

The Geometry of Helical Mesh

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Introduction

In 1961 I presented a paper, "Calculating Conjugate Helical Forms," at the semi-annual meeting of the American Gear Manufacturers Association (AGMA). Since that time, thousands of hobs, shaper cutters and other meshing parts have been designed on the basis of the equations presented in that paper. This article presents the math of that paper without the formality of its development and goes on to discuss its practical application.

Most gearing texts quickly get through the fundamental law of gear tooth action and start concentrating on the involute. While involute geometry is undeniably the most important aspect of modern gearing, it will be useful to examine the general geometry of parts in mesh with each other in order to understand the generating process for involute gears, straight-sided splines, impeller rotors, sprockets and any other toothed form that runs with a mate or that is hobbled, shaped, ground or shaved. The math presented here is applicable to any toothed form, internal or external, meshing on either parallel axes or crossed axes. This article intends to explore:

1. How to calculate a mating form to mesh with any known form.
2. How to overcome calculation problems in computer-generated forms.
3. How the fundamental law of gear tooth action applies to crossed-axis gearing.
4. How to calculate the path of contact in three dimensions.
5. Why generated forms may have unwanted "fill-in" and "round-over."
6. How to use mesh math to solve common design and manufacturing problems.

The math in this article may be used directly without having an understanding of underlying concepts. However, it is based on a theorem and its corollary, both of which are so simple in concept that

they can be understood without vigorous proof:

Theorem A. To assure conjugate action between a helical form and a rack, the surfaces of these forms should come into contact along a single line when placed in stationary mesh.

Corollary. If each of two helical forms in stationary mesh with each other makes line contact with the opposite surfaces of an infinitesimally thin rack form inserted between them, the helical forms will have conjugate action with each other.

The term "line" is used in the gearing sense—actually a curve in space, single-valued in x , y and z . Only in involute gearing does this curve resolve to a straight line. The term "conjugate" means meshing with on-going contact at a constant velocity ratio. Using this theorem reduces a dynamic analysis to a static one.

Understanding Theorem A Without Heavy Math

Consider a helical gear in stationary mesh with a rack. (This could be any rack and gear, but, for simplicity, make it an involute rack and gear.)

Mentally mark a reference axial position where a transverse cross section of the gear contains the gear tooth and rack tooth in contact at the pitch point. (The common contact line for Theorem A is a segment of the involute generatrix passing through this point. The generatrix is the straight line that unwinds from the base helix like a taut string and generates the involute tooth surface.)

Now rotate the gear through a small angle, driving the rack with it. Move the transverse cross section axially to a new reference position where the same gear tooth and rack tooth are again in contact at the pitch point.

Obviously, everything about this new configuration in the vicinity of the pitch point is indistinguishable from the previous configuration, which indicates that as the

TABLE I — GLOSSARY	
Angle Between Axes	— For two gears in mesh, this is the algebraic sum of the two helix angles.
Array	— The collection of coordinate points that describe a form.
Helix Angle	— The angle between the gear axis and any rack element. The helix angle has the same sign as the lead. The rack element that contacts the gear at the pitch point is tangent to the helix of the gear at its pitch radius.
Lead	— The axial advance in tracing any helix of a gear tooth through 360°. By convention a left-hand lead is negative.
Line-Arc Geometry	— A form made up of a series of arcs and/or lines all connected end to end.
Normal	— As an adjective: being at right angles. As a noun: a straight line perpendicular to a surface.
Normal Circular Pitch	— The normal distance from a rack element on one rack tooth to a similar element on the adjacent rack tooth.
Pitch Point	— The tangent point viewed in the transverse plane of the gear where a circle on the gear (Gear Pitch Circle) rolls without slipping along a line on the rack (Rack Pitch Line).
Rack Element	— For a rack constrained to move in a horizontal plane, any straight horizontal line on the rack surface.
Slope	— The tangent of the slope angle.
Slope Angle	— The angle from the positive X axis to a line that is tangent to a form at a given point.
Spur Part	— A gear or other toothed part that has a helix angle of zero.

gear rotates, the shape of the contact line remains unchanged, but traverses along the rack elements as the mesh advances.

