

# Application of Statistical Stability and Capability for Gear Cutting Machine Acceptance Criteria

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## Introduction

Machine tool manufacturers supplying machines to the gearing world have been in existence for many years. The machines have changed, and so has the acceptance criteria for the machines. Before the 1980s, the criteria for virtually all machine acceptance was either a supplier's standard test job or the supplier producing one or two of the customer's parts to within a print tolerance.

It wasn't until about 1984 that The Gleason Works of Rochester, NY, was required to perform a capability analysis for machine acceptance on a cylindrical hobbing machine. Capability requirements for bevel machine acceptance did not occur

until several years after that. Today virtually every customer requires a capability study on at least one parameter for at least one type of part.

Since the introduction of capability requirements for machine acceptance, we have seen the goal post move. Initially the requirement was for a Cp or Cpk of 1.33 using a 6-sigma analysis. Now we have seen requirements of a 1.67 or 2.0 Cp or Cpk with a 6-, 8- or 10-sigma analysis on tolerances that have been tightened from the original tolerance!

This can cause some real headaches for the machine tool supplier, and that is why it is very important for the supplier to understand true machine and process capability before agreeing to any capability requirement.

In 1991, the quality and supplier assessment staffs at Chrysler, Ford and General Motors worked under the auspices of the Automotive Division of the American Society for Quality Control (ASQC) Supplier Quality Requirements Task Force in collaboration with the Automotive Industry Action Group (AIAG) to put together the *Statistical Process Control (SPC) Reference Manual* (Ref. 1).

The same group developed the *Measurement Systems Analysis (MSA) Reference Manual* (Ref. 2). Many companies, such as Delphi Automotive Systems, have developed their own statistical qualification requirements based on both the SPC and MSA reference manuals.

This paper will reference the SPC and MSA manuals and the Delphi Specification SD-002 for much of the material presented.

With this paper, we hope to review some of the basics of Statistical Process Control (SPC) and provide a better understanding of its application as it relates to a machine runoff.

## Customer Agreement, Data Collection & Distributions

Before conducting a machine runoff that has a capability study tied to it, it is very important that both the customer and the machine tool supplier agree on the parameters to be measured and on

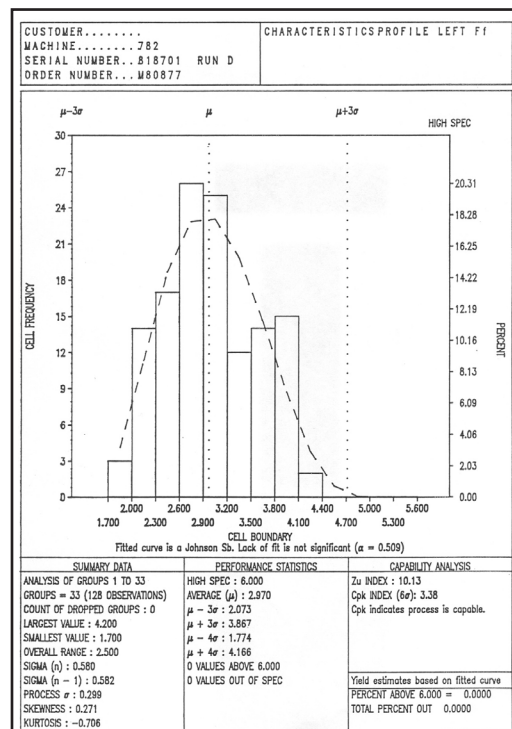


Figure 1—Histogram and distribution.

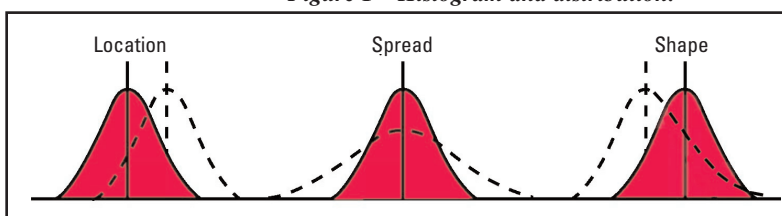


Figure 2—Distributions.

what the tolerance and capability requirement is for machine acceptance.

Once these are established, the parts are usually produced on the machine in a continuous run without interruption under the conditions agreed to. The parts are then inspected on equipment that has passed a GR&R study (more on GR&R later) and the resultant data analyzed.

There are a number of tools that are used today to analyze the data collected, from spreadsheet software that can be tailored using built-in statistical functions to software designed specifically for statistical analysis, such as the popular MINITAB package.

Once collected, the data can be organized, analyzed, interpreted and presented (Fig. 1). Besides the average, range and the data's other statistics, the standard deviation can be calculated.

Think of the standard deviation as the statistical spread or dispersion from the mean of the data collected. There are several methods that can be used to calculate the standard deviation, designated by the Greek letter  $\sigma$  (sigma). One method is to use the individual values of a process characteristic, and another method estimates the standard deviation using the average range from a subgroup analysis and a factor, designated as  $d_2$ . (See below for further discussions on subgroups.)

No two parts are exactly alike because every process contains some source of variability. While individual values may be different, as a group they can form a pattern that can be described as a distribution.

Distributions are characterized by a location (the typical value), a spread (the span of values) and a shape (the pattern of variation). See Figure 2.

The causes of variation in a distribution are referred to as either "common causes" or "special causes." The term "assignable causes" is often used in place of "special causes."

The type of variation preferred in any distribution is that of common causes. When common causes exist (also referred to as random variations), the process is said to be "in control" and the process's output is stable and predictable over time, as depicted in Figure 3.

Distributions with special or assignable causes of variation are not stable over time. When present, assignable causes will produce changes in the distribution, and if they are not removed, the process output cannot be predicted (Fig. 4).

If a process is in control (a predictable distribution), the number of in-specification parts can be estimated. As long as the process remains sta-

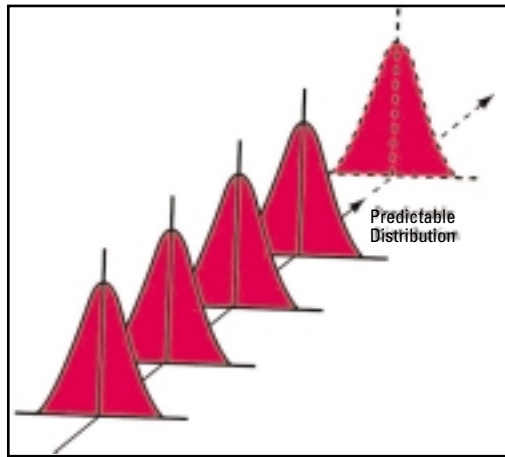


Figure 3—Predictable distribution.

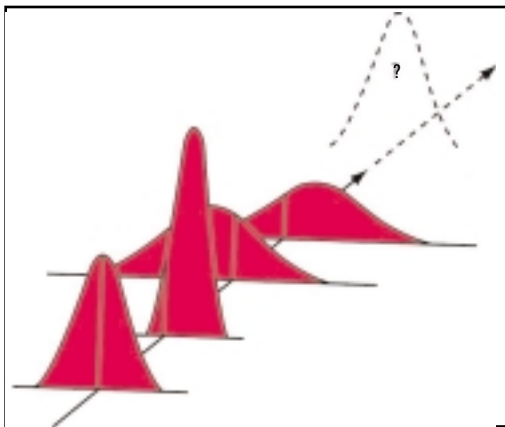


Figure 4—Unpredictable distribution.

ble and the distribution does not change in location, spread or shape, it will continue to produce the same distribution of in-specification parts.

#### Control Charts: $\bar{X}$ and R Charts

In the 1920s, Dr. Walter Shewhart of Bell Laboratories developed what is known as the control chart to make the distinction between controlled and uncontrolled variation due to common (random) and special (assignable) causes. Control charts for variables are powerful tools that can explain process data in terms of both spread (piece-to-piece variability) and location (process average).

