Vectors in Gear Design

Clifford M. Denny

Terms

- D_j journal diameter
- F force vector
- M module
- m tooth loading force displacement
- R moment arm vector
- r_f journal friction radius
- ε efficiency
- μ coefficient of friction
- ϕ_n normal profile angle
- ϕ_o operating pressure angle
- ω angular velocity

Appendix

This article presupposes a knowledge of vector equations. An appendix will be posted at *www.geartechnology.com/vectors.htm*. This appendix contains a more basic introduction to the equations used in this article with explanations of their origins and supporting figures.

Introduction

Friction weighs heavily on loads that the supporting journals of gear trains must withstand. Not only does mesh friction, especially in worm gear drives, affect journal loading, but also the friction within the journal reflects back on the loads required of the mesh itself.

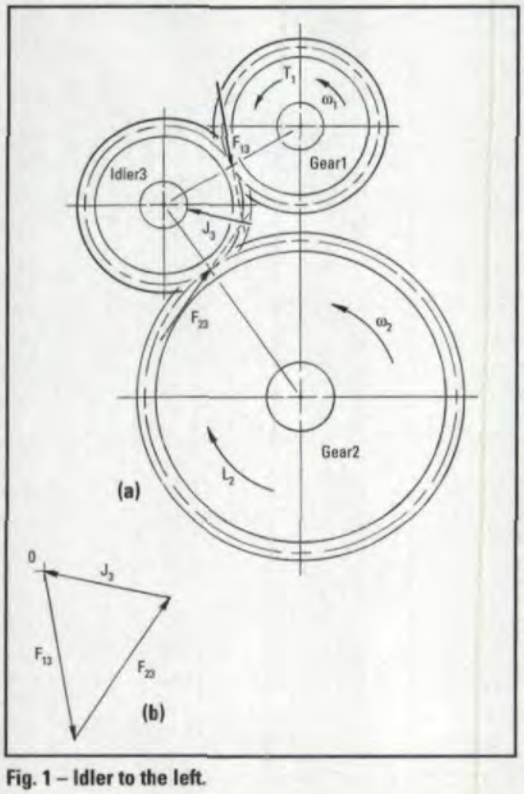

Fig. 1 – Idler to the left.

Several simple problems using principles of elementary mechanics that illustrate these concerns are provided. These examples present the designer with analytic and graphical methods that focus on problems that could arise in a design. The designer can then work around and eliminate such problems before they are magnified many times in reality.

Two- and three-dimensional problems are included. These involve spur, helical, and worm gears.

Two-Dimensional Problems

These problems can be solved with vectors alone in the same way that one may draw using a CAD program or drawing board. Most simply, this can be illustrated in the frictionless case:

Problem 1

Idler placement[2]. Depending on the direction of power flow, Figure 1 shows Gear 1 driving Gear 2 through Idler 3. Gear 1 rotates in the positive direction (anti-clockwise). The torque T_1 that drives Gear 1 is also positive (anti-clockwise). The equilibrating load torque L_2 restraining Gear 2 will be negative (clockwise).

Tooth loading vectors will lie on the individual lines of action of the specific meshes involved. Considering the idler in Figure 1a, its journal reaction force passes through the journal's center and the point of intersection of the two tooth-loading forces acting on this idler. The magnitudes of tooth loading forces are known from the gear geometries and applied torque. As the idler is in equilibrium, the three forces acting upon it must add vectorially to zero, as shown in the force vector diagram in Figure 1b. J3's magnitude is thereby found. J3 is the journal load on Idler 3; F13 is the tooth load of Gear 1 on Idler 3; F23 is the tooth load of Gear 2 on Idler 3.

Figure 2a shows Idler 4 in mesh on the opposite side of the gear train. Following the same procedures just discussed, the direction of Idler 4's journal load P4 is found. Vector addition determines its magnitude shown in Figure 2b.

For the conditions of this example, it is apparent that the location of Idler 3 is decidedly superior.

Problem 2

Idler with Friction. Figure 3a is a 2:1 reduction drive through an idler. The driver and idler

Clifford M. Denny

is an engineering consultant involved in the design of plastic gearing. He is a graduate engineer with a BSME degree from Georgia Institute of Technology, Atlanta, GA, and an MSME degree from Purdue University, W. Lafayette, IN. His experience covers 30 years in the design and development of office products including typewriters and ink-jet, wire-matrix and laser printers. This includes 29 years with IBM and 5 years with Lexmark Corporation, where he first became involved in the design of plastic gearing for laser printers. More recently, he has taught courses in plastic gear design and has presented papers on gear related subjects at various technical events. He is a consultant member of the AGMA and chairman of the AGMA Plastics Gearing Committee.

