The Effect of Manufacturing Microgeometry Variations on the Load Distribution Factor and on Gear Contact and Root Stresses

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Management Summary

Traditionally, gear rating procedures consider manufacturing accuracy in the application of the dynamic factor, but only indirectly through the load distribution are such errors in the calculation of stresses used in the durability and gear strength equations. This paper discusses how accuracy affects the calculation of stresses and then uses both statistical design of experiments and Monte Carlo simulation techniques to quantify the effects of different manufacturing and assembly errors on root and contact stresses. Manufacturing deviations to be considered include profile, lead slopes and curvatures, as well as misalignment. The effects of spacing errors, runout and center distance variation will also be discussed.

Introduction

Gear rating formulas have numerous design factors that are intended to create realistic evaluations of the stress levels encountered by a gear pair. The dynamic factor, however, is the only factor that has manufacturing accuracy directly included in its evaluation. This paper discusses most of the other factors in the AGMA rating procedure (Ref. 1) that might be influenced by manufacturing accuracy and then uses load distribution analysis to assess the effects of profile, lead and spacing deviations on root stresses, contact stresses and load distribution factors. The dynamic factor has received ample study in the past (Ref. 27), so it will not be further investigated here. As part of the presented analysis, a procedure is provided for obtaining an acceptable microgeometry design that is relatively insensitive to manufacturing deviations and misalignment.

Manufacturing Accuracy Definitions

Prior to looking at the factors that affect tooth stresses, what is meant in this paper as "manufacturing accuracy" will be defined. In this context, manufacturing accuracy encompasses all factors in manufacturing or assembly that change the microgeometries and hence the load sharing of a tooth pair. The AGMA accuracy classification standard (Ref. 8) uses its quality number system to define quality levels for profiles, leads, runout and spacing. The accuracy of housings, bearings and support shafting, geometry changes to the surface and root geometries and variables such as center distance, backlash, outside diameter and tooth thickness are not included, but are occasionally discussed. Also not included are deviations that are measured through composite and single flank tests. However, these deviations result from the elemental variations that are considered.

Provided below is a brief discussion of factors affecting the tooth microgeometry that are studied in this paper.

Lead deviations and misalignment. These are discussed together since they have similar effects on the load distribution across the tooth face width. The effects of lead deviations, which essentially shift the load to one end of the tooth, have been discussed in many papers (Refs. 9–14). Misalignment that is at right angles to the normal contacting plane is an additive to lead slope deviation, so in this paper these effects for both the gear and pinion will be lumped together into a single variation. The AGMA accuracy standard (Ref. 8) recognizes that there are potentially two types of lead deviations—one of the linear type and the second being a curvature deviation. The linear slope deviation may be added to misalignment, while the curvature deviation is treated as a deviation in the specified lead crown.

Profile deviations. Profile deviations are often thought of as deviations of the tooth form from a true involute. But, in loaded teeth operating at the gear pair's rated load, profile modifications in the form of tip and root relief are desirable

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variations from a perfect involute. Hence, profile deviations are thought of as deviations from the specified profile shape. Again, the AGMA accuracy standard allows one to specify deviations in terms of slope and curvature. Profile deviations tend to affect tooth-to-tooth load sharing across the profile of the tooth pair.

Bias deviation. Although not spelled out in AGMA standards, this type of deviation, which is essentially a twisting of the tooth form, is identified by performing multiple profile and/or lead measurements on each measured tooth. This type of deviation, which commonly occurs when gears are finished by screw type generation grinding, also affects load sharing.

Spacing deviations. Tooth-to tooth spacing deviations may affect dynamics, but have a greater effect on tooth-to-tooth load sharing (Ref. 15). In this paper, AGMA quality number values are used and these load sharing effects are analyzed.

Runout deviations. Runou tresults from eccentricities in both the manufacture and the assembly of gears. The most common form of runout is radial runout, which manifests itself in terms of cyclic spacing deviations, cyclic changes in the profile slope and cyclic changes in the operating center distance and/or the effective outside diameter. The latter two effects slightly change the profile contact ratio of the gear pair. In the analysis of this paper, only the spacing deviation effect of profile runout is considered. Another form of runout commonly referred to as lead runout or lead wobble occurs in the face width direction. It is assumed that lead wobble effects are already included in the tolerance used for lead deviations, so no special analysis of lead wobble is performed in this paper.

AGMA rating equations. Since this study concentrates on the effects of manufacturing deviations on stresses, we first look at the current AGMA method for computing these stresses and discuss in general how each factor in these stress equations is affected by manufacturing deviations. The AGMA stress formulas (Ref. 1) for bending and durability are respectively given below.

Contact stress equation:

$$s_c = C_p \sqrt{W_t K_o K_v K_s \frac{K_m}{dF} \frac{C_f}{I}}$$

Tensile bending stress equation:

$$s_t = W_t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

where,

s_c contact stress number, lb/in^2;
C_p elastic coefficient, [lb/in^2] $^{0.5}$;
W_t transmitted tangential load, lb;
K_m load distribution factor;
K_o overload factor;
K_v dynamic factor;
K_s size factor;
C_f surface condition factor for pitting resistance;
F net face width of narrowest member, in;
I geometry factor for pitting resistance;
d operating pitch diameter of pinion, in;
s_t tensile stress number, lb/in^2;
K_B rim thickness factor;
J geometry factor for bending strength;
P_d transverse diametral pitch, in^{-1}.

