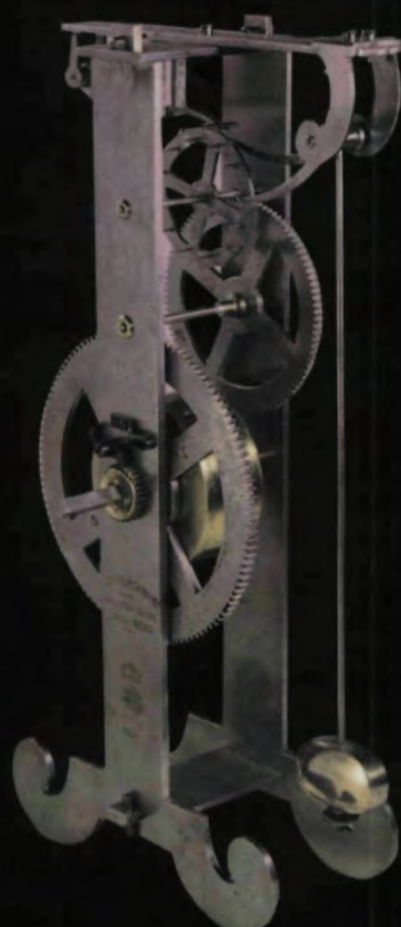
GEAR TECHNOLOGY

The Journal of Gear Manufacturing

SEPTEMBER/OCTOBER 1988



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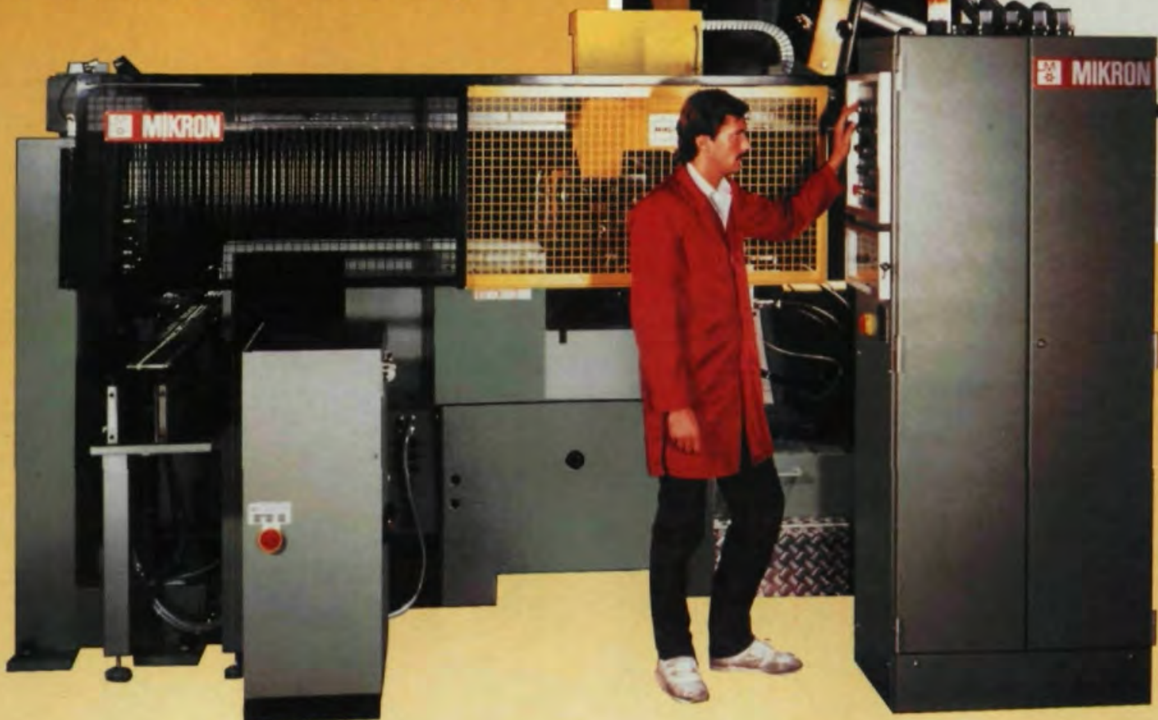

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OUR COVER

Galileo's Escapement. This model, built from a drawing of Galileo's, was constructed in Florence, Italy, in 1883 and is now on display in the Science Museum, London. Probably designed by Galileo in the 1630's, the device is one of the earliest to use a pendulum to regulate a clockwork. It is an application of the principle, discovered by Galileo, that the period of oscillation of a pendulum is the same regardless of the size of the arc made by its swinging motion. It consists of an escape wheel, pinion, pallets with crutch and the pendulum and its suspension. (Photo courtesy of Science Museum, London, England.)

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September/October, 1988 — Vol. 5, No. 5

GEAR TECHNOLOGY, The Journal of Gear Manufacturing (ISSN 0743-6858) is published bimonthly by Randall Publishing Co., Inc., 1425 Lunt Avenue, P. O. Box 1426, Elk Grove Village, IL 60007. GEAR TECHNOLOGY, The Journal of Gear Manufacturing is distributed free of charge to qualified individuals and firms in the gear manufacturing industry. Subscription rates for non-qualified individuals and firms are: $35.00 in the United States, $55.00 for foreign countries. Second-Class postage paid at Arlington Heights, IL and at additional mailing office.

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MANUSCRIPTS: We are requesting technical papers with an educational emphasis for anyone having anything to do with the design, manufacture, testing or processing of gears. Subjects sought are solutions to specific problems, explanations of new technology, techniques, designs, processes, and alternative manufacturing methods. These can range from the "How to . . ." of gear cutting (BACK TO BASICS) to the most advanced technology. All manuscripts submitted will be carefully considered. However, the Publisher assumes no responsibility for the safety or return of manuscripts. Manuscripts must be accompanied by a self-addressed, self-stamped envelope, and be sent to GEAR TECHNOLOGY, The Journal of Gear Manufacturing, P.O. Box 1426, Elk Grove, IL 60007, (312) 437-6604.

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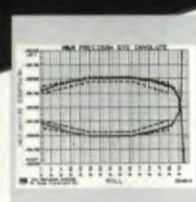

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EDITORIAL

THE 500 LB GORILLA IN THE WINDOW OF OPPORTUNITY

IMTS is back in town. From Sept. 7 through Sept. 15, the largest industrial exhibition in the Western Hemisphere will fill one of the largest exhibition centers in the world. A show of this magnitude is a little like the 500 lb. gorilla in your dining room — hard to ignore.

But like the gorilla in the dining room, IMTS raises certain ambivilent feelings. True, it's big and important, but what does it mean for our industry? The rest of the machine tool business tends to dwarf the gearing industry, and the temptation to sit the show out is strong. After all, there are other trade shows and conferences that relate more directly to our interests. Besides its bigness, what does IMTS have to offer the gear manufacturer, engineer or designer?

A lot. 1,078 companies will display the very latest machine tool products — including gear cutting and finishing equipment — from 27 nations. There is no better place to see the direction manufacturing will take in the coming decades. Running concurrently with the show is the Technology Conference, where some 200 industry experts from around the world will cover every aspect of the latest techniques in manufacturing in 48 separate half-day sessions. One whole session is devoted to modern gear making methods, and at least a half dozen other gear-related topics will be covered. That's a lot of information and opportunity all in one place at one time — information and opportunity we should not ignore.

America's industry has been through some tough times over the last decade. We have suffered from a slightly dulled competitive edge and, more critical in my opinion, a dollar/foreign-currency imbalance that no amount of competitive sharpness could overcome. Now things have changed. Currency balances are much more favorable than they have been in years, and the harsh weeding out process has left American industry leaner, meaner and capable of facing foreign competition. Suddenly our products are in demand again. Exports are growing and our factories are busy. We have been able to come up for air in our struggle to survive the competition.

But we cannot be lulled into complacency. This change for the better is a window of opportunity, not a permanent condition. Exchange rates and economic indicators are never fixed values. Politics, weather, scientific advances, all work to make the situation fluid. We have to make the best use of the good times while we have them. Not only must we repair financial damage, but also reinvest some of our higher profits in our companies, in machines, people, education, systems and information. In the years ahead, we should expect to face larger, better financed and even more strategically oriented competition. We must take advantage of this present opportunity to make ourselves ready.

IMTS is a good place to begin. Even if you're not in the market for machinery today, this is the place to seed what's out there, what the competition is manufacturing, buying and will be using in the future. A show like IMTS or the Gear Expo in Pittsburgh next year is the place to survey all the manufacturing options,

processes and techniques available to lower your costs and improve your quality.

In a chess game, the winner is the player who can plan the most moves ahead. The same is true in the highly competitive, global manufacturing world in which we operate today. Education, information and strategic planning are crucial to economic survival.

Don't let opportunities like IMTS slip by. Windows of opportunity can close as quickly as they open, and the companies that have not used their open time wisely will be shut out in the cold.

Michael Goldstein
Editor/Publisher

TECHNICAL CALENDAR

SEPTEMBER 27-29. American Society for Metals 11th Annual Heat Treating Conference, McCormick Place, Chicago, IL. Presentations on subjects including heat treating, statistical process control, new energy applications, quenching and cooling improvements. For further information, contact: ASM International, Metals Park, OH 44073. (216) 338-5151.

OCTOBER 10-12. AGMA Fall Technical Meeting. Fairmont Hotel, New Orleans, LA. Presentations from U.S. and around the world on gear materials and heat treatment, design techniques, applications technology and the latest technological developments. Contact: AGMA, 1500 King Street, Suite 201, Alexandria, VA 22314. (703) 684-0211.

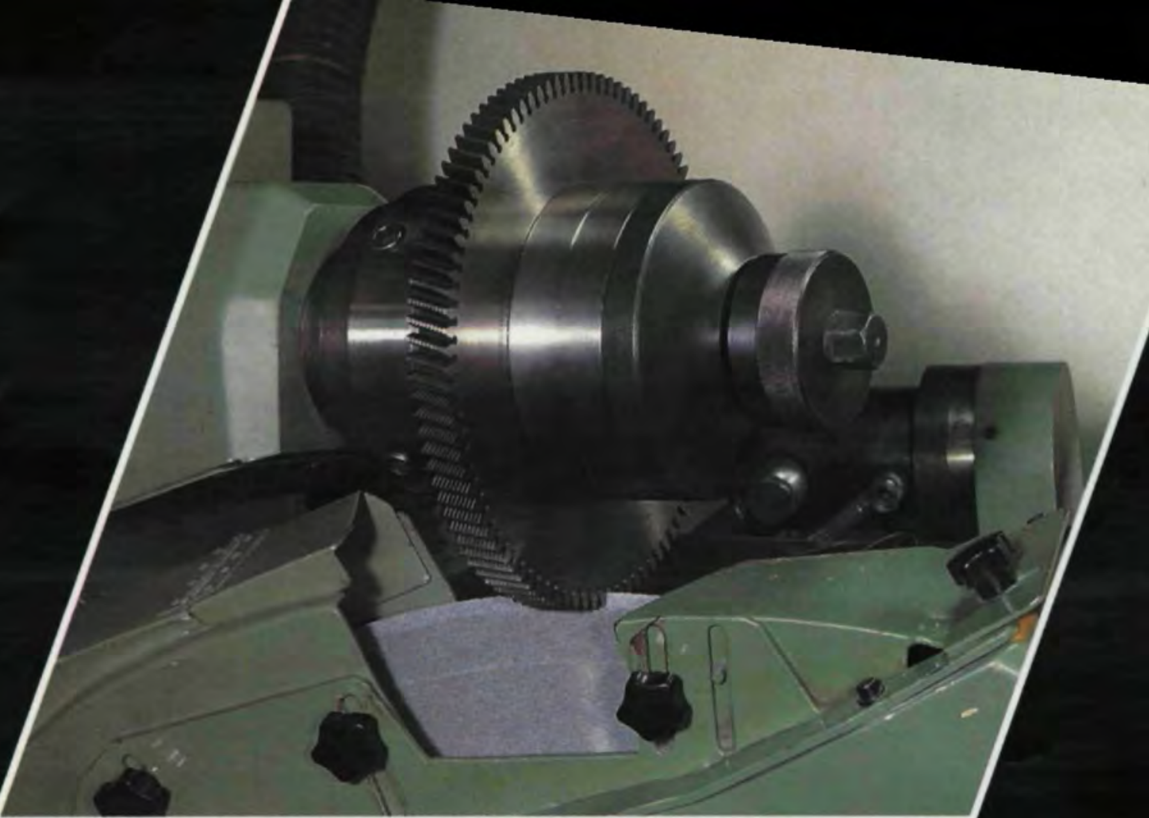
NOVEMBER 1-3. SME Gear Processing and Manufacturing Clinic, Sheraton Meridian, Indianapolis, IN. The technical conference will include papers on "Design and Selection of Hobs," "Selection of High-Speed Steel for Gear Cutting Tools", "Special Design Deburring Equipment" and other gear related subjects. Attendees will have an opportunity to meet with presenters to discuss papers and ask questions on a one-on-one basis. Tuesday evening will feature a reception and tabletop exhibits from major manufacturers. For further information, call Dominic Ahearn at SME Headquarters, (313) 271-1500 x384.

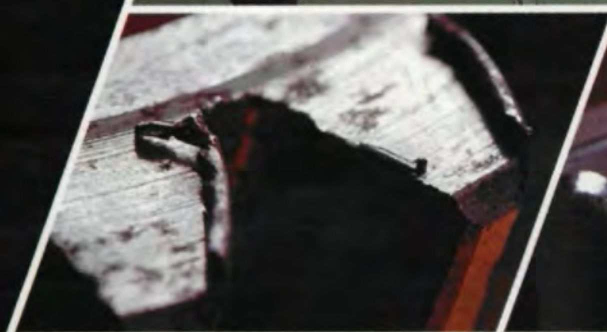
NOVEMBER 5-10. International Conference on Gearing, Zhengzhou, China. ASME-GRI and several international gear organizations are sponsoring this meeting. For more information contact: Inter—Gear '88 Secretariat, Zhengzhou Research Institute of Mechanical Engineering, Zhongyuan Rd, Zhengzhou, Henan, China. Tel: 47102. Cable 3000. Telex 46033 HSTEC CN.

NOVEMBER 8-10. American Society for Metals Near Net Shape Manufacturing Conference, Hyatt Regency, Columbus, OH. Program will cover precision casting, powder metallurgy, design of dies and molds, forging technology and inspection of precision parts. For further information contact: Technical Department Marketing, ASM International, Metals Park, OH 44073.

CALL FOR PAPERS — Tennessee Technological University for its **1st Internat'l Applied Mechanical Systems Design Conference, March 19-22, 1989, Nashville, TN.** Papers are invited on general mechanical systems subjects including strength, fatigue life, kinematics, vibration, robotics, CAD/CAM and tribology. Deadline for first drafts is Oct. 1, 1988. For further information, contact: Dr. Cemil Bagci, Dept. of Mech. Eng., TTU, Cookeville, TN 38505. (615) 372-3265.

CALL FOR PAPERS for ASME 5th Annual Power Transmission & Gearing Conference, April 25-27, 1989, Chicago, IL. Papers are invited on emerging technologies for gears, couplings, belts, chains and other power transmission devices — gear geometry, noise, manufacture, inspection, scoring, lubrication, materials, applications, efficiency, dynamics. For more information, contact Donald Borden, P.O. Box 502, Elm Grove, WI 53122.





1988 INTERNATIONAL MACHINE TOOL SHOW

ADVERTISERS' INDEX

Crowned Spur Gears: Optimal Geometry and Generation

Faydor L. Litvin, Jiao Zhang, Wei-Shing Chaing, University of Illinois, Chicago
John J. Coy, Robert F. Handschuh, NASA Lewis Research Center, Cleveland, OH

Abstract:

The authors have developed a method to synthesize the pinion crowned surface that provides a localized bearing contact and a favorable type of transmission error for misaligned gears. A method for generation of the pinion crowned surface by a surface of revolution (it slightly deviates from a regular cone surface) is proposed. Tooth Contact Analysis (TCA) programs for simulation of meshing and bearing contact for misaligned spur gears with the crowned pinion have been developed. A computer graphic program for the display of the pinion crowned tooth surface in 3-D space has also been developed.

Introduction

Involute spur gears are very sensitive to gear misalignment. Misalignment will cause the shift of the bearing contact toward the edge of the gear tooth surfaces and transmission errors that increase gear noise. Many efforts have been made to improve the bearing contact of misaligned spur gears by crowning the pinion tooth surface. Wildhaber[1] has proposed various methods of crowning that can be achieved in the process of gear generation. Maag engineers have used crowning for making longitudinal corrections (Fig. 1a); modifying involute tooth profile uniformly across the face width (Fig. 1b); combining these two functions in Fig. 1c and performing topological modification (Fig. 1d) that can provide any deviation of the crowned tooth surface from a regular involute surface.[2]

The main purpose of these methods for crowning is to improve the bearing contact of the misaligned gears, which addresses only half the problem. The transmission errors of misaligned spur gears are a main source of gear noise. According to the open literature, the influence of gear misalignment on transmission error has not been investigated. Maag's method of topological modification does not determine the relationship between the surface deviations and the transmission errors. Also, the optimal geometry for the pinion crowned surface has not been proposed.

