Impact of Root Geometry Manufacturing Deviations from a Theoretical Hob Rack on Gear Bending Stress

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Introduction

High reliability, superior efficiency, and light weight are key requirements of mechanical power transmission systems, such as automotive transmissions. The competing design requirements pose a challenge to gear designers. Rigorous engineering analysis and sophisticated computational tools might be needed to help in finding the best compromise of design parameters and product performance requirements. One important aspect to be considered during the gear design phase is the manufacturing process. Different manufacturing processes can be used to produce gears, such as hobbing, shaping, and milling (Ref. 1). Each one of them has its advantages and limitations. Gear hobbing is a cost-effective and widely used method of cutting gear teeth (Ref. 1).This generating process makes both the tooth involute flanks and root fillet. The involute flanks can be finished by a post-process such as shaving, grinding, or honing (Ref. 1). In the hobbing operation, root fillets are generated by the hob rack tip corner. The generated root fillet is not a true radius, but a trochoid form (Ref. 2). An undercut root fillet is formed when the "trochoid lies inside a line drawn tangent to the involute profile at the point of intersection of the involute and the trochoid" (Ref. 3) (Fig. 1). Under certain conditions the trochoid form may intersect the tooth involute flanks above the start of active profile, resulting in undercut (Ref. 4). Undercut is generally considered an undesired result of the generating process because it may affect load distribution and reduce gear load capacity (Refs. 3, 5). Undercut was comprehensively investigated by Su and Houser (Ref. 4), and Pedrero et al. (Ref. 5).

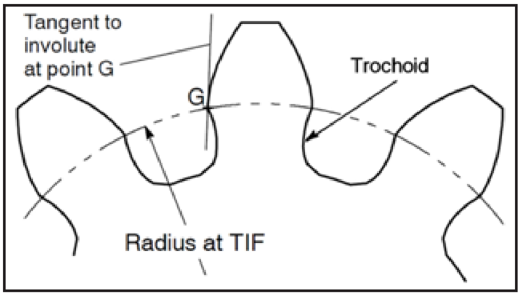


Figure 1 Root fillet shape of undercut teeth (Ref. 3).

The root fillet shape is a leading element for determining tooth bending strength of gears, which is the resistance to cracking (Ref. 6). The root fillet is particularly susceptible to fracture because it is where the highest tensile bending stresses are found (Ref. 7). ANSI/AGMA 1010 (Ref. 7) recommends the use of tools with fully rounded tips and protuberance for reducing bending stresses in the root area. Hob racks with fully rounded tip generate full fillets and reduce stress concentrations, and protuberance tools minimize risks of stress risers, such as notches or steps on the root fillet of gears that are finished after hobbing (Ref. 7). Industry standards, such as ANSI/AGMA 2101 (Ref. 6) and ISO-6336-3 (Ref. 8), provide methods for calculating bending stresses that use stress correction factors derived from basic geometry of the generating tool. However, computational tools for gear analysis, such as *WindowsLDP* (Ref. 9), should be employed for accurate bending stress prediction. Lastly, bending stresses and reliability are determined during the design phase and under the assumption that an equivalent design hob rack will be used for making the gears in production. Chaphalkar et al. (Ref. 10) pointed out gaps on gear drawings, and between design and manufacturing — which may jeopardize gear strength. In general, gear drawings contain complete specifications of involute profile, lead, and other micro-geometry parameters, but typically this is not the case for root fillet shapes, where in some instances only a minimum radius is specified in the root area (Ref. 10). As a result of that, the geometry of the hob cutter actually used to cut the gears in production may deviate from the intended hob rack, which may result in differences of root fillet shape and eventually affecting root bending stress. Also, the root fillet shape is not usually part of quality control and gear inspection, which are mostly focused on tooth contact area — such as involute form and lead (Ref. 10). The objective of this study is to investigate the impact of manufacturing deviation of root fillet

Table 1 Basic gear data

Parameters	Units	Values
Number of teeth	-	66
Module	mm	2.5
Pressure angle	deg	20
Helix angle	deg	0
Outside diameter	mm	171.40
Pitch diameter	mm	165.98
Base diameter	mm	155.97
Root diameter	mm	157.15
Face width	mm	33
Load intensity	N/mm	750