Therefore, without ever rotating the gear, we know that conjugate action would occur with the rack if we did rotate it.

By using the corollary to Theorem A, we can determine the form of a second gear meshing conjugately with a given gear without considering any rotational aspect.

TABLE II — SYMBOLS

Known part lead	L	Left hand is negative
Mating part lead	L'	Left hand is negative
Known part number of teeth	N	
Mating part number of teeth	N'	
Known part pitch radius	R	
Mating part pitch radius	R'	
Known part helix angle	α	Left hand is negative
Mating part helix angle	β	Left hand is negative
Multiplier 1	IK	-1 if either part is internal, else +1
Multiplier 2	IM	-1 if mating part is internal, else +1
Angle between axes	Σ	
Center distance	C	
Normal circular pitch	P	
Input coordinates	xt, yt	Known form defined in transverse plane
Slope at xt, yt	st	
Output coordinates	xr', yr'	Rack form
	xt', yt'	Mating form in its transverse plane
	za', ya'	Mating form in its axial plane
	xc', yc', zc'	Path of contact in rack axis system
	xc, yc, zc	Path of contact in known part axis system
	xc', yc', zc'	Path of contact in mating part axis system

TABLE III — STANDARD EQUATIONS

$P = \pi/\text{Normal Diametral Pitch}$	(1)
$R = P \cdot N / (2 \cdot \pi \cdot \cos \alpha)$	(2)
$R' = P \cdot N' / (2 \cdot \pi \cdot \cos \beta)$	(3)
$L' = (2 \cdot \pi \cdot R \cdot N' / N) \cdot \cos \alpha / \sin \beta$	(4)
$L' = IK \cdot (2 \cdot \pi \cdot R \cdot N' / N) / (\sin \Sigma - (2 \cdot \pi \cdot R / L) \cdot \cos \Sigma)$	(4.1)
$\alpha = \arctan(2 \cdot \pi \cdot R / L)$	(5)
$\beta = \arctan(2 \cdot \pi \cdot R' / L')$	(6)
$\Sigma = \alpha + \beta \cdot IK$	(7)
$C = IM \cdot (R + IK \cdot R')$	(8)

Note 1: Equation 8 is for a standard center distance. However, the pitch cylinders of two gears in mesh at crossed axes may be tangent to each other, intersect or not meet at all, so C can vary from this calculation.

Note 2: R' in Equation 3 is not used in the mathematical form development equations, and since β and C are usually determined by external requirements, R' is mainly an informational quantity.

Note 3: The lead of a spur gear is infinite. For computer purposes, the lead of a spur gear may be entered as zero, which the computer program can use as a code to actually make $1/L$ equal to zero. By the same token, the solution for $1/L'$ instead of L' (Equation 4) is usually made when calculating by computer. This avoids operations with infinity, which result in computer error.

Part I—The Math of Mesh

Parallel-axis gearing and crossed-axis gearing differ in that two helical forms in mesh on crossed axes make only point contact, while two forms in mesh on parallel axes make line contact. This math is applicable in both cases, including spur forms.

For the purposes of this article, the form of the known part is assumed to be in its transverse plane (plane of rotation). We choose the transverse plane because it will accommodate all gears from spurs to worms. A spur form cannot be defined in the axial plane.

The form is expressed as a series of coordinate points with associated slope (xt, yt, st). All of these xt, yt points taken together are referred to as the xt, yt array. In practice, of course, the form may begin by being defined (by line-arc geometry for example) in some other cutting plane and cranked along the helix by some utility computer program to the transverse plane in point-slope format. It is a fairly simple procedure to pick both points and slopes from forms defined by line-arc geometry. To keep our definitions straight, note that the known part form is the input form, and the mating part form is the output form.

