

Efficient Methods for the Synthesis of Compound Planetary Differential Gear Trains for Multiple Speed Ratio Generation

Cemil Bagci
Tennessee Technological University
Cookeville, TN

Abstract:

This article presents an efficient and direct method for the synthesis of compound planetary differential gear trains for the generation of specified multiple speed ratios. It is a train-value method that utilizes the train values of the integrated train components of the systems to form design equations which are solved for the tooth numbers of the gears, the number of mating gear sets and the number of external contacts in the system. Application examples, including vehicle differential transmission units, rear-end differentials with unit and fractional speed ratios, multi-input function generators and robot wrist joints are given.

Introduction

In a simple planetary gear train each planet gear shaft carries one gear; in a compound planetary gear train each planet gear shaft carries two or more gears connected to each other. (See the simple and compound planetary gear trains in Figs. 1 and 3, respectively.) Planet gears in a compound planetary gear train cause speed reduction and change in direction of rotation; planet gears in simple planetary gear trains merely cause change in direction of rotation. Synthesis of compound planetary gear trains can be done by the tabulation method, which utilizes relative motions of gears with respect to the planet arm. In general, it is an iterative process.

Using the equations of motion of planetary gear trains instead of the tabulation method yields a very simple, direct method for analysis as well as for synthesis of both simple and compound planetary gear trains. The equations of motion for a simple planetary gear train, such as shown in Fig. 1, are formed by writing the velocity loop-closure equations

for contact points A and C and solving them simultaneously. Given the following criteria:

$\bar{\omega}_i = \omega_i \bar{k}$ is angular velocity;

r_i is the pitch circle radius of the i^{th} gear or member; member 3 is the planet arm;

$\bar{\omega}_3 \equiv \bar{\omega}_a$; $\bar{V}_A = -V_A \bar{i}$; $\bar{V}_c = -V_c \bar{i}$;

$\bar{V}_B = -V_B \bar{i}$; $\bar{V}_{AB} = V_{AB} \bar{i}$; and $\bar{V}_{CB} = -V_{CB} \bar{i}$

where

$$V_A = r_2 \omega_2; V_c = r_5 \omega_5; V_{CB} = r_4 \omega_4 = V_{AB} \text{ and}$$

$$V_B = (r_2 + r_4) \omega_a$$

The velocity loop-closure equations are

$$\bar{V}_A|_2 = (\bar{V}_B + \bar{V}_{AB})|_4 \quad (1)$$

$$\bar{V}_C|_5 = (\bar{V}_B + \bar{V}_{CB})|_4 \quad (2)$$

where the subscripts 2, 4, 5 designate the contact points on members 2, 4, 5, respectively. Equations 1 and 2, in terms of r_i and ω_i , become

$$-r_2 \omega_2 = -\omega_a (r_2 + r_4) + r_4 \omega_4 \quad (1a)$$

$$-r_5 \omega_5 = -\omega_a (r_2 + r_4) - r_4 \omega_4 \quad (2a)$$

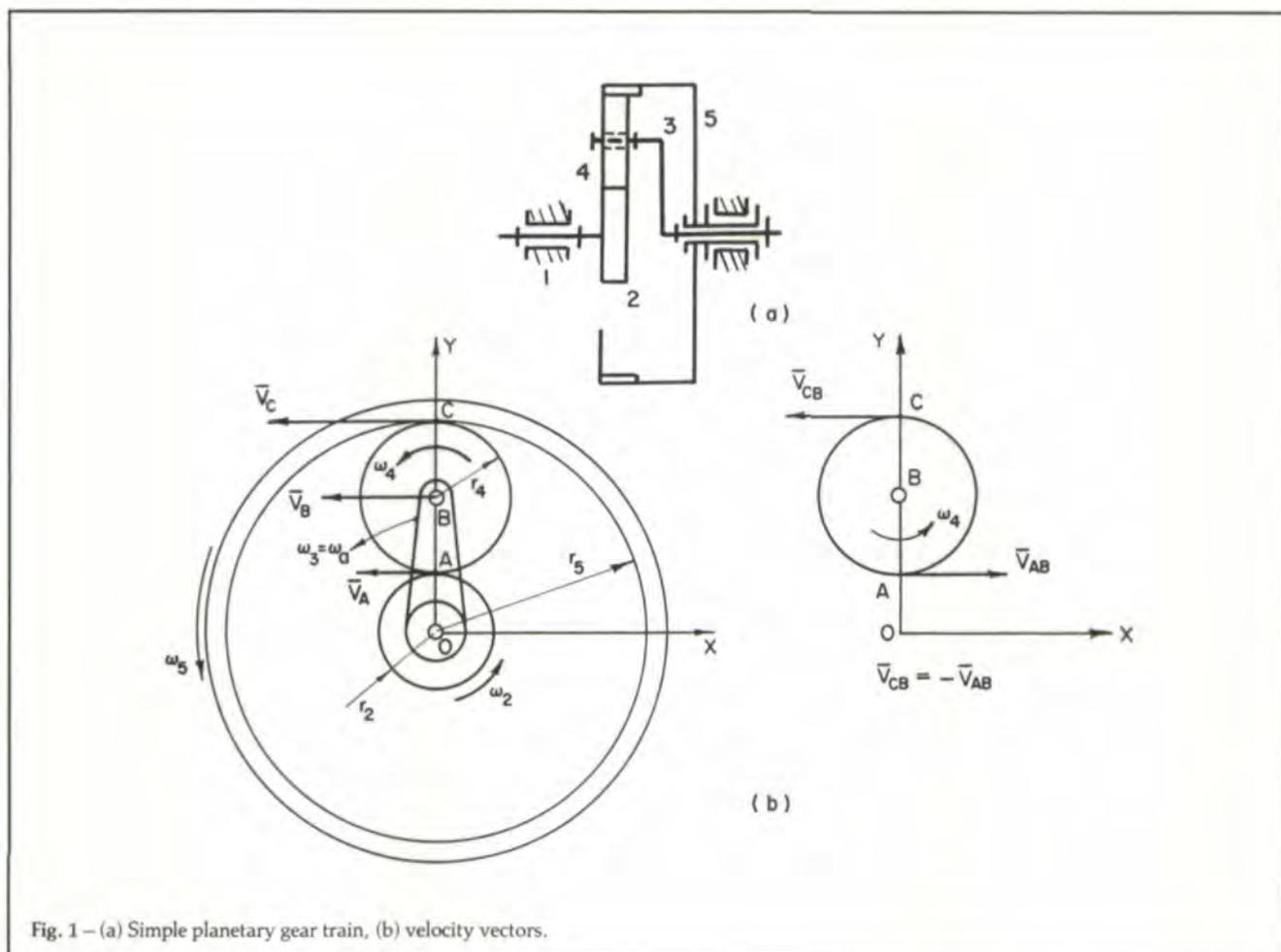


Fig. 1 - (a) Simple planetary gear train, (b) velocity vectors.

Eliminating ω_4 in Equations 1a and 2a we obtain the equations of motion relating speeds of gears 2, 5 and the planet arm as

$$r_2\omega_2 + r_5\omega_5 = 2\omega_a(r_2 + r_4) \quad (3)$$

with the constraint that

$$r_5 = r_2 + 2r_4 \quad (4)$$

Substitute r_5 from (4) into (3) and note that $r_i = N_i/2P$ and $\omega_i = \pi n_i/30$; P is the diametral pitch; n_i is the speed of the i^{th} gear in rpm; n_a is the speed of the planet arm. Then the equation of motion in terms of tooth numbers of gears becomes

$$N_2(n_2 - n_a) + N_5(n_5 - n_a) = 0 \quad (5)$$

which is written in the form of train value as

$$e = -\frac{N_2}{N_5} = -\frac{1}{K} = \frac{n_5 - n_a}{n_2 - n_a} \quad (6)$$

In this equation e is the train value formed as the ratio of the products of tooth numbers of driving gears starting with the input gear, $\Sigma PDVER$, to the products of tooth numbers of the driven gears, and $\Sigma PDVEN$, when the planet arm is considered stationary. The sign of e , however, is very important in order that no error is introduced during the analysis

and synthesis. Thus, e is defined as

$$e = (-1)^q \frac{\Sigma PDVER}{\Sigma PDVEN} \quad (7)$$

where q is the number of external contacts of the mating gears. For the gear train in Fig. 1, e is defined as

$$e = -\frac{N_2N_4}{N_4N_5} = -\frac{N_2}{N_5} \quad (8)$$

with $q=1$ for contact between gears 2 and 4. For the gear train of gears 2 to 7 in Fig. 2, e is defined with $q=3$ as

$$e = (-1)^3 \frac{N_2N_4N_5N_6}{N_4N_5N_6N_7} = -\frac{N_2}{N_7} \quad (9)$$

where gear 6 is a double length gear to mate with gears 7 and

AUTHOR:

DR. CEMIL BAGCI is professor of mechanical engineering at Tennessee Technological University, Cookeville, TN, where he teaches courses and conducts research in machine design, kinematics and robotics. Dr. Bagci studied at Technical Teachers College, Ankara, Turkey and received his undergraduate and graduate degrees from Oklahoma State University, Stillwater, OK. He is author of over 170 technical papers and the recipient of numerous research awards. He is a member of AGMA, ASME and several other technical societies.

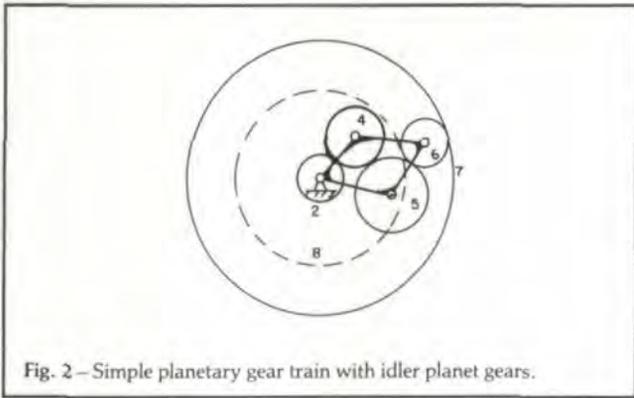


Fig. 2—Simple planetary gear train with idler planet gears.

8. For the gear train of gears 2 to 8, e is defined with $q=4$,

$$e = \frac{N_2}{N_8} \quad (10)$$

As observed in Equations 8-10, idler gears do not affect the speed ratio, but their number defines the sign of e .

Subtract Equation 1 from Equation 2 to have

$$\bar{V}_A - \bar{V}_C = 2\bar{V}_{AB} \quad (11)$$

or

$$-r_2\omega_2 + r_5\omega_5 = 2r_4\omega_4 \quad (12)$$

which, upon substituting r_i and ω_i in terms of N_i and n_i , gives the speed of the planet gear 4 as

$$n_4 = \frac{n_5N_5 - n_2N_2}{2N_4} \quad (13)$$

Either substituting e and n_5 from Equation 6 and

$$N_5 = 2N_4 + N_2 \quad (14)$$

from Equation 4 into Equation 14, or considering Equation 1a, we obtain

$$n_4 = n_a - \frac{N_2}{N_4}(n_2 - n_a) \quad (15)$$

defining n_4 in terms of n_2 . Also, either substituting e and n_2 from Equation 6 and N_2 from Equation 14 into Equation 13, or considering Equation 2a, we obtain

$$n_4 = n_a + \frac{N_2}{N_5}(n_5 - n_a) \quad (16)$$

defining n_4 in terms of n_5 .

The train value defined by Equation 6 is written in general form as

$$e = \frac{n_L - n_a}{n_F - n_a} \quad (17)$$

where n_F and n_L are the speeds of the first and last gears in the train considered, and e is written correspondingly. The

first gear tooth number in $\Sigma PDVER$ is the tooth number of the first gear of the train, whichever end of the train one starts with, and the last gear tooth number in the train is the last number in $\Sigma PDVEN$.

