 = \frac{.36397}{.90023} = .40431 \qquad \phi_2 = 22.014° \qquad \text{COS}\ \phi_2 = .92709$$

INV ϕ_2 = .020093 INV ϕ_1 = .007387

$$R_2 = \frac{2.65256}{.92709} = 2.86117$$

$$X = 2.86117 - .185 + \frac{1}{2 \times .40431}\left[5.72234\left(\frac{.2888}{5.5168} + .007387 - .020093\right) - \frac{.31416 \times 2.86117}{30}\right] = 2.5729$$

Given the center distance, number of teeth and basic rack proportions (hob proportions) of a pair of helical gears, to determine the hobbing data:

When,

ϕ_{nc} = Pressure Angle of Hob

p_{nc} = Diametral Pitch of Hob

a_c = Addendum of Hob

C_1 = Center Distance with Pressure Angle of ϕ_1

C_2 = Given Center Distance of Operation

N_1 = Number of Teeth in Pinion

R_{o1} = Outside Radius of Pinion

R_{r1} = Root Radius of Pinion

L_1 = Lead of Pinion

R_1 = Pitch Radius of Pinion

b_1 = Dedendum of Pinion

ψ_1 = Helix Angle of Generation

N_2 = Number of Teeth in Gear

R_{o2} = Outside Radius of Gear

R_{r2} = Root Radius of Gear

L_2 = Lead of Gear

R_2 = Pitch Radius of Gear

b_2 = Dedendum of Gear

ψ_2 = Helix Angle of Operation

ϕ_1 = Pressure Angle of Generation in Plane of Rotation

ϕ_2 = Pressure Angle of Operation in Plane of Rotation

p_1 = Diametral Pitch of Generation in Plane of Rotation

h_t = Total Tooth Depth of Gears

Then, Make trial calculation for lead as follows:

$$\text{COS } \psi_1 = \frac{N_1 + N_2}{2\, p_{nc}\, C_2}$$

$$L_1 = \frac{\pi N_1}{p_{nc} \text{ SIN } \psi_1} \qquad L_2 = \frac{\pi N_2}{p_{nc} \text{ SIN } \psi_1}$$

Select values for L_1 and L_2 which can be readily obtained on the hobbing machine:

Then,

$$\text{SIN } \psi_1 = \frac{\pi N_1}{p_{nc} L_1} = \frac{\pi N_2}{p_{nc} L_2} \qquad \text{TAN } \phi_1 = \frac{\text{TAN } \phi_{nc}}{\text{COS } \psi_1}$$

$$p_1 = p_{nc} \text{ COS } \psi_1 \qquad C_1 = \frac{N_1 + N_2}{2\, p_1} \qquad \text{COS } \phi_2 = \frac{C_1 \text{ COS } \phi_1}{C_2}$$

$$(R_{r1} + R_{r2}) = C_1 - 2\, a_c + \frac{C_1}{\text{TAN } \phi_1} (\text{INV } \phi_2 - \text{INV } \phi_1)$$

$$b_1 = \frac{C_2 - (R_{r1} + R_{r2})}{1 + \sqrt{\frac{N_2}{N_1}}} \qquad b_2 = \frac{C_2 - (R_{r1} + R_{r2})}{1 + \sqrt{\frac{N_1}{N_2}}}$$

Note: When smallest N is 30 or more, then, $b_1 = b_2 = \frac{C_2 - (R_{r1} + R_{r2})}{2}$

$$R_1 = \frac{N_1 C_2}{N_1 + N_2} \qquad R_2 = \frac{N_2 C_2}{N_1 + N_2} \qquad R_{r1} = R_1 - b_1 \qquad R_{r2} = R_2 - b_2$$

$$h_t = .932\, [C_2 - (R_{r1} + R_{r2})\,] \qquad R_{o1} = R_{r1} + h_t \qquad R_{o2} = R_{r2} + h_t$$

$$\text{TAN } \psi_2 = \frac{2\, \pi R_1}{L_1} = \frac{2\, \pi R_2}{L_2}$$

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Example: $N_1 = 20$ $N_2 = 60$ $p_{nc} = 5$ $A_c = .2314$ $C_2 = 9.00$