Fig. 1 shows the three coordinate systems used in this math. X, Y, Z is the system for the known part where Z is the part axis, and the X - Y plane is the transverse plane of the known part. X', Y', Z' is the system for the mating part where Z' is the axis of the mating part. The rack occupies the X'', Y'', Z'' system where Z'' is parallel to the rack elements. Note the glossary of terms in Table I, the symbol definitions in Table II and the standard equations in Table III. The standard equations provide a means for deriving various parameters from other known parameters. The helix angle of the known part is α ; that of the mating part is β , and Σ is the angle between the axis of the known part and the axis of the mating part. In Fig. 1, α and β are both illustrated as positive angles, which for this case means both the known part and the mating part have right hand leads and helix angles. The center distance between the known part and the mating part is C . The number of teeth in the known part is N , and N' is the number

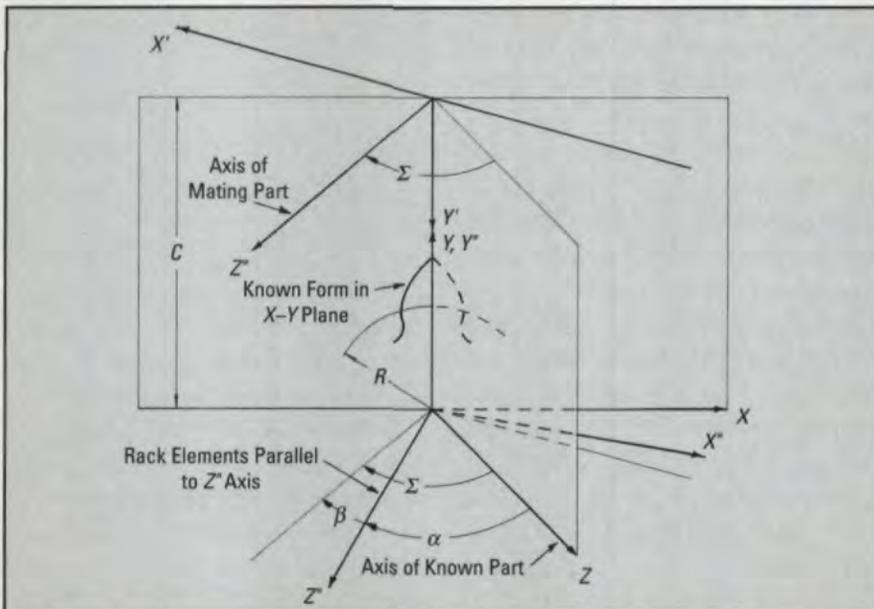


Fig. 1 — The three coordinate axis systems.

GEAR FUNDAMENTALS

of teeth in the mating part. The lead of the known part is L , and L' is the lead of the mating part. Two unit multipliers, IK and IM , provide required sign changes when either of the parts is internal. The pitch radius of the known part, R , may have any value that lies within the framework of the standard equations and that is large enough to keep Equation 11 from becoming the square root of a negative number. It is often chosen so that the pressure angle of the mating part stays above some minimum value. The sign convention of the axis systems allow the output form geometry to be fed back directly as input form geometry.

$$\phi = \arctan(1/st) \dots \dots \dots (9)$$

$$v = xt \cdot \sin\phi + yt \cdot \cos\phi \dots \dots \dots (10)$$

$$q = \sqrt{R \cdot R - v \cdot v} \dots \dots \dots (11)$$

$$x2 = v \cdot \sin\phi \pm q \cdot \cos\phi \dots \dots \dots (12)$$

$$y2 = v \cdot \cos\phi \mp q \cdot \sin\phi \dots \dots \dots (13)$$

$$\Theta = \arctan(x2/y2) \dots \dots \dots (14)$$

$$xk = xt \cdot \cos\Theta - yt \cdot \sin\Theta \dots \dots \dots (15)$$

$$yk = xt \cdot \sin\Theta + yt \cdot \cos\Theta \dots \dots \dots (16)$$

$$zk = .5 \cdot L \cdot \Theta / \pi \dots \dots \dots (17)$$

$$xr = xk + R \cdot \Theta \dots \dots \dots (18)$$

$$yr = yk \dots \dots \dots (19)$$

$$xr'' = xr \cdot \cos\alpha \dots \dots \dots (20)$$

$$yr'' = yr \dots \dots \dots (21)$$

In the X - Y plane, $x2$, $y2$ is a point where the normal to the form at the point xt , yt intersects the pitch circle. Since any line that intersects a circle has two intersection points, $x2$, $y2$ has two possible positions as determined by the \pm sign. Both of these positions are valid, but only one applies to the application at hand. In most cases, the non-applicable root applies if you change the space into a tooth and the tooth into a space.

