

Endurance Limit for Contact Stress in Gears

by
G. C. Mudd and J. M. France
David Brown Gear Co.

Synopsis

With the publishing of various ISO draft standards relating to gear rating procedures, there has been much discussion in technical papers concerning the various load modification factors. One of the most basic of parameters affecting the rating of gears, namely the endurance limit for either contact or bending stress, has not, however, attracted a great deal of attention. In view of the fact that ISO and other modern gear ratings attempt to assess the real stresses experienced by the gear teeth, it is important that the material allowable stresses are equally understood. This is particularly so when material properties are varying, as in a surface hardened layer.

This paper reports on work done examining variations in surface hardened gears and the interaction of surface, bending, and residual stresses in a field of varying hardness.

Introduction

The traditional stress analysis approach to determining the performance of a load carrying member is to evaluate the stress cycle and, by means of a Goodman diagram, determine a factor of safety for the material in question. There is no fundamental reason why a gear tooth should not be evaluated in the same way. However, for surface hardened gears, there are complications, because of the following reasons:

- The stress cycle is a combination of contact (Hertzian) and bending stresses.
- Residual stresses are present in the surface of the gear tooth.
- The resistance of the material to fatigue, which is a function of hardness, varies with depth below the surface.

The Stress Cycle

As illustrated in Fig. 1, the stress cycle, experienced by a point on the flank of a gear tooth, consists of a compressive Hertzian stress followed by a tensile bending stress.

AUTHORS:

DR. GEORGE MUDD is Director of Engineering at David Brown Gear Industries, Huddersfield, England. He has a B. Sc, C. Eng., and MI. Mechanical Engineering degrees. He began his career in 1951 in Design, moving in 1953 to Research and Development with particular emphasis on the fundamental nature of gear tooth failure in all applications.

MR. J. M. FRANCE joined David Brown Gear Industries in 1974 after spending eight years in the airframe industry. Mr. Brown holds a B. Sc. and has worked in various sections of the Engineering Division including gear development testing. Currently he is supervisor of a specialist section dealing with gear technology.

Cycle of Stress at a Point on Tooth Flank

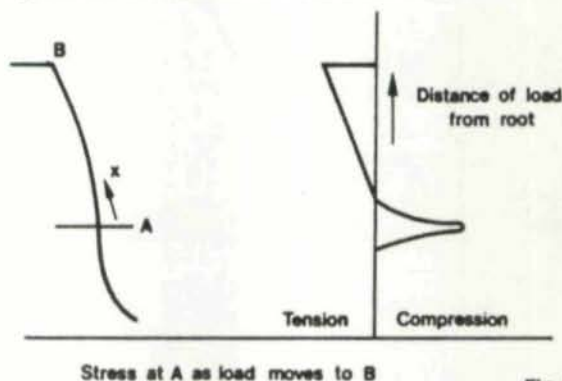


Fig. 1

Hertzian stresses are well known, and are dependant upon the relative radius of curvature of the gears at the point of contact, and vary with depth below the surface and distance from the point of application of the load. Bending stresses are traditionally associated with the tooth root bending calculations, but also have an effect on the stress cycle at the pitch line as the load moves towards the tip. In the tooth fillet, of course, the stress cycle consists of the bending stress alone, experienced once per revolution.

Residual Stress

Residual stresses may be present in all materials, but may be regarded as negligible in (through) hardened materials. In surface hardened steels, however, residual stress is induced, because of volume changes as austenite transforms to martensite during the quenching operation. As shown typically in Fig. 2, heat treatment procedures are chosen to produce a compressive residual stress at the surface, which is balanced by tensile residual stresses near the case/core interface.

The total stress state resulting from the combination of Hertzian, bending and residual stresses is, therefore, complex varying in pattern at every point from the tooth tip, to the root fillet, and from the surface to sub-surface.

Hardness Gradients

For each point, the total stress must be compared with the fatigue resistance of the material at that point, which, for a surface hardened gear, because of the hardness variation, also changes with distance from the surface (Fig. 3).