$\phi_{nc} = 14.500$ $\quad$ $\text{TAN}\ \phi_{nc} = .25862$

Trial Calculation:

$$\text{COS}\ \psi_1 = \frac{20 + 60}{2 \times 5 \times 9.00} = .88889 \qquad \psi_1 = 27.266° \qquad \text{SIN}\ \psi = .45812$$

$$L_1 = \frac{20\ \pi}{5 \times .45812} = 27.4303 \qquad L_2 = \frac{60\ \pi}{5 \times .45812} = 82.2909$$

We will select the following values for L_1 and L_2:

$L_1 = 27.500$ $\quad$ $L_2 = 82.500$

$$\text{SIN}\ \psi = \frac{20\ \pi}{5 \times 27.500} = .45696 \qquad \psi_1 = 27.1910 \qquad \text{COS}\ \psi_1 = .88949$$

$$\text{TAN}\ \phi_1 = \frac{.25862}{.88969} = .29069 \qquad \phi_1 = 16.208° \qquad \text{COS}\ \phi_1 = .96025 \qquad \text{INV}\ \phi_1 = .007796$$

$$p_1 = 5 \times .88949 = 4.44745 \qquad C_1 = \frac{20 + 60}{2 \times 4.44745} = 8.99392$$

$$\text{COS}\ \phi_2 = \frac{8.99392 \times .96025}{9} = .95960 \qquad \phi_2 = 16.3416 \qquad \text{INV}\ \phi_2 = .007994$$

$$(R_{r1} + R_{r2}) = 8.99392 - 2 \times .2314 + \frac{8.99392}{.29069} [.007994 - .007796] = 8.5372$$

$$b_1 = \frac{9.00 - 8.5372}{1 + \sqrt{60/20}} = .16938 \qquad b_2 = \frac{9.00 - 8.5372}{1 + \sqrt{20/60}} = .29340$$

$$R_1 = \frac{20 \times 9.00}{20 + 60} = 2.250 \qquad R_2 = \frac{60 \times 9.00}{20 + 60} = 6.750$$

$$R_{r1} = 2.250 - .16938 = 2.08062 \qquad R_{r2} = 6.750 - .29340 = 6.45660$$

$$h_t = .932\ [9.00 - 8.5372] = .43133$$

$$R_{o1} = 2.08062 + .43133 = 2.51195 \qquad R_{o2} = 6.45660 + .43133 = 6.88793$$

$$\text{TAN}\ \psi_2 = \frac{2\ \pi\ 2.250}{27.5} = .514079 \qquad \psi_2 = 27.207$$

The specifications for this pair of gears are as follows:

$N_1 = 20$	$R_{r1} = 2.08062$	$L_2 = 82.500$	Helix angle for hobbing = 27.1910
$R_{o1} = 2.51195$	$N_2 = 60$	$R_{r2} = 6.45660$	
$R_1 = 2.250$	$R_{o2} = 6.88793$	$C_2 = 9.00$	
$L_1 = 27.500$	$R_2 = 6.750$		

Given the proportions of an internal helical gear drive, to determine the contact ratio:

When,

R_1 = Pitch Radius of Helical Gear

R_{o1} = Outside Radius of Helical Gear

R_{b1} = Base Radius of Helical Gear

ϕ = Pressure Angle in Plane of Rotation

p = Circular Pitch in Plane of Rotation

C = Center Distance

m_p = Contact Ratio

R_2 = Pitch Radius of Internal Gear

R_i = Internal Radius of Internal Gear

R_{b2} = Base Radius of Internal Gear

Then,

$$m_p = \frac{\sqrt{R_{o1}^2 - R_{b1}^2} + C \text{ SIN } \phi - \sqrt{R_i^2 - R_b^2}}{p \text{ COS } \phi}$$

Example:

$R_1 = 1.250$ $R_{o1} = 1.4375$ $R_{b1} = 1.1746$ $\phi = 20°$ $p = .3927$

$R_2 = 3.500$ $R_i = 3.4375$ $R_{b2} = 3.2888$ $C = 2.250$

SIN ϕ = .34202 COS ϕ = .93969

$$m_p = \frac{\sqrt{(1.4375)^2 - (1.1746)^2} + (2.250 \times .34202) - \sqrt{(3.4375)^2 - (3.2888)^2}}{.3927 \times .93969} = 1.62$$