The factors that are unlikely to be affected much by manufacturing (deviations in materials are not considered in this paper) include the elastic coefficient, the overload factor and the size factor. As mentioned earlier, the dynamic factor has been studied extensively (Ref. 27) and is the only factor used in the above formulas that is currently based on gear quality. The load distribution factor certainly is affected by misalignment, profile and lead modifications. In a way, the current use of this factor does depend upon accuracy, but this use is not quantified in terms of the accuracy numbers. In the simulations of this paper, the load distribution factor is evaluated based on quality numbers. The surface condition factor is certainly affected by manufacturing, but also is not considered by AGMA in its accuracy definitions and will not be studied in this paper. Both geometry factors are subject to manufacturing tolerances, but again they are not quantified in the AGMA tolerance numbers, so will only be briefly discussed. The bending strength geometry factor is subject to changes in surface finish and shape imperfections in the root fillet region that could affect root stresses. For instance, this author once had some gears made in which the hob feed rate was varied and the teeth with the coarser feed rate were found to have lower lives based on single-tooth fatigue testing (Ref. 16). With regard to the surface durability geometry factor, it has been shown that the contact stress may increase significantly at locations on the tooth

Table 1—Helical Gear Geometry

	Pinion	Gear
Number of teeth	25	31
Normal diametral pitch (1/in)	8.598	
Normal pressure angle (deg)	23.45	
Helix angle (deg)	21.50	
Center distance (in)	3.50	
Face width (in)	1.25	1.25
Outside diameter (in)	3.360	4.110
Root diameter (in)	2.811	3.561
Standard pitch diameter, d_p (in)	3.125	3.875
Transverse tooth thickness at d_p (in)	0.1934	0.1934
Profile / face contact ratio	1.383/1.254	
Total contact ratio	2.637	
Pinion torque, lb-in	5000	

changes in the tooth form, an example being the radius of curvature change in the tip relief "break" region (Ref. 17). The rim thickness factor is subject to manufacturing to the extent that there are tolerances on the thickness value that could affect root stresses.

Baseline Microgeometry Selection

When studying microgeometry variations, one must first start with a baseline microgeometry. This, in essence, means to define the baseline profiles and leads of the design. Every gear designer has his own approach to establishing these parameters. Some designers might choose a profile so as to avoid corner tooth contact and tip interference; others might minimize transmission error and others may wish to minimize the effects of spacing deviations. When selecting lead variation, some designers use no lead crown; others select a standard amount of lead crown based on experience and still others might prefer end relief rather than lead crown. In this study, we shall use a load distribution simulation (Refs. 18–20) to select both the profile and lead modifications that will avoid corner contact and tip interference, and at the same time provide reasonable insensitivity to misalignment.

The basic gear geometry to be used in this paper is given in Table 1. The geometry is similar to one used in a previous study (Ref. 21) except that the root diameters have been adjusted slightly.

The procedure for coming up with an acceptable microgeometry is as follows:

Step 1. Identify the rough torque rating for the gear pair. For the sample gear set, we used an AGMA rating formula with approximate constants to come up with a torque rating of roughly 5,000 lb–in.

Step 2. Using this rating, a load distribution analysis with perfect involutes (shafts not included in the analysis) is performed. Figure 1 shows the load distribution at one contact position for the perfect involute analysis. One observes that there is significant contact at the tooth tips (tip interference) and at the entering corner of the tooth (corner contact). Figure 2 shows a composite plot of contact stresses that results from the analysis of many positions of contact. The stresses at the tip, root and corners are abnormally high due to the tip interference and corner contact, and also due to the fact that the radius used to compute the contact stress at the tip is much smaller than that along the tooth flank. Peak contact stresses are about 240 ksi in the corners, 200 ksi on the tip edge and 170–180 ksi in the tooth center. From Figure 3, which shows root stresses at five equally spaced locations along the root of the tooth, the peak pinion root stress is about 44 ksi. The root stresses of the gear were quite similar, so in all subsequent analyses we shall only observe the pinion root stresses.

Step 3. Identify the maximum tooth deflection, the value of which is then used as a guideline in selecting the values of tip and root relief. This deflection, which may be taken from the transmission error plot of Figure 4, is about 0.001 inch.

Step 4. For reference, determine at

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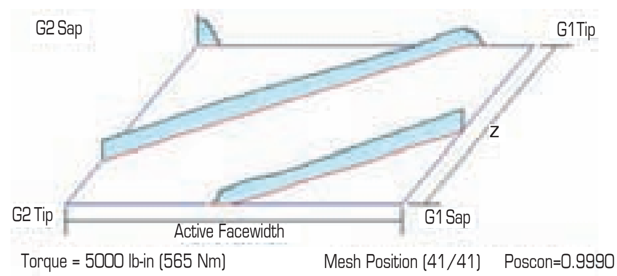

Figure 1—Load distribution of a perfect involute with 5,000 lb–in torque.

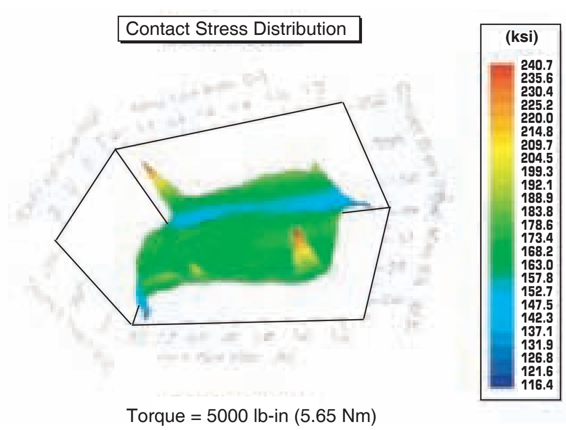

Figure 2—Contact stress distribution of a perfect involute with 5,000 lb–in torque.