Control charts should be prepared and analyzed in pairs, most commonly the  $\bar{X}$  and R charts.  $\bar{X}$ , the average of the values in subgroups, describes the location of the data. R, the range of the values within each subgroup, measures the data spread.

A sample subgroup table or data block is shown in Figure 5. The table consists of a defined number of subgroups and includes data from each subgroup. Each subgroup has a total, an average ( $\bar{X}$ ) and a range (R).

Generally, there are three to five individual parts per subgroup and 25 or more subgroups in the analysis. The frequency of the data collection

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## Data and Control Charts for Capability Study

Sub-Group #	1	2	3	4	5	6	7	8	9	10
Data	30.5	30.4	37.3	35.7	37.9	37.0	32.6	35.0	33.5	34.8
From Each	34.3	36.2	36.6	30.3	31.1	30.8	30.6	34.1	31.3	35.7
Sub-Group	36.7	31.3	36.3	37.0	31.2	34.9	34.8	36.1	30.6	37.7
	37.6	35.6	34.3	34.9	32.6	31.5	35.3	31.5	32.9	33.4
	37.5	37.5	37.0	33.1	36.9	33.9	30.8	34.5	34.7	32.9
Total	176.60	171.00	181.50	171.00	169.70	168.10	164.10	171.20	163.00	174.50
Average (x-bar)	35.32	34.20	36.30	34.20	33.94	33.62	32.82	34.24	32.60	34.90
Range (R)	7.1	7.1	3	6.7	6.8	6.2	4.7	4.6	4.1	4.8

Figure 5—Data table for a control chart.

n	2	3	4	5	6	7	8	9	10
D <sub>4</sub>	3.27	2.57	2.28	2.11	2.00	1.92	1.86	1.82	1.78
D <sub>3</sub>	*	*	*	*	*	0.08	0.14	0.18	0.22
d <sub>2</sub>	1.13	1.69	2.06	2.33	2.53	2.7	2.85	2.97	3.08
A <sub>2</sub>	1.88	1.02	0.73	0.58	0.48	0.42	0.37	0.34	0.31

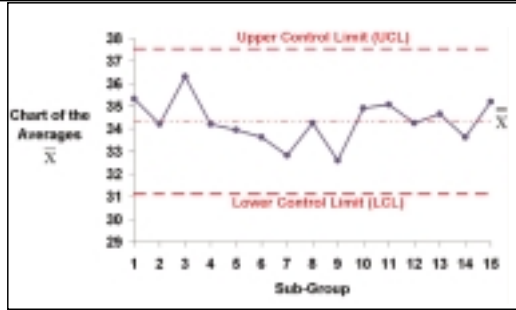


Figure 6—Control chart for averages.

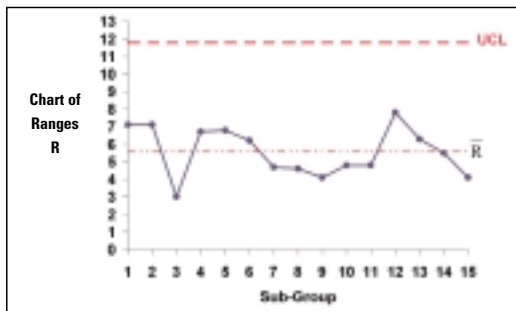


Figure 7—Control chart for ranges.

for the subgroups is determined to detect changes in the process over time. During an initial study, the frequency can be over a short period of time, or even taken with consecutive parts. This is generally the case during a machine qualification study because of the availability of parts and the time allotted to the machine runoff. In a production environment, the frequency can be hourly, several times per shift, or any feasible time frame.

From the data collected, the average range ( $\bar{R}$ ) and the process average ( $\bar{X}$ ) are calculated simply by averaging the subgroup ranges and averages. The next step in the process of creating the control charts is to calculate the control limits.

Control limits are used to show the extent by which the subgroup averages and ranges would vary if only common (random) causes of variation were present. They are based on the subgroup sample size and the amount of “within” subgroup variability reflected in the ranges. The formulas for the upper control limit (UCL) and lower control limit (LCL) for the  $\bar{X}$  and R charts follow:

$$UCL_R = D_4 \bar{R}$$

$$LCL_R = D_3 \bar{R}$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$$

The factors A<sub>2</sub>, D<sub>3</sub> and D<sub>4</sub> are constants based on subgroup size N taken from the chart shown in Table 1. Note that for subgroup sizes of less than seven, there is no lower control limit for the range chart, only an upper control limit. An interesting point to keep in mind is that the control limits have absolutely nothing to do with the tolerance of the parameter being evaluated.

Now the  $\bar{X}$  and R charts can be created with the calculated control limits by plotting the  $\bar{X}$  for each subgroup on one chart (Figure 6) and the range R for each subgroup on another chart (Figure 7). Very often, the two charts are combined.

### Stability

Now that we have the data and charts, what do we do with them?

Basically, if the process variability and average were to remain constant, the subgroup ranges and averages would vary by chance only and would not exceed the control limits. In theory, there would be no runs or trends in the data, and the subgroups would be positioned randomly around the centerline. If all of the above were the case, the process would be “in control” and stable.

To summarize, there are several criteria that can be used to determine if a process is “out of control”:

- 1.) Data points outside the control limits (Figure 8),
- 2.) Runs within the control limits (Figure 9)—seven consecutive points above or below the centerline,
- 3.) Trends (Figure 10)—seven points consistently increasing or decreasing, and
- 4.) To be stable and in control, two-thirds of the points must be within the middle one-third of the chart. The chart’s one-third band is determined by dividing the difference of the UCL – LCL by three (Figure 11).

The criteria for stability may vary depending on the customer’s specifications. There may be changes in the number of points that determine a

trend or a run, i.e. six instead of seven, or the number of subgroups specified to be within the middle third of the chart, etc. As stated earlier, agreement with the customer should be reached and understood before any trial begins.

If the process is shown to be out of control, the assignable cause or causes must be identified and eliminated. The process of troubleshooting and correcting the assignable causes in the gear manufacturing process is a subject in itself and will not be addressed in this paper.

The important point is that the process must be in control and stable before determining the process capability.

### Capability

Process capability is a measure of how well the process output meets the specified requirements (tolerances). Every process can be classified as falling into one of four cases as shown in Table 2.

The preferred situation is to have a Case 1 condition where the process is both in control and capable. As often occurs in the case of machine qualifications, a customer will allow a Case 3 condition where the process may not be in control, but is capable. For the most part, it is not a case of the process being out of control, but the fact that you do not know if the process is in control or out of control. This is generally due to the fact that not enough parts are available from the customer to perform a true stability study. If this is the case, only a test for capability is conducted at the supplier's facility.

The capability indices that are used today are Cp, Pp and Cpk, Ppk. Cp and Pp are indices of process variation that are relative only to a specification. Cpk and Ppk are indices that combine process variation and process centering (location) relative to a specification. As you will see below, the equation for Cp and Pp is the same, except for the method of calculation used for the standard deviation in the equation.