Similarly, Equations 15 and 16 can also be written in the following general forms: Where n_p is the designated speed of the planet gear, e_{FP} and e_{LP} are the train values between the first and planet gear and last and planet gear, respectively.

$$n_p = n_a + e_{FP}(n_F - n_a) \quad (18)$$

$$n_p = n_a + e_{LP}(n_L - n_a) \quad (19)$$

Note that the coefficient $[(-1)^j n_2/n_4]$ of $(n_2 - n_a)$ in Equation 15 is the train value e_{24} between gears 2 and 4; the coefficient $[(-1)^o N_5/N_4]$ of $(n_5 - n_a)$ in Equation 16 is the train value e_{54} between gears 5 and 4.

Equations 18 and 19 form bases for the tabulation method to define the speeds of planet gears. This method simply assumes all moving members are fixed to the planet arm rotating with the arm speed. Then it adds to that speed their individual speeds relative to the planet arm, as if they were ordinary gear trains whose frames were fixed to the planet arm.

As shown in the following section, Equations 17-19 are also the equations of motion for all compound gear trains, bevel gear planetary gear trains and differentials.

Equations of Motion for Compound Planetary Gear Trains

Two compound planetary gear trains are shown in Figs. 3(a) and (b). In (a) the last gear is an internal gear; in (b) it is an external gear. Equations of motion for the compound gear train are written in the same manner as for the simple planetary gear train. Thus, as shown in Fig. 4, velocity loop-closure equations for A and C are

$$\bar{V}_A|_2 = (\bar{V}_B + \bar{V}_{AB})|_4 \quad (20)$$

$$\bar{V}_C|_6 = (\bar{V}_B + \bar{V}_{CB})|_5 \quad (21)$$

which reduce to

$$-r_2\omega_2 = -\omega_a(r_2 + r_4) + r_4\omega_4 \quad (22)$$

$$-r_6\omega_6 = -\omega_a(r_2 + r_4) - r_5\omega_5 \quad (23)$$

Noting that $\omega_5 = \omega_4$ and eliminating them in Equations 22 and 23, we obtain

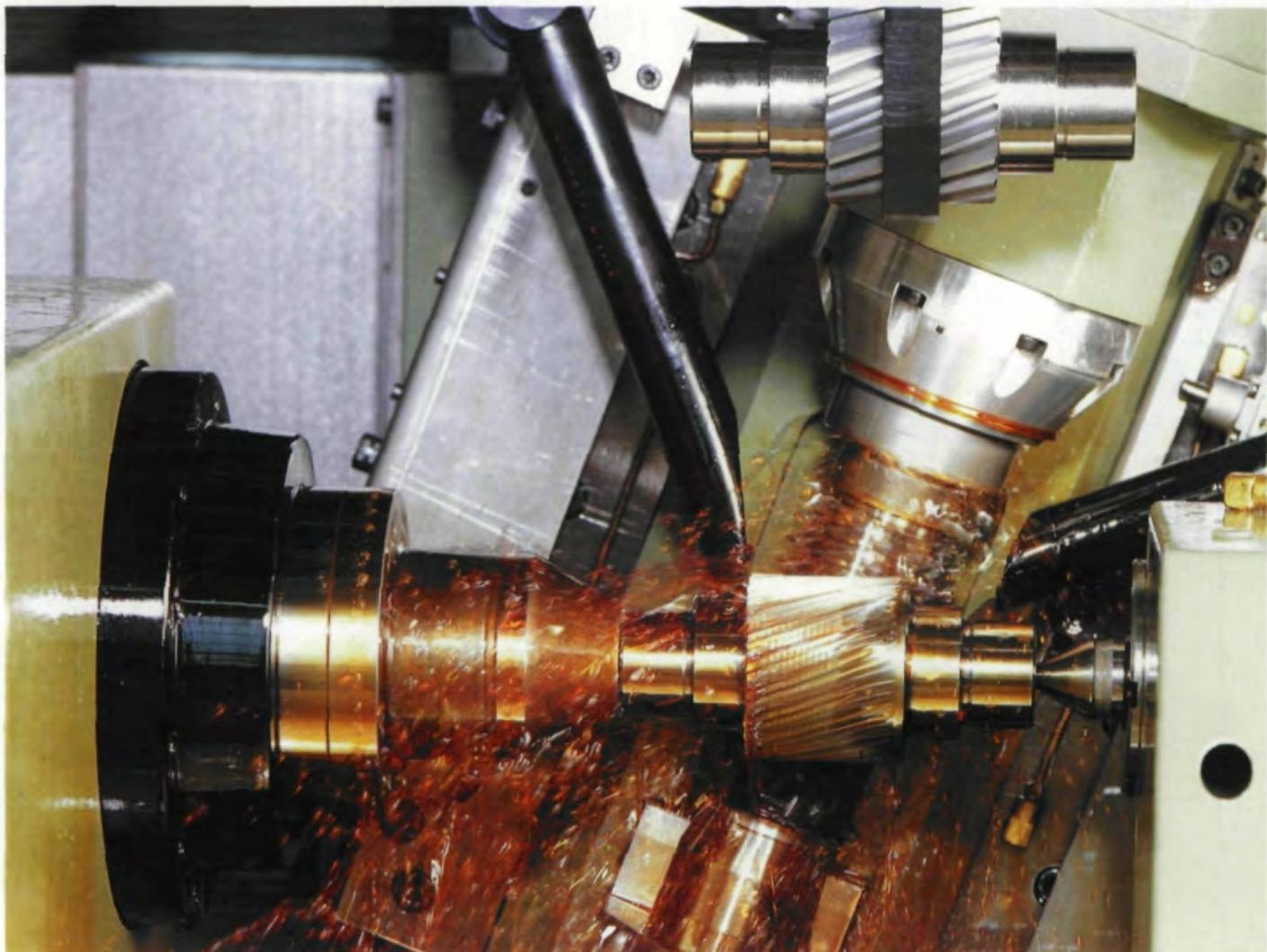
$$r_5r_2(\omega_2 - \omega_a) + r_4r_6\omega_6 = \omega_a[r_4^2 + r_4(r_2 + r_5)] \quad (24)$$

Noting the following equation of constraint

$$r_2 + r_4 = r_6 - r_5 \quad (25)$$

or P_{ij} defining diametral pitch between gears i and j ,

$$\frac{N_2 + N_4}{2P_{24}} = \frac{N_6 - N_5}{2P_{56}} \quad (26)$$



CNC Gear Hobbing Machine



A35 CNC Gear Hobbing Machine

Gear dia. (max.)	8 inch	Hob speed	2200 rpm
DP (max.)	6	Work speed	350 rpm
Shaft length	27 inch		



A25 CNC Gear Hobbing Machine

Gear dia. (max.)	4 inch	Hob speed	3000 rpm
DP (max.)	10	Work speed	1000 rpm
Shaft length	12 inch		

Mikron has produced high quality gear hobbing machines for 80 years. Based upon this tradition Mikron now manufactures a range of high performance CNC hobbing machines covering gears up to 8 inch diameter and diametral pitch 6. Quality and precision are, as always, an integral part of the Mikron product range. To ensure that our customers achieve the performance and receive the support that today's competitive environment demands Mikron sales and service are now available direct from Elgin, IL. For all sales, service and spares enquiries:



See us in Booth 6166
IMTS 90

Phone 708 888 0132
Telefax 708 888 0343

Mikron Corp. Elgin
1525 Holmes Road
Elgin, IL 60123



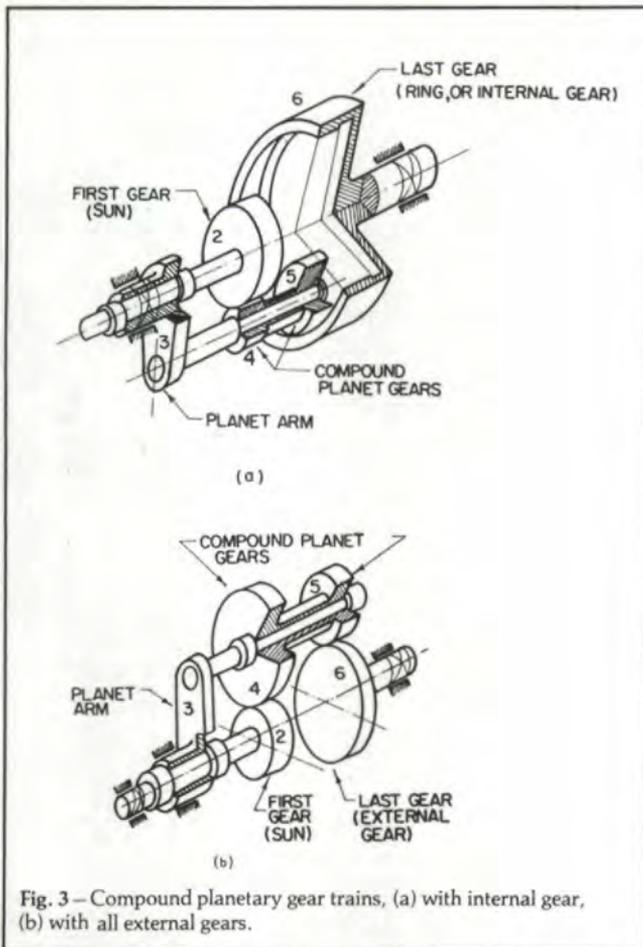


Fig. 3—Compound planetary gear trains, (a) with internal gear, (b) with all external gears.

and substituting $r_2 + r_5 = r_6 - r_4$ into Equation 24, we have the equation of motion

$$r_5 r_2 (\omega_2 - \omega_a) + r_6 r_4 (\omega_6 - \omega_a) = 0 \quad (27)$$

which, in terms of n_i and N_i , becomes

$$e = -\frac{N_2 N_5}{N_4 N_6} = \frac{n_6 - n_a}{n_2 - n_a} \quad (28)$$

This is in the form of Equation 17 with $q=1$, $n_F = n_2$, $n_L = n_6$, $N_F = N_2$, $N_L = N_6$.

The speed of the planet gear, n_4 , is defined in terms of n_2 or n_6 by Equations 22 or 23 as

$$n_4 = n_a - \frac{N_2}{N_4} (n_2 - n_a) \quad (29)$$

or

$$n_4 = n_a + \frac{N_6}{N_5} (n_6 - n_a) \quad (30)$$

which are also in the forms of Equations 18 and 19, respectively.

Caution: For ordinary gear trains in which the planet arm is fixed, $n_a = 0$ and

$$e = \frac{n_L}{n_F} = \frac{1}{S_R} \quad (31)$$

indicating that the train value for ordinary gear trains is the inverse of the speed reduction ratio S_R generated by the gear train. Hence, when a planetary gear train is driven by an ordinary gear train, the ordinary gear train must be analyzed first to determine the input speed to the planetary gear train.

Input-Output Torque Relation

The power equilibrium of a gear train defines the torque multiplication factor (or the mechanical advantage) the planetary gear train generates. Thus,

$$T_{in} \omega_{in} + T_{out} \omega_{out} = 0 \quad (32)$$

or

$$\frac{T_{out}}{T_{in}} = -\frac{n_{in}}{n_{out}} = -S_R \quad (33)$$

where the subscripts "in" and "out" designate "input" and "output", respectively. Considering the mechanical efficiency of the gear train as η_t , input torque required to generate an output torque is

$$T_{in} = -\frac{T_{out}}{\eta_t S_R} \quad (34)$$

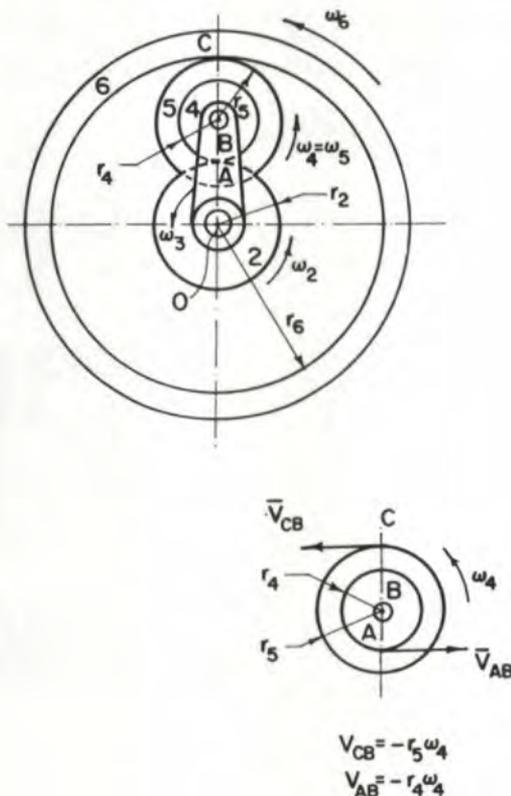


Fig. 4—Velocities in the compound planetary gear train.