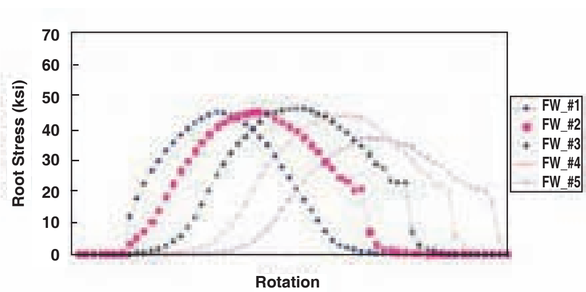

Figure 3—Pinion root stresses at five locations across the face width for a perfect involute with 5,000 lb–in torque.

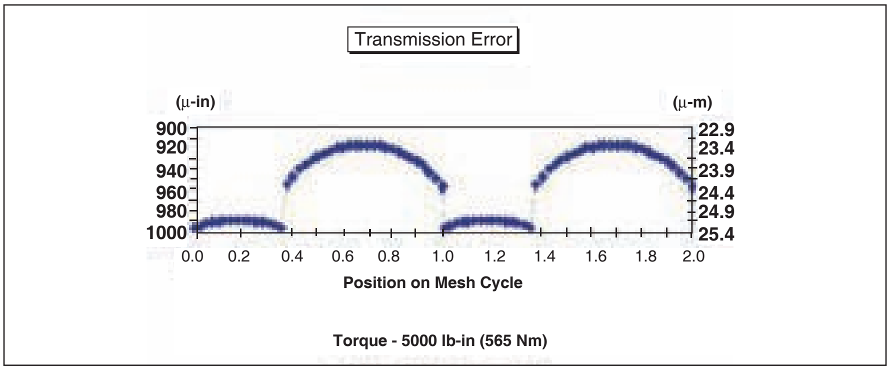

Figure 4—Transmission deviation of a perfect involute with 5,000 lb-in torque.

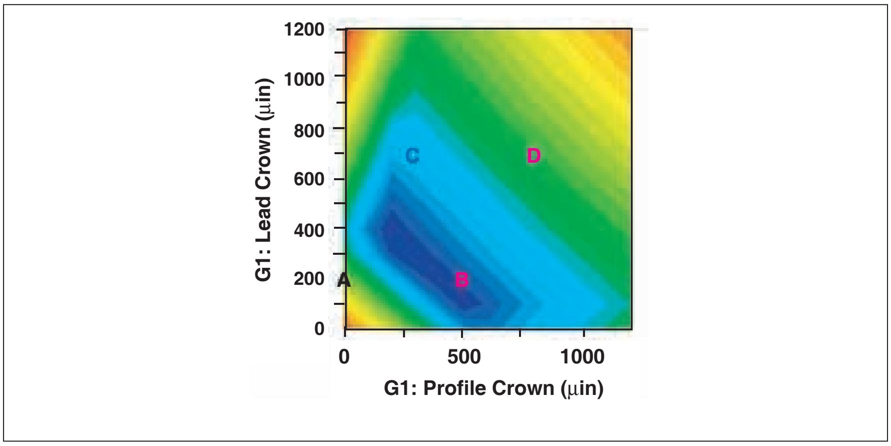

Figure 5—Effect of profile and lead crown on the peak contact stress at 5,000 lb-in torque. (A- 240 ksi; B-201 ksi; C-213 ksi; D-230 ksi)

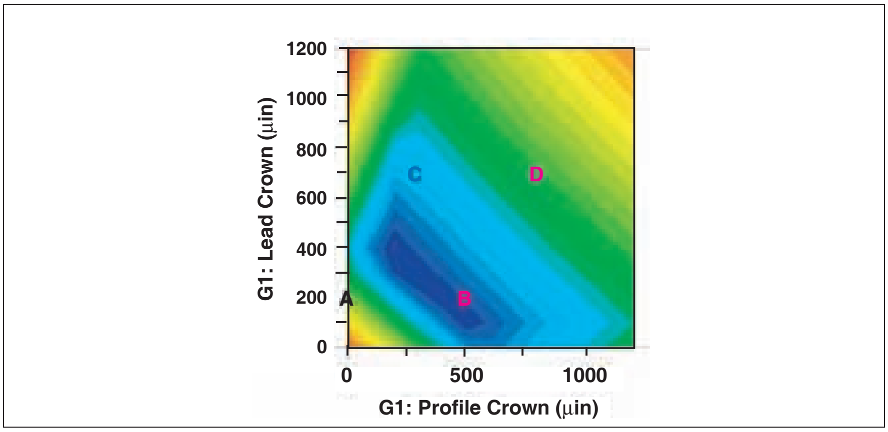

Figure 6—Effect of profile crown and lead crown on pinion root stress at 5,000 lb-in torque. (A-43 ksi; B-48 ksi; C-53 ksi; D-60 ksi)

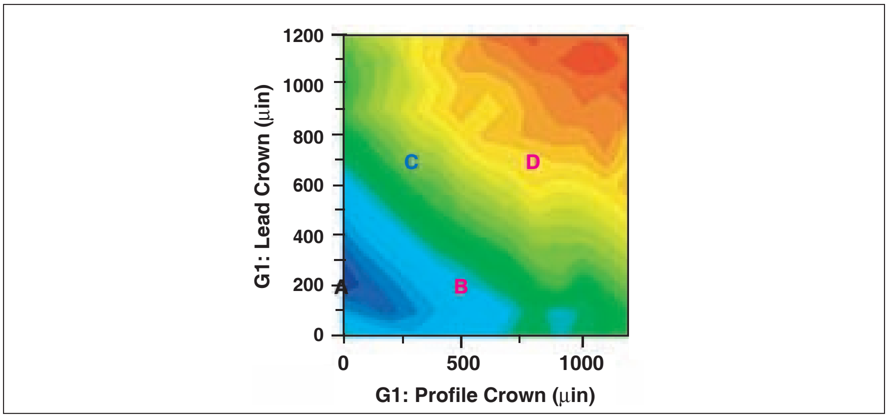

Figure 7—Effect of lead crown and profile crown on load distribution factor at 5,000 lb-in torque. (A-1.19; B-1.39; C-1.58; D-1.71)

the design load the effect of amplitude of tip relief and lead crowning on the major design parameters, namely—contact stress and root stress. For narrow face width helical gears, it has been found that both circular lead and circular profile modifications perform well in distributing the load and in reducing transmission error. Figure 5 shows the results of such an analysis for our gear pair when operating at a pinion torque of 5,000 in–lb.