As with Cp and Pp, Cpk and Ppk also have the same equation except for the method of calculation for the standard deviation. Cp and Cpk use the standard deviation ( $\hat{\sigma}_{R/4}$ ) estimated from subgroups using the average range ( $\bar{R}$ ) and the  $d_2$  factor from the chart in Table 1:

$$\hat{\sigma}_{R/4} = \bar{R}/d_2$$

Pp and Ppk use the sample standard deviation ( $\hat{\sigma}_s$ ) calculated from the individual values of the characteristic:

$$\hat{\sigma}_s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

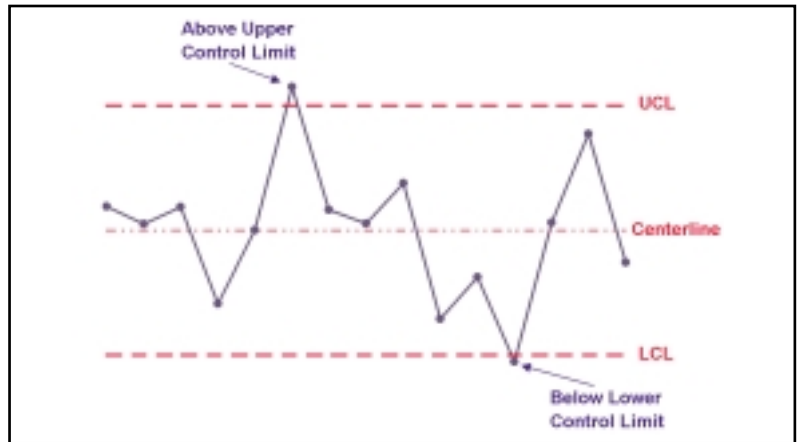


Figure 8—Subgroups outside the control limits.

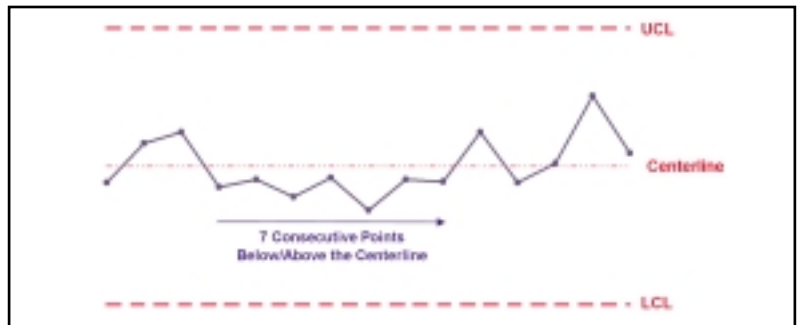


Figure 9—Consecutive points above or below centerline.

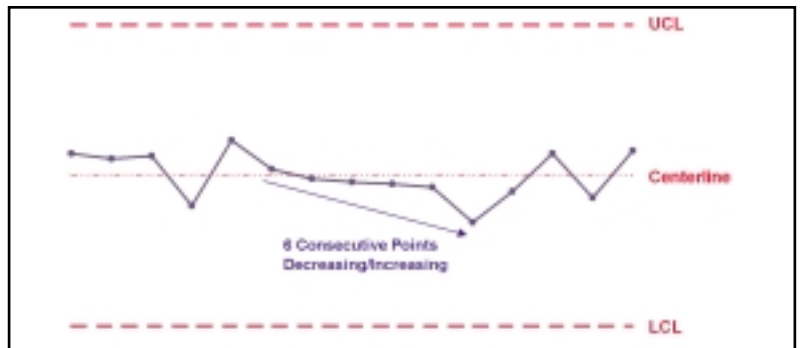


Figure 10—Consecutive points moving up or down.

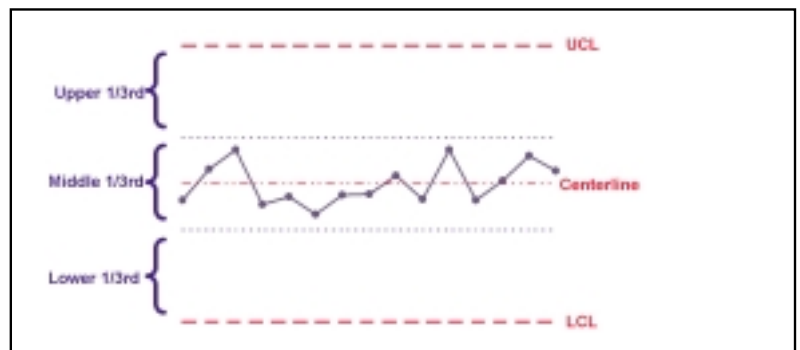


Figure 11—Subgroups within the middle third of the chart.

Meeting Requirements	In Control	Not In Control
Acceptable (Capable)	Case 1	Case 3
Not Acceptable (Not Capable)	Case 2	Case 4



where  $x_i$  is the individual value,  $\bar{x}$  is the average, and  $n$  is the total number of individuals sampled.

$C_p$  is the capability index defined as the tolerance width divided by the process capability, irrespective of process centering (location):

$$C_p = \frac{(USL - LSL)}{6\hat{\sigma}_{R/d_2}}$$

where USL is the upper specification limit and LSL is the lower specification limit.

$P_p$  is the performance index defined as the tolerance width divided by the process performance, irrespective of process centering (location).  $P_p$  should be used only to compare to or with  $C_p$  and  $C_{pk}$  and to measure and prioritize improvement over time.

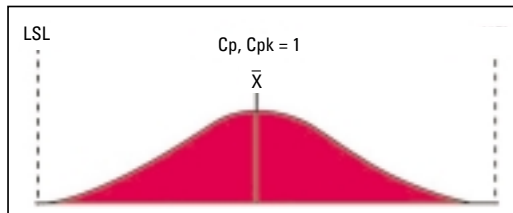


Figure 12—Data spread using all the tolerance.

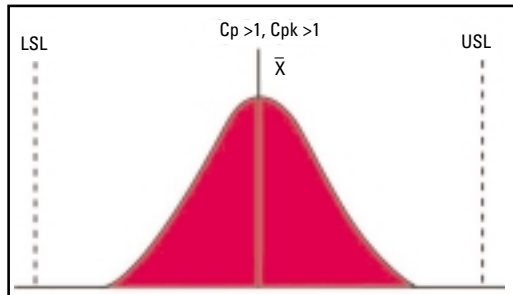


Figure 13—Distribution spread is less than the tolerance.

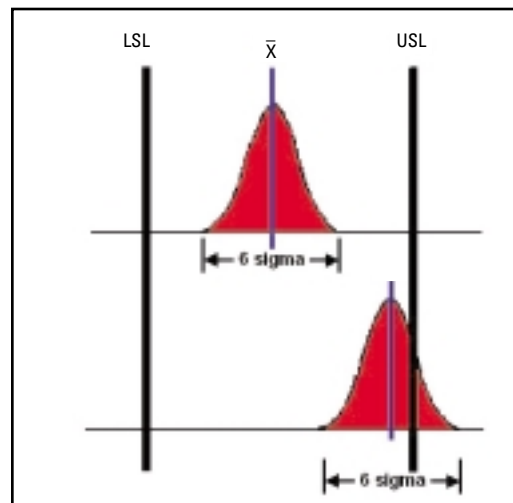


Figure 14— $C_p$  is the same for both distributions.

$$P_p = \frac{(USL - LSL)}{6\hat{\sigma}_s}$$

$C_{PU}$  is the upper capability index and is defined as the upper tolerance spread divided by the actual upper process spread.

$$C_{PU} = \frac{(USL - \bar{X})}{3\hat{\sigma}_{R/d_2}}$$

$C_{PL}$  is the lower capability index and is defined as the lower tolerance spread divided by the actual lower process spread.

$$C_{PL} = \frac{(\bar{X} - LSL)}{3\hat{\sigma}_{R/d_2}}$$

$C_{pk}$  is the capability index that accounts for process centering and is defined as the minimum of  $C_{PU}$  or  $C_{PL}$ .  $C_{pk}$  relates the distance between the process mean and the closest specification limit to half the total process spread.

$P_{pk}$  is the performance index that accounts for process centering (location) and is defined as the minimum of:

$$\frac{(USL - \bar{X})}{3\hat{\sigma}_s} \quad \text{or} \quad \frac{(\bar{X} - LSL)}{3\hat{\sigma}_s}$$

As with  $P_p$ , it should be used only to compare to or with  $C_p$  and  $C_{pk}$  and to measure and prioritize improvement over time.

