Solid Model Generation of Involute Cylindrical Gears

Wang Lixin & Huang Wenliang

<u>Nomenclature</u>

R	Reference Radius
Ra	Tip Radius
Rf	Root Radius
Rb	Base Radius
m_n	Normal module
z	Number of teeth
x_n	Normal modification coefficient
h^*_{an}	Normal addendum coefficient
c^*_n	Normal bottom clearance coefficient
α_n	Normal pressure angle on reference circle
β	Helix angle on reference circle
s_n	Normal tooth thickness on reference circle

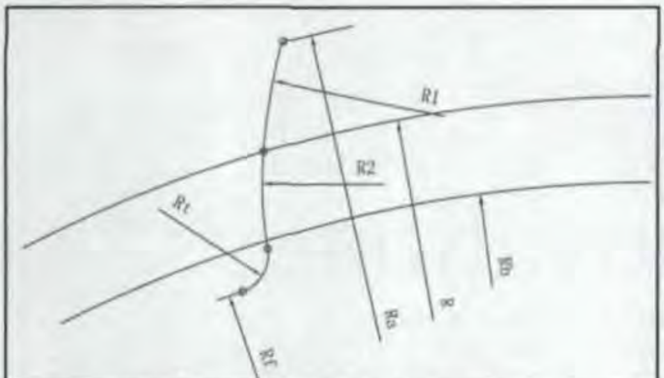

Figure 1—Three arcs substituting the tooth profile.

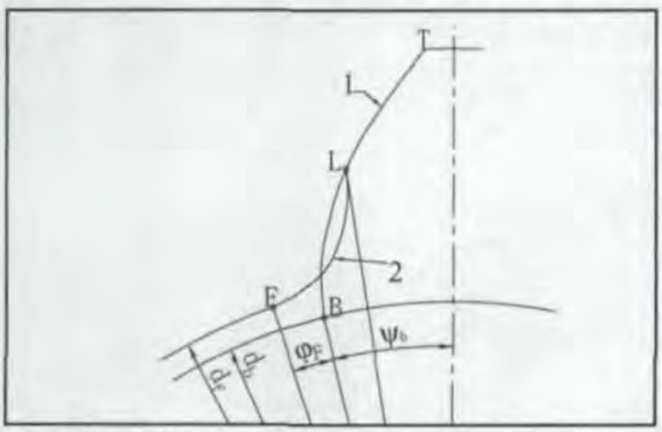

Figure 2—Tooth profile of involute gears.

This paper presents approximate and accurate methods to generate solid models of involute cylindrical gears using Autodesk Inventor 3-D CAD software.

Introduction

The key to gear solid model generating is getting the correct tooth profile. If the tooth profile is obtained, the solid modeling of cylindrical gears can be obtained easily through CAD software commands (extrusion, sweep, coil, circular pattern).

There are some resources to assist with this job. Sandeep Singal developed spur gears on Pro/E using involute equations without considering the undercut, and the root fillet is approximated with a circular curve joining the involute and root circle (Ref. 1). Deng Kai generated the accurate cylindrical gear tooth profile with the graphic solution. A cutting tool rack shape with a specific parameter needed to be drawn on the sketch status of 3-D CAD software (Autodesk Inventor) each time. Using the constraint feature, the rack was moved to simulate the real gear production procedure and the series trajectory was used to find the envelope as the gear tooth profile (Ref. 2). Another resource is the gear tooth profile included in the Autodesk Inventor's sample directory. The profile is approximately substituted by one or two arcs and a line joining the arc and root circle.

The two methods presented in this paper can generate a gear solid model directly by entering the conventional parameters of the gear only (m_n, z, α_n, β, x_n, h^*_{an}, c^*_n).

Approximate method

In many cases, gear tooth profile accuracy is not critical, such as a production demonstration, 3-D animation, etc. In those cases, we can use three arcs to substitute the involute tooth profile (Ref. 3).

In Figure 1, the gear tooth profile is formed with three tangent arcs (R1, R2, Rt).