Figure 5 is quite interesting since it shows the threshold modifications that are required in order to minimize both tip interference and corner contact stresses. The stresses at the lower left corner are abnormally high, due to the corner contact. As one increases the lead crown, these stresses drop, but soon the tip interference stresses dominate and any further increase of lead crown amplitude causes these stresses to increase. In order to totally minimize the tip interference stresses, one must apply tip relief. In this case, applying about 0.0005 in. of profile crown and about 0.0002 in. of lead crown provides a minimum contact stress of 201 ksi. As either the profile crown or the lead crown is further increased, one observes that the contact stress increases. This increase is essentially due to a focusing of the load closer to the center of the tooth.

Figure 6 shows that after a small initial amount of lead crown is applied, root stresses increase with increasing profile and lead crown. One observes that there is a slight conflict between the respective optimal microgeometries desired for root stresses and contact stresses. However, our final microgeometry is selected based on, first, insensitivity to misalignment, and second, insensitivity to all manufacturing deviations. So, at this time, this is not a big issue.

As a matter of interest, Figure 7 shows how the load distribution factor, K_m changes with microgeometry variation. The "optimum" microgeometry now has about 0.0002 in. of lead crown and no profile crown; at this condition, the load distribution factor is about 1.19. Any further misalignment or load shift the design load the effect of amplitude of tip relief and lead crowning on the major design parameters, namely—contact stress and root stress. For narrow face width helical gears, it has been found that both circular lead and circular profile modifications perform well in distributing the load and in reducing transmission error. Figure 5 shows the results of such an analysis for our gear pair when operating at a pinion torque of 5,000 in–lb.

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Step 5. Determine candidate profile crowns and perform microgeometry simulation of the interaction between lead crown and misalignment. From Figures 5 and 6 we choose profile modifications from 0.0004 to 0.0007 in. For this paper, we are using two different AGMA quality numbers, Quality A6 and A8, respectively, for the evaluation of sensitivity to misalignment. The misalignment used is essentially the root sum square (RSS) value of the AGMA lead deviations of each part as

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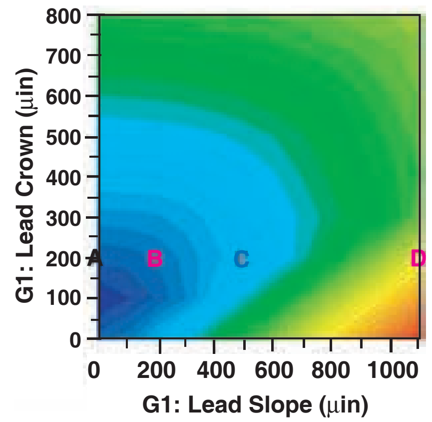

Figure 8—Interaction between misalignment and lead crown on the contact stresses with 0.0005 in. of profile crown. (A–200 ksi; B–201 ksi; C–208 ksi; D–234 ksi)

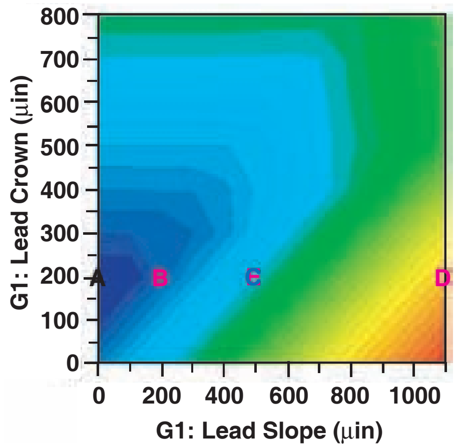

Figure 9—Interaction between misalignment and lead crown on pinion root stresses with 0.0005 in. of profile crown. (A–47 ksi; B–51 ksi; C–57 ksi; D–67 ksi)

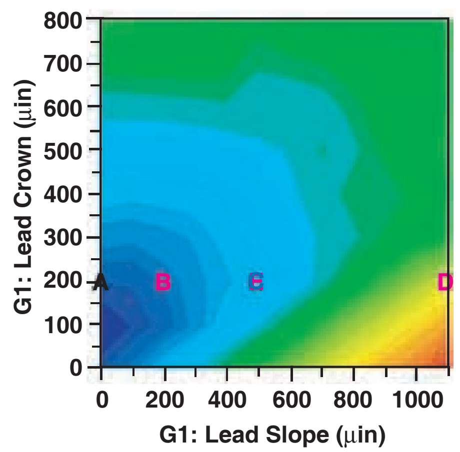

Figure 10—Interaction between misalignment and lead crown on load distribution factor with 0.0005 in. of profile crown. (A–1.41; B–1.44; C–1.53; D– 1.90)

Table 2—Standard deviation of robustness parameters		
	A6	A8
Profile slope (in)	0.000070	0.000140
Profile curvature (in)	0.000084	0.000170
Lead slope + misalignment (in)	0.000140	0.000370
Lead curvature (in)	0.000080	0.000160
Bias (in)	0.000040	0.000060

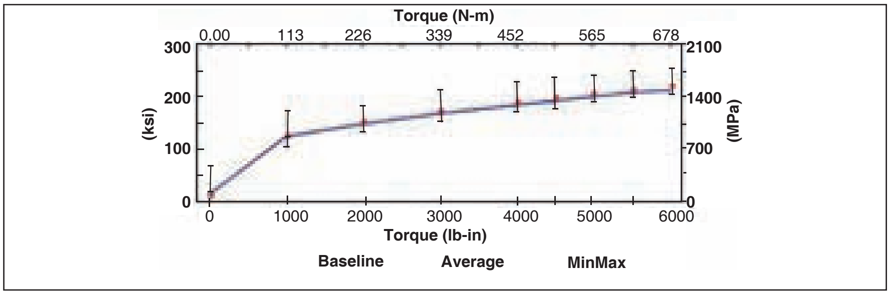

Figure 11—Peak contact stress using 100 random robustness analysis (profile crown = 0.0005 in.; lead crown = 0.0002 in.; quality = A8).