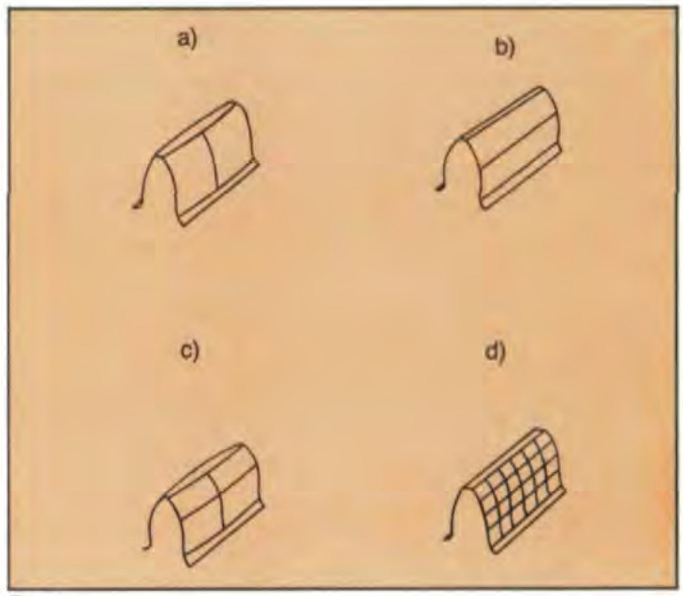


Fig. 1

The contents of this article cover the solutions to the

AUTHORS:

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MR. ROBERT F. HANDSCHUH *works for the U.S. Army AVSCOM Propulsion Directorate, NASA Lewis Research Center, Cleveland, OH, in the area of gear geometry, gear manufacturing and drive train performance. He did undergraduate work in mechanical engineering at Cleveland State University and earned an MS in mechanical engineering from the University of Toledo. Mr. Handschuh is a member of ASME and the National Society of Professional Engineers.*

following problems: optimal geometry of a pinion crowned tooth surface; new method of crowning that is based on application of a tool provided with a surface of revolution (the tool surface is slightly deviated from a cone surface); the development of TCA (Tooth Contact Analysis) programs for the determination of transmission errors for misaligned gears and their bearing contact and a computer graphic program for the display of the crowned surface and bearing contact in 3-D space. The first method of generation that provides the desired optimal pinion tooth geometry is based on the application of a computer controlled machine with five degrees of freedom. The second method for generation needs only a new tool shape and is based on application of the existing equipment.

The development of the optimal geometry of a pinion crowned gear tooth surface is based on the following considerations.

Misaligned spur gears with a pinion crowned tooth surface can provide transmission errors $\Delta\phi_2(\phi_1)$ of two types, shown in Figs. 2a and 2b, respectively. The transmission errors are determined with the equation as

$$\Delta\phi_2 = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \qquad (1)$$

Here: N_1 and N_2 are the numbers of gear teeth; ϕ_1 and ϕ_2 are the angles of gear rotation; $\phi_2(\phi_1)$ is the function that relates the angles of rotation of gears if the pinion is crowned

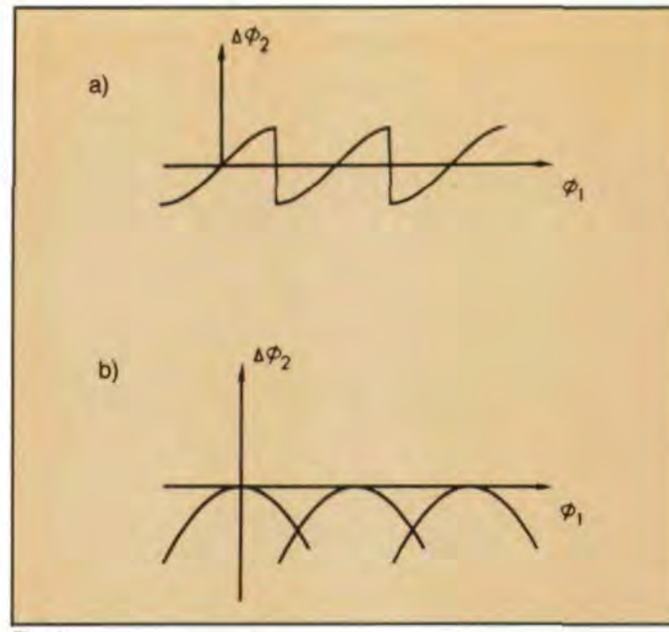

Fig. 2

and the gears are misaligned; $\phi_2^0 = \frac{N_1}{N_2}\phi_1$ is the theoretical relation between the angles of rotation of the gears in the ideal case where the gears are not crowned, not misaligned and the transmission errors do not exist. Type 1 transmission errors are not acceptable because the change of tooth meshing is accompanied by an interruption or interference of tooth surfaces. Type 2 transmission error is preferable if the level of error does not extend the prescribed limit.

At first glance crowning should be directed toward providing an exact involute shape in the middle cross section (Fig. 3). In reality, this type of crowning is not acceptable because the misaligned gears will transform rotation with transmission errors of type 1 (Fig. 2a), but not of type 2 (Fig. 2b). For this reason the authors have synthesized a specific pinion crowned tooth surface. Such a pinion, while in mesh with a gear that has a regular involute surface, is able to provide transformation of rotation with a parabolic type of transmission error function. This type of function of errors is synthesized for ideal gears that do not have any misalignment. Then, the tendency to provide parabolic transmission errors can be extended to misaligned gears and the discontinuance of meshing can also be avoided. Note that the proposed method of synthesis provides a shape in the middle cross section of the tooth that deviates in a certain way from the involute curve that is shown in Fig. 3. The longitudinal deviation from a straight line is not related to the transmission errors, but to the desired dimensions of the instantaneous contact ellipse for the gear tooth surfaces. The proposed pinion tooth surface can be generated by a plane chosen as the tool surface. The motions of the plane with respect to the pinion must be controlled by a computer, and the machine represents an automatic system with five degrees of freedom (first method for generation).

The second method of pinion crowning is based on application of a surface of revolution that slightly deviates from a regular tool conical surface (Fig. 4). Such a tool can be used as a grinding wheel or as a shaver. The motions of the tool

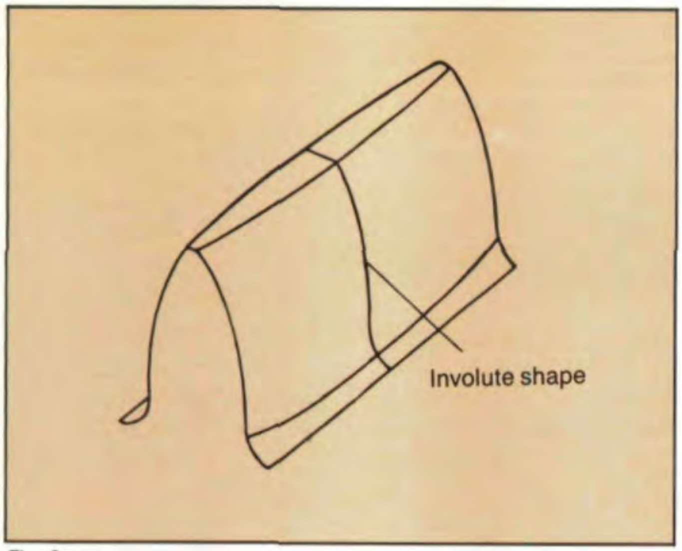

Fig. 3

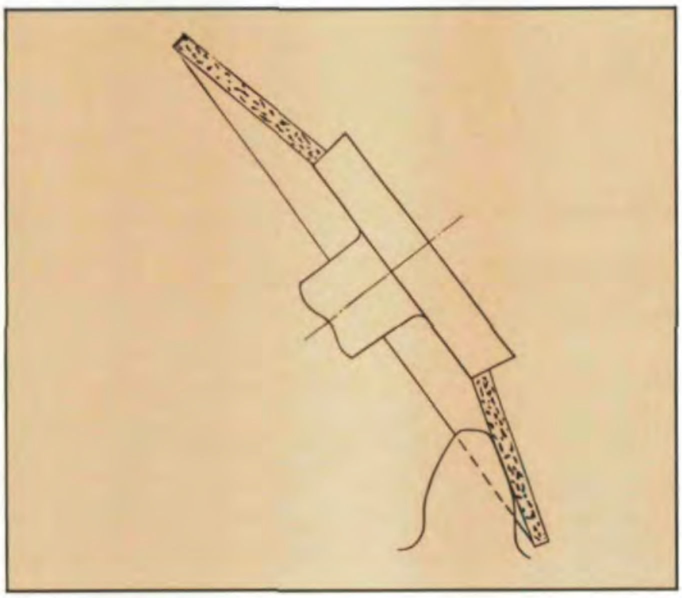

Fig. 4

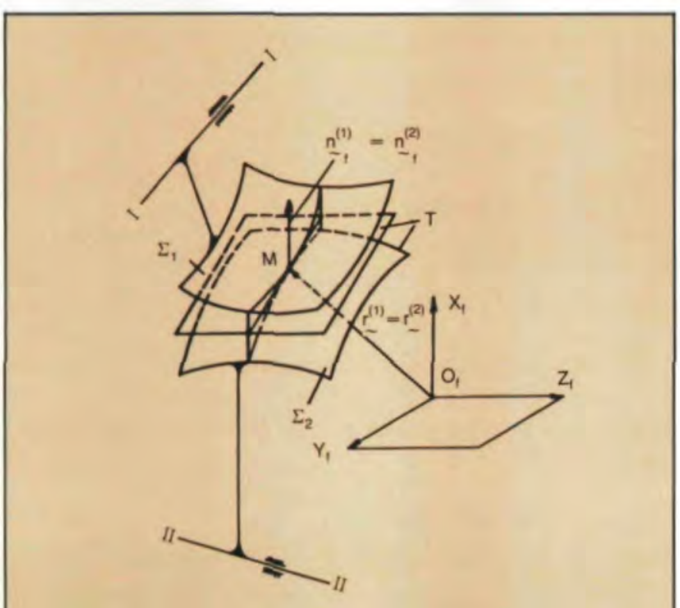

Fig. 5

and the gear being generated are related similarly to the motions of a rack-cutter and the gear. (See Section 3.) A tool with a regular conical surface can generate a pinion crowned surface whose middle cross section represents an involute curve (Fig. 3). However, this type of pinion crowned surface is not desirable because the misaligned gears provide Type 1 transmission errors (Fig. 2a). Therefore, a tool with *a surface of revolution instead of a conical surface is used.*

The evaluation of the bearing contact and transmission errors for the misaligned gears, as well as the investigation of the influence of errors of gear assembly, needs the application of TCA programs. The programs are based on the following algorithm:

(a) The contacting gear tooth surfaces are represented in a fixed coordinate system, S_f, that is rigidly connected to the gear housing (Fig. 5).

(b) The continuous tangency of gear tooth surfaces is provided if the position vectors and surface unit normals for the contacting surfaces coincide at the contact point at any instant. Then we are able to determine the path of contact on the gear tooth surfaces and the relations between the angles of rotation of the output and input gears. Knowing function $\phi_2(\phi_1)$ we can determine the deviations of this function from the prescribed linear function; i.e., the transmission errors.

(c) Due to the elasticity of the gear tooth surfaces, the surface contact is spread over an elliptical area. The dimensions

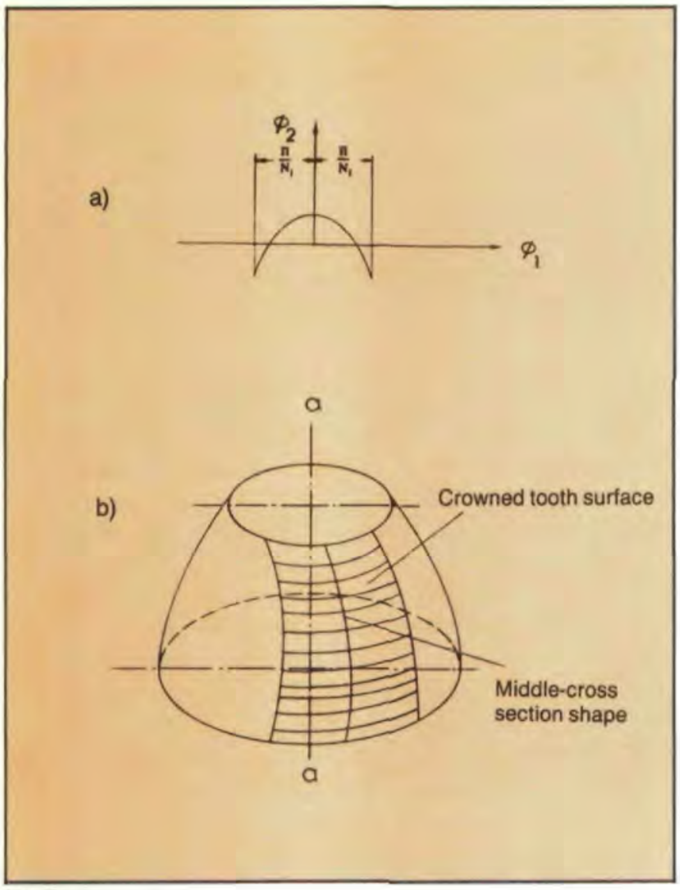

Fig. 6

and orientation of the instantaneous contact ellipse depend on the principal curvatures and the principal directions of the contacting tooth surfaces. The bearing contact is determined by the developed TCA program as the set of the contact ellipses that move over the contacting surfaces in the process of motion.

Synthesis of Pinion Crowned Tooth Surface

Consider that the gear is provided with a regular involute surface. The pinion will be provided with a crowned surface. The shape of this surface in the middle cross section is synthesized on the basis of the following considerations: Two shapes – a regular involute curve of gear 2 and the to-be-determined shape of the pinion tooth surface – are in mesh in the middle cross section. Although the gears are not misaligned, their shapes, being in mesh, must transform rotation with the function (Fig. 6a).

$$\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + \Delta\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + b - a\phi_1^2 \qquad (2)$$

Here:

$$\Delta\phi_2(\phi_1) = b - a\phi_1^2 \qquad (3)$$

$\Delta\phi_2(\phi_1)$ is a parabolic function that must satisfy the following equation:

$$\int_{-\frac{\pi}{N_1}}^{\frac{\pi}{N_1}} (b - a\phi_1^2)d\phi_1 = 0 \qquad (4)$$

Here $\frac{2\pi}{N_1}$ is the angular distance between two pinion neighboring teeth. Equation 4 states that the arithmetic average of the transmission error $\Delta\phi_2(\phi_1)$ over the interval $(-\frac{\pi}{N_1}, \frac{\pi}{N_1})$ is equal to zero.

After some transformation we obtain

$$\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + d\left[\tfrac{1}{3} - \left(\frac{N_1}{\pi}\right)^2 \phi_1^2\right] \qquad (5)$$

The magnitude of d represents the level of transmission error. Using methods of synthesis of planar gears[3] we may determine the sought-for shape of the pinion middle cross section.

The longitudinal shape of the pinion crowned tooth surface may be determined from the requirements of the contact ellipse. The authors propose representing the pinion crowned tooth surface as a surface of revolution that can be generated by rotation about its axis a-a (Fig. 6b).

The advantage of the proposed geometry of the crowned pinion tooth surface is that the gears, while misaligned, have a parabolic type of transmission error, and the discontinuance of meshing can be avoided.

An example is given below to demonstrate the results that can be obtained through use of the proposed crowning method.

Example 1

Given: numbers of teeth: $N_1 = 20$, $N_2 = 40$; diametral pitch, $P = 10 \frac{1}{in}$; pressure angle $\Psi_c = 20°$. The pinion tooth surface has been designed as a crowned surface with a parabolic transmission error maximum $d = 2$ arc seconds in the aligned condition. The developed TCA program has been applied for the evaluation of transmission errors for the following misalignments:

(i) The change of the center distance is $\frac{\Delta c}{c} = 1\%$. The gear axes are not parallel, but crossed, and the screw angle is five arc minutes. The function of transmission errors caused by the misalignments mentioned above is of a parabolic type, and it is represented in Table 1. The maximal value of transmission errors is 1.2 arc seconds.

(ii) The gear axes are not parallel, but intersected, and form the angle $\alpha = 5$ arc minutes. The function of transmission errors is of a parabolic type, and the maximal value of transmission errors is 2.0 arc seconds (Table 2).

Generation of Pinion Crowned Tooth Surface by a Tool With a Surface of Revolution

Fig. 7 shows the installment of a tool with a regular cone surface. The cone surface is tangent to a plane which is the surface of a rack cutter. We can imagine that the two tools – a cone and a rack cutter – being rigidly connected, generate a crowned pinion surface and a regular gear involute surface, respectively. In the process of generation the rack cutter and the cone perform a translational motion, while the pinion and the gear rotate about their axes (Fig. 8). The rotation of the cone about its axis, c-c, is not related to other motions that have to be provided for the tooth surface generation. The angular velocity in the rotational motion of the cone depends on the desired velocity of cutting. The tool for the crowning of the pinion can be designed as a grinding wheel or as a shaver. The opposite sides of the pinion tooth are generated separately.

The described process of the crowning of the pinion by a regular cone provides an involute shape for the pinion tooth surface in its middle section. The crowned pinion and the involute gear, if they are not misaligned, can transform rotation without transmission errors, and their bearing contact can be localized. However, the misaligned gears will transform rotation with Type 1 transmission errors (Fig. 2a). To avoid the discontinuance of tooth surfaces that occurs at the change of teeth in meshing, a surface of revolution must be used instead of a cone surface. This surface slightly deviates from a regular cone surface and its application for crowning provides Type 2 transmission errors (Fig. 2b). Also,

Table 1
Function of Transmission Errors

ϕ_1 in degrees	−3	0	3	6	9	12	15
$\Delta\phi_2$ in arc seconds	−0.71	0.00	0.40	0.50	0.33	−0.09	−0.74

Table 2
Function of Transmission Errors

ϕ_1 in degrees	−10	−7	−4	−1	0	2	5	8
$\Delta\phi_2$ in arc seconds	−2.02	−0.92	−0.24	0.01	0.00	−0.16	−0.75	−1.78

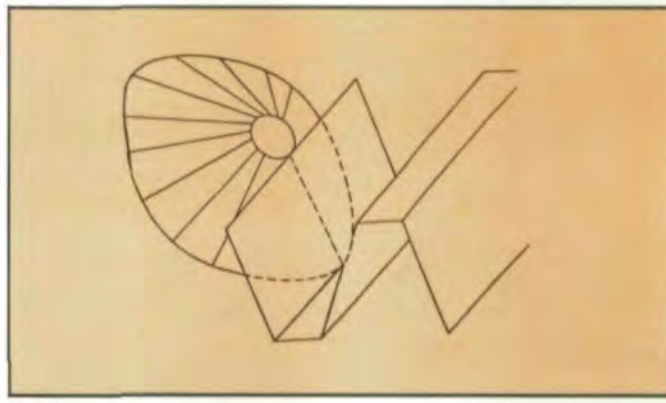

Fig. 7

knowing the topology of the pinion crowned surface that is generated by the surface of revolution, we can generate it by a plane. The conditions of gear meshing and their bearing contact have been simulated by the TCA program that has been developed by the authors. An example is shown below to demonstrate the concepts developed in this article.