have 20 teeth each. The driven gear has 40. These are module 1.0, 20° pressure angle gears. They operate on standard centers. The idler's journal is 6 mm in diameter. Friction in both the mesh and the pivot are considered, and the idler's own throughput efficiency is found. The coefficient of friction μ is 0.4. The displacements m, due to mesh friction, of the resultant driving forces are[2]:

$$m = [\pi M \cos(\phi_n) / (2 \cos(\phi_o))] \mu$$

$$m = [\pi M / 2] \mu = [\pi \cdot 1.0 / 2] \cdot 0.4 = 0.63 \text{ mm}$$

The displacement m is along the line of centers <u>into</u> the driven gear from the pitch point. The journal friction radius r_f is[1]:

$$r_f = (D_j / 2) \sin[\operatorname{atan}\mu]$$

$$r_f = 3 \cdot \sin[\operatorname{atan} 0.4] = 1.1 \text{ mm}$$

The resultant vector of the mesh forces lies tangent to the friction circle on the side that opposes motion.

Here, the force vector of Gear 1 on Idler 3 is known in both magnitude (*applied torque is known—here it's 1.0*) and direction (*gear train geometry is known*). As friction is included, only the placement and direction of the restraining force of Gear 2 on Idler 3 is known here. Idler 3's rotation determines the placement of the journal loading vector; the intersection of the two tooth loading vectors determines its direction.

Of these three vectors, only one is known in both magnitude and direction; the other two are known only in their direction. This is enough to find their magnitudes vectorially as shown in Figure 3b.

The journal force's direction depends on the friction considered. Direction 'a' is for no friction, 'b' is for journal friction alone, 'c' is for mesh friction alone, and 'd' is for both. For each journal force direction, a different idler's output force magnitude is found.

Friction then reduces the magnitude of the idler's output force below that of the input force driving it. The idler's throughput efficiency is 100 times the ratio of these two forces.

The efficiency $\varepsilon = (F_{23} / F_{13}) \cdot 100$.

The table below shows the effect of friction in its various combinations on the throughput efficiency of this idler. μ_m is the coefficient of sliding friction in the tooth mesh; μ_j is that of the journal.

Case	μ_m	μ_j	ε
a	0.0	0.0	100
b	0.0	0.4	91
c	0.4	0.0	87
d	0.4	0.4	79

Problem 3

Speed increasing and decreasing drives. In figure 4, the small gear has 18 teeth; the larger

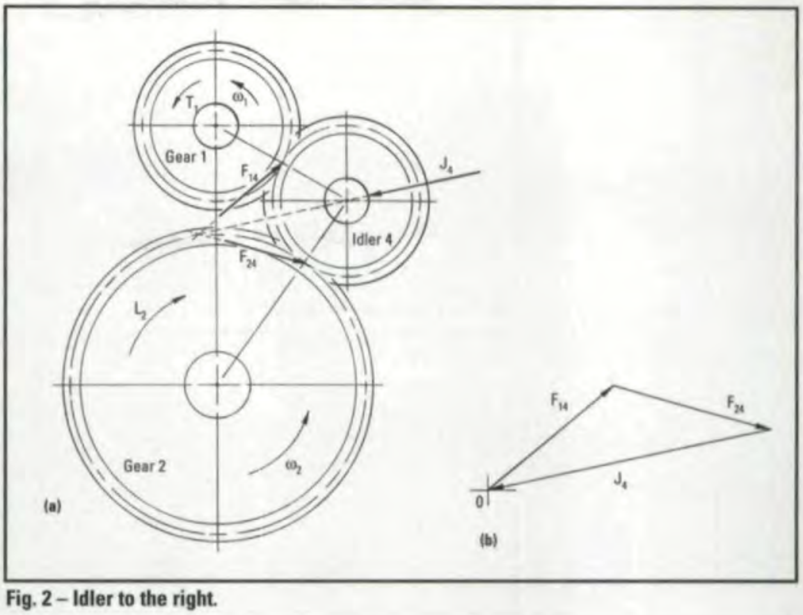

Fig. 2 – Idler to the right.