Capability can also be expressed in terms of a ratio. CR is the capability ratio equal to the reciprocal of  $C_p$ . The performance ratio PR is equal to the reciprocal of  $P_p$ .

The following graphical examples may make the concept of the capability indices easier to understand.

Figure 12 depicts a  $C_p$  of 1, since the statistical spread is equal to the tolerance. Also, since the average is exactly in the middle of the tolerance, the  $C_{pk} = 1$

Figure 13 depicts a  $C_p$  greater than 1, since the statistical spread is less than the tolerance. Since the average is exactly in the middle of the tolerance, the  $C_{pk}$  value will be equal to the  $C_p$  value.

Figure 14 depicts the same distribution in two different locations relative to the specification limits. In the top distribution,  $C_p$  and  $C_{pk}$  are both greater than one since the 6-sigma spread is less than the tolerance.  $C_p$  and  $C_{pk}$  are also equal to each other since the average is at the middle of the tolerance. In the distribution on the bottom, the  $C_p$  value is exactly the same as the top distribution, but the  $C_{pk}$  is some value less than 1

since part of the distribution falls outside the USL. With the mean of the distribution located where it is, some percentage of parts will fall outside the USL.

### Double-Sided and Single-Sided Tolerances

Double-sided tolerances or bilateral tolerances are those which define a nominal dimension along with a plus or minus tolerance. Bilateral tolerances tend to generate distributions that are normal. Typically a gear size over pins, wires or balls would be bilateral.

Single-sided or unilateral tolerances have a single limit tolerance. A zero-based dimension unilateral tolerance has zero as the inherent target value. Typically gear runout, pitch variation, etc. are unilateral tolerances. Unilateral tolerances by nature tend to generate distributions that have a visible amount of skewness or non-normality. Single-sided tolerances are calculated using the Cpk or Ppk indices as described above.

Some customers use different methods in handling the data to calculate the capability index for unilateral tolerances. One example is demonstrated in the Delphi specification SD-002 (Ref. 3). A mirror image transformation is used to “normalize” the data set. The data is ordered from the smallest values to the largest values. When there is an odd number of data points, the median is the middle value of the ordered data. When there is an even number of data points, the median is the average of the two middle values of the ordered data set. The transformation is made by first removing all the data points that fall above the median for a minimum specification and below the median for a maximum specification. For each remaining data value, a corresponding value is created equally distant from the median on the median’s opposite side. Standard techniques are then used to calculate a “trial” standard deviation from the mirrored data set.

Using  $\sigma_{\text{trial}}$ , all values that exceed the (median +  $3\sigma_{\text{trial}}$ ) are excluded and  $\bar{R}$ ,  $\hat{\sigma}$  and  $\sigma$  are recalculated using the modified data set.

### Gage Repeatability and Reproducibility

Every parameter that is subjected to a capability study will require some type of gage or instrument to measure the value of the parameter. How do we know the value we are measuring is actually what the gage says it is? How do we know the gage is good enough to make the measurement so we can rely on the reading and use the result in the capability analysis? Gage repeatability and reproducibility (GR&R) procedures have been developed to assess the statistical properties of gages.

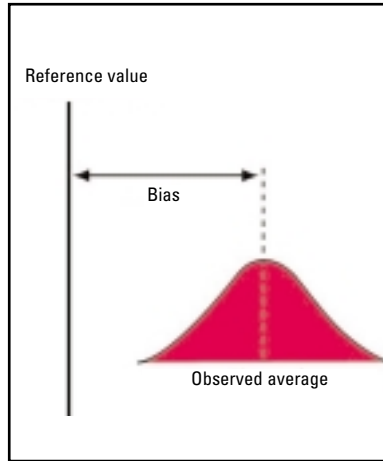


Figure 15—Gage bias.

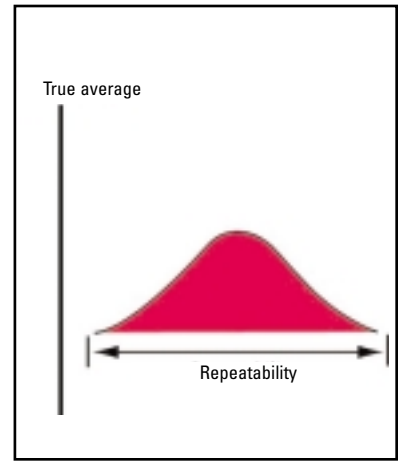


Figure 16—Gage repeatability.

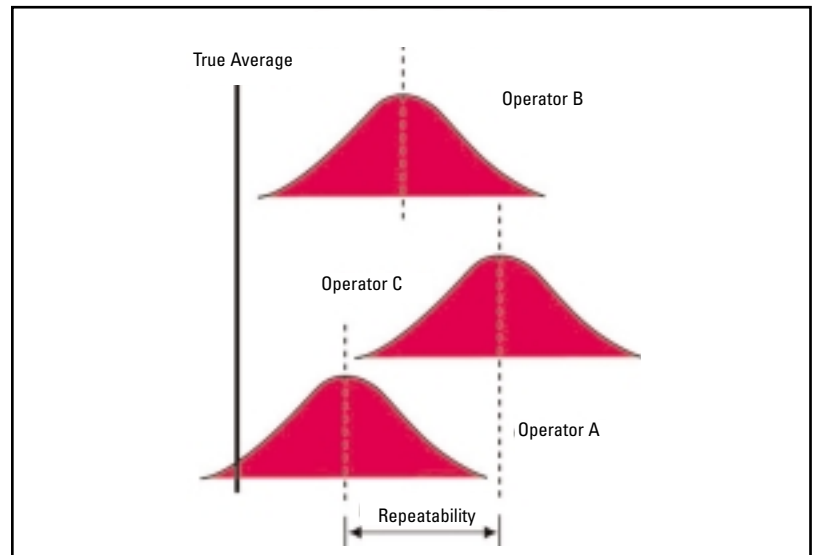


Figure 17—Gage reproducibility.

Before a gage is used for a capability study, it should be evaluated to determine its performance.

Before we discuss the different methods of conducting a GR&R, the following are definitions of a number of characteristics of any gage system.

Gage bias (Figure 15) is the difference between the observed average and the reference value. Bias is sometimes referred to as accuracy, but the term accuracy is not recommended as an alternative to bias.

Gage repeatability (Figure 16) is the variation in measurements obtained with one measurement instrument when used several times by one operator measuring the identical characteristic on the same part.

Gage reproducibility (Figure 17) is the variation in the average of the measurements made by different operators (appraisers) using the same gage when measuring identical characteristics of the same part.

Gage stability (Figure 18) or drift is the total variation in the measurements obtained with a measurement system on the same parts when measuring a single characteristic over an extended time period.

Gage linearity (Figure 19) is the difference in the bias values through the gage's expected operating range.

Parts	Number of Operators			
	2	3	4	5
1	1.41	1.91	2.24	2.48
2	1.28	1.81	2.15	2.40
3	1.23	1.77	2.12	2.38
4	1.21	1.75	2.11	2.37
5	1.19	1.74	2.10	2.36
6	1.18	1.73	2.09	2.35
7	1.17	1.73	2.09	2.35
8	1.17	1.72	2.08	2.35
9	1.16	1.72	2.08	2.34
10	1.16	1.72	2.08	2.34

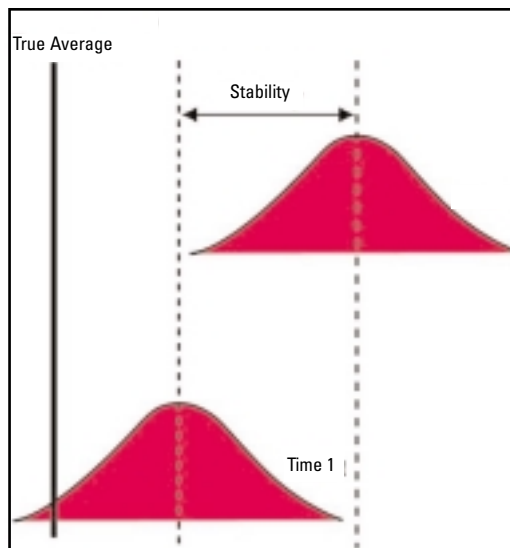


Figure 18—Gage Stability.