Example 2

The input is the same as in Example 1. The pinion tooth surface is crowned by a surface of revolution with the following parameters (Fig. 9): $\Theta = 20°$, $R = 500''$. The misalignment of gears has been simulated and the transmission errors have been evaluated by the developed TCA program with the following results. The gear axes are not parallel, but crossed, and the screw angle is $\alpha = 10$ arc minutes. The change of the center distance is $\frac{\Delta c}{c} = 1\%$. The function of transmission error is of a parabolic type, and the maximal value, 0.35 arc seconds (Table 3).

The gear axes are intersected and form an angle, $\alpha = 10$ arc minutes. The function of transmission error is of a parabolic type and its maximal value is 0.34 arc seconds (Table 4).

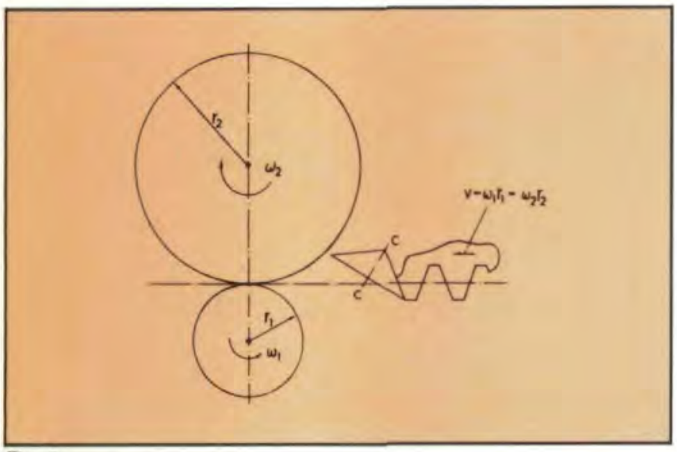

Fig. 8

Table 3
Function of Transmission Errors

ϕ_1 in degrees	−11	−8	−5	−2	0	1	4	7
$\Delta\phi_2$ in arc seconds	−0.26	−0.10	0.00	0.02	0.00	−0.02	−0.14	−0.33

Table 4
Function of Transmission Errors

ϕ_1 in degrees	−11	−8	−5	−2	0	1	4	7
$\Delta\phi_2'$ in arc seconds	−0.33	−0.15	−0.03	0.01	0.00	−0.02	−0.11	−0.20

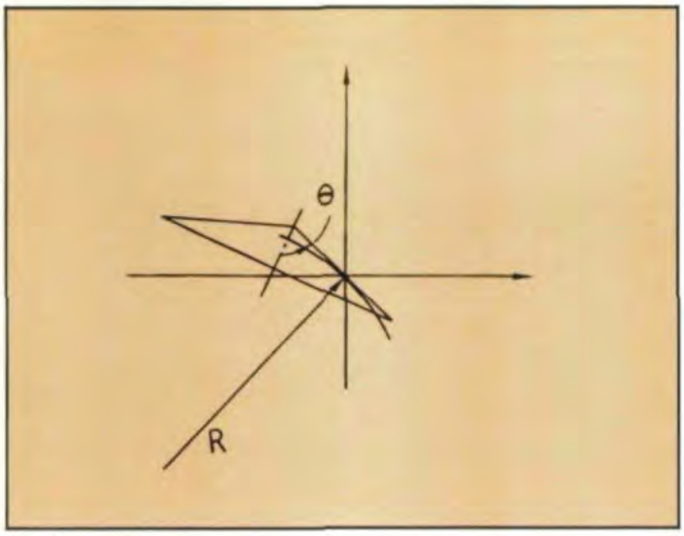

Fig. 9

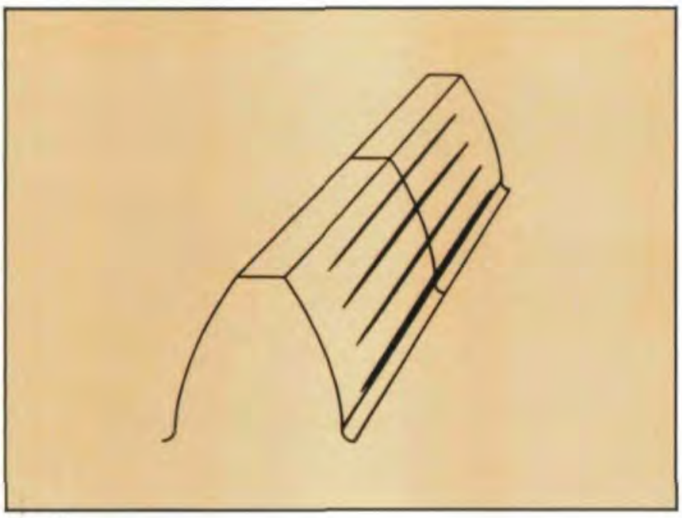

Fig. 10

Display of Analytical Results by Computer Graphics

A computer graphics program was developed to represent in 3-D space the pinion crowned tooth surface and the motion of the contact ellipse in the process of meshing.

The graphics program is based on the analytical solutions that have been obtained from the TCA program. It produces the intricate high resolution picture on a graphic terminal and laser printer. Fig. 10 shows the results of the computer graphic program that represents the pinion crowned surface and the location and orientation of the contact ellipses.

Conclusion

The authors developed:

- A method for the synthesis of a pinion crowned surface that provides a localized bearing contact and a limited level of transmission error of a parabolic type. This approach provides favorable conditions of meshing and contact for misaligned spur gears.
- A method for generation of a pinion crowned tooth surface by a surface of revolution that slightly deviates from a regular cone surface. This method can be applied for crowning by grinding and shaving.
- TCA programs to simulate the meshing and bearing contact of misaligned spur gears with the crowned pinion and to investigate the influence of misalignment on the transmission errors.
- A computer graphic program to display in 3-D space the pinion crowned tooth surface and the location and orientation of the contact ellipses.

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Calculation of Optimum Tooth Flank Corrections for Helical Gears

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Abstract:

Stricter requirements are being imposed on heavy duty gears in terms of running behavior and load carrying capacity. Furthermore, the expanding techniques in the field of gear manufacturing are demanding the development of suitable designing methods for tooth flank corrections. In order to calculate these corrections, the spatial stress and deformation state in the mesh has to be determined. This article reports on the further development of a finite element calculation method into an optimizing and designing system for flank corrections on spur and helical gears.

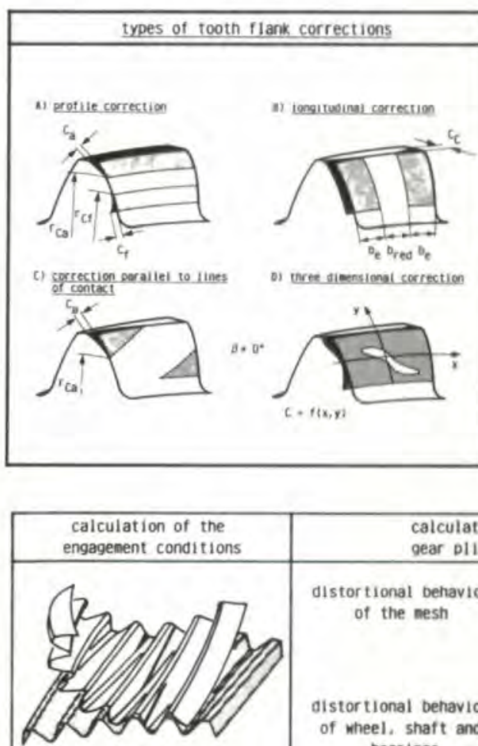
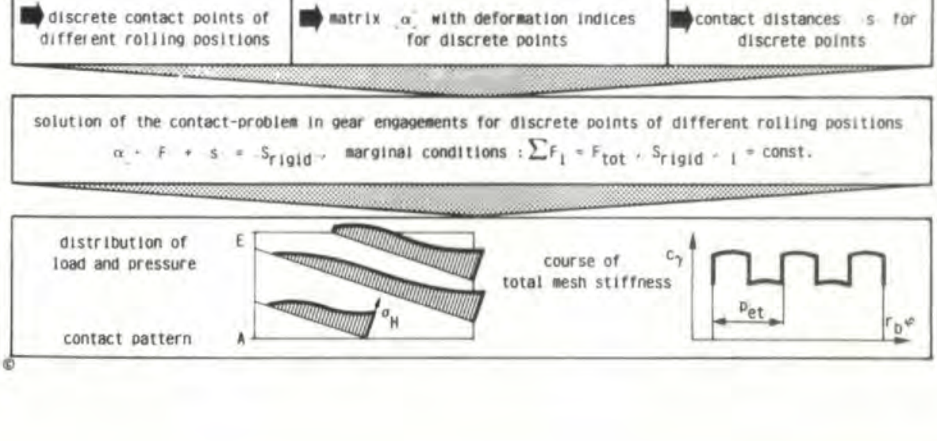


Fig. 1 – (upper left) Objectives of tooth flank corrections:
- reduction of pressure peaks
- decrease of displacement sensivity
- diminishing of engagement shocks
- lessening of parametric excitation

Fig. 2 – (below) Basic principles of contact analysis of loaded gears.



Main Influences on the Bearing Capacity and Running Behavior of Gears

The load carrying behavior of gears is strongly influenced by local stress concentrations in the tooth root and by Hertzian pressure peaks in the tooth flanks produced by geometric deviations associated with manufacturing, assembly and deformation processes. The dynamic effects within the mesh are essentially determined by the engagement shock, the parametric excitation and also by the deviant tooth geometry.[1]

The engagement shock results from a displaced starting point of engagement due to rotational deviations or pitch errors within the gear system. This transferred start of engagement is located outside the plane of action. Here deviations occur in the value and direction of the normal velocity components of the contacting tooth flanks; thus, vectorial difference

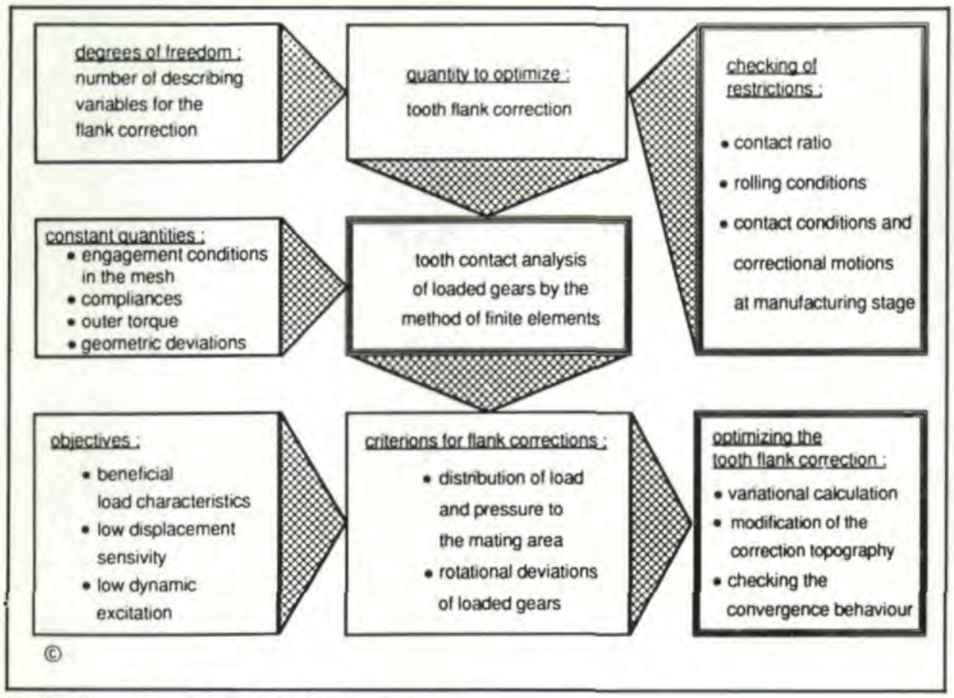


Fig. 3 – Optimization of tooth flank corrections.

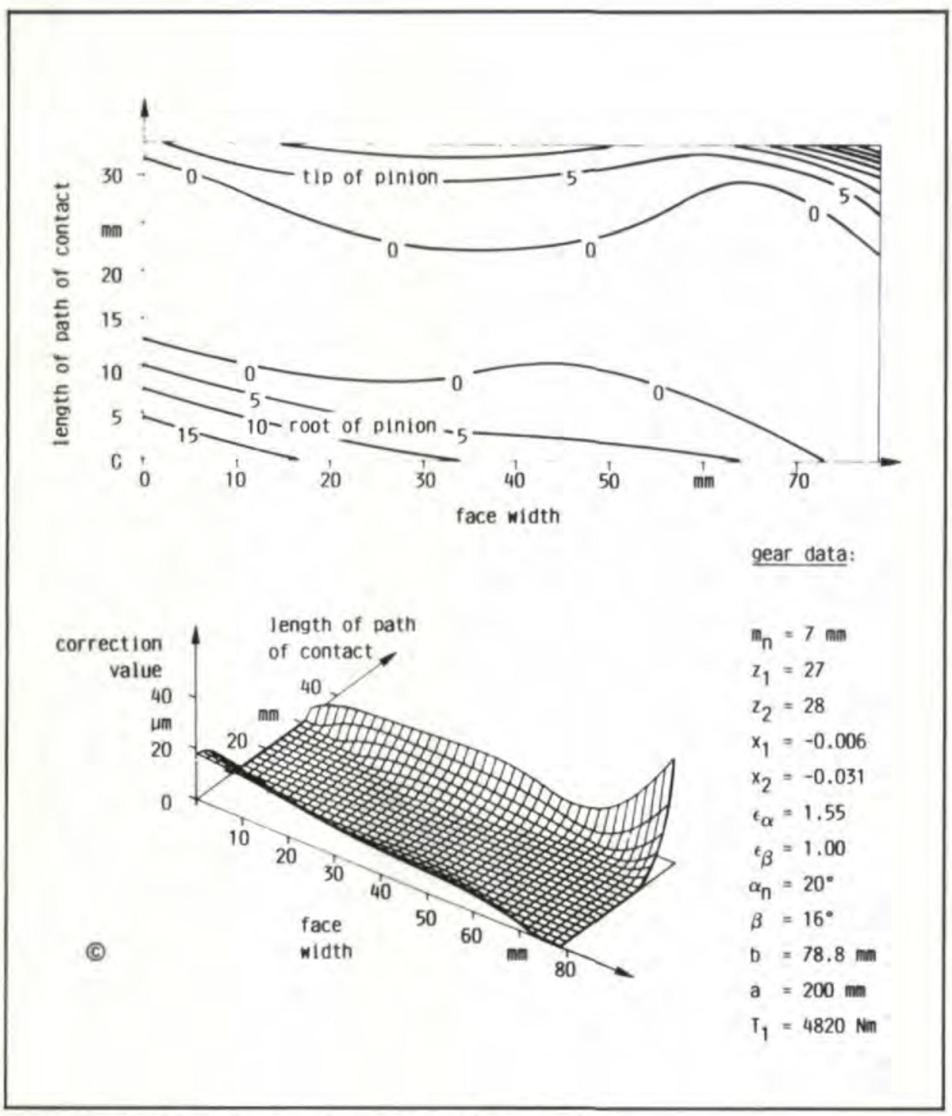


Fig. 4 – Optimized three-dimensional correction with basic gear data.

brings about an unwanted impact speed and shock activation.[2]

The parametric excitation is a consequence of the changing number of mating teeth and the contact line movement over the tooth flank during engagement. As a result, a periodic modulation of the mesh stiffness follows and induces rotational deviations and activates unwanted vibrations, even when the outer load for the gear is constant.[3]

Geometry of Flank Corrections for Helical Gears

The flank geometry of helical gears can be modified in various ways with respect to manufacturing requirements and the differential improvements on the bearing capacity and dynamic behavior. Fig. 1 gives an overview of the principle correction forms.

Flank corrections in the direction of tooth depth as shown in Fig. 1a are simply carried out by involute tip or root reliefs. In this case, the part of the flank to be taken back consists of a corrected involute profile, which is defined by a modified base circle diameter and by the intersection point with the uncorrected involute curve. This is achieved by tools having a basic rack system with altered profile angles or by using a modified working pitch diameter at generation. Another form of profile correction which has a smooth transition to the original, uncorrected involute curve is manufactured by tools with crowned basic rack profiles. Profile corrections are mostly used to decrease the engagement shock and the in-

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DIPL.–ING. GEORG MAUER *received his degree in mechanical engineering at the Technical University of Aachen and is currently working as a scientific assistant in the gear research group of WZL.*

volved strain and noise.

Longitudinal flank corrections in the direction of the face width (Fig. 1b) are used to attain a low displacement sensitivity and to avoid strain peaks occuring due to displaced positions of the gear axis (twisting, tilt) or helix deviations of the flanks.

For helical gears, a correction running parallel to the lines of contact is advantageous. (See Fig. 1c.) The intersection line of the corrected and uncorrected area is identical to a line of contact between the mating teeth. The maximum correction value is found at the end or starting point of the engagement. The corrections are carried out by helical involute areas which are mathematically defined by a modified base circle and an additional incremental change made in the helix angle. This form of flank correction has the advantage of producing a smaller loss in the contact ratio, so that the geometric correction may have a greater rolling length and more beneficial normal vector conditions.