Given the proportions of a pair of helical gears (external or internal), to determine the face contact ratio:

When,

F = Face Width

p = Circular Pitch in Plane of Rotation

ψ = Helix Angle

m_f = Face Contact Ratio

Then,

$$m_f = \frac{F \text{ TAN } \psi}{p}$$

Example:

$F = 1.500$ $p = .3927$ $\psi = 30°$ TAN ψ = .57735

$$m_f = \frac{1.500 \times .57735}{.3927} = 2.20$$

Given the proportions of a pair of helical gears (external or internal), to determine the total contact ratio:

When,

m_p = Contact Ratio

m_f = Face Contact Ratio

m_t = Total Contact Ratio

Then,

$$m_t = m_p + m_f$$

Example:

$m_p = 1.59$ $m_f = 2.20$

$m_t = 1.59 + 2.20 = 3.79$

TOOTH ROOT STRESSES . . .
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estimated), the amount of crowning should be chosen in such a way that when applying the service load, the lowest root stresses will be the result. This criterion is satisfied when the *product*

$$K_c = K_{F\beta-c} \cdot Y_\gamma \cdot K_{F\beta-f}$$

reaches a minimum.

As an example this optimization is performed for the test gears in Fig. 18. One can see that the curve for K_c has a flat minimum in the area of small crowning values (near gear set B). This result seems to be plausible because of the very stiff test rig.

It should be noted that the optimization method introduced here is only based on the tooth root stresses and should only be used if tooth breakage is the critical failure criterion. An optimization for contact stresses may be quite different and usually provides a guide to higher amounts of crowning.

Summary

By strain gauge measurements of spiral bevel gears, the influence of lengthwise crowning and relative displacements between pinion and gear on tooth root stresses was investigated. It was found that the crowning effects the load distribution over the lines of contact and the load sharing between pairs of teeth meshing simultaneously. For both influences a quantitative description could be derived.

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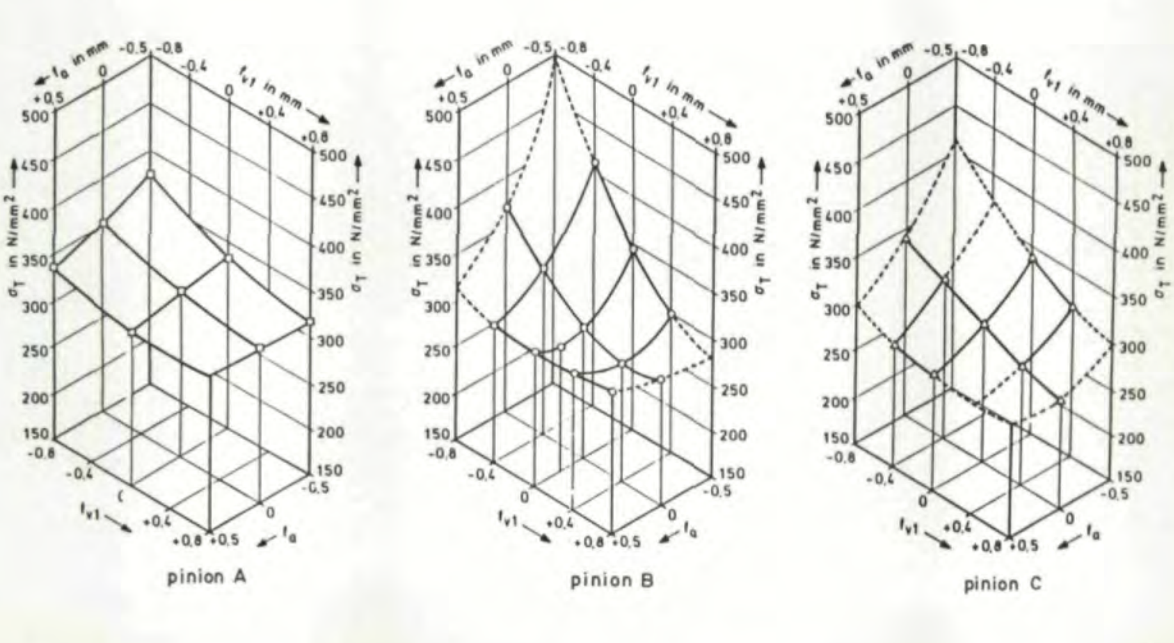

Fig. 16 – Influence of combined displacements on the maximum root stresses $\sigma_{T\ max}$ at the pinions. (Amount of crowning, see Fig. 2.)