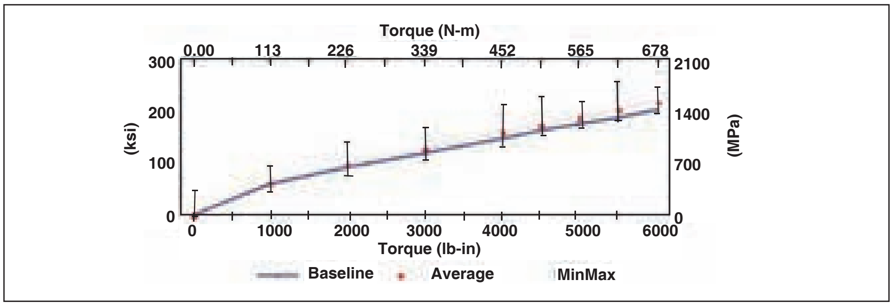

Figure 12— Pinion root stress using 100 random robustness analysis (profile crown= 0.0005 in.; lead crown = 0.0002 in.; quality = A8).

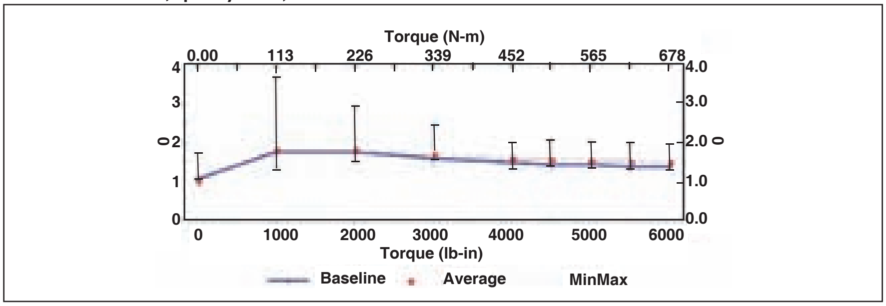

Figure 13— Load distribution factor using 100 random robustness analysis (profile crown = 0.0005 in.; lead crown = 0.0002 in.; quality = A8).

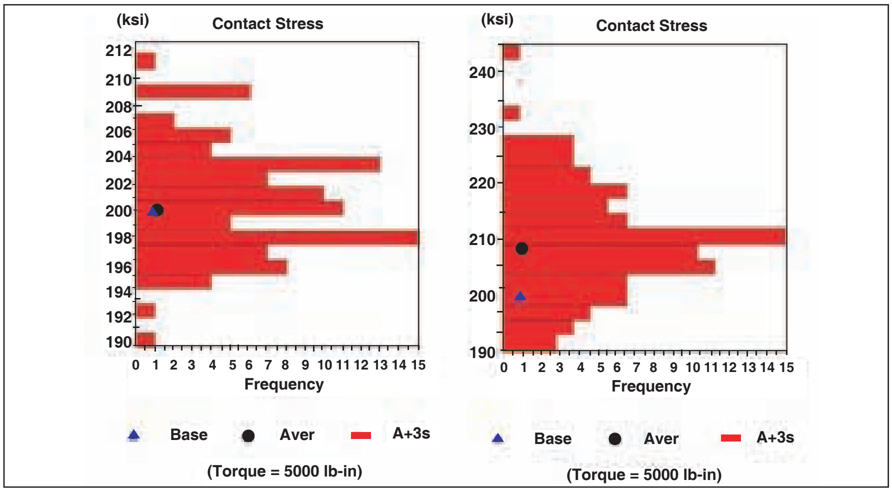

Figure 14—Frequency distribution of peak contact stress using random robustness analysis at 5,000 lb-in torque (profile crown = 0.0005 in.; lead crown = 0.0002 in. A6 on left, A8 on right).

0.0002 in., the load distribution factor of quality A6 gears at the maximum misalignment is 1.53, while the same factor for A8 quality gears is 1.90.

Robustness Analysis

A Monte Carlo type robustness analysis (Refs. 22–23) for quality A6 and A8 manufacturing deviations is performed next. In this analysis, one assumes that each manufacturing variable has a Gaussian distribution and each variable is randomly sampled from this distribution for each load distribution simulation. In this case, load distribution simulations of 100 randomly sampled sets of manufacturing variables are performed for each of the selected profile and lead crown combinations. Here, the standard deviations of each variable come from the AGMA accuracy standard (Ref. 8), but in practice, the manufacturer could establish these standard deviations through product audits. When using the AGMA tolerance values, the tolerance is assumed to be six standard deviations in width. A shaft misalignment tolerance must be added to the lead slope tolerance in order to account for misalignment. Since bias is not included in the AGMA tolerances, a relatively small bias value relative to the other factors was used in the simulation. Table 2 shows the standard deviations of the tolerance values that were used.