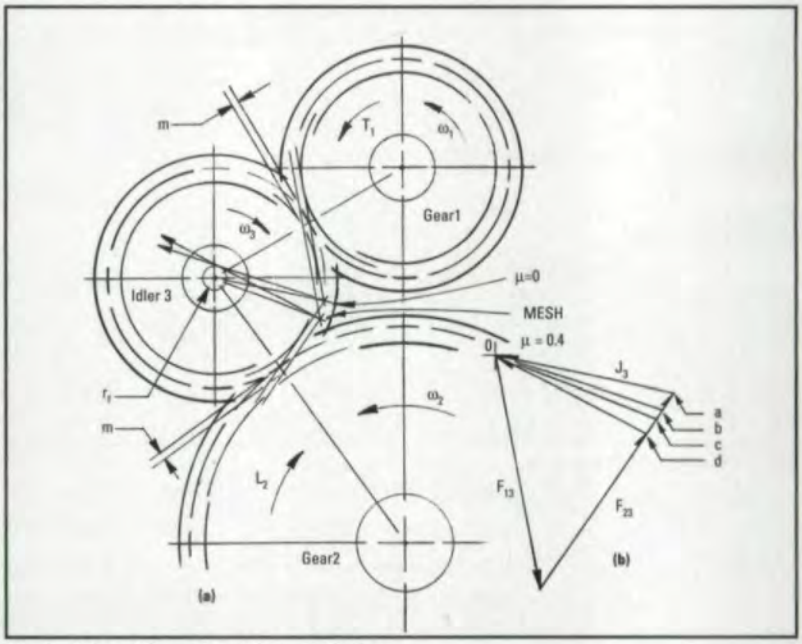

Fig. 3 – Idler with mesh and journal friction.

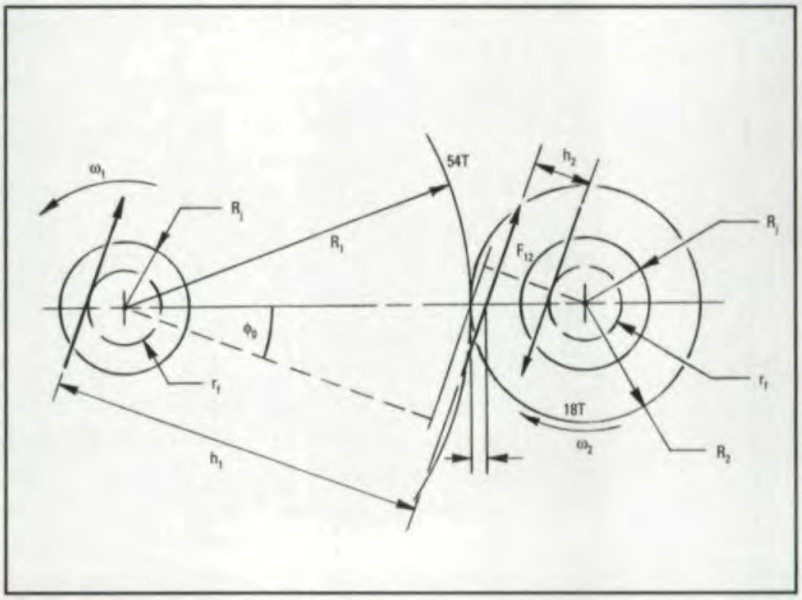

Fig. 4 – Speed increasing drive.

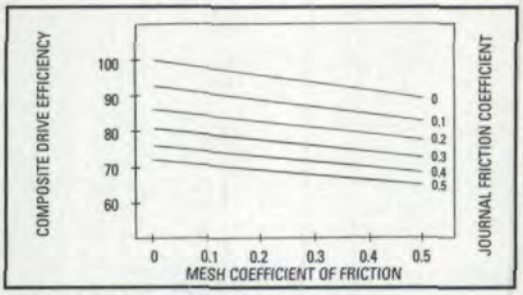

Fig. 5 – Speed decreasing drive.

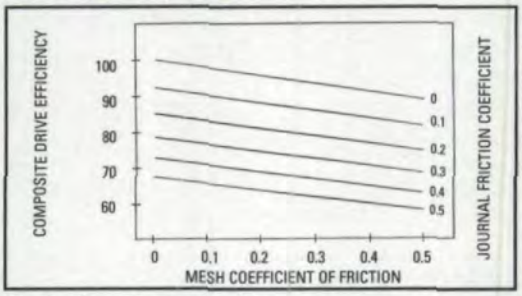

Fig. 6 – Speed increasing drive.

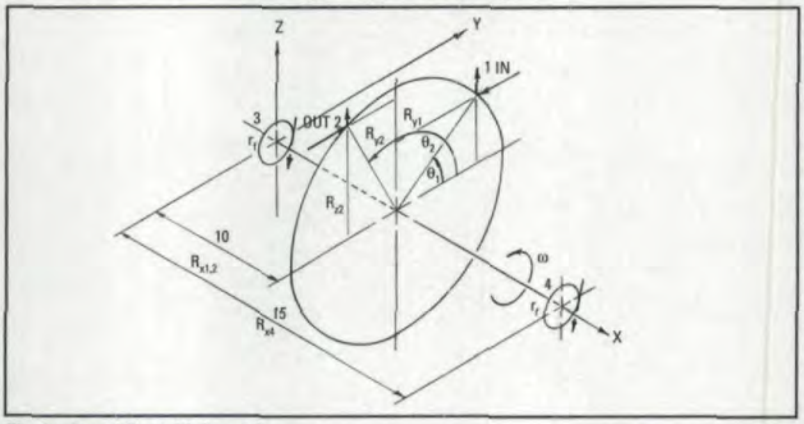

Fig. 7 – Spur idler with journals.

one has 54. These are 1.0 module, 20° pressure angle gears operating on standard centers. To simplify matters for illustrative purposes, both the motivating source and the load are pure torque. Therefore all vector directions will be parallel to the line of action in the mesh.