In Part II we will discuss selection of roots and root tracking.

In the known part axis system (X , Y , Z), xk , yk , zk is a point where the known form contacts the rack form when placed in stationary mesh in accordance with Theorem A.

Point xr , yr is on the rack as seen in the X - Y plane.

Point xr'' , yr'' is on the rack in the rack's own X'' - Y'' plane. The xr'' , yr'' array defines the complete rack form.

$$ym' = C \cdot IM - yr \dots \dots \dots (22)$$

$$xm' = (IK \cdot R \cdot N'/N - ym' \cdot$$

$$\cos\beta / \cos\alpha) / \tan(\phi - \Theta) \dots \dots (23)$$



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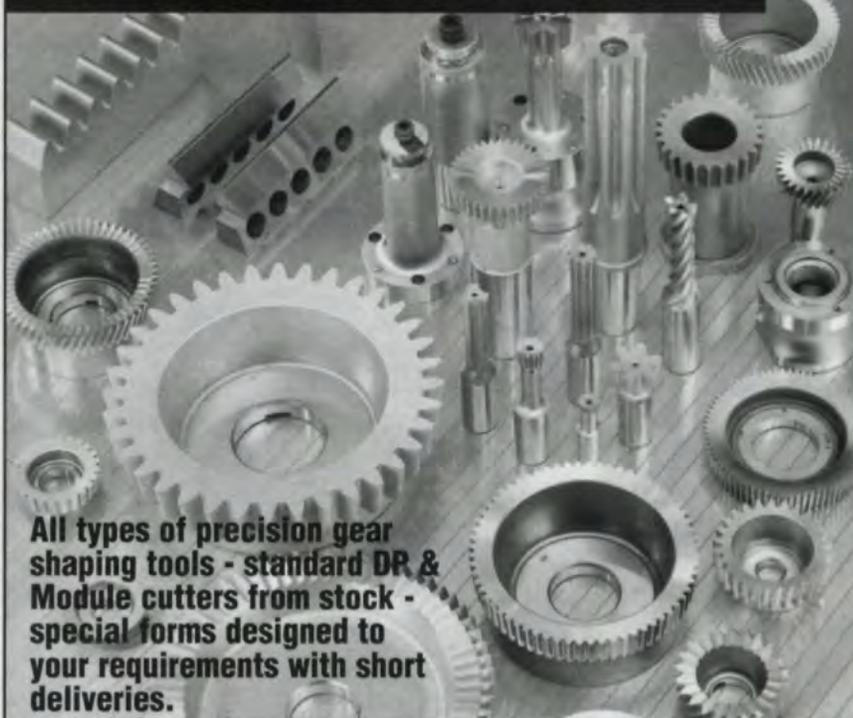
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$$zm' = IK \cdot (-xr' - xm' \cdot \cos\beta) / \sin\beta \dots (24)$$

$$\tau = \arctan(xm'/ym') -$$

$$(xm' \cdot \cos\beta / \cos\alpha + xr') \cdot$$

$$IK \cdot N / (R \cdot N') \dots (25)$$

$$za' = IK \cdot \tau \cdot .5 \cdot L / \pi \dots (26)$$

$$ya' = \sqrt{xm' \cdot xm' + ym' \cdot ym'} \dots (27)$$

$$xt' = IK \cdot ya' \cdot \sin\tau \dots (28)$$

$$yt' = IK \cdot ya' \cdot \cos\tau \dots (29)$$

The transverse pressure angle of the known part at the point of contact with

the rack is $(\phi - \Theta)$. Point xm', ym', zm' is in the mating part axis system (X', Y', Z'), where the mating form contacts the rack form when placed in stationary mesh in accordance with the corollary to The-orem A. This same point cranked along the helix to the transverse plane of the mating part is xt', yt' . Cranked along the helix to the axial plane of the mating part, it is za', ya' . This math works even when the mating part is a

spur form. In this case, calculations for ya', xt' and yt' are valid, but za' in Equation 26 goes to infinity. For this reason, the xt', yt' array should be considered the primary definition of the output form.