Gear train manufacturers⁽¹⁾ suggest that the mechanical efficiency of two well-lubricated, mating precision gears is about 0.98; for two and three stage speed reductions, it is about 0.97 and 0.96, respectively. Therefore, if there are R sets of gears in a gear train, its mechanical efficiency may be approximated by

$$\eta_t = (0.98)^{[1+0.5(R-1)]} \quad (35)$$

Reduction in efficiency due to energy loss in bearings should also be considered.

Example of Analysis of a Compound Planetary Gear Train

Following is the analysis needed to find the input torques required for each output shaft operation. The gear train shown in Fig. 5 is used to lift 10⁶lbf load where the lift ropes wrap around 10" diameter drums. Mechanical efficiency of the system is 0.93. The input shaft of two-lead-worm rotates at 1000 rpm.

Gears 2 and 3 form an ordinary gear train generating the input speed n_4 of the planetary gear train. Hence, $n_4 = n_F$ must be determined first. It is $n_F = 100/3$ rpm.

Since the train value equation (No. 17) contains three shaft speeds, it can only be solved for one speed when the other two speeds are specified. Then, in the planetary gear train of Fig. 5, we must find a sub-planetary gear train receiving two input speeds. Since in this arrangement, gear 9 is fixed, and $n_9 = 0$, the sub-planetary gear train we are

looking for is of gears 4, 5, 6, 7, 8 and 9 with $n_L = n_9 = 0$. This train generates the speed of the planet arm, which is one of the output speeds. Thus, for this train we have

$$e_{4-9} = (-1)^2 \frac{N_4 N_6 N_8}{N_5 N_7 N_9} = \frac{n_9 - n_a}{n_4 - n_a}$$

With the tooth numbers shown it reduces to

$$0.00766 = \frac{-n_a}{100/3 - N_a}$$

and

$$n_a = -0.260587 \text{ rpm}$$

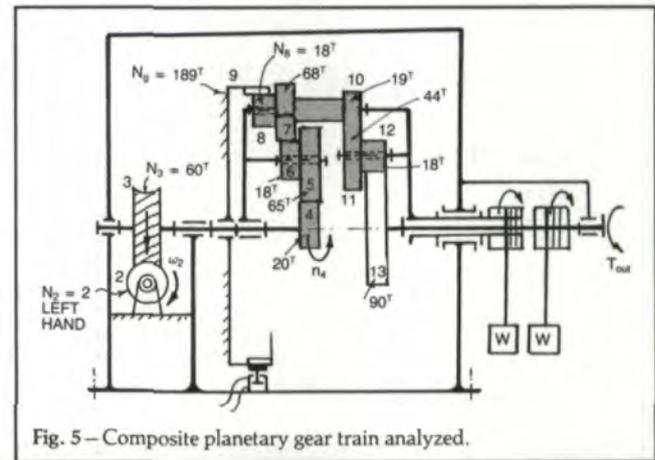


Fig. 5 - Composite planetary gear train analyzed.

I N E X P A N S I O N A R B O R S A
P I C T U R E I S W O R T H A 1 0 0 0 W O R D S



Splined Arbor

guehring

AUTOMATION INC.

W227 N6193 Sussex Rd.
P.O. Box 125 Sussex, WI 53089
(414) 246-4994
Fax (414) 246-8623

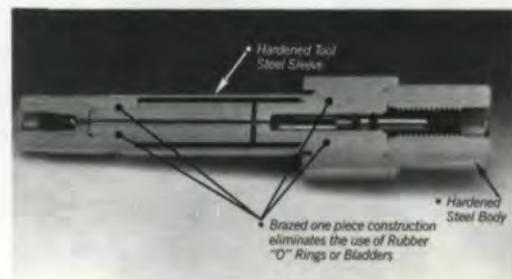


- RUPTURE PROOF
May be fully expanded with no part... Allows a true inspection of the arbor/chuck with no inspection rings or plugs.
- INCREDIBLE ACCURACY
.000080" TIR or less for most applications.
- EXTREME HOLDING POWER
Up to 7200 PSI.
- AUTOMATIC OR MANUAL ACTUATION
Adaptable to most machines.
- NO HYDRAULIC SLEEVE SEALS
Eliminates leakage, increases accuracy, virtually maintenance free.
- SPLINED VERSIONS
Also available.

Now you can see the difference in expansion arbors and chucks. Eliminate losses and down time caused by accidental actuation or leaking seals that effect products of lesser quality.

HDT manufactures a superior self-contained hydraulic work holding/tool holding system designed for your needs.

DON'T SETTLE FOR LESS THAN HDT — COMPARE OUR FEATURES



Representative inquiries invited.
CIRCLE A-7 ON READER REPLY CARD

KLINGELNBERG

**Puts it all together
with exciting, new
products to keep
you competitive!**

Gearmakers worldwide face customer demands for highest quality and on-time delivery at fiercely competitive prices. Meeting these demands requires greater flexibility, productivity and the ability to integrate new technology. Recognizing your ultimate goals – customer satisfaction and profitability – Klingelnberg global technology is ready with the most advanced CNC Spiral Bevel Gear Generators and Grinders, and the most complete line of CNC Gear Checkers available... anywhere.

We've put it all together:

Spiral Bevel Gear Applications

- KNC 40/60 fully CNC controlled Generators
- AMK Large Gear Generators
- W800CNC (Wiener System) Grinder
- CNC Inspection Equipment
- Quench Presses

Worm and Rotor Grinding Applications

- HNC Series of CNC controlled Grinders
- CNC Inspection Systems

Parallel Axis Gear Applications

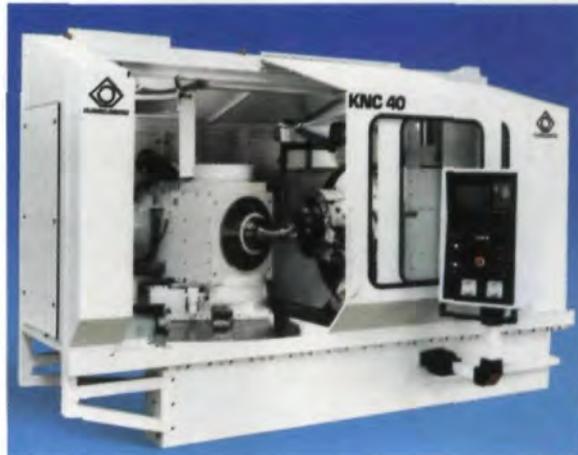
- CNC controlled gear Inspection Systems
- SNC Series of CNC Hob Sharpeners
- Gear Cutting Tool Inspection Systems
- Hurth Product Line: CNC Shavers, Hard & Fine Finishers, Shaving Cutter Grinders and Deburring/Chamfering Equipment.



GEARMAKING!

The KNC-40/60 Gear Generators

These new, fully CNC, 8 axis gear generators produce Spiral Bevel Gears up to 24" O.D. Perfect for small batch or intermediate level production runs, the KNC Series generates Spiral Bevel Gears of any geometry by continuous or single indexing operation. Its program storage capacity for 250 different workpieces and unlimited storage capacity with a DNC interface, makes it the most flexible gear generator you can buy.



The W800CNC Gear Grinder (Wiener System)

With leading edge technology, the W800CNC becomes the universal Spiral Bevel Gear grinder for any gear geometry. Perfect for ground gear quality applications up to 31.5" O.D., the grinder offers great versatility, short set-ups and superior accuracy.



The PNC Series Gear Checkers

Compact, new CNC controlled gear checkers provide fully automatic measuring of gears up to 80" O.D. Fast operation, high accuracy, excellent documentation and a wide variety of software modules, form the perfect gear checking package.



For the latest information on Klingelberg systems that satisfy customer demands, for increased productivity and profitability, contact our representative. Or, write to Klingelberg Gear Technology Inc., 15200 Foltz Industrial Parkway, Strongsville, OH 44136. Phone: 216/572-2100; FAX: 216/572-0985.



KLINGELBERG

...Puts it all together.

The planet arm is rotating in the direction opposite that of gear 4, generating a speed reduction ratio of

$$S_{R_1} = \frac{n_2}{n_a} = -3837.498$$

The second sub-planetary gear train in the system is of gears 4, 5, 6, 7, 10, 11, 12 and 13. Its equation of motion is

$$e_{4-13} = (-1)^4 \frac{N_4 N_6 N_{10} N_{12}}{N_5 N_7 N_{11} N_{13}} = \frac{N_{13} - n_a}{N_4 - n_a}$$

or

$$0.00703 = \frac{n_{13} + 0.260587}{100/3 + 0.260587}$$

$$n_{13} = -0.024422 \text{ rpm}$$

and

$$S_{R_2} = \frac{n_2}{n_{13}} = -40946.73$$

Speeds of gears 5, 7 and 12 are determined using Equations 18 and 19 as

$$n_5 = n_a - \frac{N_4}{N_5} (n_4 - n_a) = -10.5972 \text{ rpm}$$

$$n_7 = n_a + \frac{N_4 N_6}{N_5 N_7} (n_4 - n_a) = 2.4756 \text{ rpm}$$

$$n_{12} = n_a - \frac{N_{13}}{N_{12}} (n_{13} - n_a) = -1.4414 \text{ rpm}$$

Input torque required to lift the load by the planet arm shaft is

$$T_{in_1} = \frac{-5 \times 10^6}{(0.93) (S_{R_1})} = 1401 \text{ in-lbf}$$

To lift the load by the shaft of gear 13 it is

$$T_{in_2} = \frac{-5 \times 10^6}{(0.93) (S_{R_2})} = 131.3 \text{ in-lbf}$$

Bevel Gear Planetary Gear Trains

The value of e is determined as shown above. Its sign, however, must be determined by releasing all the gears, retaining the planet arm fixed, and observing the direction in which the last gear rotates relative to the direction of the

input rotation. A right hand rule can be followed. Thus, consider the planetary gear train of a robot joint shown in Fig. 6, where for the planetary gear train of gears 2, 4, 5

$$e_{2-5} = -\frac{N_2 N_4}{N_4 N_5} = \frac{0 - n_a}{n_2 - n_a} = -\frac{1}{4}$$

and

$$n_a = n_2/5$$

generating speed reduction.

For the planetary gear train of gears 2, 4, 6, 7

$$e = -\frac{N_2 N_6}{N_4 N_7} = \frac{n_7 - n_a}{n_2 - n_a} = -2.3935$$

and

$$n_7 = -1.7148 n_2$$

generating a speed increase (over drive ratio).

A bevel gear planetary gear train with only sun and planet gears is shown in Fig. 7. The equation of motion for the train is given by Equation 19 as

$$n_4 = n_a - \frac{N_2}{N_4} (n_2 - n_a)$$

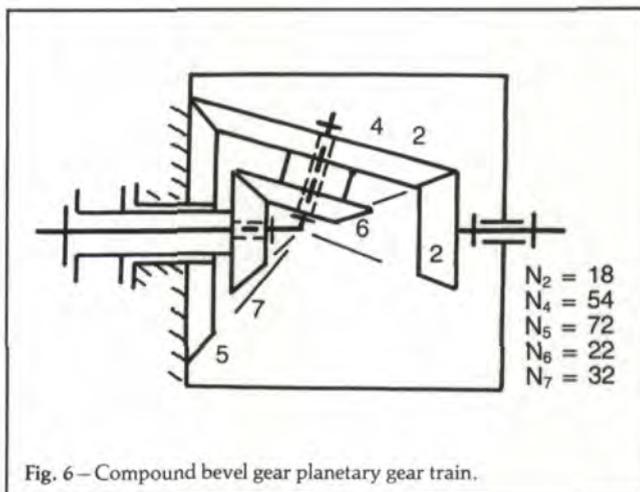


Fig. 6 - Compound bevel gear planetary gear train.