Figures 11–13 respectively show the effects of torque on the contact stresses, root stresses and load distribution factor for quality A8 gear pairs having 0.0005 in. of profile crown and 0.0002 in. of lead crown. The mean of the 100 robustness simulations is shown as the solid line; the baseline stress values are the dashed line; and the deviation bands indicate the maximum and minimum values at each of the loads. The deviation bands for each of the loads adjacent to the 5,000 lb–in load are roughly the same. This justifies using an estimate of the rated load in establishing deviation bands.

Figures 14–16 show, respectively, the distributions within the 5,000 lb-in. deviation band for the contact stresses, pinion root stresses and the load distribution factor for each of the quality levels. It is noted that the bands, as expected, are much wider for the lower quality level and the mean values also are higher for the lower quality level.

Tables 3 and 4 show summaries of the statistical data evaluations for the 5,000 lb–in. load. The first two columns provide the respective amplitudes of the profile and lead crowns in tenths of thousandths of an inch. The next three columns respectively show the mean contact stress, the standard deviation for that stress, and the value of the mean plus three standard deviations. The latter quantity is felt to provide an estimate of the expected maximum value of the worst-case combinations of gear accuracy deviations and provides a better number for making comparisons than does the true maximum value of the 100 runs. The next three columns provide similar data for the peak pinion bending stress, and the final three columns provide similar assessments of the load distribution factor.

Because of the danger of corner contact and tip interference, profile crowning was not reduced beneath 0.0004 in. However, if one wishes to totally avoid corner contact, the sum of the profile crown and lead crown must exceed 0.001 in. Also of interest is

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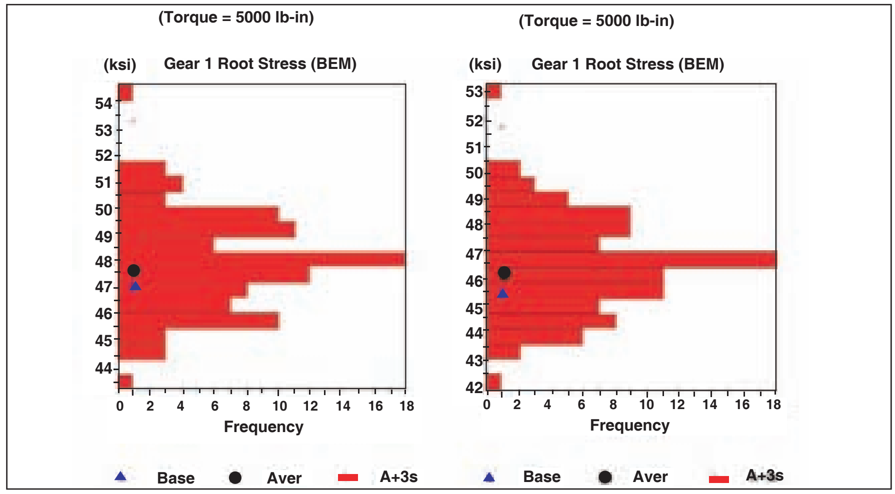

Figure 15—Frequency distribution of peak pinion root stress using random robustness analysis at 5,000 lb–in torque (profile crown = 0.0005 in.; lead crown = 0.0002 in. A6 on left, A 8 on right).

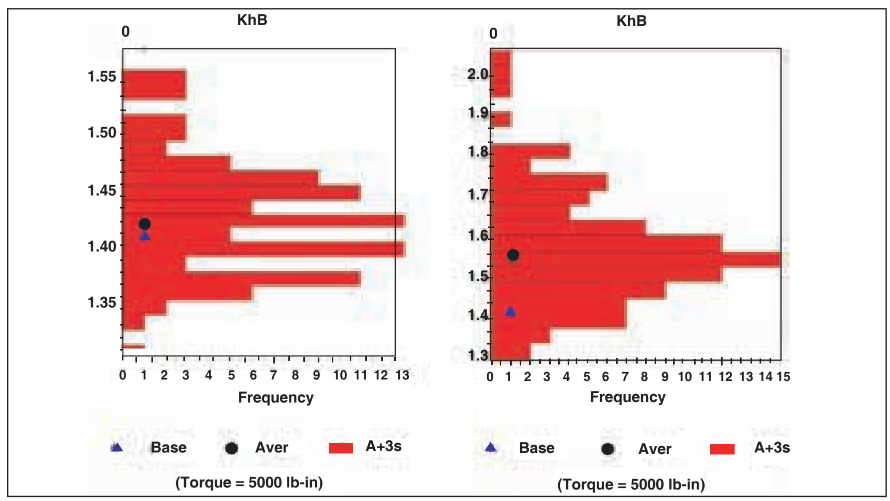

Figure 16—Frequency distribution of the load distribution factor using random robustness analysis at 5,000 lb–in torque (profile crown = 0.0005 in.; lead crown = 0.0002 in. A6 on left, A8 on right).

Table 3—Summary data for A6 quality
(P = profile crown, L=lead crown, both in tenths of thousandth of an inch; stresses in ksi)

P	L	s_c	s	s_c+3s	S_t	S	S_t+3s	K_m	S	K_m+3s
4	2	196	4.1	209	46.3	1.8	51.7	1.38	.05	1.54
4	3	199	4.9	214	46.8	1.8	52.2	1.41	.06	1.59
5	2	200	4.2	213	47.7	1.9	53.3	1.42	.05	1.57
5	4	206	5.1	222	49.6	2.0	55.7	1.51	.07	1.70
7	2	208	4.2	220	50.7	1.8	56.2	1.51	.05	1.66
9	0	222	6.5	241	56.0	2.9	64.6	1.67	.09	1.95
8	4	218	4.8	232	54.4	2.1	60.6	1.64	.06	1.81

Table 4—Summary data for A8 quality
(P = profile crown, L=lead crown, both in tenths of thousandth of an inch; stresses in ksi)

P	L	s_c	s	s_c+3s	S_t	S	S_t+3s	K_m	S	K_m+3s
4	3	204	8.2	228	48.9	3.5	59.3	1.51	.13	1.91
5	2	209	9.9	238	50.7	3.9	62.5	1.55	.15	1.98
5	3	207	8.1	232	50.3	3.5	60.9	1.53	.11	1.87
6	3	211	8.1	235	51.7	3.6	62.4	1.56	.10	1.87
9	0	233	13.5	274	58.6	5.3	74.5	1.81	.20	2.40
7	3	214	8.0	238	53.2	3.6	63.9	1.59	.10	1.88
8	4	219	8.1	243	55.0	3.5	65.6	1.64	.09	1.90

that the minimum transmission error is achieved when the sum of the profile and lead crown are equal to about 0.0009 in. The rows are oriented such that the sum of profile and lead crowns is lowest at the top and increases as one reads down the table.