Note that Equations 17 and 24 are included for three-dimensional completeness but, along with Equation 26, are not required in the math for the final output form. Avoiding these three equations will remove most possibilities of dealing with division by zero in the computer.

For computer applications, Equations 14 and 25 should use an expanded arctan routine [ATAN2(x2, y2), for example], so that the resultant angle will lie within the full range of $\pm 180^\circ$ instead of the usual $\pm 90^\circ$.

Part II—Solving Calculation Difficulties

Point $x2, y2$ always has two possible positions corresponding to the \pm condition in Equations 12 and 13. (Note that when “+” is used in (12), “-” is used in (13) and vice versa.) Experience has shown that there is not an easy way to pre-select which sign to use in choosing one of these positions, so we calculate both of them and compare them with our model for the application at hand.

In order to apply root tracking logic, the array of points xt, yt must be in consecutive order from the shoulder on one side of the form to the shoulder on the other side. Use the following algorithm to select the correct root:

1. Select the first point xt, yt with slope st . Set IFIRST=1.
2. If Equation 11 becomes the square root of a negative number, there is no output point corresponding to this input point, so go to Step 6. Otherwise, calculate both positions of the point $x2, y2$ and both positions of the point xr, yr .
If IFIRST equals 0, go to Step 4.
Otherwise, calculate the distance, DIST1, from xt, yt to the first position of $x2, y2$. Calculate the distance, DIST2, from xt, yt to the second position of $x2, y2$. Distance calculation is the simple “square root of the sum of the squares” used to find the hypotenuse of a right triangle.
3. If DIST1 is less than DIST2, then choose the first position of $x2, y2$ and xr, yr as the correct values. Go to Step 5.

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Otherwise DIST2 is less than DIST1, so choose the second position of x_2 , y_2 and x_r , y_r as the correct values. Go to Step 5.

4. Calculate the distance, DIST1, from XRSAV, YRSAV to the first position of x_r , y_r . Calculate the distance, DIST2, from XRSAV, YRSAV to the second position of x_r , y_r . Go to Step 3.

5. Save the chosen position of point x_r , y_r as XRSAV, YRSAV. Calculate and save x_t' , y_t' in a point array. If all points in the x_t , y_t array have been processed, go to Step 7. Set IFIRST=0.

6. Select the next point x_t , y_t with slope st . Go to Step 2.

7. Exit algorithm.

This tracking logic works best when the output points x_t' , y_t' are spaced closely together (about .001"). Because output spacing is seldom the same as input spacing, sometimes the spacing of the input points x_t , y_t will need to be decreased in order to achieve satisfactory output spacing. Also, whenever the input tooth form has a sudden change in slope, such as the corner of a tooth where the outside radius intersects the tooth flank, that corner point should be entered several times with the same x_t , y_t values, but with slope (st) values ranging from the slope of the curve at the outside radius to the slope at the flank. This is because a single input point can generate a series of different output points. A similar situation occurs when replacing the corner point with a very small corner radius.

After the output form has been calculated, it invariably needs further enhancement to be useful. Some of the calculated output coordinate points may need to be thrown out because they form a closed loop or zigzag. Part V goes into the significance of these discarded points.

Although it is possible to graphically display the output directly as a series of points and to manually unselect the extraneous points, it is much easier to use a curve fitting utility program to fit the points into line-arc geometry. Then another utility program can trace through this geometry and eliminate the loops and zigzags. Fig. 2 shows a known form which is to be processed to calculate its mating form. This is the tooth space of an ordinary 20-tooth pinion. The mating form is a single tooth of a 75-tooth gear.

Fig. 3 shows the mating form directly after curve fitting and again after elimination of loops and zigzags.

A zigzag occurs whenever the generation goes through a brief reversal before continuing on. The zigzag is usually not visible except at high magnification and would not ordinarily affect the display or usage of the form. However, if line-arc geometry containing a zigzag is presented to a computer program for further processing or to an NC machine, it will be perceived as an error condition. Fig. 4 shows a highly magnified section of a form containing a zigzag and again with the zigzag removed. Removal is accomplished by finding the intersection point after the reverse leg of the zigzag is discarded and joining the form at that point.