The correction forms described above are to a certain extent limited in optimizing the running behavior and load carrying capacity, because the flank corrections do not meet the gear geometry and mating conditions adequately. These conditions can be met by using three dimensional (often called topologic[4]) flank corrections (See Fig. 1d.), which are characterized by variable correction forms in both directions of tooth depth and face width.[5] There are, however, various marginal conditions which have to be taken into consideration, these being the continuity of differentials, minimum contact ratio, rolling conditions and manufacturing processes.[1]

Optimizing Method Based on Finite Element Calculations

To obtain the required starting position to optimize flank corrections, one has to return to the basic principles of contact analysis of loaded gears illustrated in Fig. 2. The continuous, spatial load deformation problem is replaced by a system of discrete contact points, defined by a matrix with deformation indicies [α], a load vector {F} and a vector containing contact distances {s}, which describes the geometric flank deviations or corrections. The equation system is then set up for different rolling positions and is solved by considering the marginal conditions concerning the total load and the rigid body

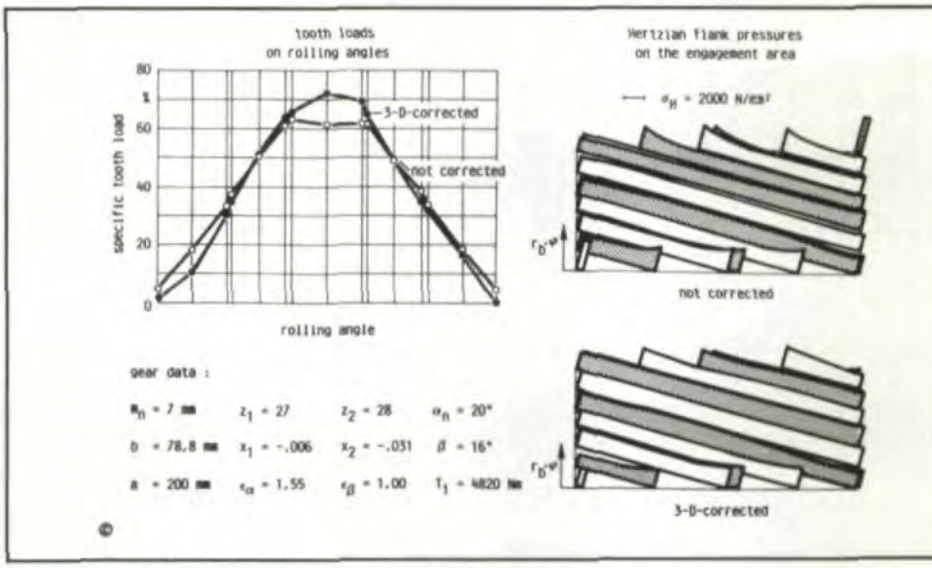


Fig. 5 – Calculated tooth loads for a full cycle of meshing contact.

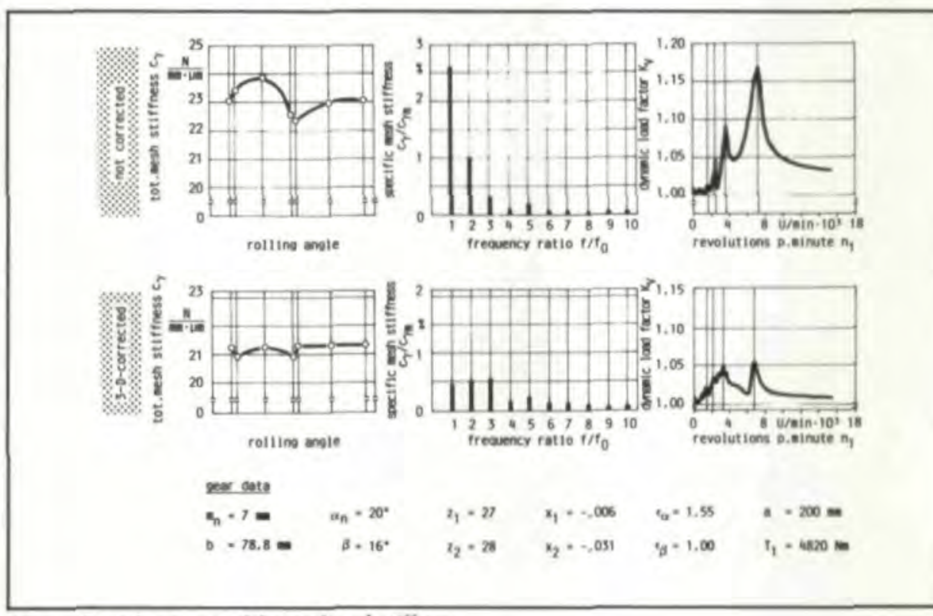


Fig. 6 – Periodic courses of the total mesh stiffness.

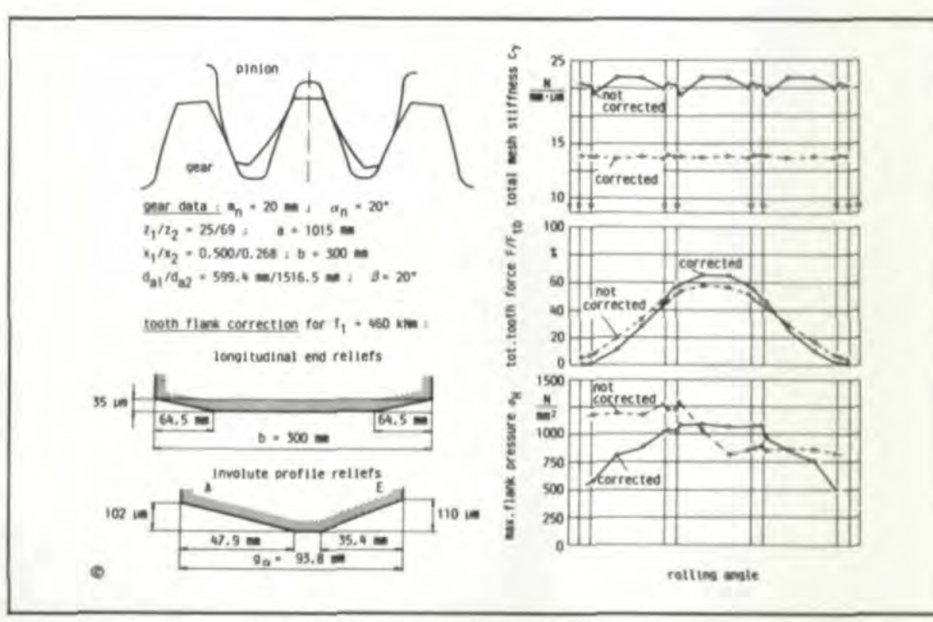


Fig. 7 – Samples of geometrically simple correction forms.

displacement.[6] The results provide information on the load and pressure distribution on the field of action and the shape of the rotational deviation curves, which is influenced by the alternating mesh stiffness. It becomes clear, therefore, that load distribution and mesh stiffness are direct functions of the correction topography. The pliability indicies $[\alpha]$ are calculated by finite element structures of the mesh and analytical models of shafts and bearings.

This contact analysis is the central starting point to optimize tooth flank corrections. (See Fig. 3.) By varying the correction topography, a beneficial load and pressure distribution on the flanks, a low displacement sensitivity and a lower dynamic activation can be achieved.

The contact and deformation analysis, as described above, presents all the criteria necessary to assess a particular correction with regard to the various objectives. The mathematical variation provides information regarding the trends and dependent variables of the topography and the target objectives, so that the correction can be improved in a stepwise fashion. This numeric process has to be repeated several times after checking that all the restricting marginal conditions are met.

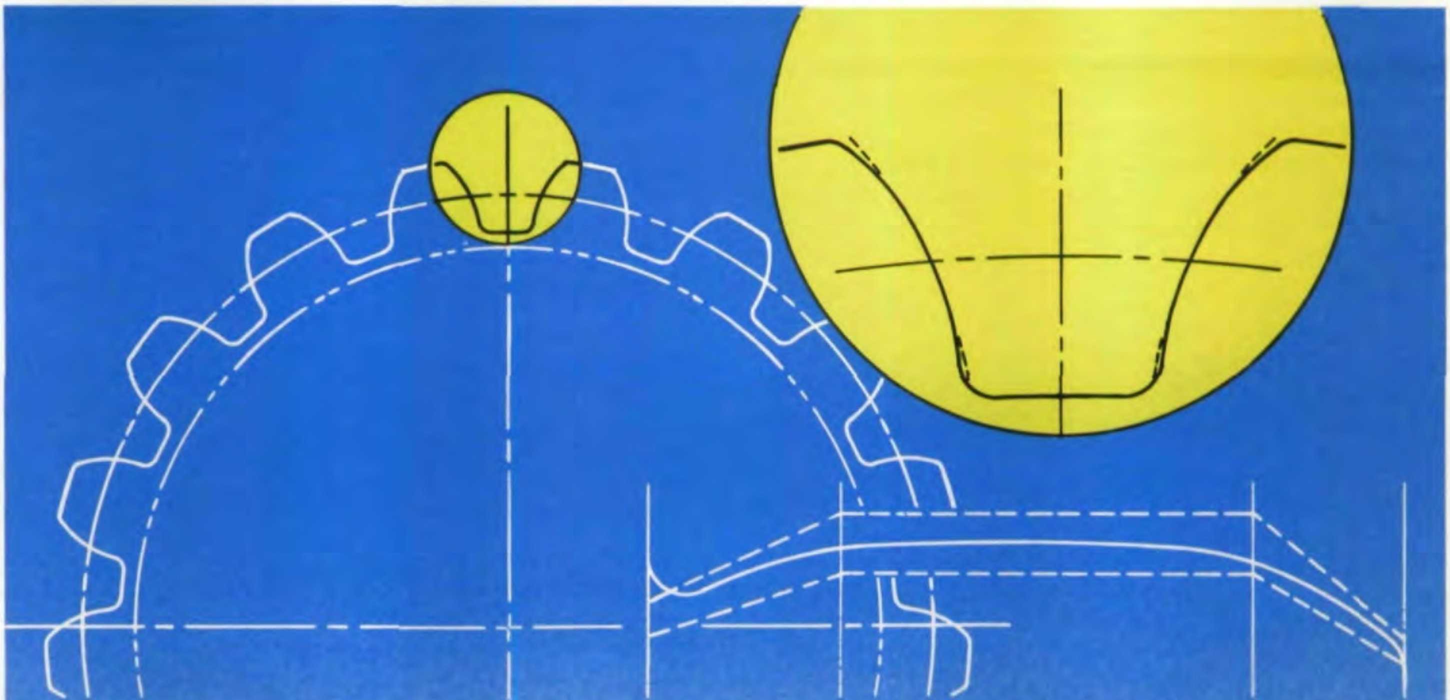
Other research developments at WZL concern the simulation of grinding processes for corrected tooth flanks, whereby correction movements and contact conditions are analyzed. Control data for NC grinding machines are also generated.[7]

Calculated Examples

The following section contains the calculation results of two flank corrections for typical helical gears in industrial use. Fig. 4 shows an optimized three-dimensional correction with the basic gear data. The correction values are drawn up on the plane of action with a maximum relief of approximately 40μm. In the middle of the engagement area an uncorrected region ensuring a contact ratio greater than 1.0 can be seen.[1] The correction topography is observed to run parallel to the original involute flank at the approach contact, which is favorable for lower excitation produced by engagement shocks.

The calculated tooth loads for a full cycle of meshing contact are illustrated in Fig. 5. A lower force level at the start of engagement and a shallower load reception gradient for the corrected gear can be seen clearly. The pressure distribution on the plane of action depicts lower strains for the corrected gear, and the pressure peaks have been successfully reduced where the contact lines end at the root of one of the mating teeth.

In Fig. 6 the periodic courses of the total mesh stiffness are drawn up on one base pitch. The specific range of variation which is characteristic for the intensity of the parametric excitation could be reduced by the three-dimensional flank correction from 3.3% to 0.9%. The simulation of the dynamic behavior of the gear at various rotational speeds shows that the maximum dynamic load factor K_v could be decreased from 1.17 to 1.06. This is achieved by depressing the maximum resonance peaks as well as the first and second order Fourier coefficients of the



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mesh stiffness course.

Clear improvements of the dynamic and bearing behavior can be achieved also by geometrically simple correction forms. (See Fig. 7.) Here the results of the tooth contact analysis of a loaded helical gear for the corrected and the uncorrected case are illustrated. The tooth flank correction consists of longitudinal end reliefs and two involute profile corrections. The relief lengths and values were optimized in the method described previously.

The corrected gear shows several optimized characteristics. These are

- a much smoother mesh stiffness curve with a smaller specific variational range,
- a reduction of the force levels at the beginning and end of engagement,
- a lower load reception gradient,
- a well-balanced and lower pressure course.

The unmodified area on the middle flanks of the corrected gear is sufficient to ensure that a minimum contact ratio of 1.10 is achieved; thereby guaranteeing a smooth operation, free from kinematic and dynamic disturbances.

Conclusions

A method is given for optimizing various forms of tooth flank corrections with regard to the load carrying capacity and the running behavior of spur or helical gears. Two examples for corrected gears confirm that it is possible to optimize flank corrections at the designing stage and to make empirical attempts unnecessary.

Even when flanks are corrected by simple geometric modifications which do not require advanced manufacturing methods, the gear characteristics can be improved so that an increased transferable power and a reduction in noise emission is achieved. A more complex correction topography allows for an optimum conformation with the gear geometry and the engagement conditions, while, at the same time, keeping the correction topography within manufacturing restrictions.

It becomes clear that this new calculation makes it possible to influence all essential effects which determine the dynamic and load carrying behavior in a specific way.

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Enhanced Product Performance Through CBN Grinding

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Abstract:

The extraordinary physical and thermal properties of CBN abrasives are a primary factor in the generation of beneficial residual compressive stresses. These residual stresses have a favorable influence on the fatigue life of components ground with CBN abrasives. This article suggests that the effects of CBN grinding should be factored into the original design of highly stressed, fatigue-prone drive train components.

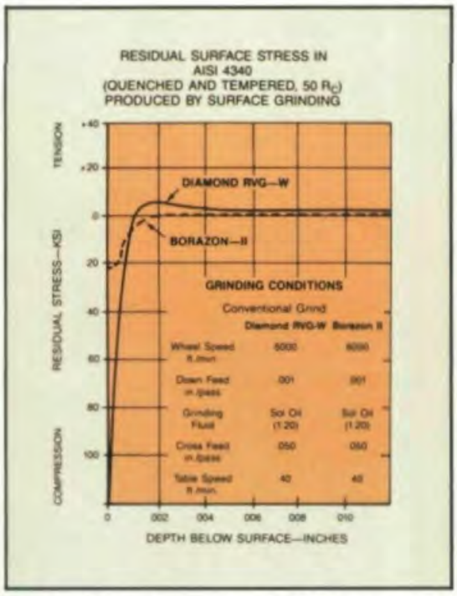

Fig. 1a – Residual stresses in SAE 4340 after grinding with CBN and diamond.

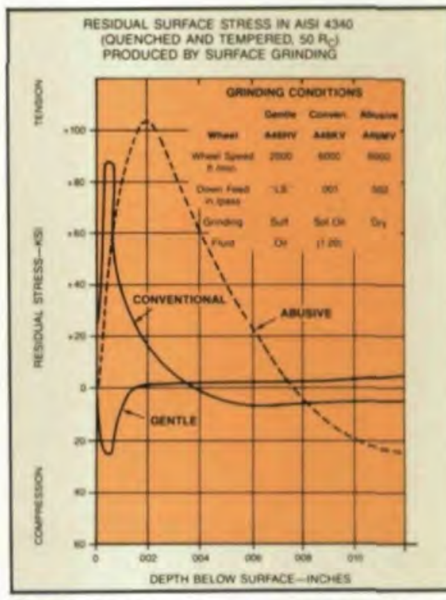

Fig. 1b – Typical residual stresses in SAE 4340 after grinding with alumina.

Introduction

Modern manufacturing processes have become an ally of the product designer in producing higher quality, higher performing components in the transportation industry. This is particularly true in grinding systems where the physical properties of CBN abrasives have been applied to improving cycle times, dimensional consistency, surface integrity and overall costs. Of these four factors, surface integrity offers the greatest potential for influencing the actual design of highly stressed, hardened steel components.

The purpose of this article is to review both the empirical studies and theoretical analyses which substantiate the surface integrity characteristics inherent with CBN grinding. In addition, some important new empirical findings of direct interest to this subject are presented and discussed.

Background

Significant evidence empirically substantiates the fact that grinding with CBN abrasives can produce a significant level of compressive residual stress in the surface of hardened steels. Most bibliographies include the work of Navarro,[1] the first to establish that both CBN and diamond abrasives produce residual compressive stresses.

Navarro's key results are shown in Figs. 1a, b and c. Figs. 1a and 1b contrast typical residual stress distributions found after grinding with CBN, diamond and aluminum oxide. The positive consequences of CBN grinding are illustrated in 1c, where the bending fatigue characteristics of AISI 4340 samples ground with three different abrasives are compared. In recent years, several other studies [2-5] have served to substantiate the work first reported by Navarro.

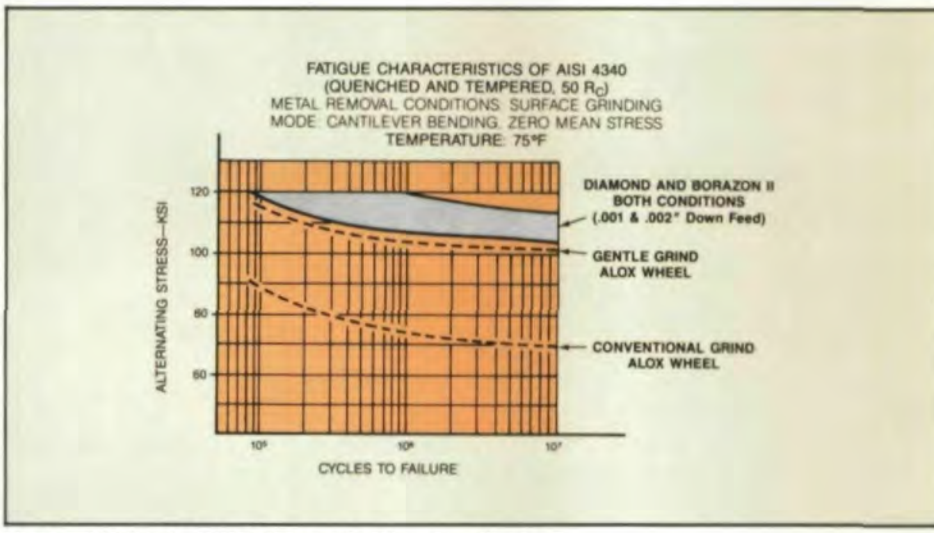

Fig. 1c – S-N data for SAE 4340 ground with various abrasives.