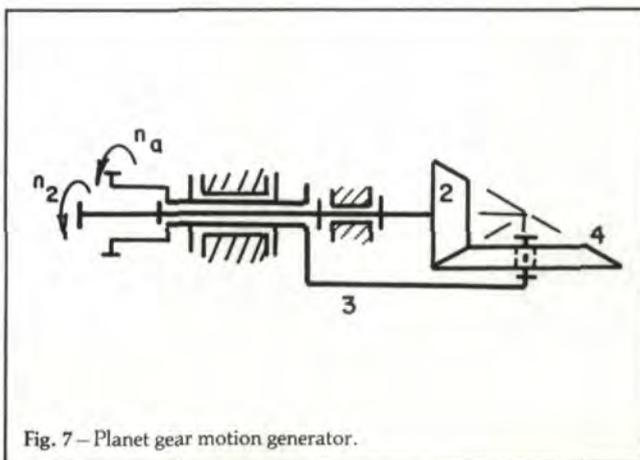


Fig. 7 - Planet gear motion generator.

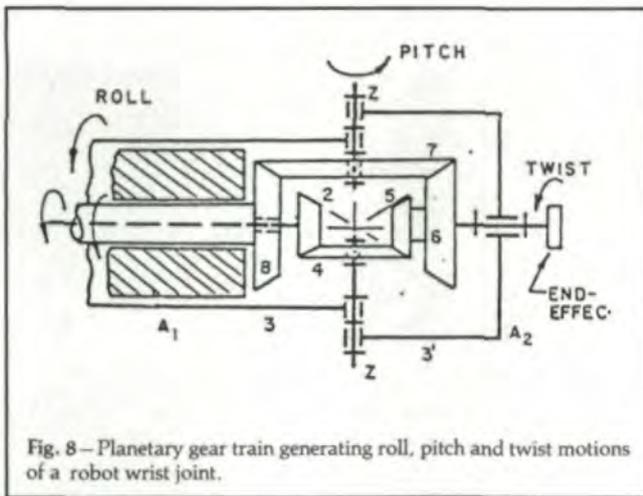


Fig. 8 - Planetary gear train generating roll, pitch and twist motions of a robot wrist joint.

Robot Wrist Joint Using Planetary Gear Trains

Fig. 8 shows another bevel gear planetary gear train used in robot wrist joints. It has three input shafts, shafts of arm A_1 , gears 2 and 8. Arm A_1 rotates the joint to reposition the z-z axis about which the arm A_2 rotates when gear 2 or 8 or both rotate. The shaft of gear 5 is the end-effector. With $n_{A_1} = 0$,

$$n_4 = -n_2 \frac{N_2}{N_4} \quad (36)$$

and

$$n_7 = n_8 \frac{N_8}{N_7} \quad (37)$$

provide input to the planetary gear train of gears 4, 5, 6, 7 and arm A_2 . Its train value is

$$e_{4-7} = -\frac{N_4}{N_5} \frac{N_6}{N_7} = \frac{n_7 - n_{A_2}}{n_4 - n_{A_2}} \quad (38)$$

which defines speed of arm A_2 . Substituting n_4 and n_7 from Equations 36 and 37, n_{A_2} is defined in terms of the input speeds as

$$n_{A_2} = \frac{1}{1 + \frac{N_4 N_6}{N_5 N_7}} \left[n_8 \frac{N_8}{N_7} - n_2 \frac{N_2 N_6}{N_4 N_5} \right] \quad (39)$$

The speed of the end-effector is

$$n_5 = n_{A_2} - \frac{N_4}{N_5} (n_4 - n_{A_2}) \quad (40)$$

As observed in Equation 39, in order to retain $n_{A_2} = 0$, n_2 and n_8 must satisfy

$$n_8 = \frac{N_2 N_6 N_7}{N_4 N_5 N_8} n_2 \quad (41)$$

Synthesis of Compound Planetary Gear Trains

As observed in the examples for analysis of planetary gear trains, Equation 17 can be solved to generate only one shaft speed by a sub-planetary gear train. The objective of synthesis is to find the number of mating gears in the train, their tooth numbers and the number of external contacts q to yield the value of e . The three speed reductions a compound gear train can generate with single input speed are

$$S_{R1} = \frac{n_F}{n_a} = \frac{e-1}{e}, \quad n_L = 0 \quad (42)$$

$$S_{R2} = \frac{n_F}{n_L} = \frac{1}{e}, \quad n_a = 0 \quad (43)$$

$$S_{R3} = \frac{n_L}{n_a} = (1-e), \quad n_F = 0 \quad (44)$$

The split input case with two inputs forms the fourth type of speed generation. Inversion generating S_{R2} is a reverted ordinary gear train. S_{R1} and S_{R2} are large speed reductions. S_{R3} is the smallest reduction. Depending on the sign of e , either S_{R1} or S_{R2} is the forward and the largest reduction. For example, if $e < 0$, $S_{R1} > 0$ and largest, $S_{R2} < 0$ and intermediate; if $e > 0$, $S_{R2} > 0$ and largest, $S_{R1} < 0$ and intermediate. S_{R3} is always the smallest forward reduction when $e < 1$.

During the synthesis one should keep in mind that the tooth ratio between any two mating spur gears and straight bevel gears should be retained below 8 to assure proper contact, sufficiently large contact ratio and low noise level. Larger tooth ratios may be tolerated for internal and helical gears due to increased contact ratio.

Example: Let us synthesize a compound planetary gear train to generate a speed reduction ratio of +577 from the planet arm shaft with $n_L = 0$. By Equation 17

$$e = \frac{n_L - n_a}{n_F - n_a} = \frac{-577}{1 - \frac{1}{576}} = -\frac{1}{577}$$

It can be factored into

$$e_1 = -\frac{1}{576} = -\left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{1}{8}\right) = (-1)^3 \frac{N_2 N_5 N_7 N_9}{N_4 N_6 N_8 N_{10}}$$

or

$$e_2 = -\left(\frac{1}{8}\right)\left(\frac{1}{6}\right)\left(\frac{1}{12}\right) = (-1)^3 \frac{N_2 N_5 N_7}{N_4 N_6 N_8}$$

The last gear in e_1 , gear 10, must be an internal gear; in e_2 , gear 8 must be an external gear to generate $q = 3$. These two



MORE BITES FOR YOUR BUCK

Fellows — always on the cutting edge of technology.

For nearly a century the Fellows name has been synonymous with the finest quality gear shaping cutters available. It still is!

We produce a wide variety of cutters, with and without TiN coating, all computer designed to provide optimum performance for your most exacting requirements.

Fellows cutters include all types — spur and helical, both in commercial and

precision grade... disk, deep counterbore, taper shank, non-involute profile, herring-bone, and internal. We also offer a broad range of shaving cutters and master gears.

And... to make good things even better, we have a wide variety of **STANDARD STOCK CUTTERS** available on a one-day basis as well as a full line of **HOBS** and **DISPOSABLE CUTTERS**. In addition we offer complete resharpener, recoating, and

regrinding services for all types of cutters.

When it comes to quality cutting tools... be sure you get what you pay for! Call us today.

FELLOWS[®]

FELLOWS CORPORATION
PRECISION DRIVE, P.O. BOX 851
SPRINGFIELD, VT 05156-0851 USA
TEL: 802-886-8333, FAX: 802-886-2700

See us at IMTS '90, Booth #6128.

CIRCLE A-9 ON READER REPLY CARD

gear trains are shown in Figs. 9 and 10, respectively. In Fig. 9 let

$$N_2=20, N_4=120, N_5=20, N_6=80, N_7=24, N_8=72, N_9=24, N_{10}=192$$

In terms of center distance vectors

$$\bar{C}_{25} = (r_2 + r_4) \bar{U}_{25}$$

$$\bar{C}_{57} = (r_5 + r_6) \bar{U}_{57}, \bar{C}_{79} = (r_7 + r_8) \bar{U}_{79},$$

and $\bar{C}_{29} = (r_{10} - r_9) \bar{U}_{29}$, where \bar{U}_{ij} is the unit vector.

$$|\bar{C}_{29}| \leq |\bar{C}_{25} + \bar{C}_{57} + \bar{C}_{79}| \geq \bar{C}_{29} \quad (a)$$

must be satisfied when mounting the planet gears on the planet arm. (See Fig. 9b.) Considering the same diametral pitch for all the gears, Equation (a) demands that, depending on the locations of shafts of gears 5 and 7,

$$N_2 + N_4 + N_5 + N_6 + N_7 + N_8 \geq N_{10} - N_9$$

$$N_2 + N_4 + N_5 + N_6 - N_7 - N_8 \leq N_{10} - N_9$$

and

$$N_2 + N_4 - N_5 - N_6 + N_7 + N_8 \leq N_{10} - N_9$$

must be satisfied.

In Fig. 10 let

$$N_2=20, N_4=160, N_5=20, N_6=120, N_7=18, N_8=216$$

which must satisfy

$$|\bar{C}_{27}| \leq |\bar{C}_{25} + \bar{C}_{57}| \geq |\bar{C}_{27}| \quad (b)$$

which demands that

$$N_2 + N_4 - N_5 - N_6 \leq N_7 + N_8$$

and

$$N_2 + N_4 + N_5 + N_6 \geq N_7 + N_8$$

must be satisfied.

To generate this speed reduction an ordinary planetary gear train will require $N_L/N_F=278$. With $N_F=20$, $P=6$, one notices the unbelievable gear size: $N_L=5520$, $d_L=920''$. This defect with ordinary planetary gear trains is minimized by forming series-connected ordinary planetary gear trains.

The planet arm and the planet gears must be balanced, which is commonly achieved by mounting symmetrical planet gears.

Multiple Speed Reduction Generation

Synthesis of a compound planetary gear train is performed in two forms:

Form 1. All the desired speed reductions are generated simultaneously, without requiring shifting for gear arrangements within the gear train.

Form 2. Only one desired speed reduction is generated at a time, requiring shifting of gears, releasing and activating clutches.

The second form of synthesis yields costly systems, since it requires complex mechanisms, connections and clutches to shift the gears and change the shafts. In general it requires a series connection in which the output of one unit is used to drive the next unit.

Example for Form 1 Synthesis

Let us synthesize a compound planetary gear train to generate the speed reduction ratios, 10, -4 and 2.5 simultaneously.