Criterion of Failure and Endurance Ratio

Over a period of years, disc tests have been carried out at D.B.G.I. as a means of comparing material performance, using a variety of materials (refs. 1 to 3). One of the limitations of tests conducted with similar sized discs has been, that

Typical Residual Stresses

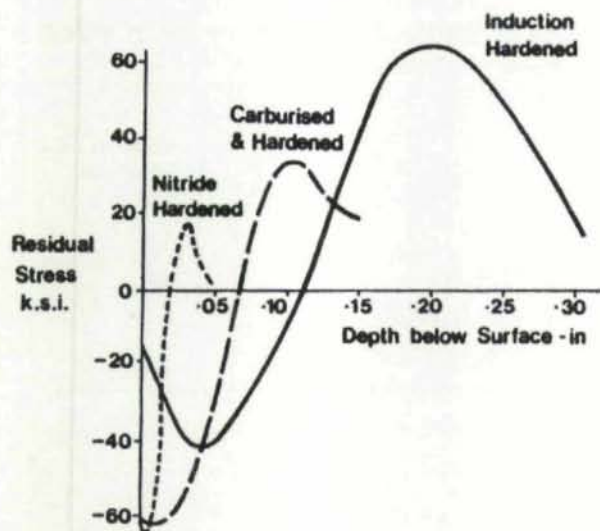


Fig. 2

Typical Hardness Gradients

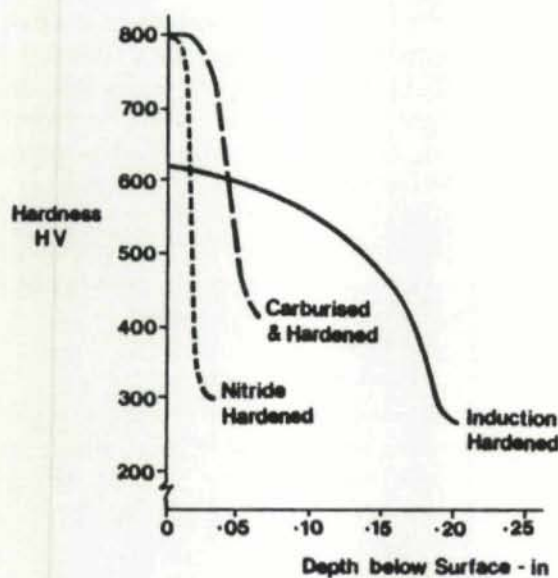
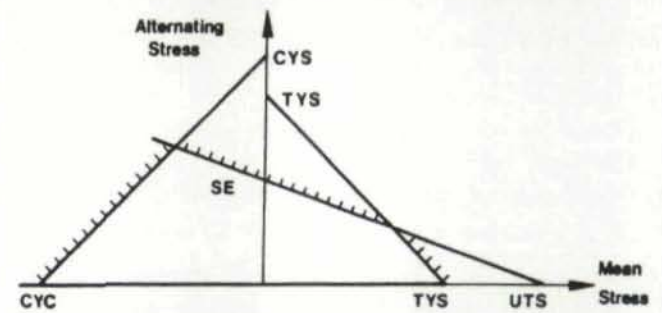


Fig. 3

for surface hardened materials, the results could not reflect the effect of radius of curvature on the performance; but these results were of great value in providing the information for carrying out an evaluation of the effectiveness of several alternative failure criteria. By comparing the predictions with the disc tests results, a judgement was made as to the criterion which most closely predicted the actual results. The combination of criterion of failure and endurance ratio, which gave the best correlation between theory and test results, was a Direct Stress criterion using an endurance ratio of 0.5. The Goodman diagram used in the evaluation is shown in Fig. 4.

Goodman Diagram used in Analysis



----- Indicates boundary of permissible stress

Nomenclature:-

CYC Compressive yield strength TYS Tensile yield strength
SE Endurance limit (0.5 UTS) UTS Ultimate tensile strength

Fig. 4

Use of HTZ to Evaluate Endurance Limit for Contact Stress

A computer program, HTZ, has been written to analyze the total stress field experienced by a gear tooth and compare this with the material properties at points on and below the surface of the tooth flank and fillet. A description of program HTZ, with a typical output, is shown in Appendix 1.