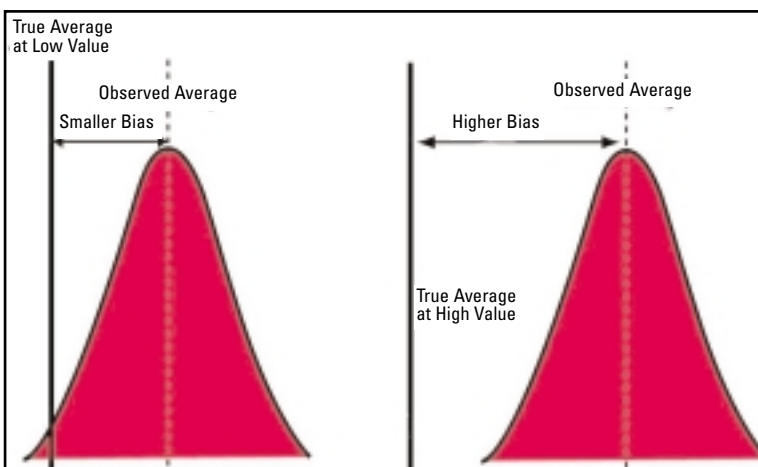


Figure 19—Gage Linearity.

## GR&R Techniques—Long and Short Studies

There are several techniques that can be used to perform a gage study. The two most widely used are the range method (short study) and the average and range method (long study).

The range method or short study will provide a quick approximation of the gage variability. It will not distinguish the variability between repeatability (equipment variation) and reproducibility (appraiser variation), but only provide an overall picture of the measurement system.

Typically, the short study will require only two operators (appraisers) and five parts. Each operator measures each part once. The range for each part is the absolute difference between the measurements obtained by the operators. The sum of the ranges found and the average range  $R$  is calculated. The total measurement variability is found by multiplying the average range by  $5.15/d_2$ . The  $d_2$  factor can be found in a table for the distribution of the average range for two trials and five parts (Table 3). It is interesting to note that there are some tables where  $d_2$  values are based on the number of parts times the number of appraisers, where other tables use only the number of parts.

The long study (average and range method) will provide an estimate of both the repeatability (equipment variation) and reproducibility (appraiser variation) for a measurement system.

The number of appraisers, trials and parts may vary, but typically 10 parts that represent the actual or expected range of process variation are numbered and used with three appraisers. Each appraiser checks the 10 parts in random order two or three times each.

For the short study, the calculated GR&R value is generally expressed as a percentage of the tolerance of the parameter being measured. It can also be expressed as a percentage of the process variation, if it is known. For the long study, the GR&R value can be expressed as a percentage of the total variation measured in addition to the tolerance or process variation.

The information in Table 4 can be used as a general guide for acceptance criteria of a percentage GR&R study. Generally speaking, if the percentage GR&R is 10% or less, the gage will be acceptable. If it is 10% to 30%, the gage may be acceptable based on the importance of the application. If the GR&R is more than 30%, the gage is determined to be unacceptable.

It should be noted that it is an acceptable practice to factor out the gage error when making the

capability calculation. The calculated standard deviation is actually made up of the process standard deviation and the standard deviation of the gage. They are related by the following formula:

$$\sigma^2_{\text{process \& gage}} = \sigma^2_{\text{gage}} + \sigma^2_{\text{process}}$$

Since we know the total standard deviation (consisting of the process and the gage) and we know the gage standard deviation, we can solve for the process standard deviation and use the process standard deviation in the capability calculations:

$$\sigma_{\text{process}} = \sqrt{\sigma^2_{\text{(process \& gage)}} - \sigma^2_{\text{gage}}}$$

The appendix contains a sample of a short study and a long study GR&R. The long study sample uses a spreadsheet set up to follow the example in the MSA reference manual.

#### Summary

The understanding and application of the SPC and GR&R techniques presented in this paper is essential for a successful machine runoff. When properly applied, much can be learned about the process and machine.

As you may have already concluded, the techniques described in this paper are not limited to just gear cutting machines. They can be applied to any parameter for any process.

To demonstrate the techniques, case studies from actual machine runoffs for cylindrical and bevel gear cutting machines are provided.

#### Case Studies

For cylindrical gear applications, the parameter most often measured and evaluated is the tooth size, usually by measurement over pins or wires. Equipment such as the Mahr Diamar and Unite-A-Matic tooth size checkers are preferred over using hand micrometers because the tooth checkers have a much better GR&R than that of micrometers. Other parameters that have been evaluated are lead and profile average, lead and profile variation, runout and spacing parameters.

For bevel gear applications, the parameter most often measured and evaluated is also the tooth size, usually by measurement with a ding-ing ball gage supplied by the customer. Other parameters inspected are flank form errors, runout and spacing parameters.

A typical machine runoff today generally consists of a 10-part mini-run prior to any extended runs. The mini-run serves to verify targeting of the size and verify that all other parameters are

GR&R Percentage	Measurement System
Less than 10%	Acceptable
10% to 30%	May be acceptable based on importance of application, gage cost, etc.
More than 30%	Unacceptable—measurement system needs improvement

within specifications.

Often a customer will require mini-run results prior to his visit for the machine runoff. If possible, a full run is made producing the required number of parts prior to the customer's visit under the exact conditions requested by the customer. This helps to eliminate any surprises that might occur during a run that would take place for the first time in the customer's presence.

Most machines have some type of temperature compensation system to allow for machine growth as the machine warms up. The extended runs allow an opportunity to verify that the correct temperature compensation factor is being used.

As stated previously in the paper, although technically required to show stability before capability, most customers will specify a run of anywhere from 25 to 125 continuous parts without stoppage for a machine runoff and only the capability is calculated from the inspection results. It is not uncommon to repeat the capability study in the customer's facility after the machine is shipped.

#### Appendix 1: Subgroup Stability—X-Bar and R-Bar Charts

The case study in Appendix 1 is a hobbing machine runoff with a requirement to prove stability, then capability, on the tooth size parameter for a plastic worm gear. The data is shown in a spreadsheet created to analyze the specific customer requirements with respect to stability and capability. The size over balls was measured with a special gage provided by the customer. Prior to using the gage, a GR&R was conducted and it was found to be acceptable for use.

The customer required the hobbing machine to be warmed up by rotating the spindles for 8 hours prior to beginning the run. The size was targeted to within 0.005 mm of nominal, and a total of 654 worm gears were run continuously in automatic mode without interruption.

Twenty subgroups of three parts per subgroup were selected for analysis throughout the run. There were 18 cycles with cutting between each three-piece subgroup. In other words, three out of every 21 parts were selected for the analysis.

Offsets to the size were allowed during the run, but could not be made on two consecutive sub-



groups. Note that only the last four digits of the gage measurement were entered in the spreadsheet. The results for stability and capability are summarized in the top table of Appendix 1. Note the cells in the right-hand column indicating a "Pass" or "Fail." The X-bar and R-bar charts for the data are also shown in the appendix. In the X-bar chart, note the relationship between the actual specification limits and the control limits.

#### **Appendix 2: Pp and Ppk Capability Data**

Appendix 2 data is presented in another spreadsheet created to analyze basic statistical data with respect to a capability analysis. The table provided shows 11 of the 15 parameters that were required for the analysis. The data is from an actual bevel gear cutting machine runoff for a truck application requiring face-hob-cutting of a 41-tooth gear.