The largest reduction is generated by S_{R1} or S_{R2} . In a compound planetary gear train it is a preferred trade to retain the

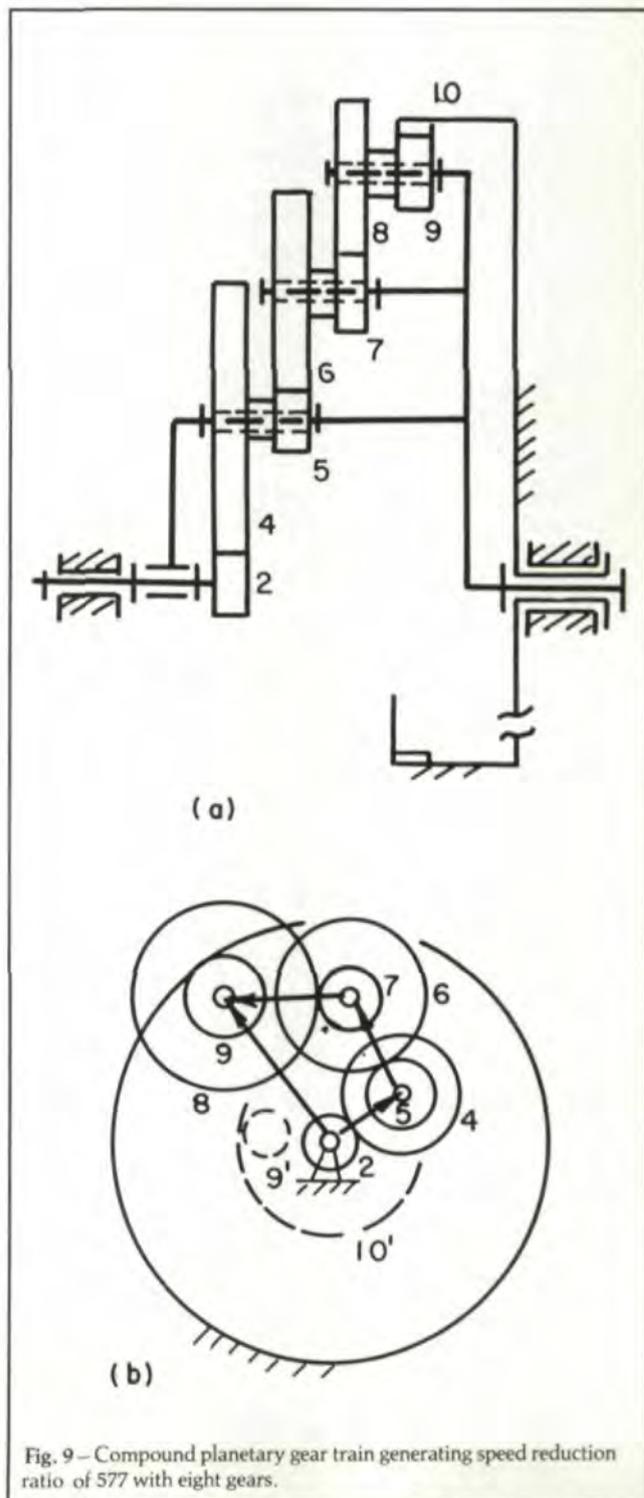


Fig. 9 - Compound planetary gear train generating speed reduction ratio of 577 with eight gears.



Let us quote on ALL of your broaching needs

KINGSFORD BROACH & TOOL INC.

Specializing in FLAT, FORM, KEYWAY, CARBIDE, SERRATION BROACHES, SPLINES AND ROUNDS. Standard keyways in stock at all times. Blanks for most sizes ready . . . off-sizes are also available. Send for brochure containing keyway chart and price list.

RECONDITIONING & SHARPENING FIXTURES • BROACH PULLERS HOLDERS • METRIC SIZES ENGINEERING SERVICES

P.O. Box 2277 • Kingsford MI 49801
906/774-4917 FAX 774-6981
CIRCLE A-10 ON READER REPLY CARD

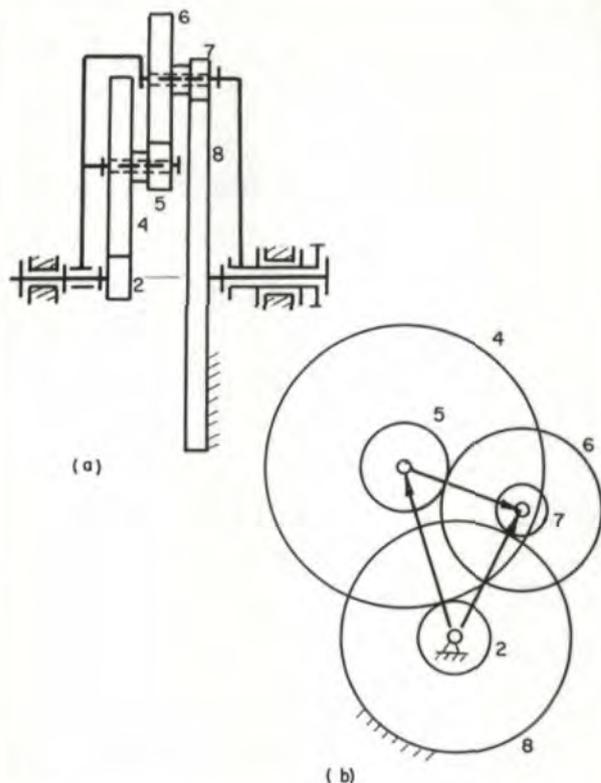


Fig. 10 - Compound planetary gear train generating speed reduction ratio of 577 with six gears.

RAF

QUALITY GEARS UP TO AGMA 15

GEAR GRIND FACILITY - COMPLETE GEAR BOXES
LONG RUN PRODUCTION - SINGLE GEARS

AMERICAN STANDARD

SPURS - SPIRALS - BEVELS - HELICALS
WORMS - WORM GEARS - SEGMENTS
SPLINES - CLUSTERS - SHAFTS

INTERNAL & EXTERNAL



SPECIALISTS
SPIRAL
BEVEL

STATE OF THE ART HEAT TREAT FACILITY ON PREMISES

RAF INDUSTRIES INC.

EXECUTIVE OFFICES
181 Greenwood Avenue
Midland Park, New Jersey 07432
201-445-2413

MANUFACTURING FACILITY
Crestwood Industrial Park
Mountain Top, Pennsylvania 18707
717-474-5440

CIRCLE A-11 ON READER REPLY CARD

planet arm active. So let S_{R1} in Equation 42 generate speed reduction 10. Hence,

$$10 = \frac{e_1 - 1}{e_1} \quad (a)$$

and

$$e_1 = -\frac{1}{9}$$

or

$$e_1 = \frac{1}{\frac{-10}{1 - \frac{1}{10}}} = -\frac{1}{9} \quad (b)$$

Using four gears, the first two having external contact, we form

$$e_1 = \left(-\frac{1}{3}\right) \left(\frac{1}{3}\right) = \left(-\frac{N_2}{N_4}\right) \left(\frac{N_5}{N_6}\right) \quad (c)$$

which states that gear 6 is an internal gear. This portion of the gear train is shown in Fig. 11(a). Considering $N_2=1$, $N_4=3$, $N_5=1$, $N_6=3$, the constraint equation

$$a(N_2 + N_4) = b(N_6 - N_5) \quad (d)$$

must be satisfied, where

$$a = \frac{1}{2P_{24}}, b = \frac{1}{2P_{56}}$$

To use the same diametral pitch

$$\begin{aligned} a(1+3) &= b(3-1) \\ 2a &= b \end{aligned} \quad (e)$$

Letting $a=1, b=2$, we have

$$1 + 3 = 6 - 2 \quad (f)$$

defining $N'_2=1, N'_4=3, N'_5=2$, and $N'_6=6$. Now, both sides of (f) are multiplied by the same number to define the actual tooth numbers. Multiply by 40 (or 20, 24, 30, etc.) to have $N_2=40, N_4=120, N_5=80, N_6=240$.

Using the same planet arm, the other two speed reductions are generated. Thus, by Equation 18, and noting that

$$-\frac{N_2}{N_4} = -\frac{1}{3}$$

must also exist in the sub-gear trains, speed reduction ratio (-4) is generated by

$$e_2 = \left(-\frac{N_2}{N_4} \right) e_B = \frac{-\frac{1}{4} - \frac{1}{10}}{1 - \frac{1}{10}} = -\frac{14}{36} = \left(-\frac{1}{3} \right) \left(\frac{14}{12} \right) \quad (g)$$

where

$$e_B = \frac{14}{12}$$

Its positive sign requires another internal gear, gear 8, or two sets of external gears. Choosing the latter,

$$e_B = \frac{14}{12} = \left(\frac{2}{2} \right) \left(\frac{7}{6} \right) = \left(\frac{N_7}{N_8} \right) \left(\frac{N_9}{N_{10}} \right) \quad (h)$$

Tooth numbers in (h) must satisfy

$$N_2 + N_4 \leq N_7 + N_8 + N_9 + N_{10}$$

and

$$N_2 + N_4 + N_7 + N_8 \geq N_9 + N_{10}$$

Hence,

$$160 \leq C(2 + 2 + 7 + 6) \leq 240$$

and

$$160 \geq C(7 + 6 - 2 - 2)$$

and

$$9 \leq C \leq 17$$

letting $C=10, N_7=20, N_8=20, N_9=70$ and $N_{10}=60$.

Gear Grinding Specialists

Reishauer RZ 300E

Electronically Controlled Gear Grinders

Commercial & Precision Gear Manufacturing to AGMA Class 15 including...

- Spur
- Helical
- Internal
- Pump Gears
- Splines and Pulleys
- Serrations
- Sprockets
- Grinding to 12-1/2" Diameter
- Hobbing to 24" Diameter
- O.D. and I.D. Grinding, Gear Honing w/Crowning, Broaching, Keyseating, Turning and Milling, Tooth Chamfering and Rounding

Supplied complete to print Finishing operations on your blanks Grind teeth only



941G MILITARY RD. BUFFALO, NY 14217

FAX (716) 874-9003 • PHONE (716) 874-3131

CIRCLE A-12 ON READER REPLY CARD

THE LEADER IN

GEAR DEBURRING

See us at Booth E-1548 ITMS-1990



"ZERO-SETUP"

- Redin Model 24-Universal gear chamfering/deburring machine — 2 models.
- Eliminates costly manual machine setup.
- C.N.C. controlled — 8 or 10 axis.
- Part program storage — 1000 p/p.
- Program loading — M.D.I. or Disc.
- Single or doublehead machines.

1817 - 18th Ave.

Rockford, IL 61104

815-398-1010 FAX 815-398-1047

DEALERS WELCOME!

REDIN CORPORATION

CIRCLE A-13 ON READER REPLY CARD

This sub-train expands the gear train to the form shown in Fig. 11(b), gears 7 and 8 being the same size.

Speed reduction ratio (+2.5) is generated by

$$\left(\frac{-N_2}{N_4} \right) e_c = \frac{\frac{1}{2.5} - \frac{1}{10}}{1 - \frac{1}{10}} = \frac{3}{9} = \left(-\frac{1}{3} \right) \left(-\frac{1}{1} \right)$$

where

$$e_c = -\frac{1}{1} = -\frac{N_{11}}{N_{12}}$$

gear 12 being an external gear. Since

$$N_2 + N_4 = 160 = N_{11} + N_{12} = d(1+1)$$

$$d = 80, \text{ and}$$

$$N_{11} = N_{12} = 80$$

The final form of the gear train is shown in Fig. 11(c). The gear train may drive three units with the three speeds it generates, or only one unit may be driven by simply using an external clutch coupling or shaft splines shifted axially as shown in Fig. 11(c).

It should be noted that the second speed reduction (-4) could also be generated, forming a gear train that uses $n_7 = n_2$ and the planet speed (1/10) of the first reduction as the split inputs, where the planet arm drives the last gear.

Thus,

$$e_2 = \frac{\frac{1}{10} + \frac{1}{4}}{1 + \frac{1}{4}} = \frac{7}{25} = \left(-\frac{2}{5} \right) \left(-\frac{7}{10} \right) = \left(-\frac{34}{85} \right) \left(-\frac{49}{70} \right) = \left(-\frac{N_7}{N_8} \right) \left(-\frac{N_9}{N_{10}} \right)$$

yielding a second compound planetary gear train with $N_7 = 34$, $N_8 = 85$, $N_9 = 49$ and $N_{10} = 70$, which satisfy $(r_7 + r_8) = (r_9 + r_{10})$ with $P_{79} = P_{9,10}$. In this case the planet arm of the first unit must drive gear 10, requiring a more complex shaft arrangement.

Speed reduction 2.5 may be generated similarly using split inputs $n_{11} = n_2$ and $n_L = 1/10$. Thus,

$$e_3 = \frac{\frac{1}{10} - \frac{4}{10}}{1 - \frac{4}{10}} = -\frac{1}{2} = -\frac{40}{80} = -\frac{N_{11}}{N_{13}}$$

yielding a simple planetary gear train as the third unit with $N_{11} = 40$, $N_{12} = 20$, $N_{13} = N_{13} = 80$. The new solution has three planetary gear train units with three planet arms and more complex shaft connections. It may be a much larger and more costly large drive system compared to the solution in Fig. 11c.

As noted, Form 1 synthesis is a *stepwise synthesis* for simultaneous speed reduction generation.