Using the criterion of failure selected above, the program has been used to analyze a large selection of hypothetical and real gears. The following hardening processes were considered:

1. Through hardened
2. Carburised and hardened
3. Nitride hardened
4. Induction hardened

The effect of varying the core hardness (within limits typical for each process) and of varying the case depths has also been investigated.

Previous work (Ref. 8) has indicated that the limiting contact stress for gears is a function of the dimensionless parameter - relative radius of curvature/module (ζ_c/m_n). This parameter has been used in the presentation of the theoretical results and good correlation was again found. This is shown in Fig. 5 and Fig. 6.

In the case of surface hardened gears, a further complication is introduced by the possible variation in case depth. The actual case depth used on a gear may be chosen for geometrical, or economic reasons, or as a limitation of the process

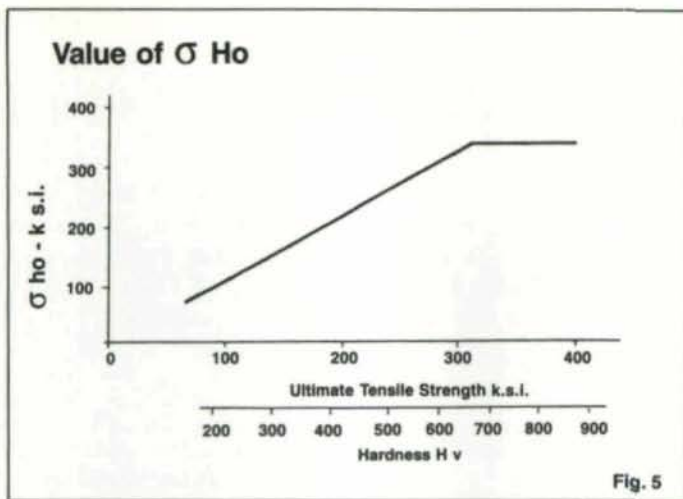


Fig. 5

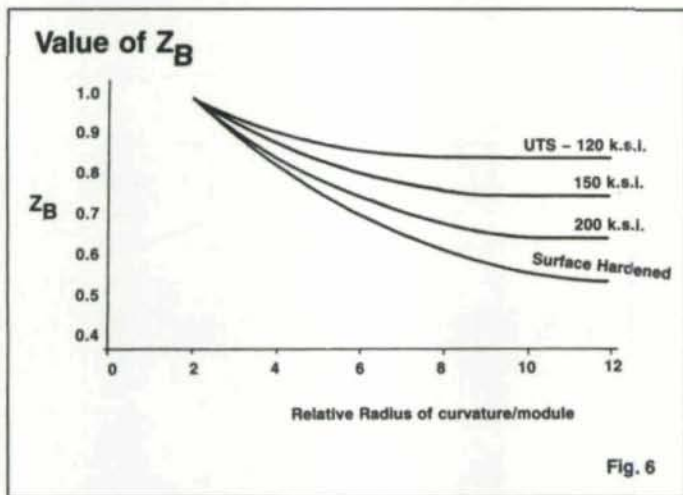


Fig. 6

used. The gear examples were, therefore, computed with different values of effective casedepth and the data generated used to define:

- A 'limiting depth' (i.e. the effective casedepth beyond which a further increase in casedepth does not produce further increase in failure load), and
- The reduction in failure load resulting from effective casedepths less than the limiting depth. This applies in particular to large module nitrided gears where the limiting casedepth cannot normally be achieved.

The limiting depths were found to be:

Steel	Limiting Casedepth
655M13 carburised and hardened	0.16 x normal module
722M24 nitride hardened	0.20 x normal module
817M40 induction hardened	0.32 x normal module

Effective casedepth is defined as the depth at which the hardness falls below 500 HV. The value of 0.16 x normal module for carburised and hardened steel was found to be remark-

ably close to the historical values, which have proved to work well in practice.