Both journal diameters are 10 mm. For Gear 1 driving Gear 2,

$$h_1 = (R_1 + m)\cos\phi_o + r_{f1}$$
$$h_2 = (R_2 - m)\cos\phi_o - r_{f2}$$

The efficiency equation becomes:

$$\varepsilon = [(h_2 R_1)/(h_1 R_2)] \cdot 100.$$

For Gear 2 driving Gear 1,

$$h_1 = (R_1 - m)\cos\phi_o - r_{f1}$$
$$h_2 = (R_2 + m)\cos\phi_o + r_{f2}$$

The efficiency equation becomes:

$$\varepsilon = [(h_1 R_2)/(h_2 R_1)] \cdot 100.$$

Figures 5 and 6 show that the speed decreasing drive has greater efficiency than the speed increasing drive. Full ranges of friction coefficients are shown from 0 to 0.5. Journal friction here just happens to have a more severe effect on the efficiency than does the mesh friction. This is not true in all cases.

Three-Dimensional Problems

Equilibrium conditions are observed in three dimensions just as they were in two dimensions. Simply stated, the summation of moments is zero, and the summation of forces is zero. In solutions to these problems, all moments and all forces are resolved into their orthogonal 'x', 'y' and 'z' components and handled mathematically.

The appendix[4] shows the use of 3x3 determinants to find the vector components of moment or torque[3]. Also, the equations for the vector components of force are given. These all could have been used in the solution of the two-dimensional problems as well.

Problem 4

Spur Idler on a Shaft. Figure 7 shows an idler gear integral to a shaft supported by two bearings. The operating pitch circle radius is 30 mm. The operating pressure angle is 20°. The input pitch point p_1 is at 35° from the positive x-y plane. The output pitch point p_2 is at 115° from this x-y plane. The gear's pitch points are at x = 10 mm. Bearing B_3 is at x = 0 mm; bearing B_4 is at x = 15 mm. The input torque is + 400 Nmm.

Spur gear sets have no x components of force here. Using the equations set forth in the appendix,

$$F_{y1} = -\{400/[(30-m)\cos 20°]\}\sin(35°+20°) = -11.62$$
$$F_{z1} = \{400/[(30-m)\cos 20°]\}\cos(35°+20°) = +8.14$$

The sign of m here is negative, as this is the point of power input. The sign of m would be positive at points of power output. As mesh friction is ignored in the example, m = 0.

Input point:

$$R_{x1} = 10\text{mm}$$
$$R_{y1} = (30-m)\cos 35° = 24.57\text{mm}$$
$$R_{z1} = (30-m)\sin 35° = 17.21\text{mm}$$

Output point:

$$R_{x2} = 10\text{mm}$$
$$R_{y2} = (30+m)\cos 115° = -12.68\text{mm}$$
$$R_{z2} = (30+m)\sin 115° = 27.19\text{mm}$$

Using 3x3 determinants, the resultant torques from forces at the various points are:

$$\begin{vmatrix} i & j & k \\ 10.0 & 24.57 & 17.21 \\ 0.0 & -11.62 & 8.14 \end{vmatrix} = T_1$$

$$\begin{vmatrix} i & j & k \\ 10.0 & -12.68 & 27.19 \\ 0.0 & F_{y2} & F_{z2} \end{vmatrix} = T_2$$

$$\begin{vmatrix} i & j & k \\ 0.0 & 0.0 & 0.0 \\ 0.0 & F_{y3} & F_{z3} \end{vmatrix} = T_3$$

$$\begin{vmatrix} i & j & k \\ 15.0 & 0.0 & 0.0 \\ 0.0 & F_{y4} & F_{z4} \end{vmatrix} = T_4$$

Solve the determinants:

i	j	k
400	–81.38	–116.23
$-12.68F_{z2}-27.19F_{y2}$	$-10\,F_{z2}$	$10\,F_{y2}$
0	0	0
0	$-15\,F_{z4}$	$15\,F_{y4}$

Sum the components:

$$i:\quad 400 \quad -27.19\,F_{y2} \quad -12.68\,F_{z2} = 0$$
$$j:\quad -81.38 \quad -10\,F_{z2} \quad -15\,F_{z4} = 0$$
$$k:\quad -116.23 \quad +10\,F_{y2} \quad +15\,F_{y4} = 0$$

Also, F_{y2} and F_{z2} are related by virtue of their position at 115°. Furthermore, as this is the output point on the gear, these forces produce a net negative torque to counteract the driving positive 400-Nmm torque. Hence, the 20° pressure angle here takes on the negative sign.

$$F_{y2}\cos[115-20°] + F_{z2}\sin[115°-20°] = 0$$
$$-0.087\,F_{y2} + 0.996F_{z2} = 0$$