Example for Form 2 Synthesis

In this case three gear trains in series will be formed with different shaft connections for each speed reduction generated. Since there are three speeds to be generated, three products of speed ratios are formed using the forms of speed reductions in Equations 42-44. One can form 729 combinations of three products of S_{R1} , S_{R2} and S_{R3} , considering each unit causing speed reduction. Noting that the inverse of a speed reduction is an overdrive condition, with the possibility of two overdrive units out of three units, 243 other combinations are possible.

Let us generate speed ratios 10, -4 and 2. Choose the following products:

$$\left(\frac{e_1 - 1}{e_1} \right) \left(\frac{e_2 - 1}{e_2} \right) \left(\frac{1}{e_3} \right) = 10 \quad (a)$$

$$\left(\frac{e_1 - 1}{e_1} \right) \left(\frac{1}{e_2} \right) \left(\frac{1}{e_3} \right) = -4 \quad (b)$$

$$\frac{(e_1 - 1)}{e_1} \left(\frac{1}{e_2} \right) (1 - e_3) = 2 \quad (c)$$

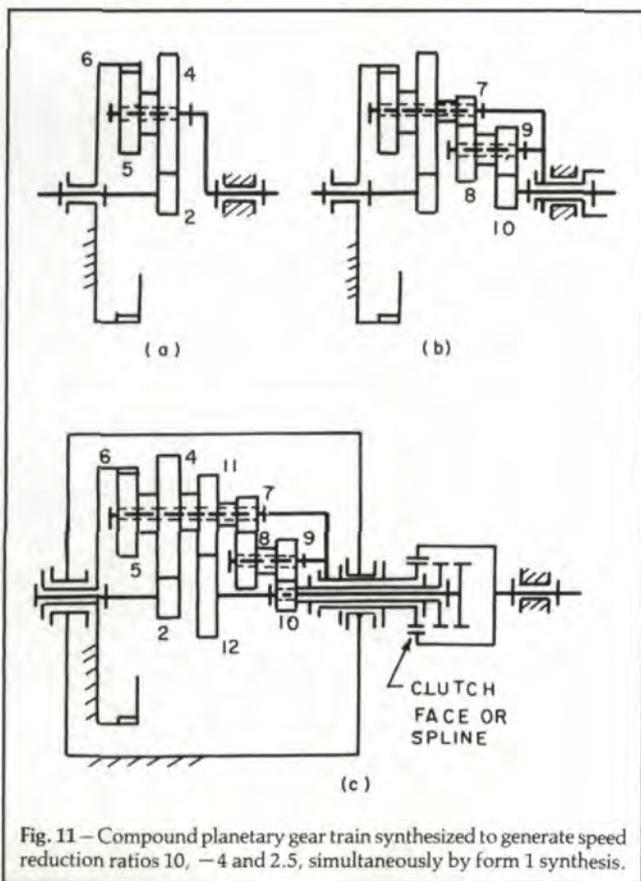


Fig. 11 - Compound planetary gear train synthesized to generate speed reduction ratios 10, -4 and 2.5, simultaneously by form 1 synthesis.

See us at Booth #5163.
IMTS 90
 The world of manufacturing technology
 September 5-13, 1990 • Chicago, Illinois, USA

CIMA KANZAKI

Your Source for World Class Gear Shaving

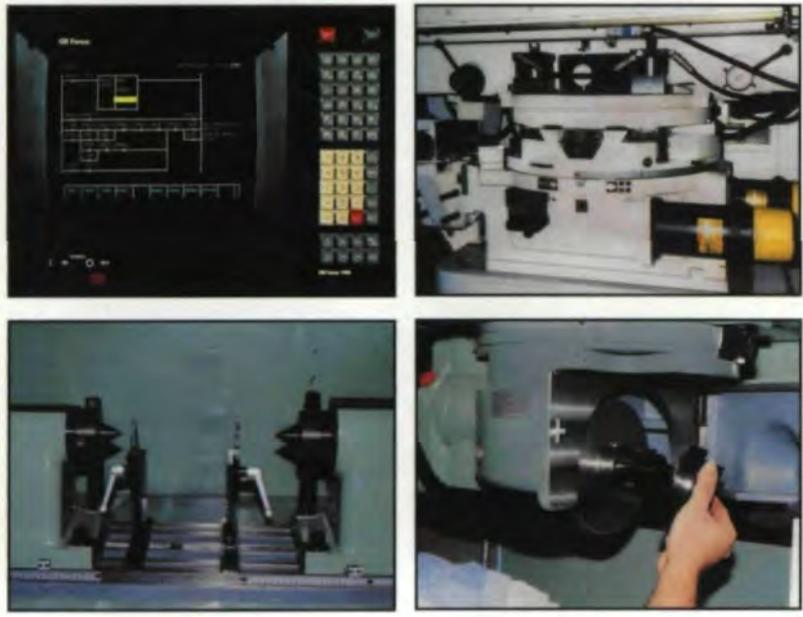


GSF 400 CNC 6

When you order a CIMA Kanzaki Gear Shaver, you get World Class features... and the heavy-duty construction you require for rigorous gear-making conditions. There's no need to compromise accuracy either... as the CIMA Kanzaki gear shaver provides repeatable quality levels of $\pm .00004$ " on workpieces to 18" O.D.

CIMA Kanzaki World Class features include state-of-the-art CNC controls from FANUC, General Numeric or your favored source. The GSF-400 CNC-6 features 6-axis capability, quick cutter change, heavy duty tailstocks and patented table design, complementing standard features designed to do one thing... deliver years of trouble-free operation. CIMA Kanzaki will build a gear finishing system to meet your exact needs. Need a semi- or fully-automatic tool changer or automatic load/unload system for in-line operation? Our application engineering staff in Richmond (VA) specializes in custom designs and their installation.

World Class Features...The Big Difference



Great American gearmakers deserve World Class Finishing equipment that **REDUCES CYCLE TIMES, IMPROVES QUALITY and INCREASES SHOP PROFITABILITY.**

Ask our sales representative for further details or contact:

CIMA USA, Division of GDPM, Inc.
 501 Southlake Blvd.
 Richmond, VA 23236
 Phone: (804) 794-9764
 FAX: (804) 794-6187
 TELEX: 6844252



CIMA USA

Global Technology with a U.S. Base

where the last gear of unit 1 is fixed, its planet arm drives the first gear of unit 2, whose planet arm drives the first gear of unit 3, whose arm is fixed, and its last gear generates $S_{R1} = 10$ in (a). Unit 1 does not change its fixed shaft condition. In (b) and (c) the arm shaft of unit 1 drives the sun gear of unit 2, whose last gear drives the first gear of unit 3 in (b); it drives the last gear of unit 3 in (c), the first gear being fixed. Several clutches and shaft coupling units are needed to form the shaft arrangements in this case. Solving (a), (b) and (c) simultaneously one finds

$$e_2 = -1.5$$

and

$$e_3^2 - e_3 - 0.5 = 0$$

or the two values of e_3 as

$$e_{31} = 1.3661$$

$$e_{32} = -0.3661$$

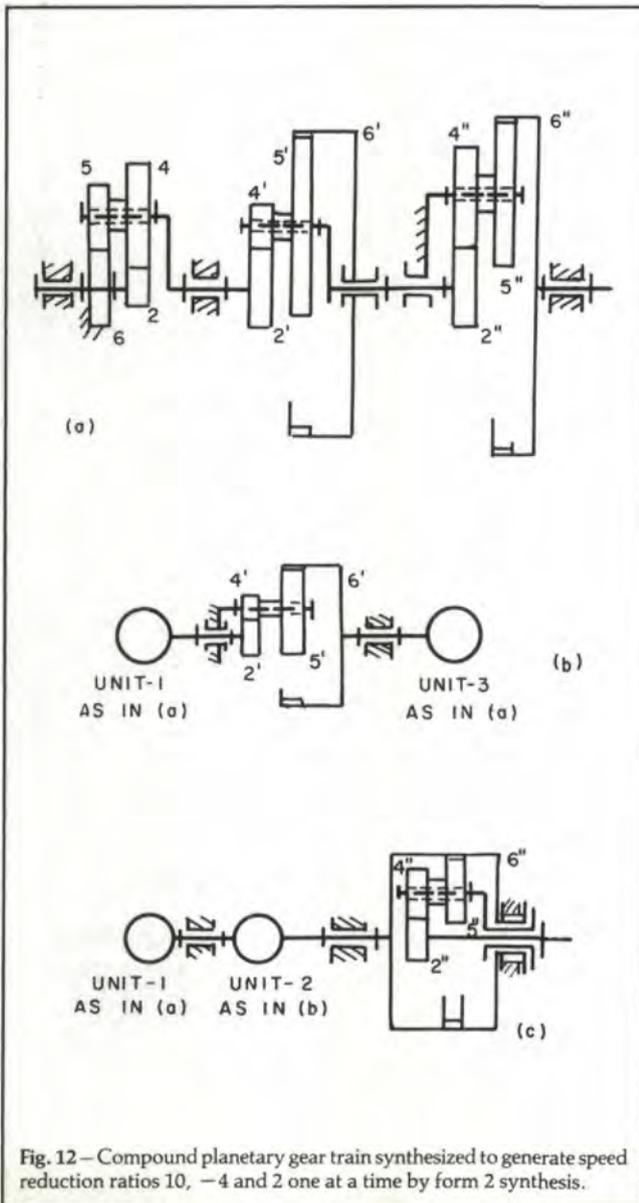


Fig. 12—Compound planetary gear train synthesized to generate speed reduction ratios 10, -4 and 2 one at a time by form 2 synthesis.

Then,

$$e_{1i} = \frac{1}{1 + 4 e_2 e_{3i}}, \quad i=1,2$$

leading to two solution gear trains with

$$e_{11} = -0.1390, \text{ and } e_{12} = 0.3129$$

The number of gears, their tooth numbers and the value of q for each solution unit is determined independently of the others. Thus, for unit 2

$$e_2 = -\frac{N'_2 N'_5}{N'_4 N'_6} = -1.5 = \left(\frac{-3}{1}\right) \left(\frac{1.5}{3}\right)$$

and N'_6 must be an internal gear. Satisfying

$N'_2 + N'_4 = N'_6 - N'_5$, $a(3+1) = b(3-1.5)$,
 $a=3$, $b=8$; $N'_2 = 54$, $N'_4 = 18$, $N'_5 = 72$, $N'_6 = 144$ for unit 2. Values e_1 and e_3 are satisfied approximately.

Consider the second solution. For unit 1

$$e_{12} = 0.3129 = \frac{1}{3.1959} = \left(-\frac{1}{2}\right) \left(\frac{1}{1.6}\right) = \frac{1}{3.2}$$

$$\left(\frac{-N_2}{N_4}\right) \left(\frac{-N_5}{N_6}\right)$$

in which gear 6 is an external gear, satisfying

$$a(N_2 + N_4) = b(N_5 + N_6)$$

$$N_2 = 26, N_4 = 52, N_5 = 30, N_6 = 48.$$

For unit 3

$$e_{32} = -0.3661 \cong -\frac{1}{2.7315} \cong \left(\frac{-1}{1.3}\right) \left(\frac{1}{2.1}\right) =$$

$$\left(\frac{-N''_2}{N''_4}\right) \left(\frac{N''_5}{N''_6}\right)$$

in which gear 6 is an internal gear, and satisfying

$$a(N''_2 + N''_4) = b(N''_6 - N''_5)$$

$$N''_2 = 110, N''_4 = 143, N''_5 = 230, N''_6 = 483.$$

The three gear trains with their shaft connections to generate the three speed ratios are shown in Figs 12(a), (b) and (c). With the approximated e_3 and e_1 , the system generates:

$$10.01, -4.004, \text{ and } 2.039.$$

Observing the sizes of gears in unit 3, and noting the existence of three planet arms and several clutch and coupling

units and shifting or actuating mechanisms within the system in form 2 synthesis, synthesis as in form 1 is more economical and advantageous. Other products of e_{32} should be searched. A smaller drive with a larger speed error may be formed. For example, try

$$e_{32} \cong \left(-\frac{1}{1.7}\right)\left(\frac{1}{1.6}\right) = -0.36765 = \left(-\frac{10}{17}\right)\left(\frac{10}{16}\right) \\ = \left(-\frac{N_2''}{N_4''}\right)\left(\frac{N_5''}{N_6''}\right)$$

and $N_2'' = 30$, $N_4'' = 51$, $N_5'' = 135$, $N_6'' = 216$, generating the speed reduction ratios as 9.9547, -3.98188, 2.00216.