However, little work has been done to increase our fundamental understanding of the mechanism by which these residual compressive stresses are generated. Production grinding of steel components in the metalworking industries has been dominated by the use of aluminum oxide abrasives for over 80 years. Therefore, the total context of our thinking about the grinding process is oriented around aluminum oxide abrasive grains and their specific properties. For example, "grind-

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ing" will generally leave a residual tensile stress in the surface of a workpiece, such as AISI 4340, unless special precautions are taken to avoid this phenomenon.[6] It is also generally agreed that these residual tensile stresses are created because the "grinding" process rapidly increases the temperature of the ground surface, which is then subjected to various rates of quenching, depending upon the specific nature of the operation. This process can lead to the generation of both untempered and over-tempered martensite as well as surface cracking. Expensive and tedious procedures, such as low stress grinding (LSG), nital etching or shot peening, may have to be employed in certain cases to overcome these inherent problems.

That the crucial importance of the specific thermal properties of aluminum oxide grain have been overlooked is understandable; but in view of the growing importance of CBN grinding, comparison with the extraordinary thermal properties of CBN abrasives must now be made.

Selected physical properties of CBN and aluminum oxide abrasives are shown in Table I. The density (ϱ), thermal conductivity (k), and specific heat (c) are listed along with the calculated value of thermal diffusivity. Thermal diffusivity (a thermophysical property, $k/\varrho c$), is the ratio of heat conducted versus the heat absorbed in a body.[7] In transient situations, such as the grinding process, a high value means that much heat is transmitted through the abrasive relative to the heating of the abrasive itself. As reflected in Table I, the thermal diffusivity of CBN is almost two orders of magnitude greater than that of aluminum oxide.

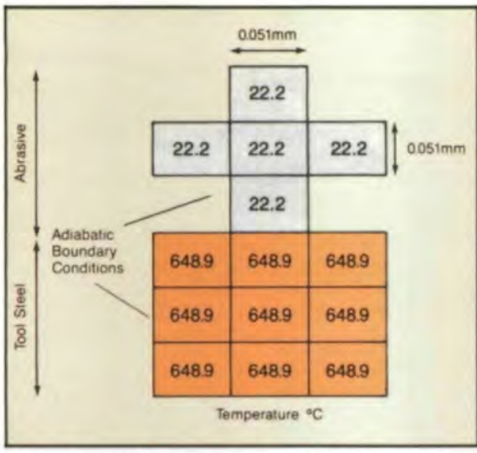
In order to investigate the signficance of these properties on temperatures generated in the workpiece surface, Shaw and Ramanath [8] have determined the fraction of grinding energy (R) going into the workpiece to be

$$R = \frac{1}{1 + \left\{\dfrac{k\varrho c \text{ abr.}}{k\varrho c_{\text{work}}}\right\}^{0.5}}$$

Thus, the fraction of heat generated in the grinding process which flows down into the work is governed by the ratio of the products, $k\varrho c$ of abrasive/$k\varrho c$ of work.

For aluminum oxide abrasive, the calculation shows $R = 0.76$ and for CBN, $R = 0.37$. These writers demonstrate that this difference in R is sufficient for the aluminum oxide grinding process to generate temperatures well in excess of the softening temperature of steel, while CBN grinding will not reach such temperature. This analysis strongly suggests that the aluminum oxide grinding process subjects the workpiece surface to a severe thermal disturbance in addition to the normal mechanical process of chip formation. CBN grinding, on the other hand, may only subject the workpiece surface to the normal mechancial disturbance with minimal thermal disturbance.

Johnson [9] illustrates the importance of the extraordinary differences in thermodynamic properties by use of a simple finite element analysis. In a typical grinding process, any single abrasive grain will be in contact with the work for only 80 microseconds. The analysis examines the temperature distribution in both an abrasive grain and a steel workpiece during the 80 microseconds after a grain at room temperature is placed in contact with a steel workpiece at 648°C. It must be noted that Johnson has not in any way attempted to simulate the complex heat generation and



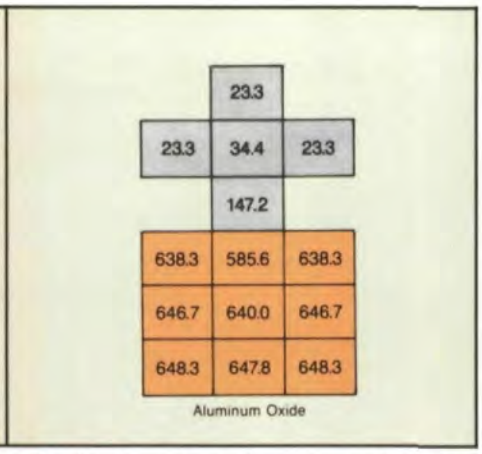


Fig. 2a – Finite element analysis model of heat transfer between workpiece and abrasive.

Fig. 2b – Comparison of temperature distribution after 80μsec of contact between workpiece and abrasive for three types of abrasives.

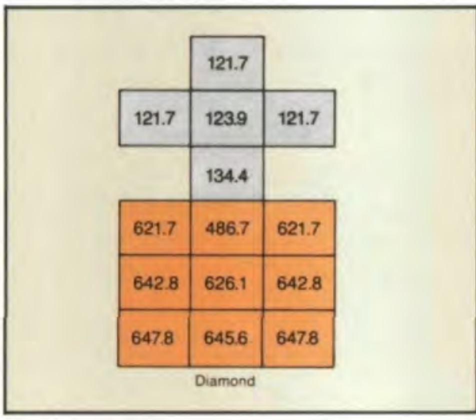


Fig. 2b

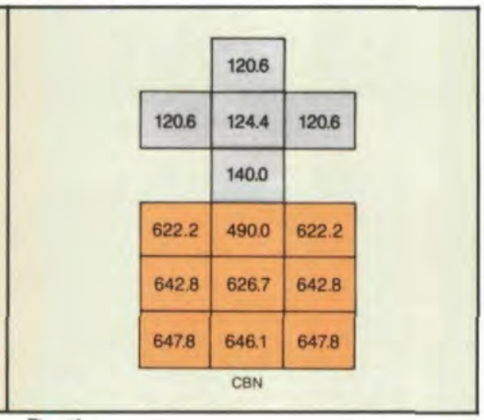


Fig. 2b

Table I
SELECTED PHYSICAL PROPERTIES OF CBN AND ALUMINA ABRASIVES

Property/Units	BORAZON* CBN	Aluminum Oxide	Ratio
Formula	BN	Al_2O_3	
Knoop Hardness, (kg/mm.^{2})	4500	2100	2:1
Density, (gm/cm^3)	3.45	3.97	1:1
Thermal Cond. @ (298°K),(W/m°K)	1300	35	37:1
Specific Heat @ (298°K),(J/kg°K)	506.2	774.9	2.3
Therm. Diff. @ (298°K),(m^2/s) × 10^5	74.4	1.14	65:1

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Table II
DESCRIPTION OF GRINDING WHEEL AND CONDITIONS USED IN SAMPLE PREPARATION

GRINDING WHEEL	**Size:** 12 in. × 1 in. × 5 in. **Bond:** Phenolic Resin **Abrasive:** Borazon* CBN Type II-100/120 Mesh **Concentration:** 100
GRINDING CONDITIONS	**Machine:** Cincinnati Universal Cylindrical **Grind Mode:** Plunge **Coarse Rate:** 0.060 IPM **Fine Rate:** 0.002 IPM **Wheel Speed:** 5500 SFPM **Work Speed:** 55 SFPM **Coolant:** ADCOOL #3 (1:20)

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heat transfer dynamics of the actual grinding process. The initial conditions are shown in Fig. 2a, while the temperature distribution after 80 microseconds is illustrated in Fig. 2b. This study graphically reinforces the concept developed by Ramanath and Shaw that heat is forced down into the work in the aluminum oxide case, but can flow up into the abrasive grain itself in CBN grinding.

Dodd and Kumar[10] have also studied this problem using yet a different analysis. Their work suggests that 63% of the heat generated in aluminum oxide grinding goes down into the work, while in CBN grinding, only 4% goes into the work. They have concluded that chip formation takes place at a much lower temperature in the case of CBN grinding than in the case of aluminum oxide grinding.

While none of these studies in and of themselves conclusively prove that the extraordinary thermal properties of CBN abrasives are solely responsible for the residual compressive stress phenomenon, they clearly establish that these properties *are the predominant factors.*

Experimental Investigation

Most investigators have conducted empirical residual stress studies using some form of plain surface grinding and flat workpiece specimens. This investigation will utilize cylindrically shaped specimens. In addition, most studies concentrate on the residual stresses obtained on only two or, at the most, three sample surfaces for a given combination of abrasive grain and grinding conditions. This study will utilize a total of 93 individual workpiece samples, all ground with the same CBN wheel specification. This investigation is comprised of the following steps:

1. Sample preparation – Two sets of steel cylinders, 1" in diameter by 5" long, have been carburized, heat treated and quenched to produce a hardness of Rc 62-64. A total of 40 cylinders of SAE 8620 and 51 cylinders of SAE 4620 have been accordingly prepared. Each sample cylinder has been cylindrically plunge ground with a one inch wide wheel containing CBN abrasive. The details of this procedure are shown in Table II. Fig. 3 illustrates the dimensions and configuration of the finished samples.

2. X-ray diffraction residual stress analysis – The surfaces of the samples prepared in Step 1 above have been analyzed

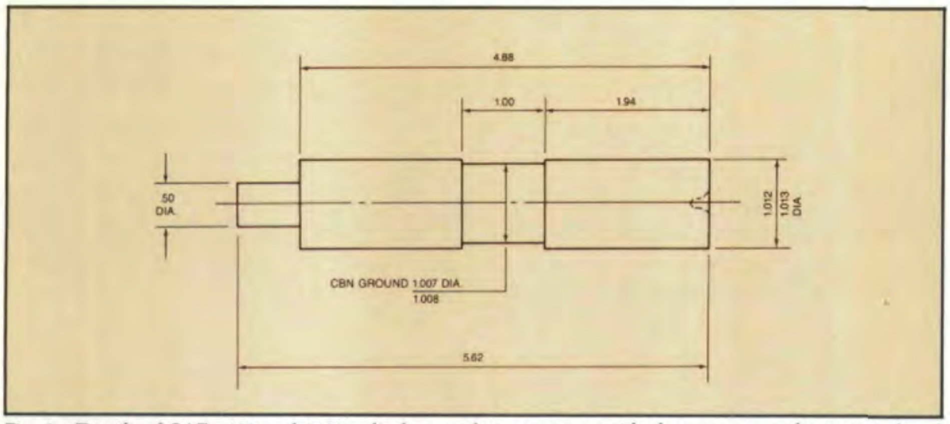

Fig. 3—Details of SAE 4620 and 8620 cylinders used to compare residual stresses — as-heat treated and CBN ground.

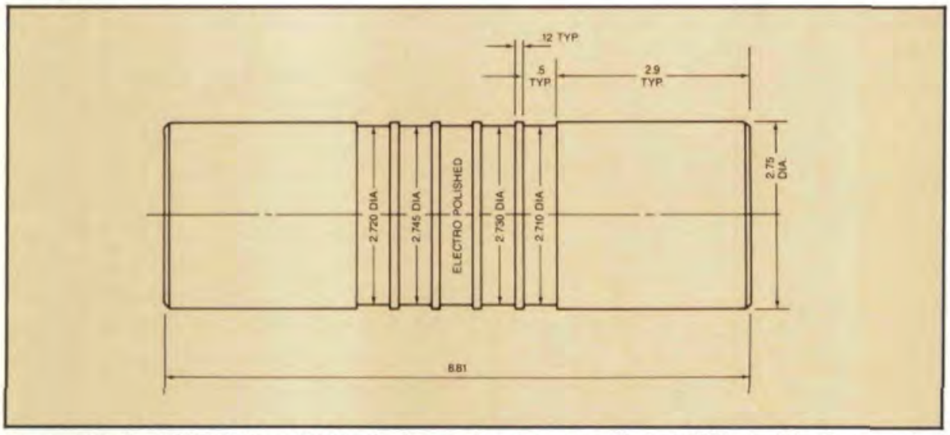

Fig. 4—Details of SAE 4620 and 8620 cylinders used to determine effect of CBN grind depth on residual stresses.

using established techniques of x-ray diffraction analysis. This analysis has been conducted on both the unground and the ground surfaces of each sample. Peripheral and longitudinal stresses have been selectively measured on the unground surfaces. The peripheral stresses in the direction of grinding and longitudinal stresses at right angles to the grinding direction have been measured on the CBN ground surfaces.

3. Effect of grinding depth on residual stresses — The amount of material to be removed in production grinding operations cannot always be precisely controlled. The slight distortions in components which have been heat treated can lead to variations in grinding depths in such cases; thus, determining how residual stress will vary as material is removed from the unground, heat treated surface is important. Therefore, another set of cylindrical samples have been plunge ground to a range of finished diameters in order to determine this effect. The configuration of such samples is illustrated in Fig. 4. The grinding conditions used to produce these *surfaces are the same as shown in Table II* except for the use of a 0.5" wide wheel.

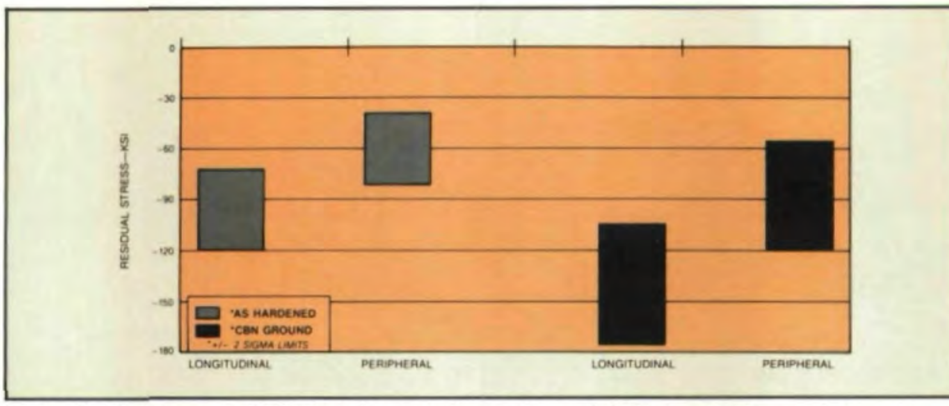

Fig. 5 – Residual stresses in SAE 4620 before and after grinding with CBN abrasives (51 samples).

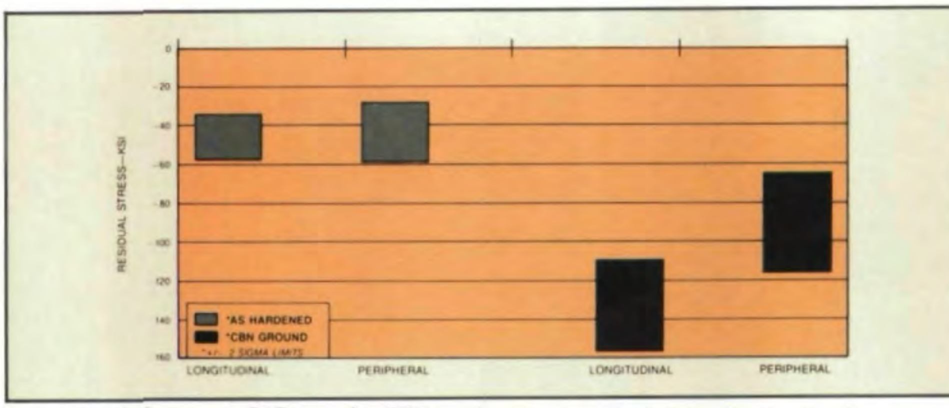

Fig. 7 – Residual stresses in SAE 8620 after CBN grinding to various depths below hardened surface.

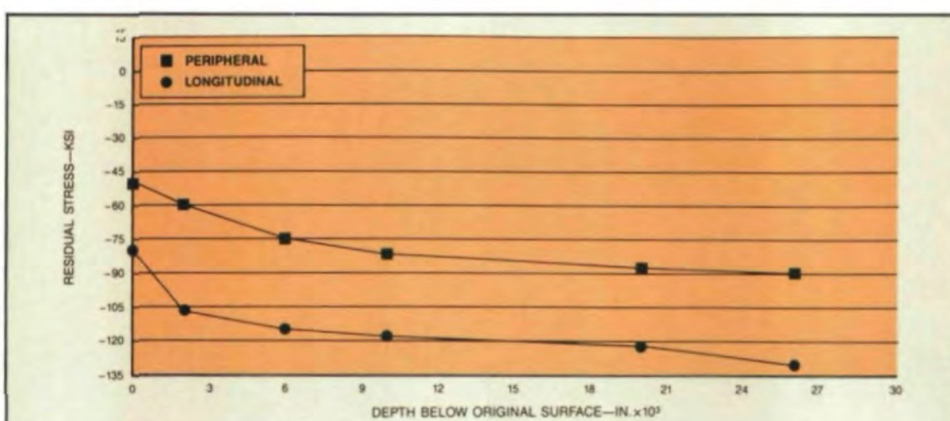

Fig. 6 – Residual stresses in SAE 8620 before and after grinding with CBN abrasives (40 samples).

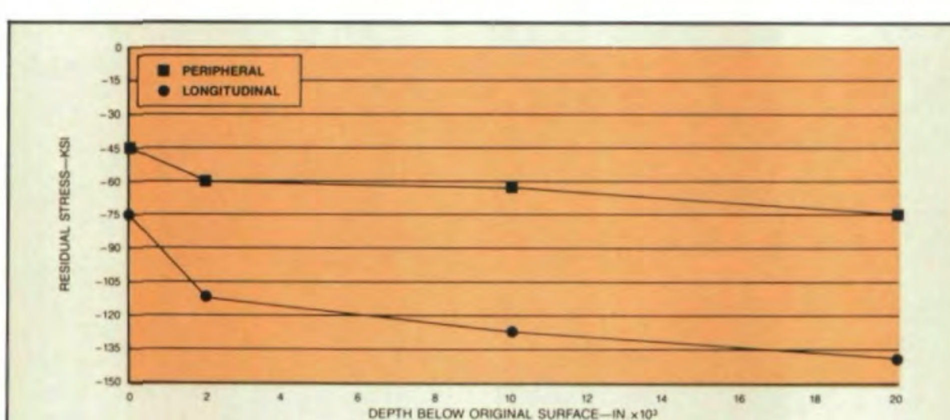

Fig. 8 – Residual stresses in SAE 4620 after CBN grinding to various depths from as-hardened surface.