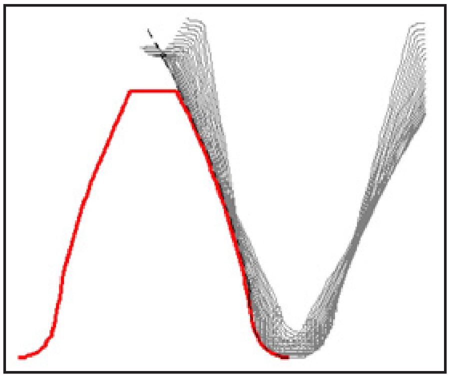


Figure 2 Gear tooth generated by a hob rack.

shape from the intended hob rack on bending stresses. Three gear samples were brought to the gear lab for root fillet inspection. The root fillets were scanned on a gear measurement machine. The measured root fillet data were imported into *WindowsLDP* (Ref. 9) for accurate bending stress calculation using a custom finite element model option available on the program. The results were compared to bending stresses from the intended hob rack. The differences were quantified and causes of deviations were identified. The overall results of this study emphasize the importance of a closed-loop approach of gear design and manufacturing to assure designed root fillet shapes are attained in production, and gears meet the design intent.

Bending Stress from Design Generating Rack

Several methods for calculating bending stress are available through industry standards and gear programs. Some of the key elements of determining the maximum tensile stress at the tooth root are the load distribution of meshing teeth and the generated root fillet shape. The *WindowsLDP* program (Ref. 9), developed by the Gear and Power Transmission Research Laboratory of The Ohio State University is a well-known gear program widely used on gear research projects and in the gear industry. It uses a finite element model for calculating bending stresses accurately, which was validated experimentally and well documented (Refs. 11–12). The program was used to calculate bending stress of the selected case study.

Gear data was entered to *WindowsLDP* along with hob rack geometry that was used for designing the gears. Basic gear geometry information is provided in Table 1. Figure 2 shows the generated tooth in the program. All analysis was done at a single load condition and under no misalignment. The default program setting was used for number of mesh cycle positions and multiplier across the face width, that is, 21 and 4, respectively.

The bending stress calculation was done and the maximum principal tensile root stress from the finite element stress analysis was recorded. Figure 3 shows the bending stress results. The picture on the left depicts a cross section of the gear where the highest bending stress was found along the root fillet. The picture on the right shows the principal stress distribution across gear face width and root fillet. The arrow points to the location of maximum tensile stress.

Bending Stress from Measured Root Fillet Shape

Three samples of the gear case study were inspected. Root fillet shapes were measured on a Wenzel gear measurement machine model WGT400 using Wenzel's *TShaft* program (Ref. 13) (Fig. 4).

The entire tooth was scanned, and 50 points were measured between the start of active profile and root radius. An example of the inspection report of root fillet measurement is shown (Fig. 5).

The measured root fillet points were loaded into *WindowsLDP* as an external file through the user defined XY shape

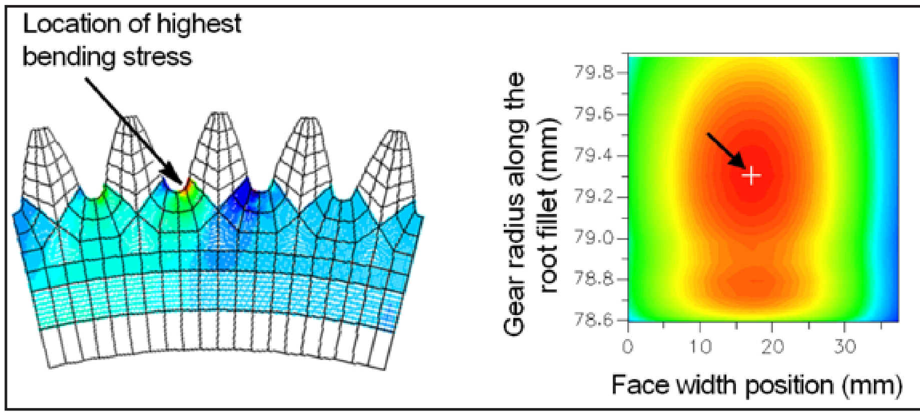


Figure 3 Example of bending stress distribution.