The parameters evaluated were tooth size, spacing (Fp and fp) on both the concave and convex flanks, runout (Fr) on both flanks, and the errors (measured in microinches) on the four corners of the tooth flank. The corners are designated as Toe Top, Toe Root, Heel Top and Heel Root. All data, including the tooth flank form error, were taken from the output of a CMM inspection machine.

The data presented for the tooth form error was measured to a master gear with zero error on the corners. Note that the flank form corner data for the convex side of the tooth is not shown in the table in order to cut down on the data presented.

The run of 35 gears was made without stoppage on a warm machine. Verification of process stability was not required for the runoff. Note that the spreadsheet is set up to enter the specification limits, the type of tolerance (unilateral or bilateral), and the Pp and/or Ppk requirement. In addition to calculating the capability results, the spreadsheet also reports a "Pass" or "Fail." The spreadsheet is also set up to yield a run chart for each of the parameters. The run charts for the accumulated pitch (Fp) and the runout (Fr) are shown in the appendix. Both the data from the concave and convex sides are shown on one chart. Run charts are an excellent visual for comparing the data to the tolerance, and any irregularities—such as flyers, trends, runs, etc.—can be seen immediately.

#### **Appendix 3: Short Study GR&R**

The range method or short study GR&R shown in Appendix 3 is presented in a spreadsheet. The data provided is from a machine runoff of a CNC test machine, measuring the first harmonic of mesh in arc seconds of a set of auto-

motive bevel gears. Two operators (appraisers) and five parts were used for the short study. Each operator measured each part once, and the range for each part (the absolute difference between the measurements obtained by the operators) was calculated.

The sum of the ranges (0.24) and the average range,  $\bar{R}$ , (0.048) is calculated. The total measurement variability or GR&R (0.2078) is found by multiplying the average range (0.048) by 5.15/1.19, where 1.19 is the  $d_2$  factor for five parts, two operators. The GR&R expressed as a percent of the 5 arc second tolerance yields a GR&R percentage of 4.2%, which is an excellent result.

As stated previously, the short study provides a quick approximation of the gage variability, and it does not distinguish the variability between repeatability (equipment variation) and reproducibility (appraiser variation), but only provides an overall picture of the measurement system.

#### **Appendix 4: Long Study GR&R**

The long study (average and range methods) shown in Appendix 4 is also presented in a spreadsheet set up to duplicate the sample provided in the MSA reference manual. The data provided is from a machine runoff of a CNC test machine, measuring the first harmonic of mesh in arc seconds of a set of automotive bevel gears. Three operators (appraisers) and 10 parts were used for the long study. Each operator inspected each of the 10 parts three different times.

The spreadsheet shows each of the operators' measurements, and the results are displayed in the last several rows of the spreadsheet. The resultant GR&R is 0.44648. As stated earlier, the long study also yields the estimate for the repeatability or equipment variation (0.44108) and the reproducibility or appraiser variation (0.06294). The results of this study are also expressed as a percentage of the total part variation (5.19602), yielding a GR&R percentage of 8.59%, which passes the criteria for a good gage. The results also could have been expressed as a percentage of the tolerance or of the process variation. ⚙

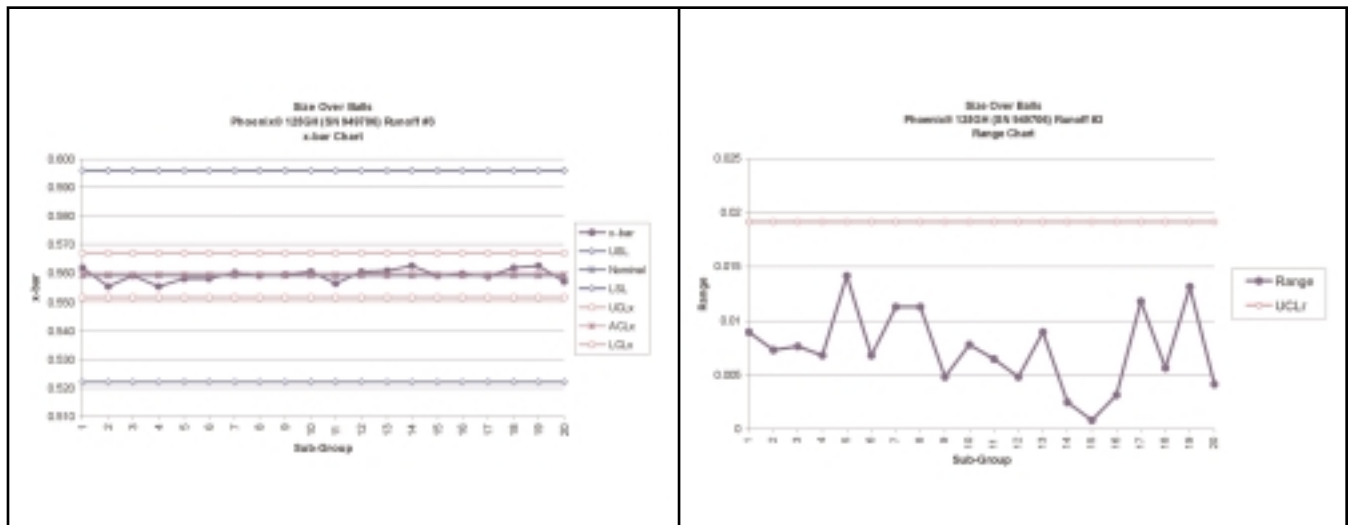
#### **References**

1. Chrysler Corp., Ford Motor Co. and General Motors Corp. *Statistical Process Control—SPC—Reference Manual*, 1992.
2. Chrysler Corp., Ford Motor Co., and General Motors Corp., *Measurement Systems Analysis—MSA—Reference Manual*, 1992.
3. Delphi Saginaw Steering Manufacturing Equipment Statistical Qualification Requirements SD-002, *Saginaw Steering Systems*, Issued 1 March 1993, Revised 7 January 2002.
4. Juran, J.M., and Frank M. Gryna, Jr. *Quality Planning and Analysis*, McGraw Hill, 1980.
5. Juran, J.M. *Quality Control Handbook*, 3rd Edition, McGraw Hill, 1979.