Results

The results of this work are detailed as follows:

1. Residual stress analysis – The unground and ground SAE 4620 results in ±2 sigma limit bars illustrated in Fig. 5. Analysis of these results using the student "T" test for significance reveals that CBN grinding has had the following effects on the residual stress levels of the ground samples:

4620 – 95% confidence that the longitudinal stress is increased from −96250 PSI to −148600+/−5900 PSI, and the peripheral stress from −61800 PSI to −88120+/−5200 PSI.

8620 – 95% confidence that the longitudinal stress is increased from −45200 PSI to −133300+/−3600 PSI, and the peripheral stress from −43300 PSI to −92700+/−3900 PSI.

2. Effect of grinding depth on residual stress – The effect on both the SAE 8620 and SAE 4620 ground samples are shown graphically in Figs. 7 and 8. In the case of both workpiece types, the consequences of CBN grinding clearly have been to further increase the residual compressive stress as the total grinding depth is increased.

Discussion

The general character of the residual stresses which result from CBN grinding in this work is in agreement with previously reported work. This investigation, however, has shown clearly that the residual compressive stresses created by CBN grinding are additive to the inherent residual compressive stresses found on as-heat treated surfaces (Figs. 5, 6, 7, 8). This is a completely new finding which should trigger further investigation.

Recently we obtained samples of automotive transmission gears ground with an electroplated CBN wheel. Five gears were selected at random during the course of grinding a run of approximately 250,000 gears. The midflank residual stress in the radial direction has been analyzed, and the results of these analyses are shown in Figs. 9a and 9b. The residual stress distribution is developed to a depth of .008" below the flank surface. The results show a remarkable consistency of compressive residual stress level at the flank surface. The small subsurface profile variations in the residual stress distributions will be due to variations in the heat treated profiles. Overall, these results again confirm the in-

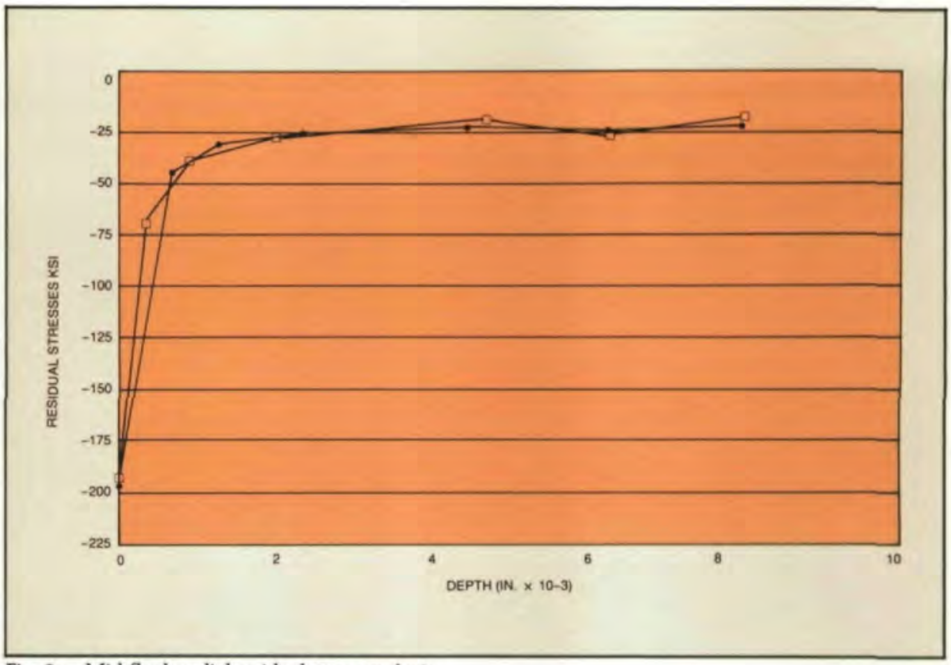

Fig. 9a – Mid-flank radial residual stress analysis.

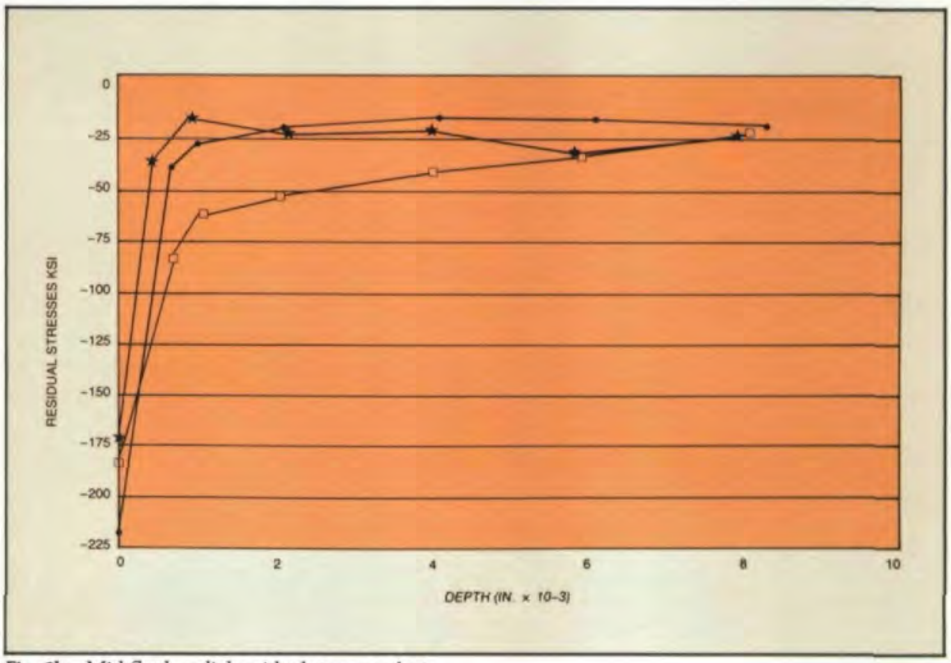

Fig. 9b – Mid-flank radial residual stress analysis.

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Fig. 10

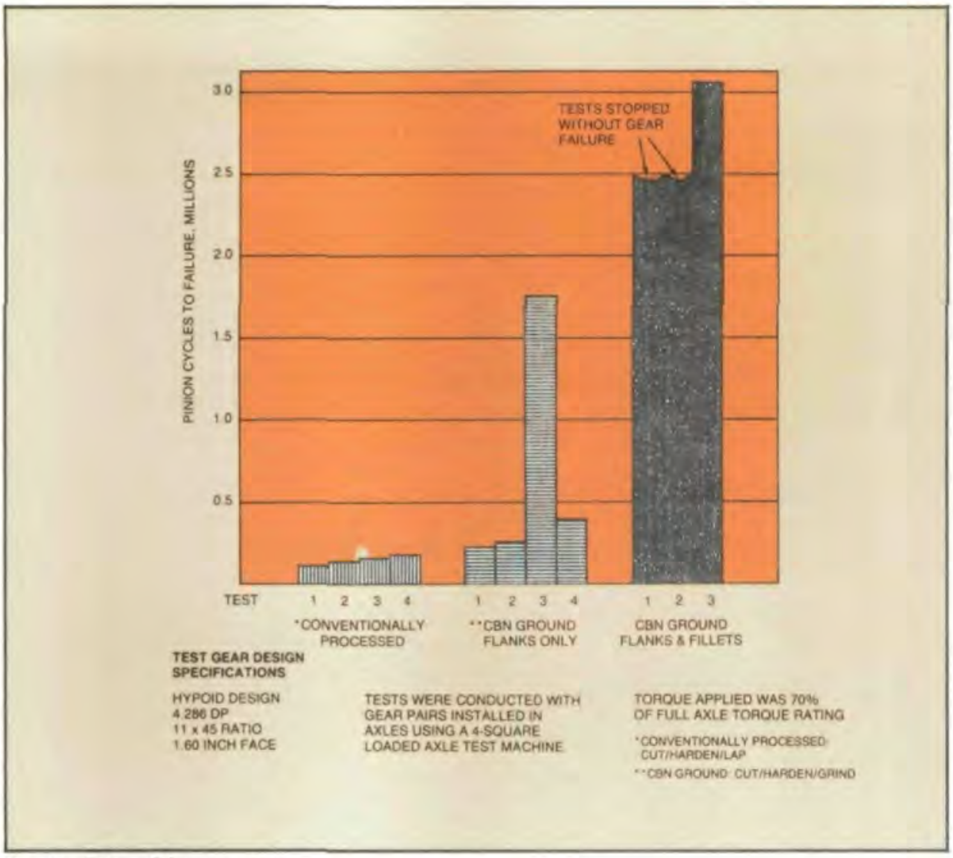

Fig. 10 – Fatigue life comparison.

herent capacity of CBN to produce significant levels of favorable residual stresses in gear components.

All of these findings tend to support the results of four-square fatigue testing of CBN ground gears reported by Kimmet.[11] This work found significant improvement in fatigue life of gears with CBN ground flanks and fillets over conventionally hardened and lapped gear sets (Fig. 10). In drawing these conclusions, Kimmet placed emphasis on the benefits of gear-to-gear uniformity and precision which result in more uniform load distribution in power transmission. In some cases, this aspect of CBN grinding may be as important or even more important than the residual stress benefits.

Yokagawa[12] has also reported increased wear resistance of the CBN ground surfaces of hardened steels over similar surfaces ground with aluminum oxide.

Honda Motor Co. has reported that the use of CBN grinding has made it possible to design and manufacture final drive gears of reduced weight and which operate at lower noise levels. These attributes derive from both the increased uniformity of tooth form and beneficial residual stresses.

Conclusions

This investigation has established that:

- CBN grinding of carburized and hardened parts can impart additional residual compressive stresses to the part surface. These stresses may be from as little as 30% greater to as much as 250% greater than the heat treated surface stresses.
- CBN grinding develops an increase in compressive residual stresses as grinding progresses from an as-heat treated surface into the case. The generation of compressive stresses has been shown in Figs. 7 and 8 to be independent of the amount of case depth removed at equivalent hardness. CBN grinding will also remove oxides, carbides and bainite from the surfaces.

The major thrust of these findings is that the beneficial effects of CBN grinding should be considered in the original design of drive train gear components. Coupled with the ability to remove all inherent distortion from the previous heat treatment process, one can also consider reducing backlash, root clearance and root configuration for ultimate beam strength. Such grinding processes offer the design engineer the confidence of knowing exactly what design loads a gear can withstand and optimizing gear size and weight in overall design to take advantage of this knowledge.

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BACK TO BASICS...

Involutometry

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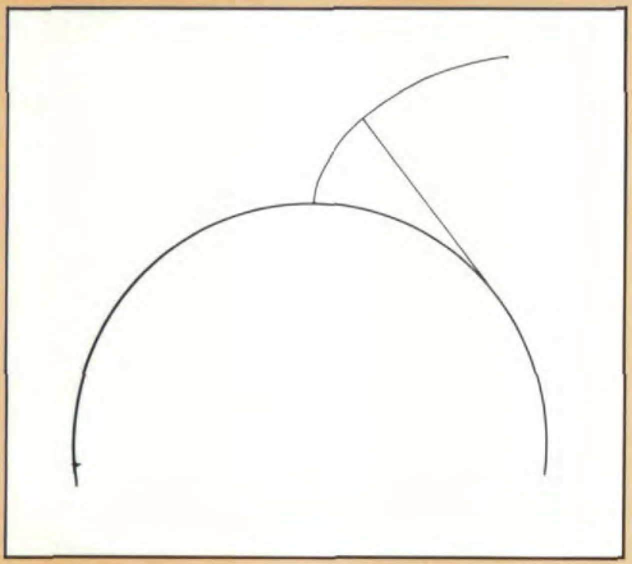

Fig. 1 – Involute curve.

Involute Curve Fundamentals

Over the years many different curves have been considered for the profile of a gear tooth. Today nearly every gear tooth uses an involute profile. The involute curve may be described as the curve generated by the end of a string that is unwrapped from a cylinder. (See Fig. 1.) The circumference of the cylinder is called the base circle.

Following are specific features of the involute curve.

- A perpendicular to the involute surface is always tangent to the base circle.
- The involute surface is a uniform rise cam (equal rise per increment of rotation).
- The radius of curvature of an involute surface is equal to the length of the tangent to the base circle.
- The same force applied perpendicular to the involute surface of a gear tooth at different tooth radii results in the same torque on the gear.

Because the involute surface is a uniform rise cam, any involute surface imparts uniform angular motion when operating against any other involute surface. To illustrate further, consider two pulleys with a crossed-belt drive as shown in Fig. 2. If we place knots in the string, then the knots will generate involute curves as they leave one circle and approach the other circle as shown in Fig. 3. If the knots are spaced close enough, there will always be knots on an involute surface when they are not on the circle. These generated curves can then be considered as teeth on the gear.

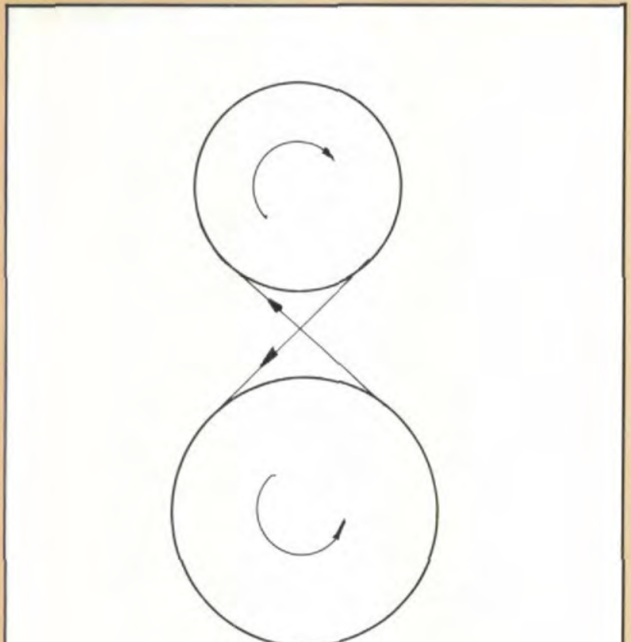

Fig. 2 – Crossed belt drive.

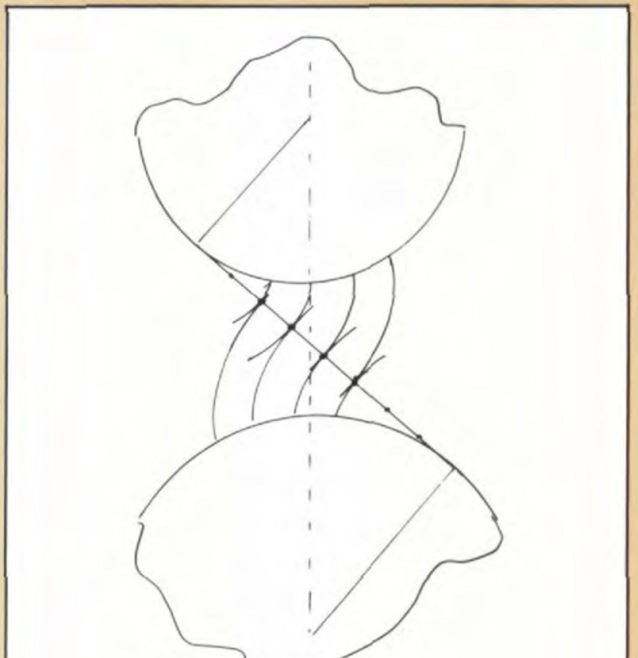

Fig. 3 – Multiple involute curves.

Of course, there are other considerations. The distance between knots must add up to an integer around the base circle, and the knots must be far enough apart to provide a structure for the tooth. When these criteria are met, the distance between knots is called the base pitch. Any two spur gears that have the same base pitch can be meshed together. (See Fig. 4.)

Just as the diameters of the two pulleys in the crossed-belt drive establish the ratio of the two shafts, likewise, the diameters of the base circles establish the ratio of the two gears.

The contact between two involute surfaces of two gears always occurs on a line that is tangent to the two base circles. This tangent line is called the "line of action". (See Fig. 5.) This phenomenon results from the from fact that a perpendicular to the involute surface is tangent to the base circle.

The line of action will cross the center line between the two gears. This intersection is called the pitch point. If we draw circles on each gear using the gear centers and passing through the pitch point, we have pitch circles. These pitch circles may be considered as cylinders which would roll with each other by friction with no slippage. The distance between the center lines of gear teeth on either pitch circle is called the circular pitch. If we divide the circumference of the pitch circle by the number of teeth in the gear, we get the circular pitch. These circles are important in the calculation of tooth thicknesses because it is here that the two tooth thicknesses, plus the backlash, must add up to the circular pitch.

Each gear has two distinct pitch circles to be considered. The gear will have one pitch circle with its cutter and another pitch circle with the mating gear. If it is the middle gear in a string of three gears, it may have a different pitch circle with each mating gear.

The pitch circles of mating gears will change as the center distance is changed. If the center distance is increased, the pitch circles will get larger, and the circular pitch will increase with the same tooth thicknesses as before. The backlash will also increase. A gear does not have an operating pitch diameter until its mating gear and center distance are specified.

Since gear teeth must make contact along the line of action, and since the contacting forces are parallel to this line, there is a force which acts to push the two gears' centers apart. The angle the line of action makes with a perpendicular to the center line of the gears is called the pressure angle. The pressure angle is defined as

$$\text{Pressure angle } \alpha = \text{Acos} \frac{\text{Base radius}}{\text{Pitch radius}} \tag{1}$$

Just as the pitch radius with another gear is unique, so the pressure angle with another gear will be unique.