Furthermore, the summation of forces in the several directions are zero:

$$-11.62 + F_{y2} + F_{y3} + F_{y4} = 0$$
$$8.14 + F_{z2} + F_{z3} + F_{z4} = 0$$

The final simultaneous equation matrix becomes:

$$\begin{vmatrix} 27.19 & 12.68 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 15 \\ 10 & 0 & 0 & 0 & 15 & 0 \\ -0.087 & 0.996 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{vmatrix} \begin{vmatrix} F_{y2} \\ F_{z2} \\ F_{y3} \\ F_{z3} \\ F_{y4} \\ F_{Z4} \end{vmatrix} = \begin{vmatrix} 400 \\ -81.38 \\ 116.23 \\ 0 \\ 11.62 \\ -8.14 \end{vmatrix}$$

The resultant forces then become:

$F_{y2} = 14.14$ N $\quad F_{z2} = 1.23$ N

$F_{y3} = -0.84$ N $\quad F_{z3} = -3.13$ N $\quad F_3 = 3.24$ N

$F_{y4} = -1.67$ N $\quad F_{z4} = -6.25$ N $\quad F_4 = 6.47$ N

Problem 5

Spur Idler on a Shaft with Journal Friction Alone. If the journal diameters are 12 mm, and if the coefficient of friction is 0.4, the bearing friction radius is $r_f = 6.0 \sin(\text{atan } 0.4) = 2.22$ mm. Refer to Figure 7.

Now, there will be y and z components of the bearing moment arms to include. These are related to the bearing load components as $|R_y F_y| = |R_z F_z|$.

The frictional moment arm directions in the journals depend on the shaft rotational direction and directions of the journals' load components as follows:

$$\text{sign}(R_z) = \text{sign}(\omega) \cdot \text{sign}(F_x)$$
$$\text{sign}(R_y) = -\text{sign}(\omega) \cdot \text{sign}(F_z)$$

The resultant torques from forces at the various points now may be written:

$$\begin{vmatrix} i & j & k \\ 10.0 & 24.57 & 17.21 \\ 0.0 & -11.62 & 8.14 \end{vmatrix} = T_1$$

$$\begin{vmatrix} i & j & k \\ 10.0 & -12.68 & 27.19 \\ 0.0 & F_{y2} & F_{z2} \end{vmatrix} = T_2$$

$$\begin{vmatrix} i & j & k \\ 0.0 & R_{y3} & R_{z3} \\ 0.0 & F_{y3} & F_{z3} \end{vmatrix} = T_3$$

$$\begin{vmatrix} i & j & k \\ 15.0 & R_{y4} & R_{z4} \\ 0.0 & F_{y4} & F_{z4} \end{vmatrix} = T_4$$

Sum the component torques:

i	j	k
400	–81.38	–116.23
$-12.68F_{z2}-27.19F_{y2}$	$-10\,F_{z2}$	$10\,F_{y2}$
$R_{y3}F_{z3}-R_{z3}F_{y3}$	0	0
$R_{y4}F_{z4}-R_{z4}F_{y4}$	$-15\,F_{z4}$	$15\,F_{y4}$

The simultaneous equation matrix becomes:

$$\begin{vmatrix} 27.19 & 12.68 & R_{z3} & -R_{y3} & R_{z4} & -R_{y4} \\ 0 & 10 & 0 & 0 & 0 & 15 \\ 10 & 0 & 0 & 0 & 15 & 0 \\ -0.087 & 0.996 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{vmatrix} \begin{vmatrix} F_{y2} \\ F_{z2} \\ F_{y3} \\ F_{z3} \\ F_{y4} \\ F_{z4} \end{vmatrix} = \begin{vmatrix} 400 \\ -81.38 \\ 116.23 \\ 0 \\ 11.62 \\ -8.14 \end{vmatrix}$$

Solve by iteration:

1. Set the R values to zero, and solve for the F's.
2. Based on the F values, solve for the R's according the following equations and include in the second iteration. Continue until the R and F values converge.

The signs of the bearing reaction force components and the direction of rotation determine those of the friction radius as previously explained. Subsequent equations for these latter components accommodate their signs and magnitudes.

$$R_{y3}^2 + R_{z3}^2 = r_f^2$$
$$R_{y4}^2 + R_{z4}^2 = r_f^2$$
$$R_{y3} = -(|\omega F_{z3}|/\omega F_{z3})\, r_f \sqrt{[F_{z3}^2/(F_{y3}^2+F_{z3}^2)]}$$
$$R_{z3} = +(|\omega F_{y3}|/\omega F_{y3})\, r_f \sqrt{[F_{y3}^2/(F_{y3}^2+F_{z3}^2)]}$$
$$R_{y4} = -(|\omega F_{z4}|/\omega F_{z4})\, r_f \sqrt{[F_{z4}^2/(F_{y4}^2+F_{z4}^2)]}$$
$$R_{z4} = +(|\omega F_{y4}|/\omega F_{y4})\, r_f \sqrt{[F_{y4}^2/(F_{y4}^2+F_{z4}^2)]}$$