$R = {}^1/_2\, m_n z/\cos\beta$

$Ra = R + m_n(h^*_{an} + x_n)$

$Rb = R\cos(\alpha_t)$

$Rf = R - m_n(h^*_{an} + c_n^* - x_n)$

$R1 = \rho' m_n$

$R2 = \rho'' m_n$

$$Rt = \frac{c^* m_n}{1 - \sin\alpha_n}$$

where: $\rho' = \dfrac{z}{2}\sqrt{1 - \cos^2\alpha_n \dfrac{z-1}{z+1}}$ $\qquad \rho'' = \dfrac{z^2 \sin^2\alpha_n}{4\rho'}$

The centers of both arcs R1 and R2 are located on the base circle, and their tangent point is located on the reference circle. Arc Rt is tangent to the arc R2 and the root circle at the same time.

Accurate Method

The Combination of the tooth profile: The cutting tool and manufacturing method generate the shape of the gear tooth profile and affect the theoretical design result. The accuracy of the gear tooth profile comes from the cutting tool and the manufacturing method.

Two parts, Involute 1 and Root Fillet Curve 2, form the gear tooth profile (see Fig. 2). Involute 1 (from Point T to L) is generated by the linear (involute) part of the rack-shaped cutting tool (or gear-shaped cutting tool). There is a standard equation to describe it. Fillet Curve 2 (from point L to F) is formed by the tip of a cutting tool, and there is no standard equation to describe it. Point T is located on the tip circle. Point F, which is the intersection and tangent point between the root circle and root fillet curve, is located on the root circle. Point L is the intersection point of involute and root fillet curves.

Determining the Root Fillet Curve of Tooth Profile

Wu Jize pointed out that root fillet curve can be divided into three classes: offset of prolate involute, offset of epicycloids and arc (Ref. 4). I.A. Borotovskii analyzes the root fillet curve equation generated from different cutting tools in detail, evaluating both the rack- and gear-shaped cutting tools with tips filleted and chamfered.

In fact, the gear mentioned in the design phase and the virtual prototyping is the theoretical gear. The geometric parameters of gears can be calculated using a gear design handbook. An accurate tooth profile can be obtained by using the generating method. In the generating method, we use a standard rack cutting tool that corresponds to the helical gear being generated (m_n, z, α_n, β, x_n, h^*_{an}, c^*_n). The parameters of the cutting tool are α_t, s_n, h_a. When $\beta = 0$, the gear is a spur gear (Fig. 3).

Based on the gear parameters m_n, z, α_n, β, x_n, h^*_{an}, c^*_n, we have the following relation of the gear:

$s_n = (0.5\pi + 2x_n tg\,\alpha_n)m_n$; $tg\alpha_t = tg\alpha_n/\cos\beta$ $mt = m_n/\cos\beta$

$d = m_n z/\cos\beta$ $d_b = d\cos\alpha_t$ $tg\beta_b = tg\beta\cos\alpha_t$

$h_a = (h_{an}^* + x_n)m_n$ $h_f = (h_{an}^* + c_n^* - x_n)m_n$

The central angle between the involute start point B and the symmetrical line of the tooth is ψ_b: $\psi_b = \frac{s_n}{m_n z} + \mathrm{inv}\alpha_t$.

The central angle between point B and F is φ_F (see Fig. 2)

$$\varphi_F = \frac{h_f \sin\alpha_n}{0.5 m_n z \cos\alpha_n} - \mathrm{inv}\alpha_t$$

According to Figure 4, the point on the Fillet Curve 2 corresponds to the manufacturing instantaneous angle μ_w, and $\alpha_t < \mu_w < 0.5\pi$, so we have:

$$y = h_f \quad (1)$$

$$my = y/\sin\mu_w \quad (2)$$

$$l = my\cos\mu_w \quad (3)$$

$$\varphi_y = \varphi_F + 2l/d \quad (4)$$

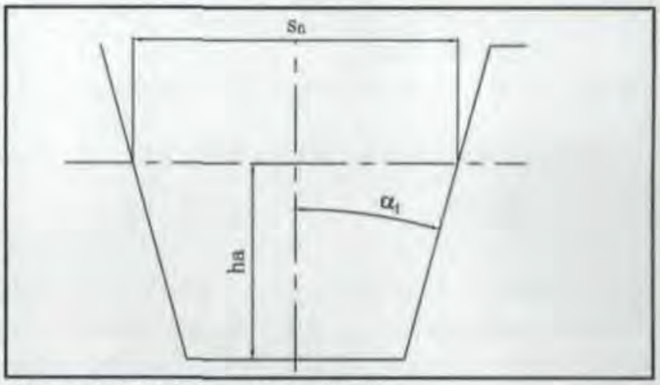

Figure 3—Template cutting tool rack.