Readers should form solution gear trains for e_{31} and e_{11} .

Differential Gear Trains

When a planetary gear train is driven with two inputs to generate the third speed (split input drive), it is in general called a "differential gear train". Some differential gear trains were already synthesized in the foregoing compound planetary gear train examples. Here, we will see how a differential with two inputs can be synthesized to generate the sum of two variables as a mechanical computer. Rewrite Equation 17 for the planet arm as

$$n_a = A n_L - B n_F \quad (45)$$

where

$$A = \frac{1}{1-e}, \quad B = \frac{e}{1-e}$$

A planetary gear train can be synthesized to generate a function of the form

$$z = ax + by \quad (46)$$

by the planet arm, x and y being the input functions $n_L \equiv x$, $n_F \equiv y$ and $n_a = z$. Consider the following example.

Example: Let us generate

$$z = 2x - 4y \quad (47)$$

Comparing Equation 45 and 47 we have

$$A = \frac{1}{1-e} = 2, \quad B = \frac{e}{1-e} = 4$$

Using A or B , find e being sure that the other coefficient, B or A , is larger than its required value so that an overdrive unit is not formed. In this case, we use B to find e as

$$e = \frac{4}{5}$$

which yields

$$A = 5$$

Therefore, the input to the last gear of the train must be supplied as

$$x/m$$

where

$$m = \frac{A_{\text{formed}}}{A_{\text{desired}}} = \frac{5}{2} = 2.5 \quad (48)$$

$e = 4/5$ is generated by four external gears to supply y . Thus,

$$e = \frac{4}{5} = \left(-\frac{N_2}{N_4}\right)\left(-\frac{N_5}{N_6}\right) = \left(-\frac{36}{36}\right)\left(-\frac{32}{40}\right)$$

and $N_2 = 36$, $N_4 = 36$, $N_5 = 32$, $N_6 = 40$. The desired differential gear train is shown in Fig. 13(a), where $N_7 = 20$, $N_8 = 30$, $N_9 = 50$.

Equation 17 may also be written to generate z by the shaft of the last gear, supplying x on the arm shaft, y on the first gear shaft, as

$$n_L = D n_a + E n_2 \quad (49)$$

with $D = (1-e)$, $E = e$. In that case, to generate the function

THE GEAR DEBURRING SYSTEM



★ FLEXIBLE ★ MIL SPEC QUALITY
★ USER FRIENDLY ★ VERY FAST

JAMES ENGINEERING

11707 McBean Drive, El Monte, California

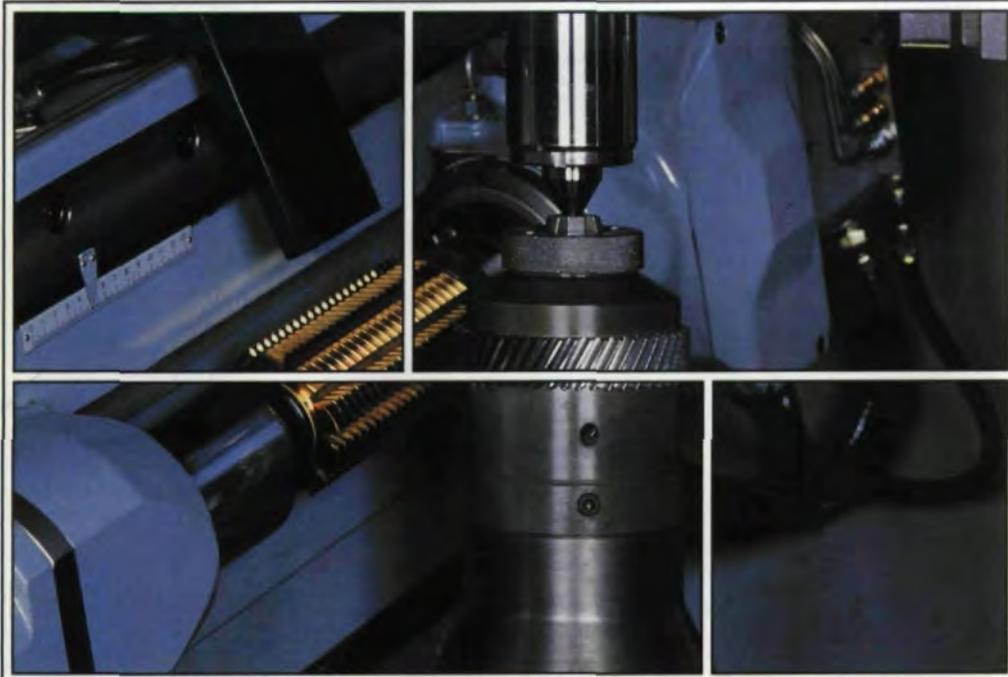
818 442-2898 • FAX 818 442-0374

See us at IMTS '90, Booth #8228.

CIRCLE A-15 ON READER REPLY CARD

SYNCHRONIZED EXCELLENCE

Kashifuji



The Kashifuji KN 150 CNC Gear Hobber. High accuracy assured through rigid construction. Full 5 axis CNC controls plus hob rotation. Excels over a wide range of high speed hobbing applications. Small foot print.

TIMELY CHANGES

Easy and fast changeover accommodates varying production lots. Handles gear sizes from 3/4" to 40". High production capacity with fast auto loaders.

Kashifuji KN-150 CNC Gear Hobbers.
Contact Kashifuji Product Manager,
Midwest Branch,
Cosa Corporation
1680 S. Livernois,
Rochester, MI 48063
Tel. 313-652-7404
Fax. 313-652-1450.



COSA CORPORATION



MACHINE TOOL DIVISION

17 Philips Parkway, Montvale, NJ 07645 Tel. 201-391-0700 Fax. 201-391-4261

CIRCLE A-16 ON READER REPLY CARD

in Equation 47, $e = -4$, $n_a = x/2.5$ and $n_F = y$. A bevel gear train generating z for this case is shown in Fig. 13 (b), where

$$e = -4 = -\frac{N_2 N_5}{N_4 N_6} = -\left(\frac{40}{80}\right)\left(\frac{20}{40}\right)$$

The reader should form another gear train using $e = -1$ supplying $n_a = x$, $n_F = 4y$ as overdrive.

Vehicle Rear End Differentials

The objective in designing a vehicle rear end differential planetary gear train is to generate $e = -1$, so that when the vehicle is making a turn without the planet arm rotating, the outer wheel goes one unit rotation forward as the inner wheel goes one unit rotation backward. Fig. 14 shows a commonly used bevel gear rear end differential, where gear 7 is driven by the universal shaft to supply the input by the planet arm. $N_2 = N_5$; N_4 is of any suitable number and $e = -1$. When gear 2 is held stationary, $n_2 = 0$, and gear 5 is lifted for balancing, one observes

$$-1 = \frac{n_5 - n_a}{-n_a}, n_5 = 2n_a$$

Vehicle rear-end differentials can also be formed using spur gears, provided $e = -1$ is maintained. Figures 15(a) and (b) show two such rear-end differentials, where planet gears

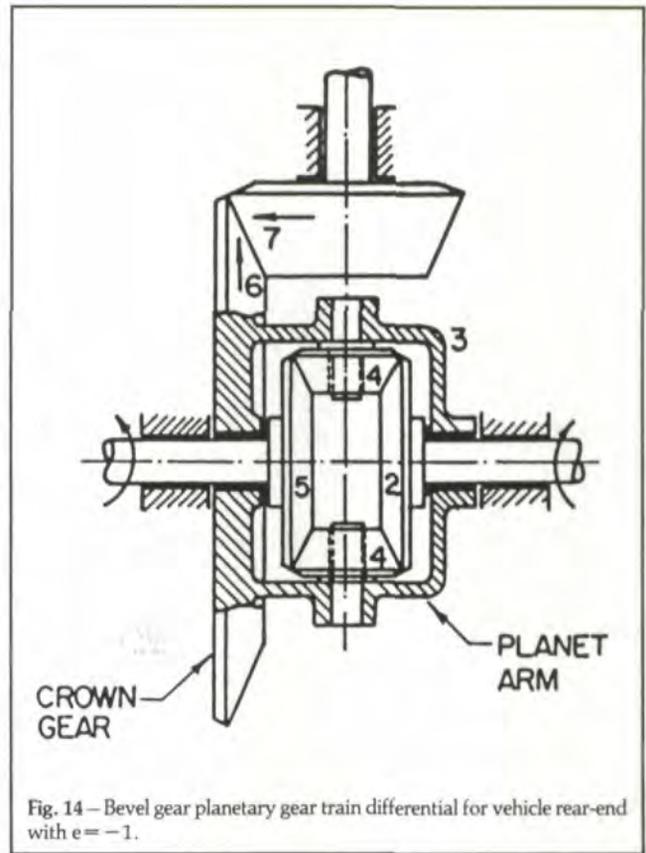


Fig. 14 - Bevel gear planetary gear train differential for vehicle rear-end with $e = -1$.

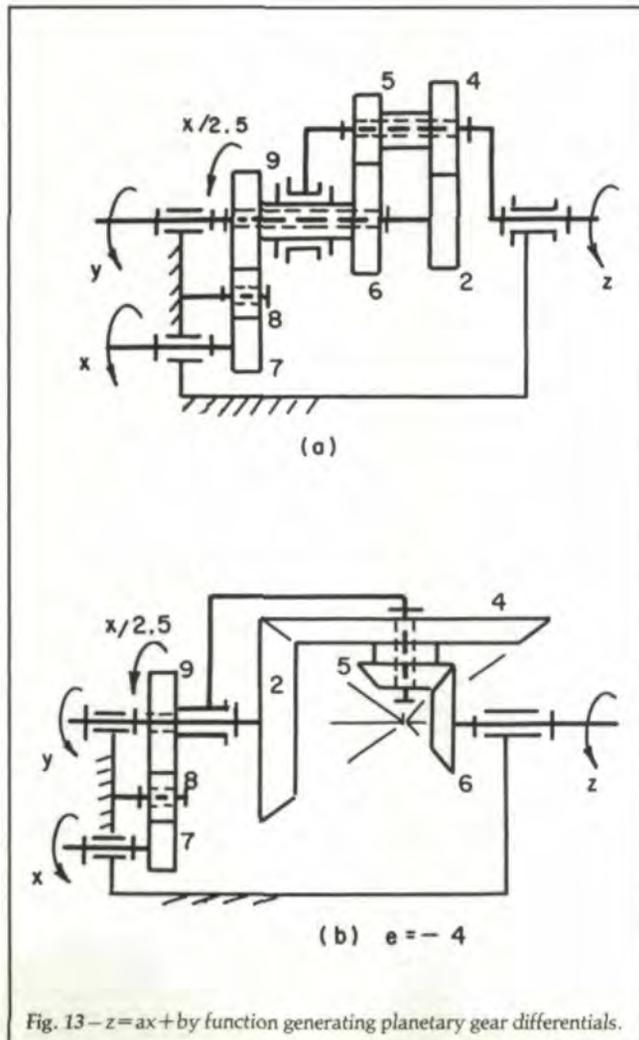


Fig. 13 - $z = ax + by$ function generating planetary gear differentials.