The reduction in the endurance limit for contact stress, with reduction in effective casedepth, is shown graphically in Fig. 7 against the parameter actual casedepth/limiting casedepth. The same graph can be used for each of the three hardening processes considered.

The endurance limit for surface stress is then calculated from the product of the values from the three graphs in Fig. 5, Fig. 6 and Fig. 7, i.e.

$$\sigma_{\text{Hoeff}} (S_{AC} \text{ in AGMA}) = \sigma_{\text{Ho}} \cdot Z_B \cdot Z_C$$

when σ_{Hoeff} = endurance limit for surface stress

σ_{Hoeff} = endurance limit for surface stress of a disc

Z_B = disc/gear correlation factor

Z_C = casedepth factor

Experimental Results

Disc Results

Disc results are presented as Hertzian stresses at the failure load calculated from:

$$\sigma_{\text{HD}} = \sqrt{\frac{F}{b\pi} \cdot \frac{E}{1-\nu^2} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)}$$

when F = force between discs (failure load)

b = width of disc

E = Young's modulus

ν = Poisson's ratio

r_1, r_2 = radii of curvature of roller and disc respectively

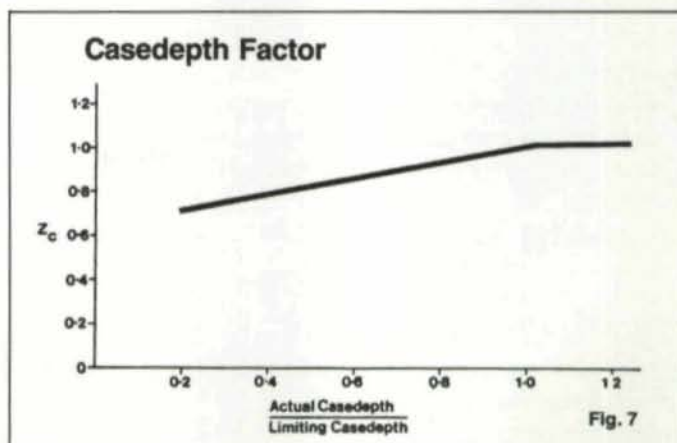


Fig. 7

Experimental and HTZ Results for Discs

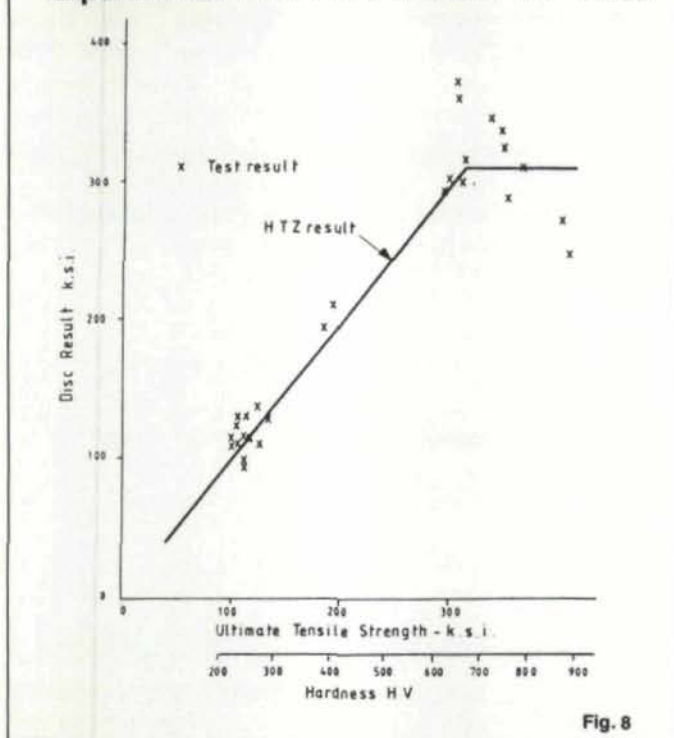


Fig. 8

The results are shown graphically in Fig. 8 with the theoretical result from HTZ for comparison.