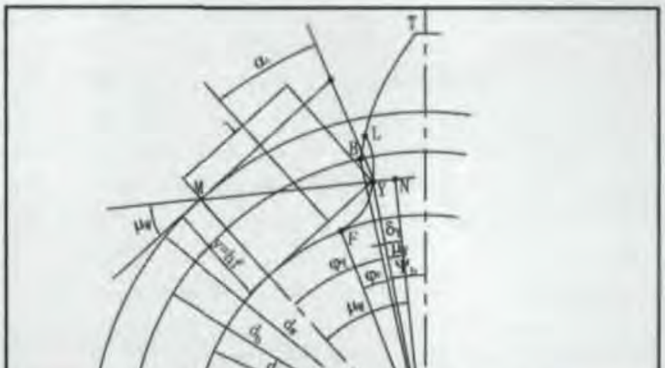
Figure 4—Determining the instantaneous mesh angle.

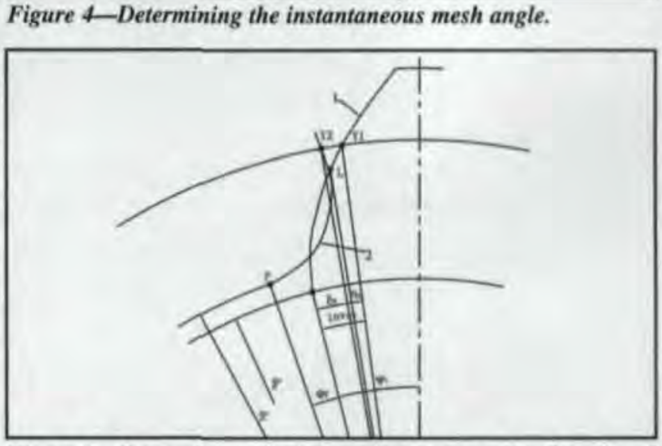
Figure 5—Determining the intersection between Involute 1 and Root Fillet Curve 2.

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$$tg\mu_y = tg\mu_w - \frac{2my}{d\cos\mu_w} \tag{5}$$

The polar angle and radius of Point Y is $\delta_y = \mu_w - \mu_y - \varphi_y$,

$$r_y = 0.5d\frac{\cos\mu_w}{\cos\mu_y}$$

The datum of δ_y is line BO. Point O, which is not shown in the figures, is the center of the gear. When the rotational direction is clockwise, the sign of δ_y is +; – stands for counter-clockwise. At point F: $\mu_y = \mu_w = 0.5\pi$; $my = y = 1$; $\delta_y = -\varphi_F$; $r_y = 0.5d_f$. Point F is the intersection of root circle and root fillet curve.

After getting the description of Root Fillet Curve 2 and knowing the equation of Involute 1, it is necessary to find the intersection Point L between Involute 1 and Root Fillet Curve 2.

Assuming an arbitrary μ_w, δ_y and r_y of point Y2 can be derived from the above formulas. The pressure angle of point Y1, located on the Involute 1, has the same radius r_y with Point Y2:

$$\cos\alpha_{ty} = 0.5d_b / r_y \tag{6}$$

Then, the central angle difference between Y1 and Y2 is: (see Fig. 5)

$$q_y = \text{inv}\alpha_{ty} - \delta_y \tag{7}$$

$q_y = 0$ is a nonlinear equation with variable μ_w. Finding the μ_w to satisfy the equation $q_y = 0$ means finding the intersection point (Y1 and Y2 are coincidental). In order to solve the equation, we introduce the Newton Iteration Numerical Method.

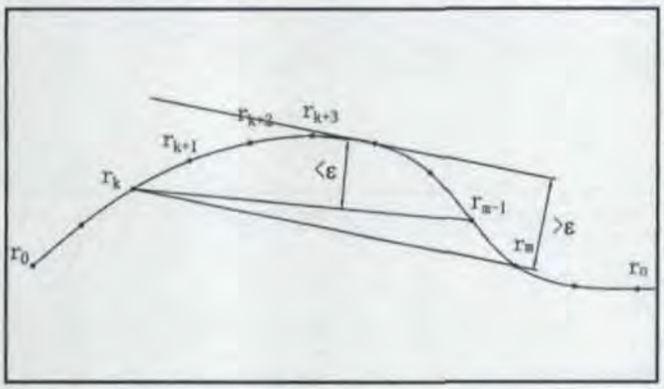

Figure 6—Equal tolerance fitting method using point choice.