For purposes of sending the output geometry back through the system as input geometry (an operation known as "play-back"), it is possible to derive from the mesh equations the slope st' corresponding to each point x_t' , y_t' of the mating part. However, in practice this is avoided simply by picking new points and slopes from the line-arc geometry provided by the aforementioned curve fitting utility program.

Part III—The Fundamental Law of Gear Tooth Action

Applied to Crossed-Axis Gearing

The pitch cylinders of two gears in mesh at crossed axes may be tangent to each other, intersect or not meet at all. This statement has two implications. First, it implies that each pitch circle has its own pitch point. Secondly, it implies that the pitch circles of parts in mesh at crossed axes do not change when the center distance changes. Both of these implications are true, which leads to a restatement of the law of gear tooth action when applied to crossed-axis gearing:

"In order for two meshing parts to have conjugate action, the common normal at any point of contact must pass through the *line whose end view in the transverse plane is a pitch point.*"

The italicized words extend the standard law into a more universal law. When the pitch cylinders are non-tangent, the common normal must therefore pass through both lines whose end views are pitch points. Visualizing this in three

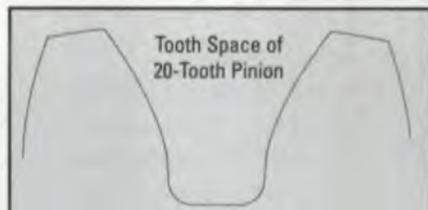


Fig. 2 — Input form.

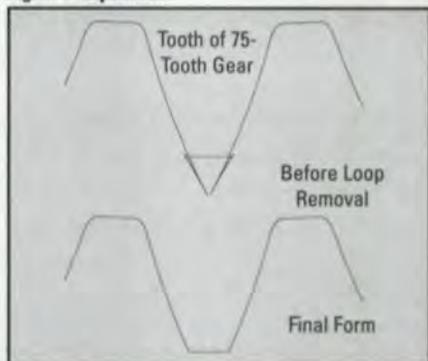


Fig. 3 — Output form.

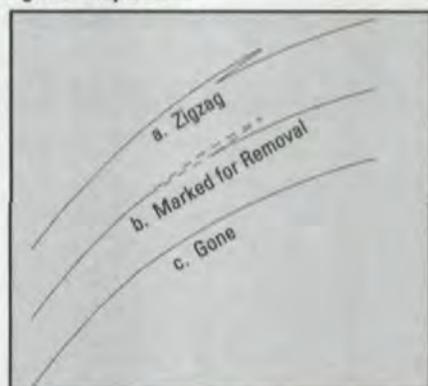


Fig. 4 — Zigzag, 250x size.

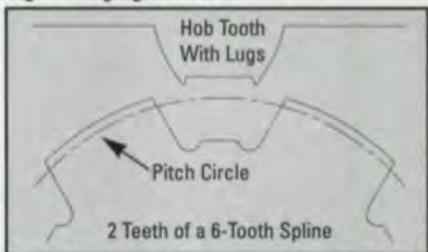


Fig. 5 — Straight-sided spline and hob tooth.

dimensions will be somewhat easier after considering the path of contact equations in Part IV.

When the pitch cylinders are tangent to each other, which is the more common case, both of the pitch points are coincident. Although the math of Part I does not require direct application of this extended law to define the mating form, nevertheless, an understanding of this geometry will help in visualizing the contact of parts running at non-standard conditions.

Two further points should be noted:

1) For parallel-axis gearing, the pitch circles of mating gears must always be tangent to each other, which means that

the pitch circles must change when the center distance changes.

2) For involute gearing, a change in center distance does not require a change in the tooth form. For any other meshing pair, a new mating form must be calculated for any center distance change, regardless of whether the parts are designed to run with tangent pitch cylinders or non-tangent pitch cylinders.

Part IV—The Path of Contact

An important part of mesh math is the calculation of how the mating parts contact each other. This can tell us not only where contact occurs in each coordinate axis system, but the limits of contact.