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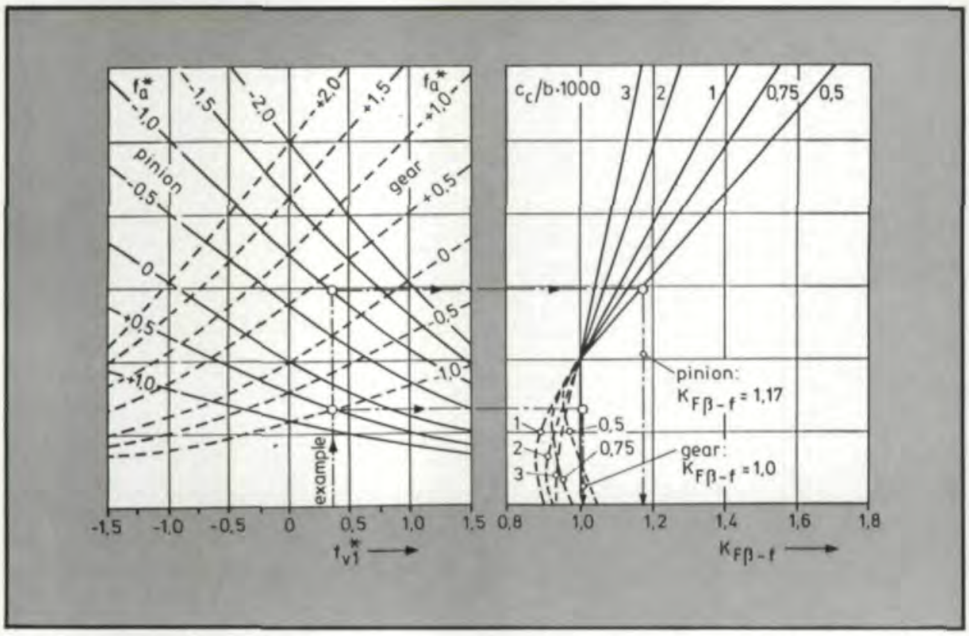

Fig. 17 – Nomogram for determining the displacement factor $K_{F\beta-f}$ ($f_{v1}^* = f_{v1}/d_{m2} \cdot 1000$, $f_a^* = f_a/d_{m2} \cdot 1000$).

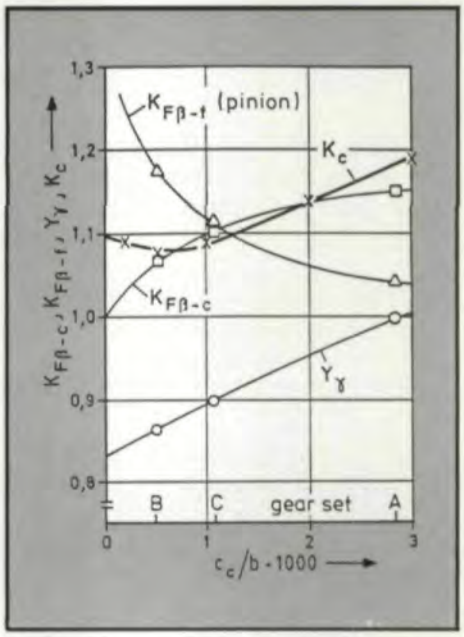

Fig. 18 – Optimization of lengthwise crowning.

In the case of relative displacements, deviations in pinion mounting distance and in offset have the strongest influence on the root stresses. A method was introduced to determine the increase or decrease of maximum stresses that have to be expected for a combination of certain values of these parameters. Further, a optimization criterion was derived that allows finding the amount of lengthwise crowning producing the lowest root stresses for a certain service condition.

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