Gear Circulator Tests

In parallel, with the theoretical analysis, a number of gear circulator tests have been performed. The first tests were to investigate the effect of the manufacturing process on tooth root bending stress. For example, grinding after heat treatment tends to reduce the compressive residual stress at the surface; shot-peening tends to produce compressive residual stress at the surface. These results have been used for comparison with the theoretical bending results from HTZ.

Before the test result can be used to calculate a bending stress, however, it is necessary to know the ratio of the peak bending moment experienced by the tooth throughout the tooth cycle, to the nominal bending moment calculated using the AGMA inscribed parabola. This was achieved using a computer program CLODA. A description of program CLODA, with a typical output, appears in Appendix 2. The misalignment was assessed from no-load contact markings and other deviations from true involute were measured. Dynamic and surface finish effects were accounted for by factors from a gear rating standard internal to David Brown Gear Industries Ltd. This gives values of dynamic factor and roughness factors similar to ISO. Step loading was accounted for by the use of Miner's Rule.

Test Details

The tests were carried out on an 8 inch centers gear circulator. Torque was locked into the system by means of an adjustable coupling, and measured using strain gauges and

telemetry equipment. Gears were 7 P_n with a 37 tooth pinion and 75 tooth wheel. Two facewidths were tested, namely 1 inch and 0.5 inch.

Test Results

Test results are, in most cases, 99% confidence levels based on five tests on identical gears. The raw results have been modified for non-uniform load and moment distribution by the factors calculated by computer program CLODA. Results are tabulated in Fig. 9 in which the theoretic result from program HTZ is also given for comparison.

Batch No.	Batch Size	Face-width	Failure Type	Test Failure Power-HP	Theoretical Power Capacity from HTZ-HP
1	5	1	Pinion Bending	647	648
2	5	1	Pinion Bending	700	648
3	4	0.5	Pinion Bending	365	324
4	2	0.5	Wheel Bending	359	330
5	1	1	Pinion Pitting	1047	965

Fig. 9

Gear Circulator Test Results

Note: The test results have been adjusted for the effect of load distribution (CLODA values), dynamic, lubricant and roughness effects. Tests to produce surface failure (pitting) are still proceeding, but early results have been used for comparison with the theory. Again, it is necessary to modify the load before calculating a Hertzian stress, in this case, by the load distribution factor as evaluated by CLODA.

Discussion of Results

The table of results (Fig. 9) shows that good agreement exists between the experimental and theoretical results for both contact and bending stress failures. For contact stress, the disc test results (Fig. 8) provided further confirmation of the theory.

The results of the effect of reduced case depth (Fig. 7), it is believed, have an importance beyond the field of gear rating only. This graph can be used as a production optimisation and when carburised pinions are meshing, for instance, with large through-hardened wheels. In such cases, the limiting case depth will not usually be required and considerable cost saving can be achieved.

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Appendix 1 - Program HTZ

The program 'HTZ' analyses the complete stress history of a disc, or a gear tooth, down the flank and in the root fillet at varying depths below the surface in the following way:

A matrix is set up consisting of ten points down the flank and six points round the root fillet, each at fifteen depths below the tooth surface.

Considering a particular point in this matrix a nominal load intensity is applied at the tooth tip and a finite element analysis performed to establish the bending stress at that point.

The same nominal load intensity is then applied at points down the flank, and using classical Hertzian theory, the contact stress at the point under consideration is calculated and combined with the bending stress previously calculated. Residual stresses, either from heat treatment processes or from finishing processes, are added to this stress pattern. Using a Goodman diagram and the stress cycle established above, the load intensity and position (for Hertzian stress) is iterated until a reserve factor of unity on fatigue life occurs at that point.

This is repeated for each point in the matrix, such that a matrix of failure load intensities can be constructed. In a surface hardened gear where the hardness varies with depth, each point in the matrix has an identifiable hardness and, therefore, a different Goodman diagram.