The resultant force and moment arm component magnitudes converge on the 4th iteration and become:

$F_{y2} = 13.39$ N $\quad F_{z2} = 1.17$ N $\quad T_{out} = -379$ Nmm

$F_{y3} = -0.59$ N $\quad F_{z3} = -3.10$ N $\quad F_3 = 3.16$ N

$F_{y4} = -1.18$ N $\quad F_{z4} = -6.21$ N $\quad F_4 = 6.32$ N

$R_{y3} = 2.18$ mm $\quad R_{z3} = -0.42$ mm $\quad r_f = 2.22$ mm

$R_{y4} = 2.18$ mm $\quad R_{z4} = -0.41$ mm $\quad r_f = 2.22$ mm

The ratio of output to input torque is 379/400 = 0.95. Journal friction alone reduced the efficiency of this idler to 95%. With mesh friction, it would be lower.

Problem 6

Helical Idler on a Shaft. Figure 8 shows the gear's orientation. Input point p_1 is 35° from the positive x-y plane; output point p_2 is 205° from this plane. The gears each have 20° operating pressure angles, and 25° left hand operating helix angles. The operating pitch radius of the input gear is 60 mm; that of the output gear is 20 mm. The input torque is 500 Nmm. Journal and mesh friction are ignored here for simplicity. Friction journal radii r_f and mesh force vector displacements m are zero.

$F_{x1} = +(500/60)\tan 25° = 3.886\ N$

$F_{y1} = -[500/(60\cos 20°)][\sin(35°+20°)] = -7.264\ N$

$F_{z1} = +[500/(60\cos 20°)][\cos(35°+20°)] = 5.087\ N$

Also,

$F_{x2}\sin(205° - 20°) + F_{y2}\cos(20°)\tan(25°) = 0$

$F_{z2}\sin(205° - 20°) + F_{y2}\cos(205° - 20°) = 0$

So,

$$F_{x2} - 5.028\,F_{y2} = 0$$
$$F_{z2} + 11.430\,F_{y2} = 0$$

The moment arms are:

$R_{x1} = 20.0 \quad R_{y1} = 49.149 \quad R_{z1} = 34.415$

$R_{x2} = 30.0 \quad R_{y2} = -18.126 \quad R_{z2} = -8.452$

$R_{x3} = 0.0 \quad R_{y3} = 0.0 \quad R_{z3} = 0.0$

$R_{x4} = 40.0 \quad R_{y4} = 0.0 \quad R_{z4} = 0.0$

The torque determinants become:

$$\begin{vmatrix} i & j & k \\ 20.0 & 49.15 & 34.42 \\ 3.89 & -7.26 & 5.09 \end{vmatrix} = T_1$$

$$\begin{vmatrix} i & j & k \\ 30.0 & -18.13 & -8.45 \\ F_{x2} & F_{y2} & F_{z2} \end{vmatrix} = T_2$$

$$\begin{vmatrix} i & j & k \\ 0.0 & 0.0 & 0.0 \\ F_{x3} & F_{y3} & F_{z3} \end{vmatrix} = T_3$$

$$\begin{vmatrix} i & j & k \\ 40.0 & 0.0 & 0.0 \\ F_{x4} & F_{y4} & F_{z4} \end{vmatrix} = T_4$$

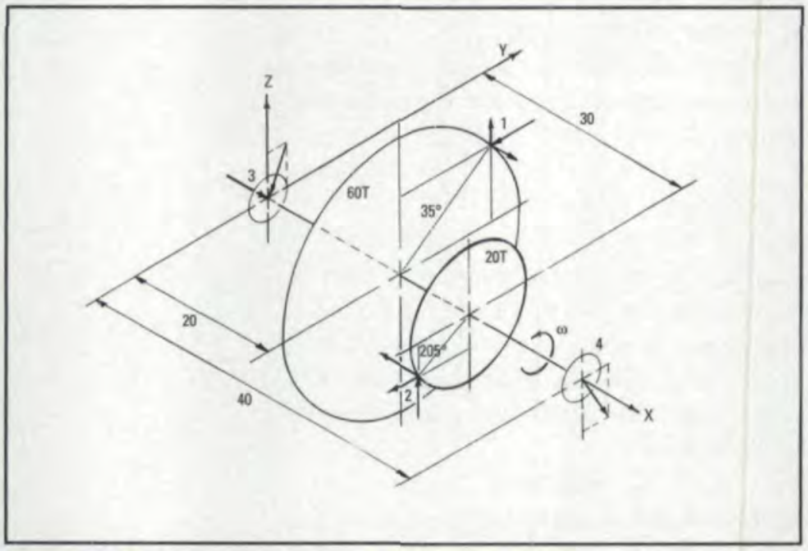

Fig. 8 – Helical idler on a shaft.