This may be confusing until one is reminded of the crossed-belt drive. The cylinders would rotate in the same manner, and the knots in the string would generate the same involutes independent of the center distance between the two cylinders. The crossover point of the line of action across the center line will result in different pitch circles, and the pressure angle will change as the center distance is changed. The force along the line of action will remain the same; it is the torque divided by the base circle radius. As we will see later, there are considerations of tooth thickness and contact ratio which restrict the

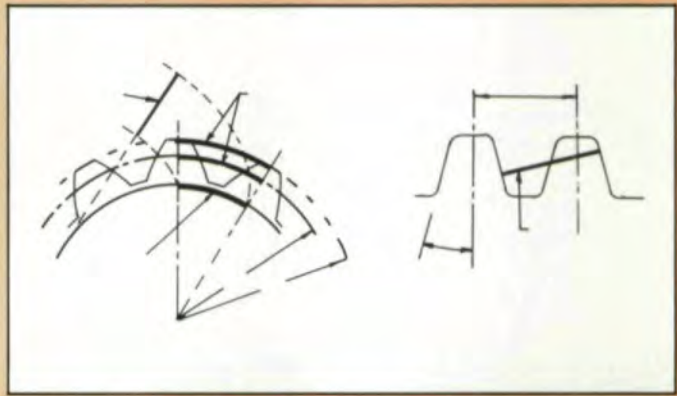

Fig. 4 — Base pitch of a gear and a rack.

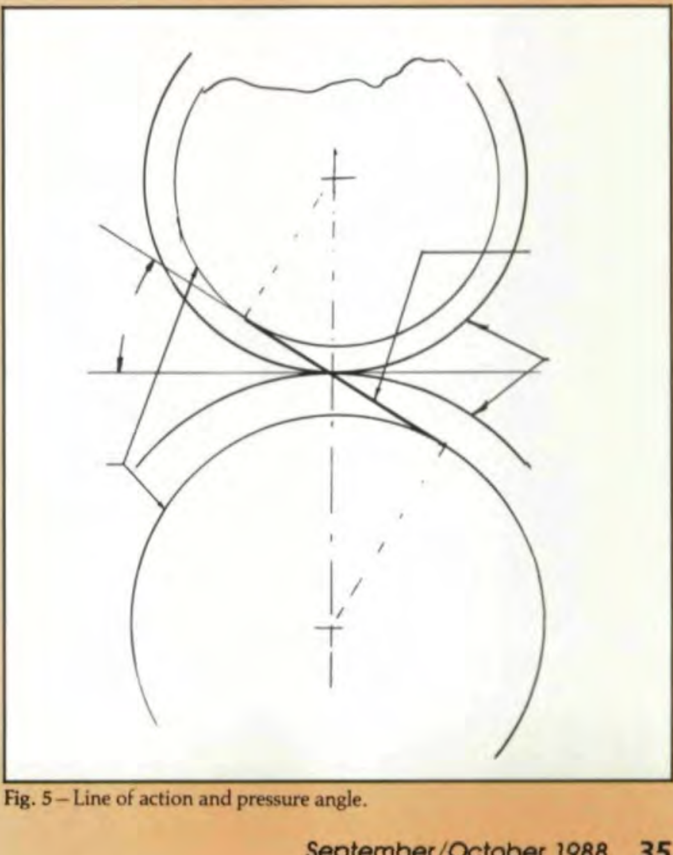

Fig. 5 — Line of action and pressure angle.

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range that the center distance may be changed.

The insensitivity of the involute action to the changing of the center distance of the gears is a distinct advantage over other tooth profiles considered in the past. As shafts deflect due to changes in load, the involute action is unchanged, assuming sufficient contact ratio still exists.

The involute profile is the path of the end of a string unwound from a base circle. The design of the gear involves establishing the ends of this curve. At the beginning of the curve there must be some sort of fillet blend to the root and the involute of the adjacent tooth. It is not good practice to plan on using the involute near the base circle because its radius of curvature is very small and changing rapidly. It is very difficult to manufacture the tooth accurately near the base circle. However, this end of the involute profile must be defined accurately to avoid problems of interference with the tip of the mating gear tooth. The "form diameter" is used to specify the root end of the involute profile.

The tooth tip end of the involute profile can either extend to the outside diameter of the gear, or if a chamfer is provided, it ends with the start of the chamfer. A chamfer is used by many manufacturers to prevent damage to the involute surface from nicks due to handling.

Helical Gears

More and more new machines use helical gears. The principal reason for this is the demand for quieter gears. Not only do we have an increased emphasis upon quiet machines, but also most machines operate at higher speeds and higher loads, both of which increase the noise made by spur gears. A pitch line velocity of 2000 feet per minute is considered by many to be the top limit for using spur gears in non-aircraft applications. The writers have observed spur gears operating quietly at pitch line velocities of 6000 feet per minute, but these were probably very accurate gears. With the normal industrial production tolerances, we can expect spur gears to be noisy above pitch line velocities of 2000 feet per minute. If this speed must be exceeded and quietness is preferred, then the design should incorporate helical gears.

We will introduce several new terms to describe physical characteristics of helical gears. The first of these is the helix angle. The helix angle is the inclination of the tooth in a lengthwise direction. The helix angle results in the gear having end thrust or an axial force during operation, and the support bearings must be able to resist this force. There is no rule for the best helix angle. It may be up to 45°.

The helix angle of the tooth depends upon the radius under consideration. Normally we quote a helix angle associated with the generating helix angle. This relates to the pressure angle of the cutter and the generating pitch radius. It is more important to work with the helix angle at the base circle. For two gears to mesh properly and operate on parallel axes, they must have the same normal base pitch and the same base helix angle.

To determine the different helix angles on a gear requires a familiarity with the term "lead". The lead of a gear may be defined as the width of a gear required for a tooth to wind one revolution around the gear. A large helix angle will result in a small lead. A spur gear may be considered as having an "infinite" lead.

In very large power applications common practice dictates the use of two helical gears, side by side, with opposite angles of the helix (hand) so that the end thrust is neutralized. Usually there is a small space between the gears to facilitate manufacture and also to give the tooth flexibility for load sharing.

Helical gears have involute tooth profiles similar to those in spur gears. Each gear will have a base circle and a line of action like a spur gear. If we observe the gear from an axial position, the gear contacts will occur along the line of action. However, an entire tooth will not make contact with the mating gear instantaneously, as does a spur gear tooth. The contact may start at the tip of one end of the tooth and proceed along the length of the tooth, with the first point of contact moving down the tooth before the other end of the tooth makes contact. The instantaneous load contact will be a diagonal line across the tooth face with a different radius as we follow the contact across the tooth. For very wide gears the pattern may be repeated several times across the gear as several teeth, with their helix angles, contact on the line of action.

With helical gears we have a second contact ratio to consider. The contact ratio expressed by the length of the zone of action divided by the base pitch is still used. The second, called face contact ratio, is a function of the helix angle and the width of the gear. For smooth action both contact ratios should exceed one.

The teeth on a helical gear have a slight "twist" in the radial direction. If a "pin" is placed in the tooth space, as is done when measuring the distance across pins to determine tooth thickness, the pin will sit on a high spot and "rock". A ball must be used for making this measurement with helical gears, which have an odd number of teeth.

To study the tooth geometry we have to consider two planes, the transverse plane and the normal plane. The gear teeth operate in the transverse plane, but the teeth are generated in the normal plane. The tooth thickness and generating pressure angle usually refer to the normal plane. The base circles, outside radius, root diameter and contact ratio are determined in the transverse plane.

To determine the tooth thickness at a second radius requires the conversion of the given tooth thickness in the normal plane to the transverse plane. The tooth thickness at the desired radius is calculated in the transverse plane. It can then be converted to the normal plane again, taking into account the change of helix angle at the desired radius.

Helical gears may be cut with the same hob that is used to cut spur gears. The hobbing machine has the ability to position the hob at an angle corresponding to the helix angle.

For gears that must be cut with a shaper because there is no clearance for hob runout, the shaper machine must be equipped with a "guide". The guide provides rotation of the cutter as it generates the teeth. Each guide has a specific lead, and the helix angle generated depends upon the diameter of the shaper cutter. A large diameter cutter will generate a larger helix angle than a small diameter. The shaper cutter must have nearly the same lead; "nearly" because the shaper cutter may still be usable even though its helix angle is up to one degree different than the helix angle being generated. The cutter has enough side relief to permit this. This should only be done for small lots.

When selecting shapers for a helical gear, the availability of a guide is most important because the cost of a guide is many times that of a shaper.

The mathematical formulae for helical gears are nearly the same as for spur gears, except for the inclusion of a function of the helix angle in the equation. The formulae are basically set up to work in the transverse plane (plane of rotation); therefore, if the tooth thickness and pressure angle are in the normal plane, they must be converted to the transverse plane.

Gear Design Mathematics

The tooth geometry of a spur or helical gear may be described by seven pieces of data. These are number of teeth, base diameter, outside diameter, form diameter, root diameter, tooth thickness at base diameter and lead (and hand).

The design of a gear involves the use of mathematical equations to obtain the above seven numbers. The mathematics involves extensive use of trigonometry. The designer must be familiar with the use of angles in radians and the involute function.

As mentioned previously, the involute curve is the path of the end of a string unwound from a cylinder.
The following relationships hold.

$$\text{Tan}\,\alpha = \frac{\sqrt{R^2 - R_b^2}}{R_b} = \frac{R_b(\alpha + \theta)}{R_b}$$

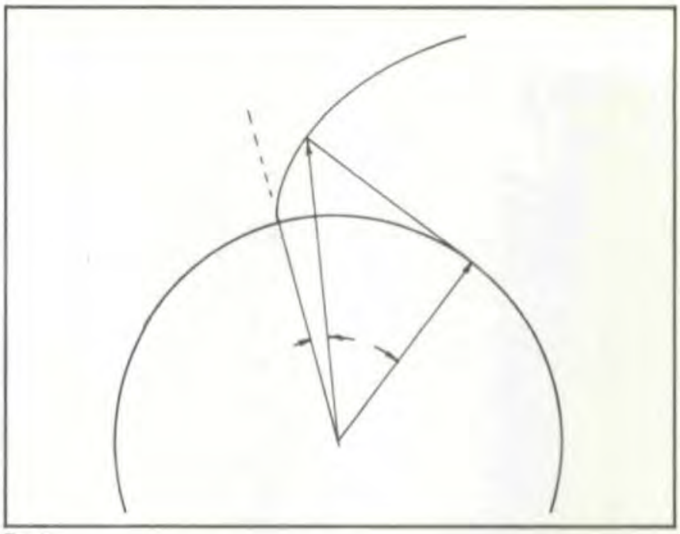
$$\text{Tan}\,\alpha = \alpha + \theta$$

$$\theta = \text{Tan}\,\alpha - \alpha = \text{inv}\,\alpha \qquad (2)$$

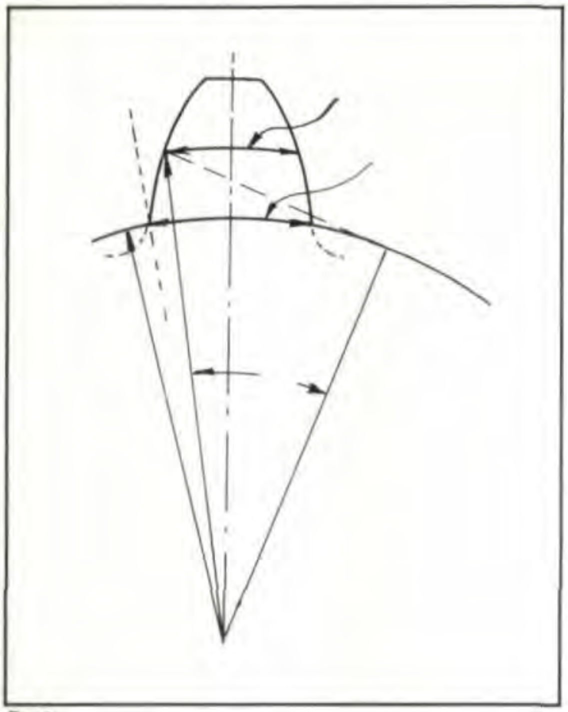
The involute function is the angular displacement of a point on the tooth from its point of intersection with the base circle. This relationship is useful for calculating the circular thickness of a tooth. When the thickness at a given diameter is desired, there is a corresponding angle or "pressure angle" of the tooth at that radius. By determining the pressure angle at a point on a tooth and then taking the involute of this angle, the angular shift of the curve from where it left the base circle is available. Now by multiplying this angle by the new radius, we have the length of arc of the shift of the tooth profile. By subtracting two times this angle from the angle on the base circle for the entire tooth, and multiplying this resultant angle by the desired radius, we get the circular tooth thickness at the desired radius. (See Equation 3.)

$$T_R = 2R\left\{\frac{T_b}{2 \cdot R_b} - \text{inv}\,\alpha\right\} \qquad \alpha = \text{Acos}\left\{\frac{R_b}{R}\right\} \qquad (3)$$

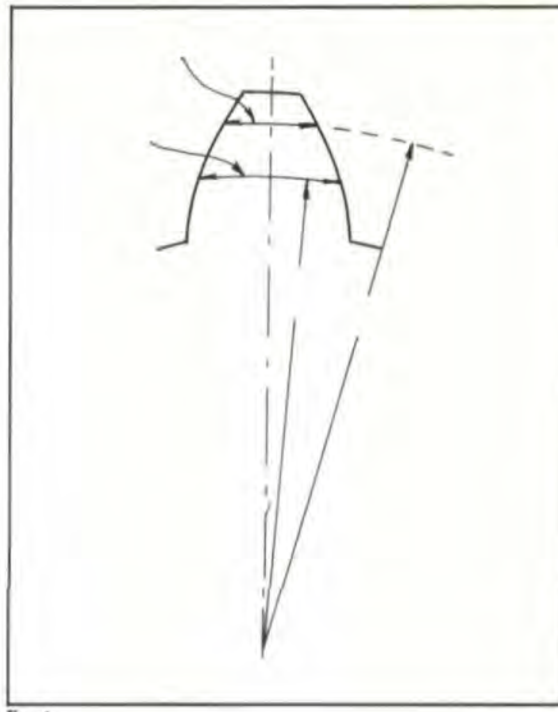
Good practice suggests use of the tooth base thickness as a fundamental dimension on the gear because finding the thickness at any other radius only requires finding one involute. If the known thickness is at some other diameter, then it is necessary to have the involute at this radius, and then use the difference in involutes to determine the difference in known tooth thickness and the desired tooth thickness.



Eq. 2



Eq. 3



Eq. 4

It is customary to use the circular dimensions for describing tooth thickness. The difference between the circular and chordal tooth thickness is very small, usually less than one-tenth of the backlash that will be provided. However, in some mathematical analyses, such as following the path of a cutter, chordal dimensions must be used to get the accuracy required.

Gear designers who work with "standard" gears usually have the tooth thickness at the pitch circle and then have to use two involutes to find the tooth thickness at another radius.

Given the arc tooth thickness and pressure angle in the plane of rotation of a helical gear at a given radius, to determine the tooth thickness at any other radius, use the following formulae.

$$\text{Cos}\,\alpha_2 = \frac{R_1 \cdot \text{Cos}\,\alpha_1}{R_2}$$

$$T_2 = 2 \cdot R_2 \left\{\frac{T_1}{2 \cdot R_1} + \text{inv}\,\alpha_1 - \text{inv}\,\alpha_2\right\} \qquad (4)$$

where

R_1 = Given radius
α_1 = Pressure angle at R_1
T_1 = Arc tooth thickness at R_1
R_2 = Radius of unknown tooth thickness
α_2 = Pressure angle at R_2
T_2 = Tooth thickness at R_2

The above formula assumes all the data is provided in the transverse plane (plane of rotation). Most helical gear data is available in the normal plane because the cutter is dimensioned in this manner. Such data as tooth thickness and pressure angle are usually in the normal plane.

The formula for the pitch diameter of a helical gear in the transverse plane is

$$PD = \frac{Z}{DP \cdot COS\beta} \qquad (5)$$

where

Z = Number of teeth
DP = Diametral pitch (normal)
β = Helix angle (generating)

Converting the normal tooth thickness to the transverse plane requires the following formula:

$$T_t = \frac{T_n}{\text{Cos}\,\beta} \qquad (6)$$

where

T_n = Normal tooth thickness
β = Helix angle
T_t = Transverse tooth thickness

Note that the helix angle must correspond to the diameter on the gear corresponding to the tooth thickness. The helix angle is different for different diameters of the tooth. The lead is constant for a gear and can be determined from a helix angle given at a given radius by the formula:

$$L = \frac{2 \cdot \pi \cdot R}{\text{Tan}\,\beta} \qquad (7)$$

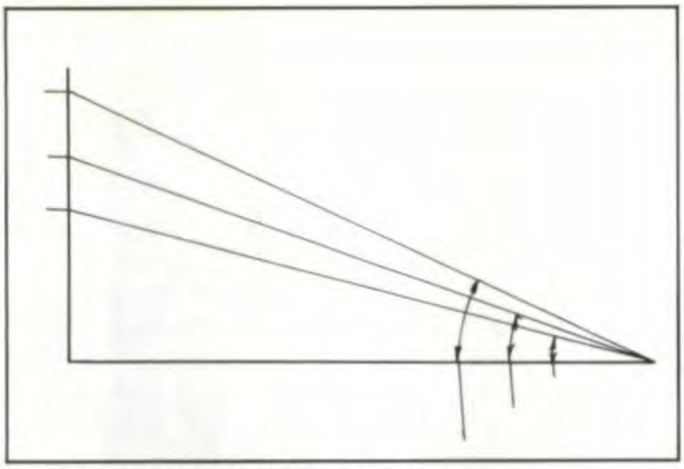


Fig. 6 – Diagram to illustrate helix angle variation.

where

R = Radius of given helix angle
β = Given helix angle
L = Lead of the gear

Once the lead is known, the following formula may be used to find the helix angle at the radius corresponding to the given tooth thickness.

$$\beta = \text{Atan}\left\{\frac{2\cdot\pi\cdot R}{L}\right\} \tag{8}$$

where

R = Radius of given tooth thickness
L = Lead of the gear
β = Helix angle for R

Fig. 6 is helpful in remembering the relationship between the lead and helix angle at different radii of the gear tooth. The lead is constant for a gear, but the circumference depends upon the radius being studied.