The failure load intensities are then converted to reserve factors by dividing throughout by their minimum (i.e. a reserve factor of unity will occur at one point in the matrix, all other values being greater or equal to unity).

The minimum reserve factor at the pitch line is then used to calculate the basic endurance limit for contact stress (SAC in AGMA) from classical Hertzian theory.

In the tooth fillet, where there is no contact, the tooth root bending stresses, including stress concentration, are used in the Goodman diagram. Again residual stresses and the hardness gradients are taken into account. In this case, the load is taken as acting at the tooth tip and by iterative procedure is used until a reserve factor of unity is achieved. Using the AGMA inscribed parabola procedure an equivalent permissible bending stress (SAT in AGMA) can be evaluated.

Input to the program consists of:

- a) gear geometry
- b) residual stress pattern
- c) material hardness gradient (if surface hardened) or ultimate tensile strength (if through-hardened)

Appendix 2 - Program CLODA and Its Use in Interpretation of Gear Tests

Description of Program

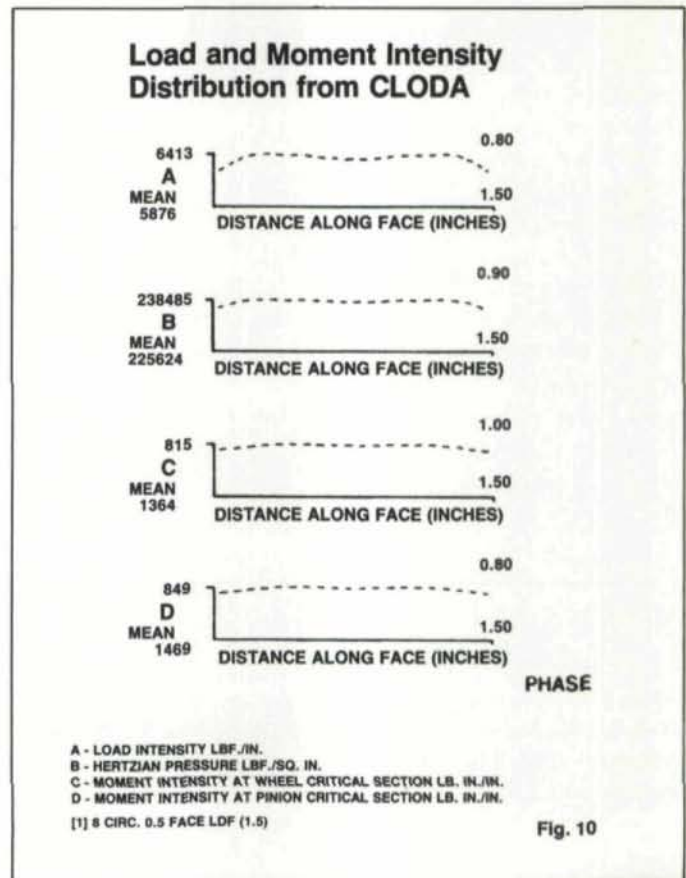
Program CLODA evaluates the distribution of load and the resultant bending moment at the critical section across a meshing spur or helical gear pair. The load distribution is calculated using the contributory flexibility of the gears, the transmitted torque and the deviation of the mesh from true involute and from true alignment. The bending moment distribution evaluated from the load distribution using our integration of Jaramillo's cantelever plate theory (ref. 5) modified by the moment-image method of Wellauer and Seireg (ref. 6).

Tooth deflections are based on a three dimensional finite element model (ref. 7) of a helical gear tooth which enabled the end effects to be evaluated. For speed of operation it was found possible to construct a modification file based on these three dimensional results which modifies the two dimensional model of each gear tooth as it is considered.

Input to the program consists of:

- a) gear details
- b) shaft sectional details
- c) operating torque
- d) alignment and pitch errors
- e) profile and helix modifications
- f) imposed deflections (due to other loads on the same shaft)

The analysis is performed for one mesh at ten positions through the engagement cycle. An option is available to graph the load intensity, moment intensity and Hertzian stress at the position at which the maximum occurs. Typical graphs are shown in Fig. 10.



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