Contact stress discussion. Much like Figure 5, the lower the sum of profile crown and lead crown, the lower the contact stress. For the A6 quality, the average contact stress was about equal to the baseline value; but for quality A8, the average was somewhat higher than the baseline level (seen from Figure 14). The standard deviations do not vary much with the selection of crowns, with the exception being the case that has zero lead crown. Here, the standard deviation was much greater than for the other modifications. The mean plus 3 standard deviation data indicates the worst case stress, and is probably the best column for comparing the different modifications. Here, we see that for the A6 quality, the P4L2 case has the lowest value and is followed quite closely by the P4L3 and P5L3 cases. The P4L3 case seems to be the best of the Quality A8 pairs, followed closely by the P5L3 case. Going from Quality A6 to A8 roughly doubles the standard deviation; the average stresses increase by 7-10 ksi and the maximum stresses increase about 20 ksi (10% of the mean stress). When taken from the baseline data, there is a peak stress increase of about 7% for the A6 quality and an increase of 14% for the A8 quality.

Pinion root stress. Again, the standard deviations are quite similar for all cases except the P9L0 case. For both quality levels, the "best" modifications are the same as those for the contact stresses. This occurrence is most fortunate since it avoids the need for a compromise to be made for these criteria. The percentage increases in root stresses from the baseline values of the A6 and A8 quality gears are 13.6% and 26%, respectively.

Load distribution factor. The best modifications for both quality levels are similar to those for the stress calculations. The percentage increases in the load distribution factors for the A6 and A8 quality levels are 12.4% and 34%, respectively.

Since the baseline gear was used to compute the percentages, the true percentage increases from the unmodified gears will be a bit higher. If the starting modifications are far afield from the "best" modifications—as occurs with the P9L0 case—it is likely that both the baseline stresses and the standard deviations will be greater than for the "best" cases. Hence, there is much to be gained by having "good" starting profile and lead modifications.

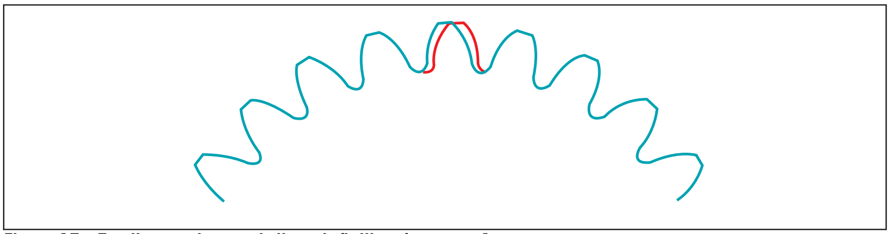

Figure17—Tooth spacing variation definition for case 1.

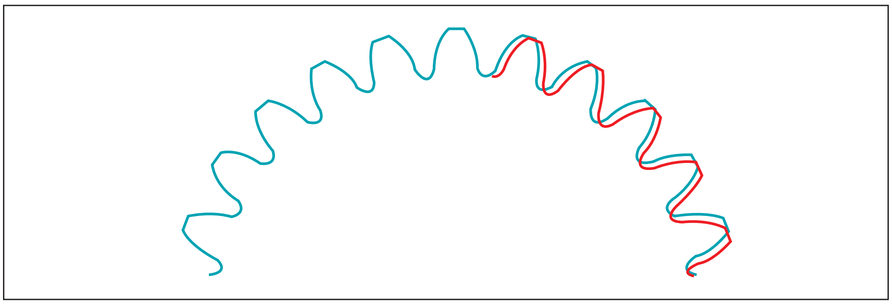

Figure 18—Tooth spacing variation definition for case 2.

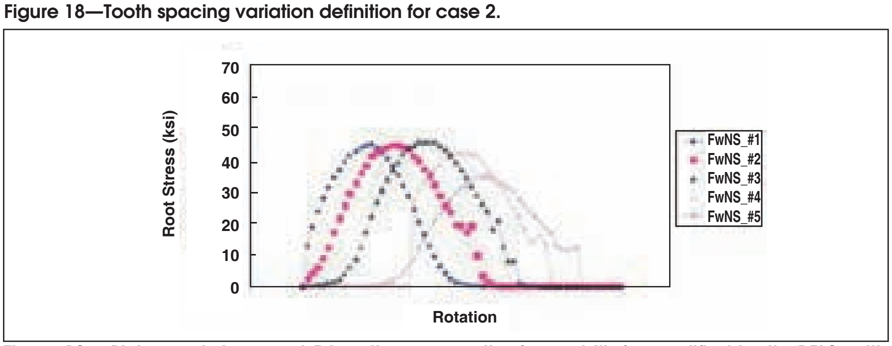

Figure 19— Pinion root stresses at 5 locations across the face width for modified teeth (P5L2) with 5,000 lb-in. torque (no spacing variations).