Assume the initial value of μ_w is α_t, convergence criteria is $|q_y| < 0.0000005$. The derivative of q_y can be derived from Equations 1–7.

$$\frac{dmy}{d\mu_w} = -\frac{y}{tg\mu_w \sin\mu_w} \tag{8}$$

$$\frac{d\varphi_y}{d\mu_w} = \frac{2}{d}\left(\frac{dmy}{d\mu_w}\cos\mu_w - y\right) \tag{9}$$

$$\frac{d\mu_y}{d\mu_w} = \left(1 - \frac{2}{d}\left(y + \frac{dmy}{d\mu_w}\cos\mu_w\right)\right)\left(\frac{\cos\mu_y}{\cos\mu_w}\right)^2 \tag{10}$$

$$\frac{dq_y}{d\mu_w} = \left(\frac{d\mu_y}{d\mu_w}tg\mu_y - tg\mu_w\right)tg\alpha_{ty} + \frac{d\mu_y}{d\mu_w} + \frac{d\varphi_y}{d\mu_w} - 1 \tag{11}$$

$$\mu_w = \mu_w - \frac{q_y}{\frac{dq_y}{d\mu_w}} \tag{12}$$

In some cases, the root of the equation $q_y = 0$ cannot be found, for example, when z is large, $Rf > Rb$, etc. In those cases, let $\mu_w = \alpha_t$ as the intersection point.

Curve Fitting Method

The equation of tooth profile formed by Involute 1 and Root Fillet Curve 2 has already been proven. Using splines to fit the involute is feasible (Ref. 6). Some CAD software has the command to generate the spline from the equation, such as Pro/E, Unigraphics, etc. But other software does not provide this kind of function. In Autodesk Inventor, create a sketch spline using a series of suitable discrete points. In the field, the discretization method of complicated curves is discussed constantly. There are three kinds of methods—equal parameter, equal step and equal tolerance—that can be applied to the curve discretization (Ref. 7). Among them, the equal tolerance fitting method is most reasonable. To realize the equal tolerance fitting method, use the point choice method.

Method description. Use tiny steps to discretize the curve and estimate the tolerance of every segment, delete the unnec-

Figure 7—Step-by-step modeling procedure.

essary points, and the tolerance in the remaining segment is approximately equal (See Fig. 6). Points $r_0, \ldots, r_k, r_{k+1}, r_{k+2}, r_{k+3}, \ldots, r_{m-1}, r_m, \ldots, r_n$ are the equal interval discrete points of a curve. ε is the fitting tolerance allowed.

The procedure in detail. For arbitrary initial point r_k (the first point is r_0), starting from r_{k+2} and being considered as a divided point r_m, find the distance d between every internal point r_i ($i = k + 1, \ldots, m - 1$) and the line $r_k r_m$, if $d < \varepsilon$. Assuming r_{k+3} is a divided point until existing a point r_i, the distance d between r_i and the line $r_k r_m$ is $d > \varepsilon$, and here $r_k r_{m-1}$ is the longest distance segment to satisfy the fitting tolerance allowed. Point r_{m-1} is saved as a qualified point and all points between r_k and r_{m-1} are deleted. Let r_{m-1} as the r_k and repeat the above procedure. We can gain all the qualified points.

Generation of Involute Cylindrical Gear Solid Model in Autodesk Inventor

In Autodesk Inventor, use VBA programming to finish the solid modeling of the involute cylindrical gear. The step-by-step procedure is as follows (see Fig. 7):

Step 1: Create closed profile of the tooth profile.

Step 2: Extrude the profile to generate a tooth (Coil the profile to generate the spur gear).

Step 3: Extrude a root circle to generate the root cylinder and join it with the tooth just created.

Step 4: Circular pattern the tooth to finish the gear.

Step 5: Use the software commands to finish the rest of the modeling, such as the central hole. ⭘

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