The study of the beam stress of the tooth and the form diameter is done in the normal plane. Doing these studies in the normal plane requires converting to a "virtual" gear. The normal plane of a helical gear is really an ellipse. The point of interest is the largest radius of curvature of the ellipse. This radius of curvature is used to construct a virtual gear.

The virtual gear is a simulated spur gear. Mathematically, the virtual gear is derived from the helical gear by the following relationships:

$$\text{Virtual pitch radius } (R_v) = \frac{\text{Pitch radius (helical gear)}}{\text{Cos}^2\text{ (Helix angle)}} \tag{9}$$

$$\text{Virtual number of teeth } (Z_v) = \frac{\text{Teeth in helical gear (Z)}}{\text{Cos}^3\text{ (Helix angle)}} \tag{10}$$

$$\text{Virtual base radius } (R_{bv}) = \text{Virtual pitch radius} \cdot \text{Cos } (PA_{normal}) \tag{11}$$

To obtain the outside diameter and root diameter, their dif-

The given pressure angle will usually correspond to the pressure angle of the cutter. To convert this into the transverse plane the following formula is available.

$$\operatorname{Tan} \alpha_t = \frac{\operatorname{Tan} \alpha_n}{\operatorname{Cos} \beta_g} \tag{12}$$

where

α_n = Normal pressure angle
β_g = Generating helix angle
α_t = Transverse pressure angle

Another calculation that is important in designing a set of gears is determining the center distance for two gears to be meshed together. Again, it is only necessary that the gears have the same base pitch for the involute action to be proper; however, there may be interference with the tips of teeth going below the form circle. The tips of the teeth may also bottom out in the root of the mating gear if the members of the pair were not designed together. If the gears are helical, they must have the same base helix angle. If they are external gears, the hand of the helixes must be opposite, assuming they turn on parallel shafts.

Given tooth proportions in the plane of rotation for two helical gears, to determine the center distance when they mesh tightly, use the following formulae.

$$\operatorname{inv} \alpha_2 = \frac{Z_1(T_1 + T_2) - 2 \cdot \pi \cdot R_1}{2 \cdot R_1 (Z_1 + Z_2)} + \operatorname{inv} \alpha_1$$

$$C_1 = R_1 + R_2 \tag{13}$$

$$C_2 = C_1 \frac{\operatorname{Cos} \alpha_1}{\operatorname{Cos} \alpha_2}$$

where

R_1 = Given radius of first gear
R_2 = Given radius of second gear
Z_1 = Teeth in first gear
Z_2 = Teeth in second gear
α_1 = Pressure angle at R_1 and r_2
α_2 = Pressure angle at meshing position
T_1 = Arc tooth thickness at R_1
T_2 = Arc tooth thickness at R_2
C_1 = Center distance for α_1
C_2 = Center distance for α_2

The previous equations are useful for determining the center distance when run with a master gear. This is a tight mesh operating condition. If two gears are to be operated together, then the desired backlash may be added to the tooth thicknesses in the equation, and the corresponding center distance derived.

ference from the pitch diameter is maintained in each plane.

Contact Ratio

If two mating gears are to provide smooth rotation, it is necessary that there be a continuing contact on the line of action. A second pair of teeth must mesh before the first set separate. This relationship is described by the "contact ratio". The contact ratio is the ratio of the length of active engagement of the gears on the line of action to the base pitch. For spur gears the contact ratio must exceed one and may exceed two for gears where quiet operation is needed.

The following figure and equations show how contact ratio is determined.

$$CR = \frac{\sqrt{R_{o1}^2 - R_{b1}^2} + \sqrt{R_{o2}^2 - R_{b2}^2} - C \cdot \sin\alpha}{P \cdot \cos\alpha} \qquad (14)$$

where

R_{o1} = Outside radius of gear 1
R_{o2} = Outside radius of gear 2
C = Center distance
CR = Contact ratio
R_{b1} = Base radius gear 1
R_{b2} = Base radius gear 2
α = Pressure angle
P = Circular pitch

With helical gears we have a second contact ratio to consider. The contact ratio expressed by the length of the zone of action divided by the base pitch is still used. The second, called face contact ratio, is a function of the helix angle and the width of the gear. For smooth action both contact ratios should exceed one. The formula for face contact ratio is

$$CR_{face} = \frac{F \cdot \tan\beta}{P} \qquad (15)$$

where

F = Gear face width
β = Helix angle at pitch point
P = Circular pitch (transverse)
CR_{face} = Face contact ratio

Generation of Gear Teeth

While we discussed the forming of the involute surface of a gear tooth as identical to the curve of a string being unwound from a base cylinder, the following is descriptive of how a gear tooth is actually generated. There are two main types of cutters used to generate gears, hobs and shapers. First consider the "shaper". This cutter closely resembles a gear. One edge is sharp and, by gradually meshing with a blank as it moves back and forth axially, it will cut a mating gear. The blank should be the appropriate diameter, and the two centers must be externally geared together with the right ratio.

A second type of cutter is the "hob". Before discussing this cutter we must discuss the "basic rack". A basic rack is a seg-

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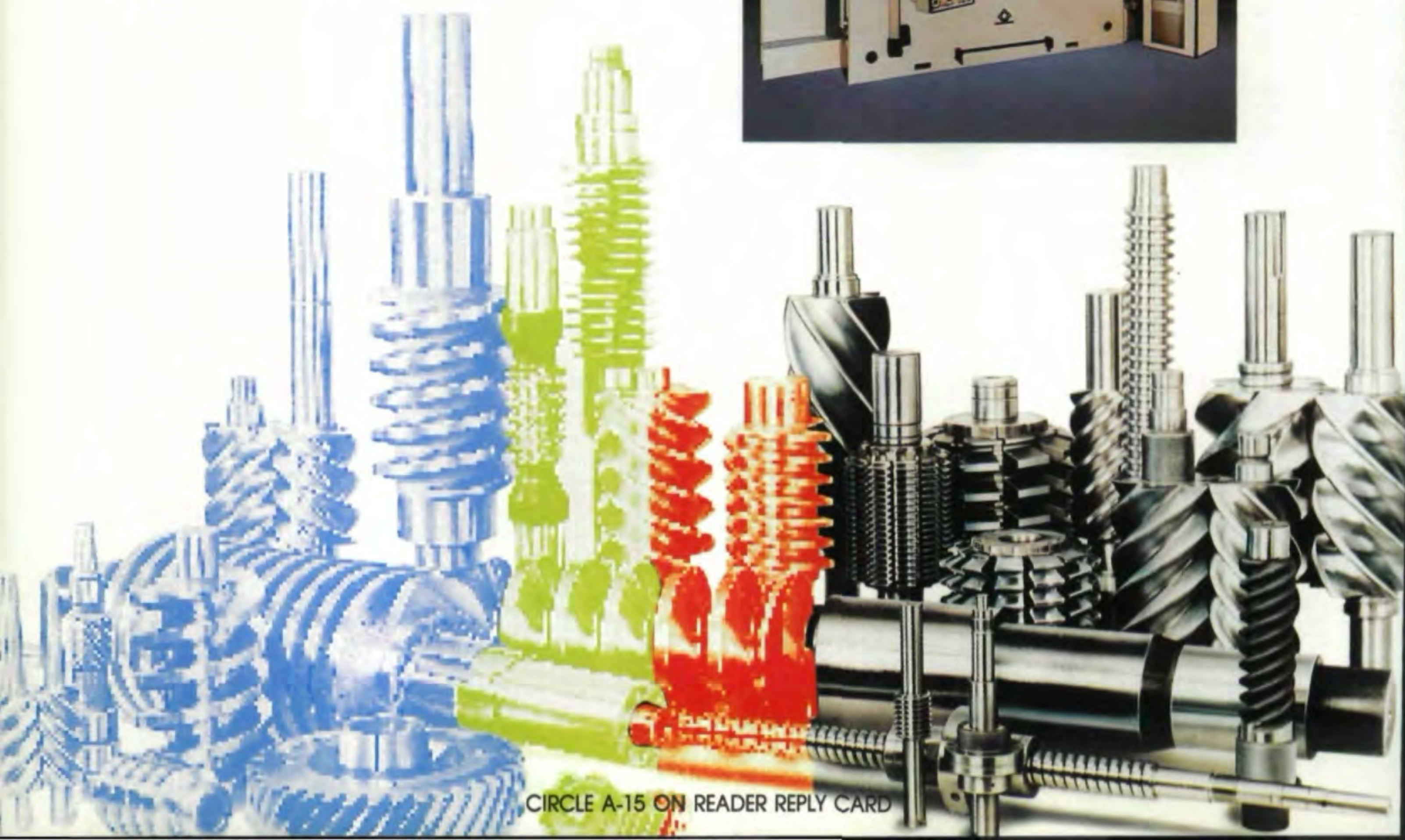

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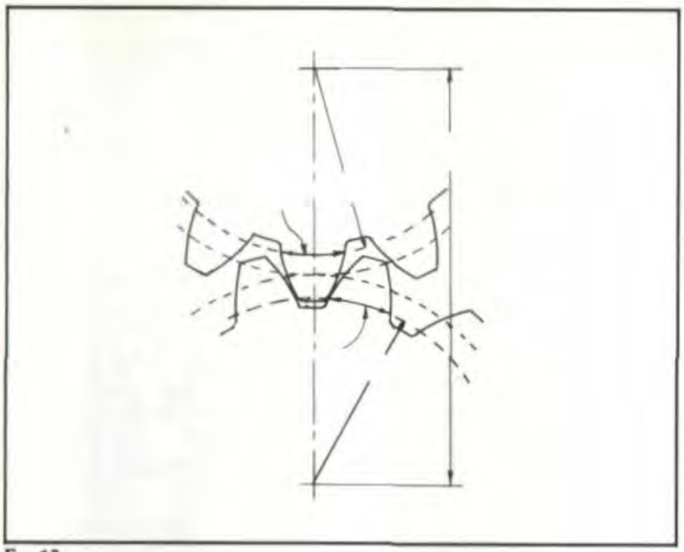
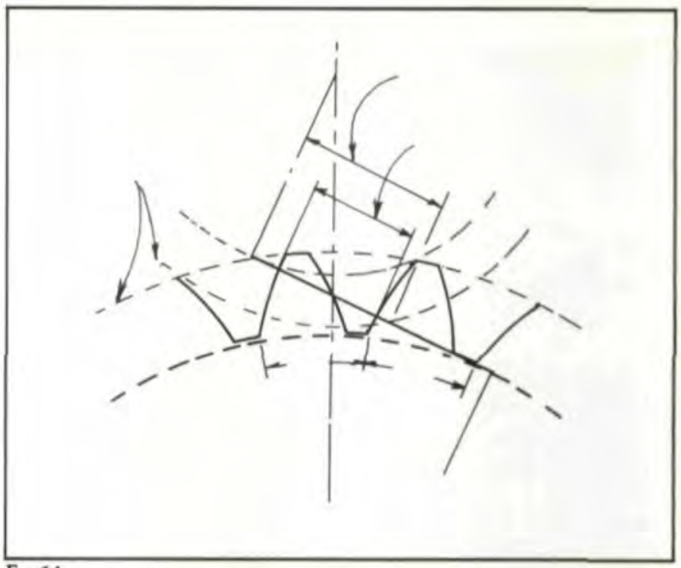


Eq. 13



Eq. 14

ment of a gear of infinite size, but with teeth the size of the gear we want to cut. As a result, the "involute" sides of the teeth are essentially straight lines. To generate a tooth, the basic rack moves in a straight line like a rack and pinion arrangement, with the cutting action being up and down axially to the gear. In reality, instead of having a very long rack, segments are arranged around a cylinder and offset to create a helix. With the axis of the cutter inclined, the teeth will contact the gear being cut parallel to the gear axis. By inclining the cutter at a different angle it can cut a helical gear. The cutting activity closely resembles a worm gear drive. Hobbing is the most popular and fastest method for cutting gears.

In our calculations it is necessary to determine the position of the basic rack with the gear being generated in order to establish the form diameter and root radius and determine

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INVOLUTOMETRY . . .

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whether the cutter will interfere with the outside diameter of the gear. Usually a gear tooth thickness is given, and the corresponding position of the basic rack is computed.

Given the tooth dimensions in the plane of rotation of a helical gear, to determine the position of a mating rack of different circular pitch and pressure angle (same base pitch), use the following formulae.

$$\sin\beta_b = \sin\beta_1 \cdot \cos\alpha_{n1}$$

$$\sin\beta_2 = \frac{\sin\beta_b}{\cos\alpha_{n2}} = \frac{\sin\beta_1 \cdot \cos\alpha_{n1}}{\cos\alpha_{n2}}$$

$$\tan\alpha_2 = \frac{\tan\alpha_{n2}}{\cos\beta_2} \qquad R_2 = \frac{R_b}{\cos\alpha_2}$$

$$X = R_2 - a + \frac{1}{2\tan\alpha_2} \cdot \left\{2 \cdot R_2 \left\{\frac{T_1}{2 \cdot R_1} + \operatorname{inv}\alpha_1 - \operatorname{inv}\alpha_2\right\} - \frac{\pi \cdot R_2}{Z}\right\} \qquad (16)$$

where

β_1 = Given helix angle at R_1
β_b = Base helix angle
α_{n2} = Pressure angle of rack
α_2 = Transverse pressure angle of rack
R_2 = Pitch radius with rack
a = Addendum of rack
Z = Number of teeth
P_{n1} = Normal pitch at R_1
β_2 = Helix angle for mating rack
α_{n1} = Normal pressure angle at R_1
α_1 = Transverse pressure angle at R_1
R_1 = Given pitch radius
R_b = Base radius
T_1 = Given tooth thickness at R_1
X = Dist. gear center to hob tip
P_{n2} = Normal pitch of rack

Base pitch must be equal ($P_{n1} \cdot \cos\alpha_1 = P_{n2} \cdot \cos\alpha_2$)

The position of the rack formula results in a figure for the root radius of the gear. The following formula will give the minimum root radius without undercut. By comparison, it can be determined if undercutting is taking place. Undercutting is undesirable. By taking the minumum root radius without undercut a new tooth thickness for the gear can be determined. By keeping one gear from undercutting, the mating gear should be checked to determine if it may be undercut as a result of shifting tooth thickness.

Given the dimensions of a hob, the helix angle and number of teeth in a helical gear, to determine the minimum root radius to avoid undercut, use the following formulae.

$$R = \frac{Z}{2 \cdot DP \cdot \cos\beta} \qquad \tan\alpha = \frac{\tan\alpha_{hb}}{\cos\beta}$$

$$R_{unc} = R \cdot \cos^2\alpha - r \cdot (1 - \sin\alpha) \qquad (17)$$

where

Z = Number of teeth in gear
α_{hb} = Pressure angle of hob
α = Pressure angle in plane of rotation
β = Helix angle
DP = Diametral pitch of hob
r = Radius of hob tooth tip
R_{unc} = Minimum root radius w/o undercut ■

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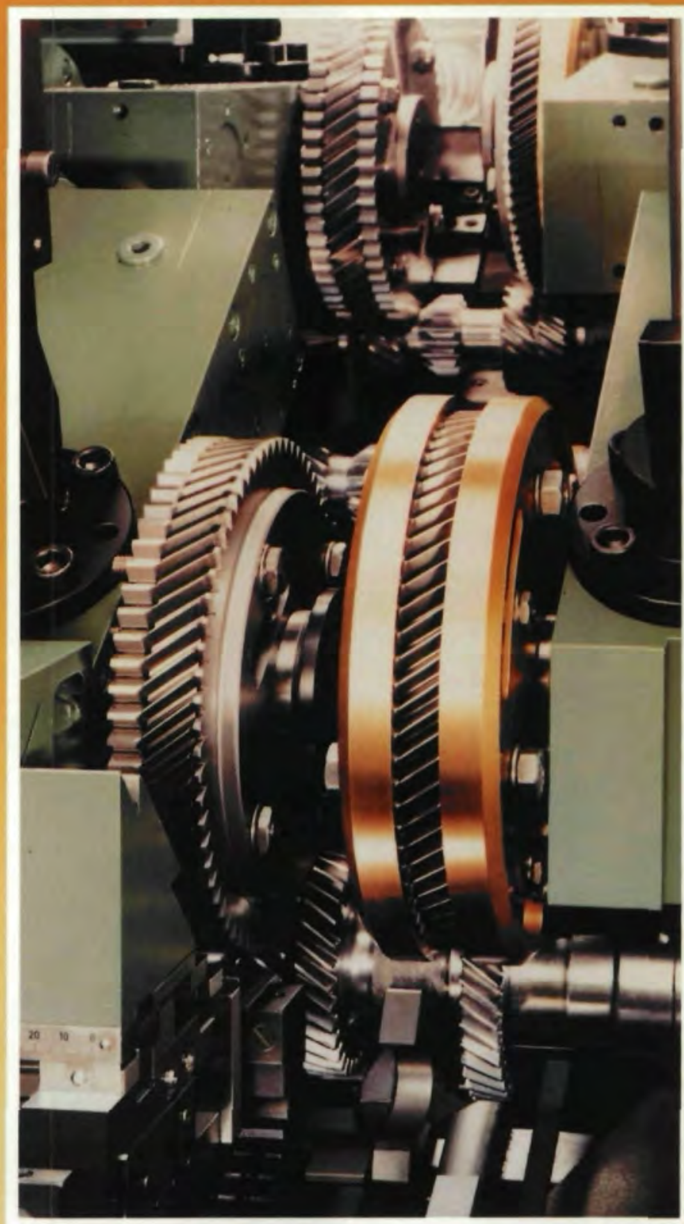

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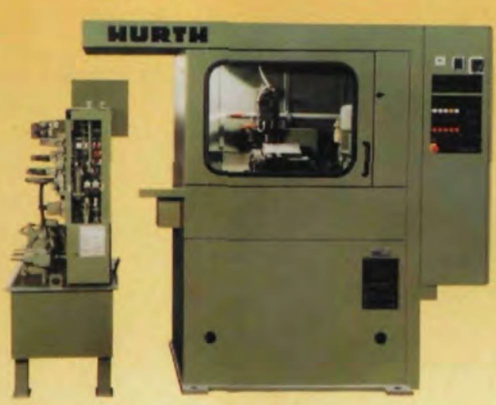

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