### Appendix 1—Subgroup stability data.

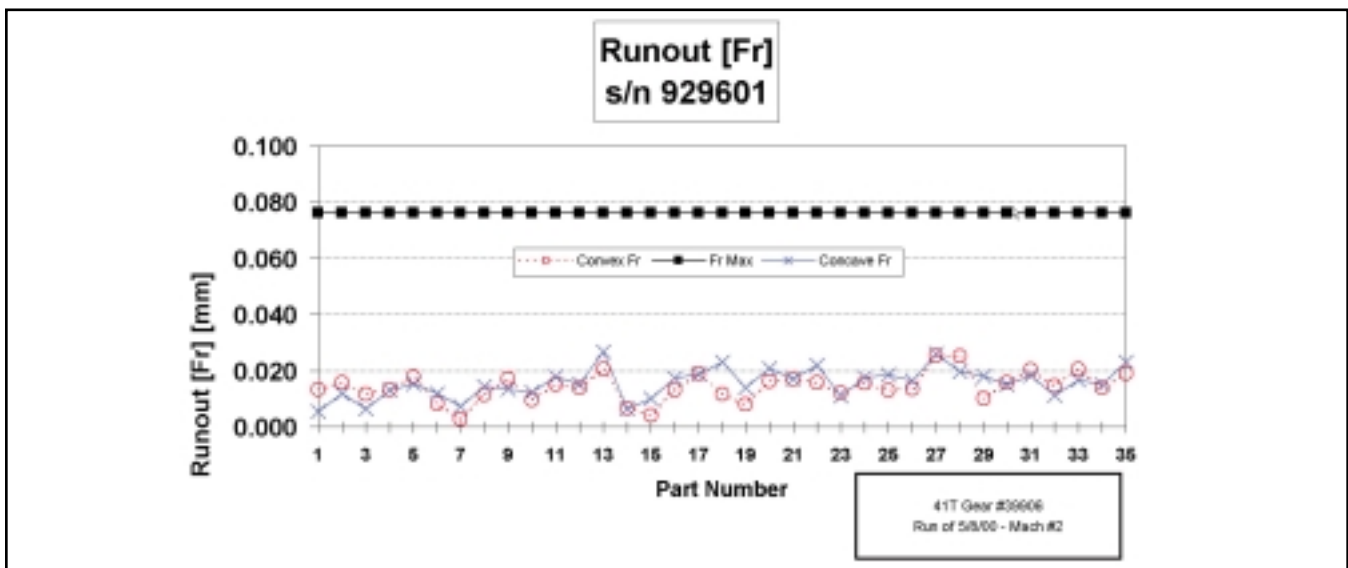
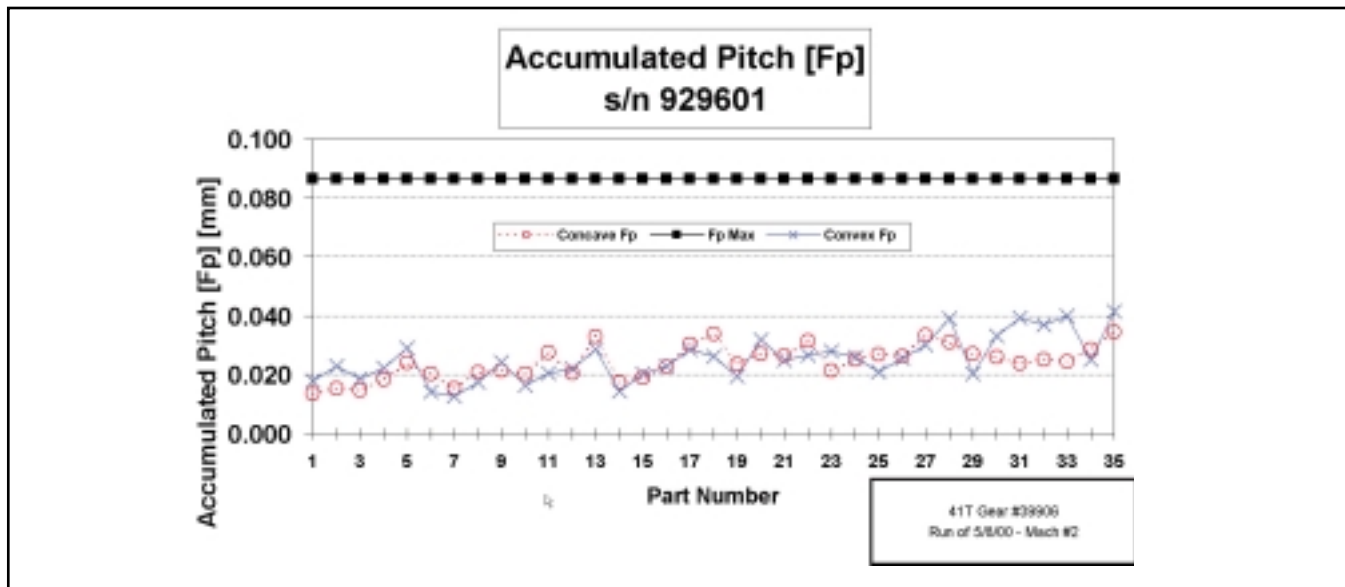
Size over Balls	Tolerances	Requirement	Value	Pass/Fail
Upper Limit	0.596	Cpk > 1.33	2.78	Pass
Lower Limit	0.522	Ppk > 1.33	2.96	Pass
Cpk	1.33	100% in CL's	100%	Pass
Ppk	1.33	> 66% in Center 1/3	70%	Pass
		No Runs (7 on one side)	0	Pass
Setup Info		No Trends (6 incr./decr.)	0	Pass
Sub-Group Size	3			
# of Blank Cycles	18		x-bar-bar	R-bar
# of Sub-Groups	20		0.55930	0.00744
Measurements/Piece	3			

Sub-Group	Piece #	Meas. #1	Meas. #2	Meas. #3	Sub-Group			Sub-Group	Piece #	Meas. #1	Meas. #2	Meas. #3	Sub-Group		
					Avg. Meas.	Average (x-bar)	Group Range (R)						Sub-Group	Avg. Meas.	Average (x-bar)
1.1	188	0.5480	0.5750	0.5610	0.56133	0.56178	0.00900	11.1	398	0.5760	0.5460	0.5540	0.55867	0.55639	0.00650
1.2	189	0.5610	0.5660	0.5725	0.56650			11.2	399	0.5490	0.5685	0.5575	0.55833		
1.3	190	0.5685	0.5485	0.5555	0.55750			11.3	400	0.5555	0.5575	0.5435	0.55217		
2.1	209	0.5670	0.5645	0.5350	0.55550	0.55528	0.00733	12.1	419	0.5705	0.5500	0.5530	0.55783	0.56039	0.00483
2.2	210	0.5600	0.5455	0.5490	0.55150			12.2	420	0.5435	0.5615	0.5830	0.56267		
2.3	211	0.5330	0.5660	0.5775	0.55883			12.3	421	0.5505	0.5705	0.5610	0.56067		
3.1	230	0.5630	0.5410	0.5705	0.55817	0.55928	0.00767	13.1	440	0.5595	0.5690	0.5510	0.55983	0.56083	0.00900
3.2	231	0.5550	0.5730	0.5630	0.56367			13.2	441	0.5730	0.5780	0.5465	0.56583		
3.3	232	0.5545	0.5760	0.5375	0.55600			13.3	442	0.5790	0.5505	0.5410	0.55683		
4.1	251	0.5565	0.5430	0.5710	0.55683	0.55522	0.00683	14.1	461	0.5600	0.5610	0.5690	0.56333	0.56250	0.00250
4.2	252	0.5610	0.5610	0.5515	0.55783			14.2	462	0.5675	0.5785	0.5440	0.56333		
4.3	253	0.5505	0.5715	0.5310	0.55100			14.3	463	0.5510	0.5785	0.5530	0.56083		
5.1	272	0.5695	0.5545	0.5515	0.55850	0.55811	0.01417	15.1	482	0.5545	0.5585	0.5660	0.55967	0.55911	0.00083
5.2	273	0.5470	0.5615	0.5440	0.55083			15.2	483	0.5735	0.5465	0.5565	0.55883		
5.3	274	0.5795	0.5465	0.5690	0.56500			15.3	484	0.5745	0.5580	0.5440	0.55883		
6.1	293	0.5665	0.5680	0.5440	0.55950	0.55822	0.00683	16.1	503	0.5645	0.5555	0.5555	0.55850	0.55972	0.00317
6.2	294	0.5700	0.5555	0.5575	0.56100			16.2	504	0.5645	0.5865	0.5340	0.56167		
6.3	295	0.5410	0.5480	0.5735	0.55417			16.3	505	0.5365	0.5805	0.5600	0.55900		
7.1	314	0.5605	0.5670	0.5360	0.55450	0.56006	0.01133	17.1	524	0.5505	0.5600	0.5575	0.55600	0.55872	0.01183
7.2	315	0.5650	0.5430	0.5715	0.55983			17.2	525	0.5345	0.5610	0.5670	0.55417		
7.3	316	0.5770	0.5525	0.5680	0.56583			17.3	526	0.5780	0.5700	0.5500	0.56600		
8.1	335	0.5650	0.5365	0.5545	0.55200	0.55906	0.01133	18.1	545	0.5595	0.5680	0.5480	0.55850	0.56189	0.00567
8.2	336	0.5805	0.5575	0.5475	0.56183			18.2	546	0.5430	0.5630	0.5830	0.56300		
8.3	337	0.5660	0.5555	0.5685	0.56333			18.3	547	0.5685	0.5475	0.5765	0.56417		
9.1	356	0.5555	0.5810	0.5480	0.56150	0.55939	0.00483	19.1	566	0.5595	0.5595	0.5450	0.55467	0.56261	0.01317
9.2	357	0.5630	0.5475	0.5695	0.56000			19.2	567	0.5860	0.5455	0.5720	0.56783		
9.3	358	0.5545	0.5525	0.5630	0.55667			19.3	568	0.5590	0.5535	0.5835	0.56533		
10.1	377	0.5515	0.5815	0.5390	0.55733	0.56039	0.00783	20.1	587	0.5690	0.5480	0.5505	0.55583	0.55711	0.00417
10.2	378	0.5390	0.5750	0.5620	0.55867			20.2	588	0.5665	0.5775	0.5355	0.55983		
10.3	379	0.5735	0.5645	0.5575	0.56517			20.3	589	0.5545	0.5365	0.5760	0.55567		