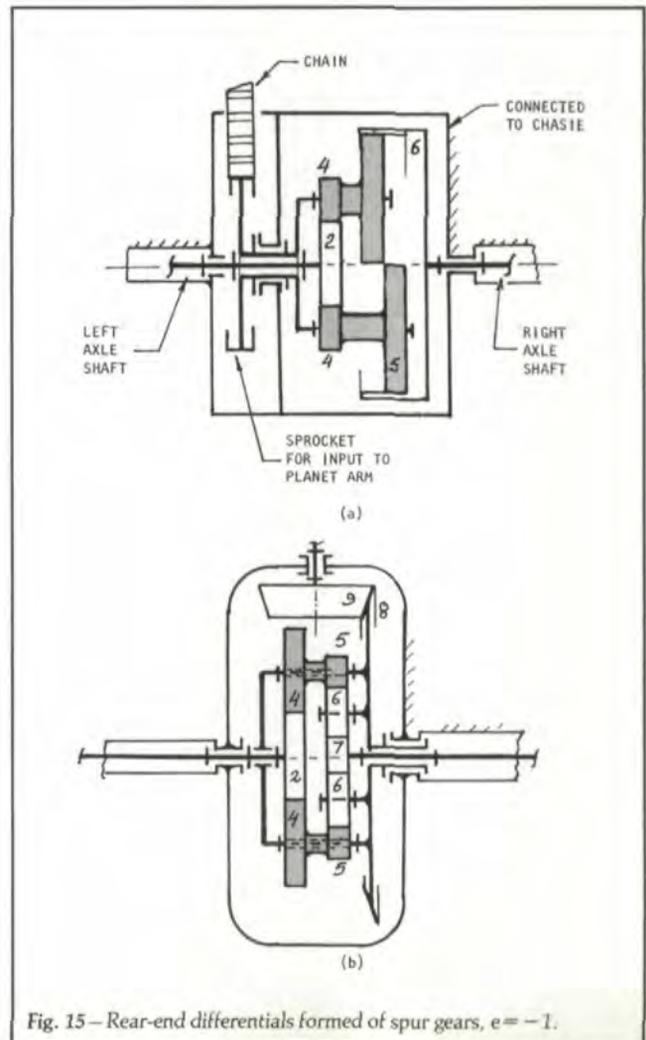


Fig. 15 - Rear-end differentials formed of spur gears, $e = -1$.

are symmetrically mounted for balancing. In (a) chain drives the planet arm; in (b) bevel gears do the same. In (a)

$$-1 = \left(\frac{-N_2}{N_4} \right) \left(\frac{N_5}{N_6} \right) = \left(\frac{-40}{20} \right) \left(\frac{60}{120} \right)$$

In (b)

$$-1 = \left(\frac{-N_2}{N_4} \right) \left(\frac{-N_5}{N_6} \right) \left(\frac{-N_7}{N_8} \right) = \left(\frac{-60}{60} \right) \left(\frac{-20}{40} \right) \left(\frac{-40}{20} \right)$$

Differentials for fractional values of e , such as $e = -0.8$ for a race car to rotate the outer wheel less than the inner wheel, are used in order to generate stabilizing traction torque that tends to retain the vehicle on the straight line. In that

case, $e = -0.8$ should be satisfied instead of -1 as done above.

Automatic Transmissions

An automatic vehicle transmission in general has two or more simple planetary gear trains whose shafts can be coupled or held stationary to form different arrangements. Input is supplied through one or two fluid couplings (torque converters). Fig. 16 shows a Hydramatic 4L60 (THM 700-R4) automatic transmission by General Motors that generates two forward and one reverse speed reductions. The skeleton of a five-speed automatic transmission with three simple planetary gear trains driven with two torque converters is shown in Fig. 17. It has three clutches at G, H, and P; four sprag overrunning clutches at J, I, N and O.

Observe its operation:

Neutral position. Small converter B-C is empty, gear 3 is held stationary and clutches G and H are open. The planet

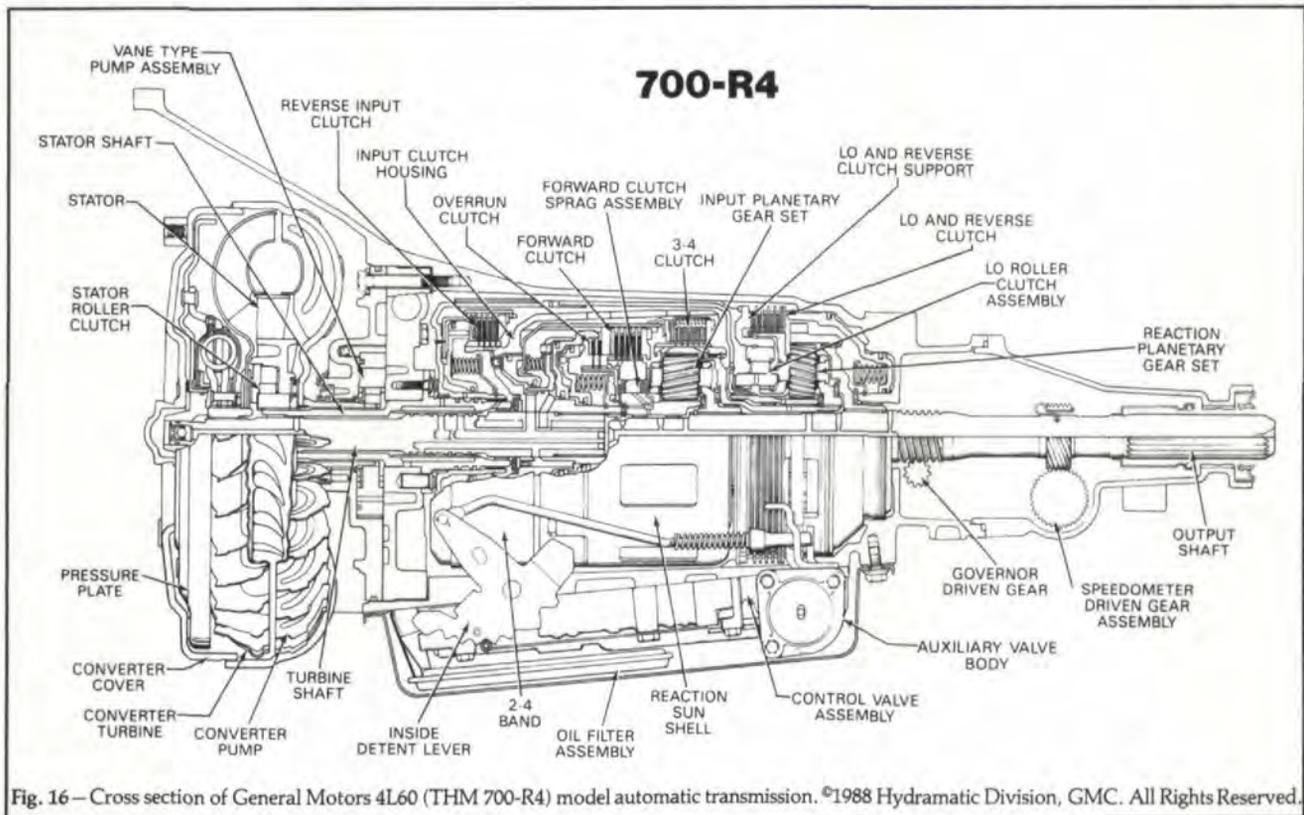


Fig. 16 - Cross section of General Motors 4L60 (THM 700-R4) model automatic transmission. ©1988 Hydramatic Division, GMC. All Rights Reserved.

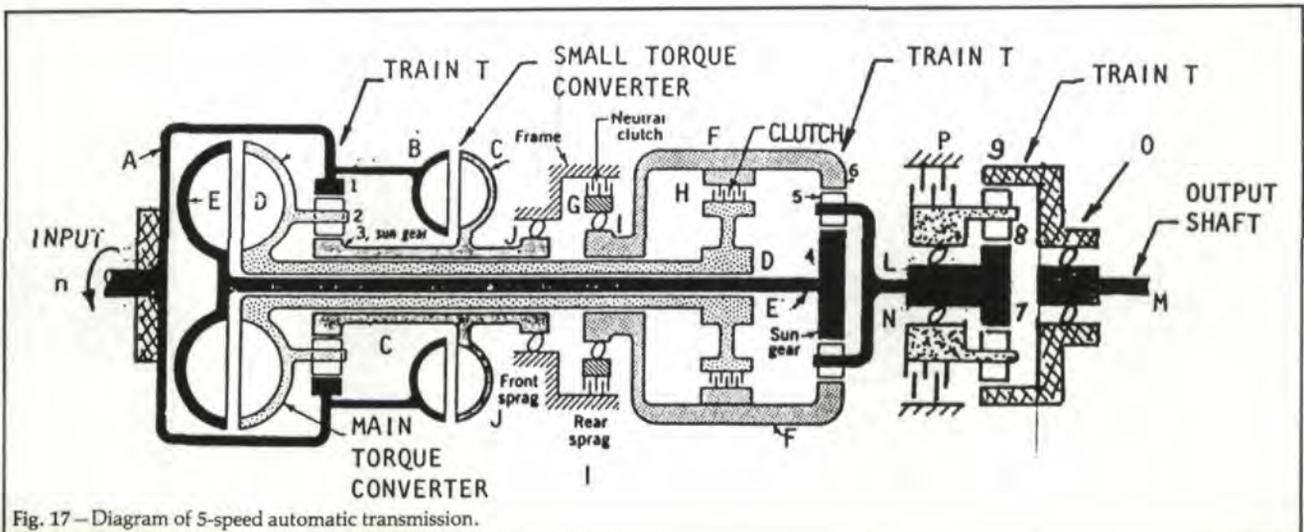


Fig. 17 - Diagram of 5-speed automatic transmission.

arm D rotates freely, $n_4 = n_D = n_6$, and shaft L idles, and $n_L = n_6$. P and O are open, N is engaged, $n_7 = n_9$ and $n_M = 0$.

Low-Low Forward Reduction. B-C is empty, D rotates E and gear 4. Clutches G, J, I retain $n_3 = n_6 = 0$. P is open; N, O are engaged. From train 1

$$e_1 = -\frac{N_1}{N_3} = \frac{0 - n_D}{n_1 - n_D}, n_D = \frac{e_1}{e_1 - 1} n_1 \quad (a)$$

for $e_1 = -1.3$ and $n_D = 0.5833n_1$.

In train 2, $n_4 = n_D$, and

$$e_2 = -\frac{N_4}{N_6} = \frac{0 - n_L}{n_4 - n_L} \quad (b)$$

with $e_2 = -0.7143$ and $n_M = n_L = 0.243n_1$.

Second Forward Reduction. H, E are open, G, I are engaged and $n_6 = 0$. Small converter B-C is full driving gear 3, $n_1 = n_3 = n_4 = n_D$; P is open, N, O are engaged, and from (b) above

$$n_M = n_L = \frac{e_2}{e_2 - 1} n_1 = 0.4167n_1$$

Third Forward Reduction. J is engaged and $n_3 = 0$. G and H are engaged, I is open, B-C is empty, P is open, N, O are engaged, and $n_6 = n_D = n_E$. Then from (a) above

$$n_M = n_L = n_D = 0.5833n_1$$

Fourth Forward Speed. In this case the entire gear train rotates as one body at the engine speed n_1 . I, J, P are open; G, H, N, O are engaged, B-C is full and $n_M = n_L = n_1$.

Reverse Reduction. B-C is full and the system is engaged as for the fourth forward speed, except that P and O are engaged and N is open. From the train value of the third gear train with $n_3 = 0$,

$$e_3 = -\frac{N_7}{N_9} = \frac{n_M}{n_1}$$

with

$$e_3 = -1/3, n_M = -n_1/3$$

Conclusions

Offered in the foregoing with illustrative examples are efficient methods of analysis and synthesis of compound planetary gear trains and planetary differentials. The methods use the train values of component planetary gear trains, and lead to very simple design processes. As expected, by introducing worm-gear sets in a compound planetary gear train, very large speed reductions can be generated with fewer number of gears in the train.

References

1. PLANETARY GEAR REDUCERS, Crichton Manufacturing Co., Johnstown, PA.
2. MACHINE DESIGN, Vol. 54, No. 18, August 12, 1982, p.55.

Don't just keep up with your competition...get ahead!



Toronto, Ontario, Canada - October 29-31, 1990 - R.S.V.P. to AGMA, (703) 684-0211

CIRCLE A-17 ON READER REPLY CARD

July/August 1990 35