When the pairs mesh on parallel axes, the path of contact does not have the same meaning as it does for crossed axes, so Equations 33–52 will not apply. Instead of single point contact, simultaneous contact occurs all across the face of the meshing parts in parallel-axis gearing, so we usually only define the path of contact in the transverse plane. Equations 15 and 16 give the contact point xk, yk in the transverse plane of the known part, which, for parallel axes, is also the transverse plane of the mating part. The following equations give the contact point in the rack's $X''-Y''$ plane (which is the part's normal plane):

$$x_c'' = xk/\cos\alpha \dots\dots\dots(30)$$

$$y_c'' = yk \dots\dots\dots(31)$$

$$z_c'' = 0 \dots\dots\dots(32)$$

For crossed-axis gearing, previous equations have established where each of the two forms contact the same rack element, but not at a coincident point. We know that part rotation will cause a contact point to move along the rack element, so it is a simple solution to find the amount of rotation, δ , defined in the transverse plane of the known part, which will bring the two points into coincidence:

$$z_k'' = z_k \cdot \cos\alpha - x_k \cdot \sin\alpha \dots\dots\dots(33)$$

$$z_m'' = z_m' \cdot \cos\beta - x_m' \cdot \sin\beta \dots\dots\dots(34)$$

$$\delta = 2 \cdot \pi \cdot (z_m'' - z_k'') / [L \cdot \cos\alpha + IK \cdot (N/N') \cdot L' \cdot \cos\beta] \dots\dots\dots(35)$$

$$x_c'' = x_r'' + R \cdot \delta \cdot \cos\alpha \dots\dots\dots(36)$$

$$y_c'' = y_r'' \dots\dots\dots(37)$$

$$z_c'' = z_k'' + .5 \cdot \delta \cdot L \cdot \cos\alpha / \pi \dots\dots\dots(38)$$

The distance on the rack element between the contact point with the known part and the contact point with the mating part when both parts are in stationary mesh in accordance with the corollary to Theorem A is $(z_m'' - z_k'')$.

Point x_c'', y_c'', z_c'' is in the rack axis system (X'', Y'', Z'') where the known part contacts the mating part after the rotational adjustment. The x_c'', y_c'', z_c'' array of points describes the path of contact of the mating pair. Further, since z_c'' is usually zero (see next paragraph), the contact path usually occupies only the $X''-Y''$ plane. This is very convenient for displaying the active mesh of a mating pair on a computer screen.

When meshing at crossed axes with tangent pitch cylinders, the term z_c'' is always zero for any form, involute or otherwise. When the cylinders are non-tangent, the center distance becomes greater or less than standard, and z_c'' becomes a nonzero constant for involute forms (plus on one side of the tooth and minus on the other). For non-involute forms, z_c'' becomes a variable. The actual value of z_c'' depends on the amount of deviation from the standard center distance and other factors. Geometrically, this deviation allows the common normal to intersect both lines whose end views are pitch points, a required condition explained in Part III. For a pair of non-involute parts meshing at a non-standard

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center distance, each pitch point moves axially during the mesh, although their end views do not change.

There are two special cases where one of the forms is helical and the other spur. In these cases either Equation 17 or Equation 24 goes to infinity.

Case 1. Known part is spur. Do not use Equations 17 or 33 or 35-38.

$$\delta = -\Theta \dots\dots\dots(39)$$

$$xc'' = xk \dots\dots\dots(40)$$

$$yc'' = yr'' \dots\dots\dots(41)$$

$$zc'' = zm'' + IK \cdot \Theta \cdot R \cdot \cos\beta/\sin\beta \dots\dots\dots(42)$$

Case 2. Mating part is spur. Do not use Equations 24 or 34-38.

$$\delta = -(xr'' + xm'')/(R \cdot \cos\alpha) \dots\dots\dots(43)$$

$$xc'' = -xm' \dots\dots\dots(44)$$

$$yc'' = yr'' \dots\dots\dots(45)$$

$$zc'' = zk'' - .5 \cdot L \cdot (xr'' + xm'')/(\pi \cdot R) \dots\dots\dots(46)$$

Finally, Equations 47-49 convert the contact point xc'' , yc'' , zc'' to the known part axis system, and Equations 50-52 convert it to the mating part axis system.