The final simultaneous equation matrix becomes:

$$\begin{vmatrix} 0 & -8.452 & 18.126 & 0 & 0 & 0 & 0 & 0 \\ 8.452 & 0 & 30 & 0 & 0 & 0 & 0 & 40 \\ 18.126 & 30 & 0 & 0 & 0 & 0 & 40 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 11.430 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -5.028 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} F_{x2} \\ F_{y2} \\ F_{z2} \\ F_{x3} \\ F_{y3} \\ F_{z3} \\ F_{y4} \\ F_{z4} \end{vmatrix} = \begin{vmatrix} 500 \\ 32 \\ 336.27 \\ -3.886 \\ 7.264 \\ -5.087 \\ 0 \\ 0 \end{vmatrix}$$

The forces become:

$F_{x2} = -11.66\ N \quad F_{y2} = -2.32\ N \quad F_{z2} = 26.50\ N$

$F_{x3} = 7.77\ N \quad F_{y3} = -5.85\ N \quad F_{z3} = -14.98N$

$F_{yz3} = 16.077\ N \quad F_{x4} = 0\ N \quad F_{y4} = 15.43\ N$

$F_{z4} = -16.62\ N \quad F_{yz4} = 22.673\ N$

Problem 7

Worm Gear Drive. Figure 9 shows a worm and gear arrangement. Only the worm's cross-sliding tooth friction will be included in the mesh, as it has a large effect in reducing the efficiency of such drives. The resultant tooth mesh force will pass through the single point of contact between the two pitch cylinders. The driven worm wheel, supporting the journals and thrust bearings with their frictional losses, is the only object under consideration here. The wheel's driven load is pure torque in this example.

The worm lead angle is 10°, which is also the operating helix angle of the gear in this 90° drive. The worm has one tooth (one start) and is left handed. The gear pitch is one module with 64 teeth at a pressure angle of 20°. Therefore, the gear's pitch radius is 32.4936 mm; the worm's pitch cylinder radius is 2.879 mm. The torque transmitted to the gear after losses is 500 Nmm. (The worm's input torque will be greater, but is outside this problem's concern.)

Were friction zero, F_N would be 16.628 N, and its components would be:

$F_x = 2.713\ N \quad F_y = -15.388\ N \quad F_z = -5.687\ N$

The following mesh forces on the gear for input torque = 500 Nmm are derived with $\mu = 0.4$.

The normal tooth load: $F_N = 17.977\ N$

The sliding friction load: $\mathcal{F} = 7.191\ N$

$\mathcal{F}$ is perpendicular to F_N and parallel to the x-y plane. Components of both $\mathcal{F}$ and F_N combine to produce the net resultant force acting through the pitch point of the mesh. Its components are:

$$F_x = 10.015\ N$$
$$F_y = -15.388\ N$$
$$F_z = -6.149\ N$$

The journals are 12 mm on each side of the gear's pitch circle, and 12 mm in diameter. Journal friction is $\mu = 0.4$. There is a thrust frictional loss acting on a radius $R_t = 9$ mm. The journal friction radii $r_f = 2.228$ mm.

The torque determinants become:

$$\begin{vmatrix} i & j & k \\ 0.0 & 0.0 & 32.494 \\ 10.015 & -15.388 & -6.149 \end{vmatrix} = T_1$$

$$\begin{vmatrix} i & j & k \\ 0.0 & -0.5 & 0.0 \\ 0.0 & 0.0 & F_{za2} \end{vmatrix} = T_{2a}$$

$$\begin{vmatrix} i & j & k \\ 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & F_{zb2} \end{vmatrix} = T_{2b}$$

$$\begin{vmatrix} i & j & k \\ -12.00 & R_{y3} & R_{z3} \\ 0.0 & F_{y3} & F_{z3} \end{vmatrix} = T_3$$

$$\begin{vmatrix} i & j & k \\ 12.00 & R_{y4} & R_{z4} \\ 0.0 & F_{y4} & F_{z4} \end{vmatrix} = T_4$$

$$\begin{vmatrix} i & j & k \\ 0.0 & -0.5 & 0.0 \\ 0.0 & 0.0 & F_{za5} \end{vmatrix} = T_{5a}$$

$$\begin{vmatrix} i & j & k \\ 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & F_{zb5} \end{vmatrix} = T_{5b}$$