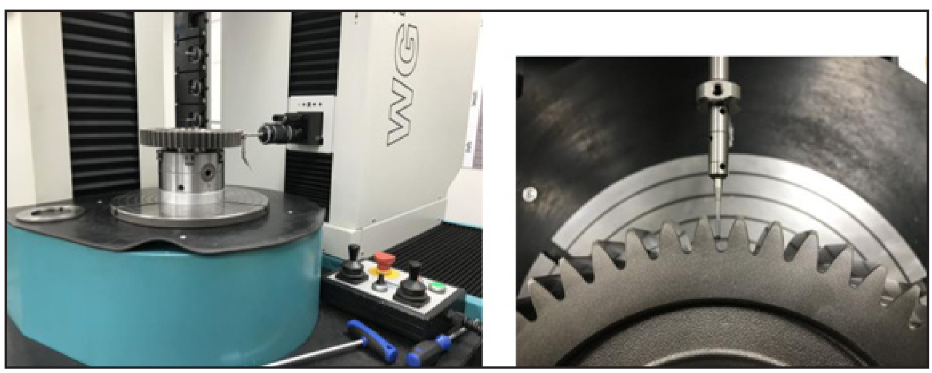


Figure 4 Gear root fillet measurement.

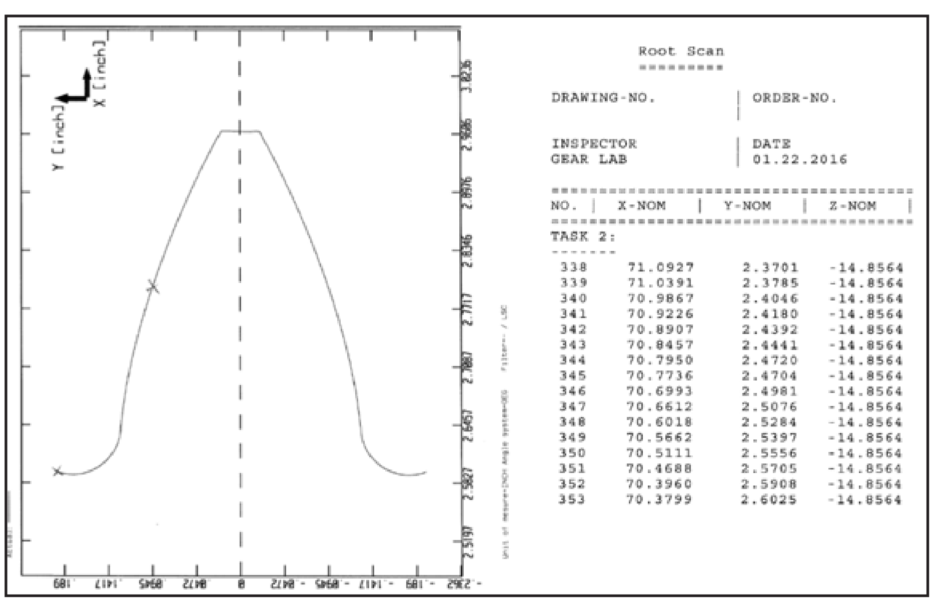


Figure 5 Example of measurement results of a gear root fillet.

option. In that case, the generated root fillet shape by the hob rack is replaced with the actual measured root fillet shape. Interpolation of measured points and finite element mesh are done automatically through a couple of program's routines. Lastly, bending stress was calculated using the finite element analysis module of the program and the maximum principal tensile root stress was recorded.

Results

Bending stress results of generated root fillet shapes by the design hob rack and actual measured root fillet shapes were compared for all three cases. The results were normalized by dividing bending stress of actual root fillet shape by bending stress of design root fillet shape. Thus, results greater than 1 mean increased stress with actual measured root fillet. The normalized results are shown in Table 2.

Table 2 Normalized bending stress results

Gear	Values
Design	1.00
Sample #1	1.22
Sample #2	1.21
Sample #3	1.13

The maximum deviation found among the three gear samples was 1.22, or 22%, over the design bending stress. Figure 6 shows a comparison of the designed root fillet and the measured root fillets. The key reasons for the differences in bending stress results are: root radius of the manufactured gears, and the root fillet curvature. Gear samples #1 and # 2 had slightly undersize root radii and smaller fillet curvatures compared to the designed root fillet shape. Gear sample #3 shows the smallest bending stress deviation to the design bending stress, 13%. Root radius of gear sample # 3 was in good agreement with the designed root radius, but root fillet curvature was smaller.