$$xc' = xc'' \cdot \cos\alpha - zc'' \cdot \sin\alpha \dots\dots\dots(47)$$

$$yc' = yr'' \dots\dots\dots(48)$$

$$zc' = xc'' \cdot \sin\alpha + zc'' \cdot \cos\alpha \dots\dots\dots(49)$$

$$xc' = -xc'' \cdot \cos\beta - zc'' \cdot IK \cdot \sin\beta \dots\dots\dots(50)$$

$$yc' = ym' \dots\dots\dots(51)$$

$$zc' = -xc'' \cdot IK \cdot \sin\beta + zc'' \cdot \cos\beta \dots\dots\dots(52)$$

Part V—Fill-In and Round-Over

Two common problems with generated forms are "fill-in" and "round-over." Fill-in occurs when you ask for a small root fillet, but because of spatial interference (those points we had to throw out back in Part II), the generating too produces a sweeping trochoidal fillet instead. This can be avoided by reducing the pitch radius so that the pitch circle passes through the root fillet, because the least amount of relative motion occurs at the pitch point. This, however, often leads to the problem of round-over. Round-over is required whenever the desired tooth form has a section where the normal to the surface does not intersect the pitch circle. It is not possible to generate a form at a point where the normal does not intersect the pitch circle (because Equation 11 becomes the square root of a negative number), so the solution must be either to increase the pitch circle or to change the curve of the tooth form (rounding over), allowing generation to occur.

The hobbing of a straight-sided spline provides a perfect example of both these problems. When the pressure angle is low (as in the case of straight-sided splines), the pitch circle must be large enough to avoid round-over, but the farther the pitch circle is from the spline tooth root, the more fill-in occurs. Often a compromise is made where the manufacturing form tolerance is used to allow incipient round-over near the tip of the spline, and lugs are added to the tip of the hob to push the fillet into a groove. Fig. 5 shows a spline form and the hob tooth that generates it.

Part VI—Using the Math of Mesh

The design of parts that mesh with each other, such as seam rollers and impeller rotors, can be optimized rapidly by generating mating forms at varying center distances and viewing the mesh graphically.

Contact limits can be calculated to find the required width of parts meshing at crossed axes. These limits also can define the generating length of a hob. Contact points and contact limits can be calculated for gear shaving cutters, which routinely mesh on crossed axes at non-standard center distances.

In addition to the obvious use of the math to calculate mating forms, it may also be used to determine the amount of error that occurs when non-involute forms mesh at changed center distances. The most common example of this is a hob designed to cut an impeller form, sprocket or a straight-sided spline. As the hob is sharpened, it runs at a closer and closer center distance than it was designed for. By running the hob form through the mesh math with a reduced center distance, the actual part form may be determined. This can be compared with the desired part form (usually by graphical overlay).

Even involute hobs are not totally free of this problem. Hobs are invariably manufactured with straight sides rather than the curvature required by the involute. When they are sharpened, they deviate even further from the theoretical curvature. This error will never be discernible except in very coarse pitch hobs or multiple start hobs because the involute curvature is so slight on single thread involute worms with a small lead/diameter ratio.

Another application is determining the range of teeth a non-involute hob or shaper cutter can cut. Chain sprockets, timing belt sprockets and pre-shaved involute gears are examples of parts which have hobs or shaper cutters designed for a particular number of teeth. Mesh math can determine the form that these tools will produce when they cut a different number of teeth. Then a graphic overlay with the theoretical form will quickly determine whether the form error is great enough to warrant a new hob or shaper cutter design.

Yet another use of mesh math is to determine how close a generated form will be to the desired form. As a case in point, consider a toothed ratchet form with a sharp root fillet which is to be produced by hobbing or shaping. The ratchet tooth form is fed into the mesh math, and the hob or shaper cutter tooth is output. Depending on several factors, there will probably be some unavoidable loss of form, which will result in having fill-in at the ratchet tooth root fillet. Because the mesh math is reversible, the hob or shaper cutter geometry form data can be fed directly back through the system to determine the actual ratchet tooth form. This often points to the simple solution of choosing a smaller pitch radius, or it can lead to a redesign of the hob as a "mutilated tooth" hob or an "alternate gash" hob.

The concepts of the mathematics of mesh are simple, but they require care in their application. Nevertheless, a minimally experienced computer programmer should be able to make productive use of these equations. ⦿

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