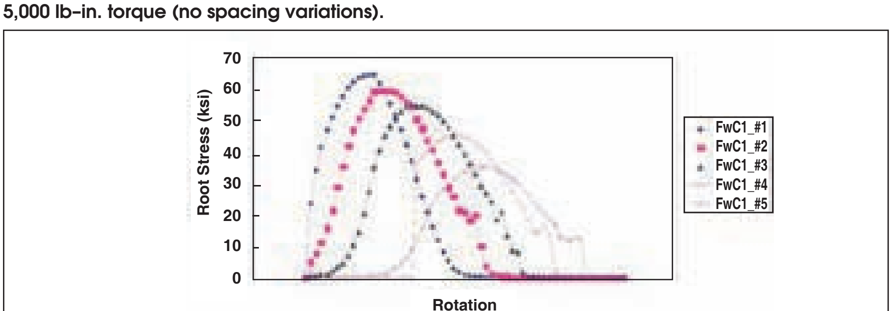

Figure 20—Pinion root stresses at 5 locations across the face width for modification (P5L2) and quality A8 spacing variation (case 1) with 5,000 lb-in. torque.

Table 5—Summary of pinion root stress using spacing deviation for the modified P5L2 helical gear pair.

	AGMA A8 0.000692 in (ksi)	AGMA A6 0.000346 in (ksi)
No spacing	45.03	
Spacing case 1 (+)	64.22	54.61
Spacing case 2 (+)	66.33	55.05
Spacing case 1 (-)	66.32	Not run
Spacing case 2 (-)	50.97	Not run

Spacing and Runout Deviations

Both of these deviations fall in the domain of the AGMA quality system, and so are evaluated using the load distribution analysis. A scheme for evaluating these deviations on both static and dynamic stresses has been performed (Ref. 15), and this method could in fact be superimposed upon the methodology described in this paper. Here, we shall only perform the static analysis. It is expected that one can simply superimpose the spacing and runout deviation effects with the lead and profile deviations, without introducing much error. So for now, we will only look at the spacing and runout deviation effects as separate cases.

Spacing Deviations. Here, we shall evaluate only the worst case tooth-to-tooth spacing deviation and will present results only for root stresses. There seem to be two possibilities for creating a worst-case scenario—the first being when only one tooth is mispositioned, and the second when many subsequent teeth are mispositioned by the deviation tolerance. Figure 17 shows the first case and Figure 18 shows the second case. In each case the position of the first tooth with an error may be positive (comes into contact early) or negative (comes into contact late). The negative condition is shown in the two figures.

These effects are simulated by essentially shifting the profiles either forward or backward, depending upon the sign of the deviation. For positive deviations, the tooth with the error comes into contact early and carries a disproportionate share of the load. The values of errors used are the sum of the square of the spacing deviations of the pinion and the gear, respectively.

Figure 19 shows the plots of root stresses for five locations across the face width (similar to Figure 3) for a modified gear tooth without spacing errors. Figure 20 shows the stresses after the addition of the spacing error, and one notes that the stresses at the edge of the tooth increase significantly. Table 5 summarizes the stress values for positive and negative errors for each case for both A6 and A8 accuracies. For quality A8 gears, the root stress increases about 45% for three of the four cases and for the A6 quality, the stress increase is still about 20%.

Runout deviations. In the simulations performed for this paper, AGMA radial runout was converted to tangential spacing errors using the tangent of the pressure angle. The runout was considered to be sinusoidal and a sinusoidal train of spacing errors was simulated. Values are not presented, but values were roughly equivalent to those of individual spacing errors of the "worst- case spacing" of the sinusoidal errors.

In converting radial runout to tangential spacing error, the following equation was used for each gear:

$$SE = \frac{f_{idT}}{2} \tan\ (\phi_t) \left(\frac{360}{N_T}\right) \qquad (3)$$

where,

SE peak effective spacing error

F_{idT} radial composite tolerance

ϕ_t transverse pressure angle

N_T number of teeth

Summary

An analysis procedure has been presented that accounts for manufacturing accuracies in evaluating contact stresses, root stresses and load distribution factors. The same procedure may be used for other design metrics such as film thickness, flash temperature and transmission error. The increases in stresses due to profile and lead deviations are certainly significant, being as high as 26% for the root stresses of the example quality A8 helical pinion. The load distribution factor increased 34% for the same pinion. Spacing variations provided even higher stress variation, with root stresses increasing by as much as 45% for the A8 spacing error. Although not shown, contact stresses and the load distribution factor also increase significantly when spacing errors are applied. Also presented in this paper is a procedure for selecting appropriate modifications that compensate for misalignment and avoid severe corner contact and tip interference. General conclusions are:

- Spacing deviations: They have a large effect on contact and root stresses, mainly due to transverse load sharing (up to 50%).
- Runout has a relatively small effect, but lower quality gears still may cause an increase in stresses.
- Microgeometry changes (profile, lead and misalignment deviations) significantly affect contact stresses (5–10%) and root stresses (10–25%).
- It is important to start a design with reasonable profile and lead modifications, since this reduces the variability in stresses due to inaccuracies.
- The best modifications do not totally eliminate corner contact or tip interference, but do reduce their effect. If one totally eliminates tip interference, stresses will be higher than those for the"best" modifications shown here.
- Although the increases in stress values due to manufacturing variability appear to be quite large relative to current rating practice, one could justify these effects as being part of the uncertainty that is part of the design factor of safety (difference between design allowable stresses and material property stresses) in the current rating practice.

Even though many manufacturing variables have been considered in this study, there are still numerous factors that are affected by manufacturing accuracy that will influence stress values. Several, such as tooth thickness, center distance variation, outside diameter variation and surface finish variation, have not been included and certainly have possibilities for future studies. Fortunately, many designers run min/max calculations for these parameters so they do in a way get considered in their evaluations.

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continued

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