Further investigation revealed that the hob cutter used to manufacture those particular gear samples had been designed with a different pressure angle, as compared to the design hob rack. The design hob rack was a short-lead hob (Ref. 2) of 17 deg. pressure angle, while the production one was a 20-deg. pressure angle hob. Also, the production hob cutter was not fully rounded at the tip, leaving a flat root condition of the gear, which can be noticed in Figure 6. The normal hob rack profile is shown (Fig. 7), along with a comparison of the two hob cutter parameters.

Discussions

Three gear samples of a case study were investigated. Root fillets were scanned using a gear measurement machine. The data was imported into *WindowsLDP* for bending stress calculation using a finite element option of the program. The results were compared to the bending stress, with the hob rack used to design the gear. Deviations from 13% up to 22% were found. Further investigation revealed differences in root radii and root fillet curvatures, which caused bending stresses of manufactured gears to be higher than the one with a hob rack of the gear design. The reasons for such differences in root fillet shape were attributed to the actual hob cutter used to manufacture the gears that had different pressure angle and tip radius, compared to the intended hob rack. The results of this paper highlight that the root fillet shape is an essential element of gear design, and emphasizes the importance

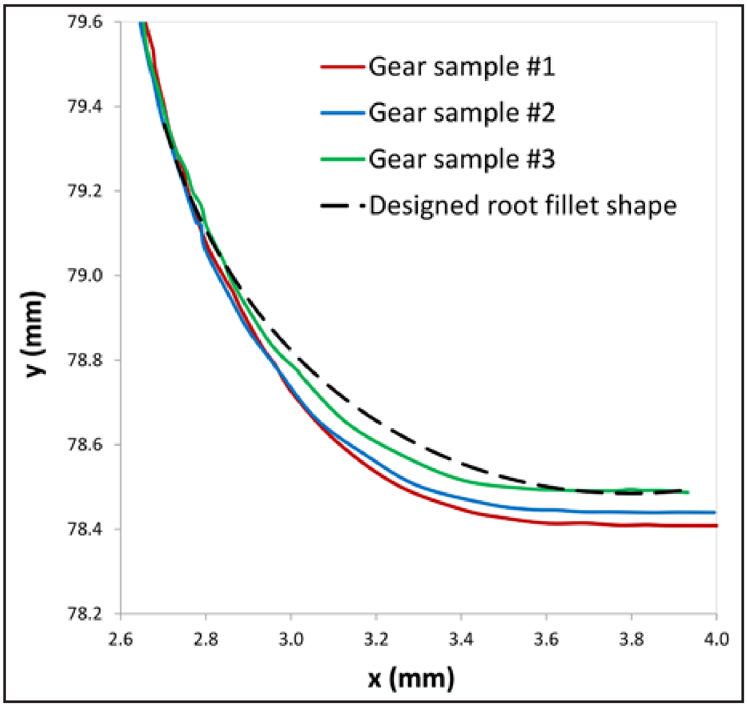
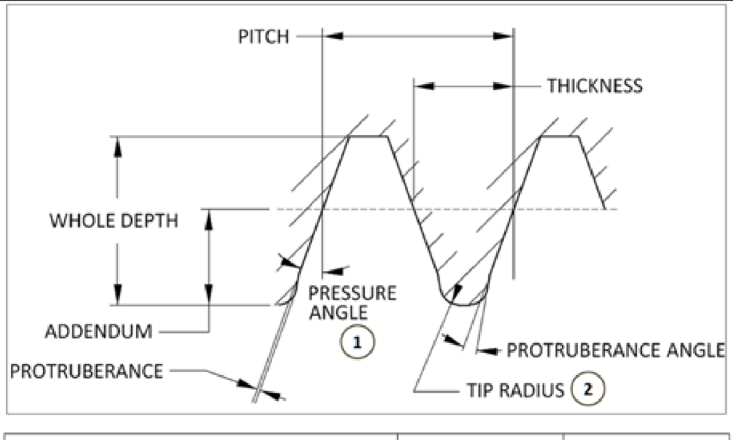


Figure 6 **Comparison of root fillet shapes.**



	Design	Production
1. Hob pressure angle (deg.)	17	20
2. Hob tip radius (mm)	1.016	0.584

Figure 7 **Comparison between design and production hob racks.**

of a complete root fillet shape specification or hob rack on gear drawings, along with a robust gear manufacturing control plan for attaining high gear reliability. Establishing a closed-loop approach of gear design and manufacturing helps assure that designed, root fillet shapes are obtained in production — and gears meet the design intent.

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