The final simultaneous matrix equilibrium equation becomes:

$$\begin{vmatrix} 0.5 & -0.5 & R_{z3} & -R_{y3} & R_{z4} & -R_{y4} & 0.5 & -0.5 & 0 \\ 0 & 0 & 0 & -12 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & -12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \mu R_t \end{vmatrix} \begin{vmatrix} F_{z2a} \\ F_{z2b} \\ F_{y3} \\ F_{z3} \\ F_{y4} \\ F_{z4} \\ F_{z5a} \\ F_{z5b} \\ F_{x6} \end{vmatrix} = \begin{vmatrix} 500 \\ 325.427 \\ 0 \\ 10.015 \\ 15.388 \\ 6.149 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

The magnitudes and directions of the journal frictional moment arm components are:

$$R_{y3} = -(|\omega F_{z3}| / \omega F_{z3})\, r_f \sqrt{[F_{z3}^2 / (F_{y3}^2 + F_{z3}^2)]}$$
$$R_{z3} = +(|\omega F_{y3}| / \omega F_{y3})\, r_f \sqrt{\{F_{y3}^2 / (F_{y3}^2 + F_{z3}^2)\}}$$
$$R_{y4} = -(|\omega F_{z4}| / \omega F_{z4})\, r_f \sqrt{[F_{z4}^2 / (F_{y4}^2 + F_{z4}^2)]}$$
$$R_{z4} = +(|\omega F_{y4}| / \omega F_{y4})\, r_f \sqrt{[F_{y4}^2 / (F_{y4}^2 + F_{z4}^2)]}$$

These insure that the respective journal loading vectors are tangent to the journals' friction circles on the side that inhibits motion.

As the driven load is pure torque here, and since the thrust frictional load is treated as a pure torque, these were treated as couples in the free body. Frictional loads here do not alter the components of the journal loading, so no iteration is required.

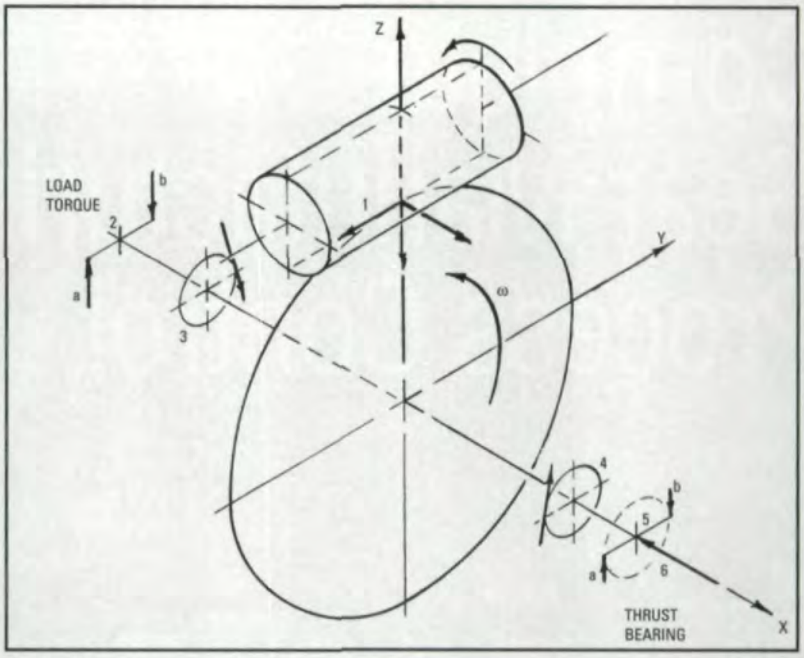

Fig. 9 – Worm gear drive with friction.

Given 500 Nmm torque input to the gear, the journal loading forces with tooth and journal friction are:

$F_{y3} = 7.7$ N $F_{z3} = -10.5$ N $F_3 = 13.0$ N
$F_{y4} = 7.7$ N $F_{z4} = 16.6$ N $F_4 = 18.3$ N

Output torque delivered and bearing losses under various conditions follow:

Results for no friction:

$$T_{out} = -500 \text{ N mm}$$
$$T_{thrust} = 0 \text{ N mm}$$
$$T_{journal} = 0 \text{ N mm}$$

Results for mesh and thrust friction only:

$$T_{out} = -464 \text{ N mm}$$
$$T_{thrust} = -36 \text{ N mm}$$
$$T_{journal} = 0 \text{ N mm}$$

Results for mesh, thrust and journal friction:

$$T_{out} = -394 \text{ N mm}$$
$$T_{thrust} = -36 \text{ N mm}$$
$$T_{journal} = -70 \text{ N mm}$$

References

1. Holowenko, A. R. *Dynamics of Machinery*, Wiley, 1955.
2. Holowenko, A. R. class notes, ME566, Purdue University, 1960.
3. Shigley, J. E. *Kinematic Analysis of Mechanisms*, McGraw Hill College Div., June, 1969.
4. Visit *www.geartechnology.com/vectors.htm* to see the Appendix.

This article is based on materials that were presented at the SAE Plastic Gears for Power Applications TOPTEC held August 26–27, 1998 Dayton, OH.