## Appendix 2—Pp and Ppk capability data.

Customer:		1000HC 36654 #39906										
Date:		May 9, 00		929601	Mach #2	41 Gear						
		side 2	side 1	side2	side 1	side 2	side 1	Conv = Concave				
		B	C	D	E	F	G	H				
Part ID	Conv Fp	Conx Fp	Conv fp	Conx fp	Conv Fr	Conx Fr	Size	ToeTop	ToeRoot	HeelTop	HeelRoot	
1	0.0136	0.0178	0.0056	0.0064	0.0056	0.0132	-0.0090	-1.6	5.5	-7.4	-4.2	
2	0.0152	0.0229	0.0083	0.0092	0.0118	0.0158	-0.0160	-0.8	5.4	-4.6	-4.1	
3	0.0146	0.0186	0.0079	0.0107	0.0064	0.0116	-0.0160	-1.7	6.2	-4.5	-0.6	
4	0.0183	0.0223	0.0062	0.0095	0.0130	0.0132	-0.0140	-0.9	5.3	-5.3	-2.5	
5	0.0241	0.0290	0.0081	0.0091	0.0153	0.0178	-0.0160	-3.4	3.9	-5.7	-2.2	
6	0.0202	0.0143	0.0069	0.0109	0.0118	0.0082	-0.0160	-1.8	5.8	-7.5	-1.2	
7	0.0153	0.0127	0.0059	0.0079	0.0074	0.0028	-0.0180	-2.6	7.3	-3.7	-1.4	
8	0.0208	0.0175	0.0074	0.0075	0.0145	0.0115	-0.0110	-4.5	3.8	-7.9	-2.7	
9	0.0214	0.0242	0.0064	0.0099	0.0135	0.0172	-0.0230	-2.5	4.0	-5.4	2.2	
10	0.0204	0.0164	0.0063	0.0071	0.0124	0.0097	-0.0230	-0.9	4.4	-5.8	2.3	
11	0.0275	0.0205	0.0108	0.0092	0.0179	0.0150	-0.0220	-9.3	-0.4	-6.4	-3.0	
12	0.0206	0.0220	0.0066	0.0095	0.0153	0.0140	-0.0220	-1.5	7.0	-4.1	2.8	
13	0.0330	0.0285	0.0112	0.0121	0.0268	0.0207	-0.0190	-2.6	3.8	-5.8	2.5	
14	0.0174	0.0144	0.0084	0.0077	0.0065	0.0067	-0.0230	-7.6	1.3	-3.6	2.7	
15	0.0189	0.0204	0.0059	0.0149	0.0101	0.0041	-0.0180	-5.6	1.9	-4.5	3.8	
16	0.0226	0.0230	0.0085	0.0135	0.0173	0.0131	-0.0180	-6.2	2.0	-4.0	1.3	
17	0.0304	0.0285	0.0087	0.0130	0.0187	0.0191	-0.0190	-8.2	0.8	-5.5	0.9	
18	0.0339	0.0263	0.0096	0.0139	0.0230	0.0117	-0.0200	-8.6	-0.7	-4.2	1.4	
19	0.0235	0.0195	0.0133	0.0137	0.0139	0.0079	-0.0270	-4.4	3.1	-3.5	4.1	
20	0.0271	0.0319	0.0070	0.0164	0.0208	0.0163	-0.0270	-2.9	5.4	-4.4	3.9	
21	0.0264	0.0249	0.0091	0.0127	0.0172	0.0170	-0.0260	-5.6	2.1	-4.5	1.2	
22	0.0315	0.0266	0.0101	0.0141	0.0220	0.0158	-0.0260	-5.7	3.2	-3.1	5.4	
23	0.0215	0.0279	0.0102	0.0171	0.0109	0.0122	-0.0240	-6.0	2.0	-3.3	3.0	
24	0.0251	0.0259	0.0090	0.0151	0.0176	0.0155	-0.0300	-4.2	2.6	-1.6	7.2	
25	0.0269	0.0210	0.0079	0.0145	0.0185	0.0130	-0.0340	-2.7	5.1	-4.0	4.1	
26	0.0263	0.0255	0.0110	0.0130	0.0164	0.0136	-0.0220	-7.6	0.4	1.0	8.0	
27	0.0334	0.0301	0.0084	0.0098	0.0260	0.0253	-0.0230	-6.5	-1.5	0.5	8.1	
28	0.0311	0.039	0.0096	0.0131	0.0196	0.0251	-0.032	-8.3	3	0.6	8.7	
29	0.027	0.0204	0.0095	0.0134	0.018	0.0101	-0.033	-6.6	1.6	-2.9	4.4	
30	0.026	0.0332	0.0114	0.0174	0.0152	0.0158	-0.031	-7.8	1.8	-2.9	5	
31	0.0237	0.0394	0.011	0.0216	0.0183	0.0203	-0.035	-3.6	4.3	0	7.3	
32	0.0252	0.037	0.0166	0.0237	0.0113	0.0147	-0.032	-10	1.5	-5.4	6.7	
33	0.0244	0.04	0.0123	0.0247	0.0166	0.0205	-0.023	-5.9	0.7	-3.2	5.7	
34	0.0285	0.0251	0.0138	0.0171	0.0152	0.0139	-0.035	-2.9	6.2	2.7	12.6	
35	0.0344	0.0415	0.0107	0.0194	0.023	0.0191	-0.027	-5.9	2.5	-1.8	4.8	
	Conv Fp	Conx Fp	Conv fp	Conx fp	Conv Fr	Conx Fr	Size	ToeTop	ToeRoot	HeelTop	HeelRoot	
Count	35	35	35	35	35	35	35	35	35	35	35	
Min.	0.0136	0.0127	0.0056	0.0064	0.0056	0.0028	-0.035	-10.000	-1.500	-7.900	-4.200	
Max.	0.0344	0.0415	0.0166	0.0247	0.0268	0.0253	-0.009	-0.800	7.300	2.700	12.600	
Range	0.0208	0.0288	0.0110	0.0183	0.0212	0.0225	0.026	9.200	8.800	10.600	16.800	
Average	0.0243	0.0254	0.0091	0.0131	0.0157	0.0143	-0.023	-4.769	3.180	-3.763	2.806	
SD	0.00572	0.00761	0.00250	0.00455	0.00524	0.00510	0.00683	2.66235	2.24458	2.45022	3.99588	
3*SD	0.01717	0.02284	0.00749	0.01366	0.01573	0.01529	0.02049	7.98705	6.73373	7.35066	11.98763	
6*SD	0.03434	0.04567	0.01499	0.02733	0.03146	0.03059	0.04098	15.97409	13.46745	14.70132	23.97526	
Upper Lim	0.0864	0.0864	0.0193	0.0193	0.076	0.076	0.076	30.000	30.000	30.000	30.000	
Lower Lim	0.000	0.000	0.000	0.000	0.000	0.000	-0.076	-30.000	-30.000	-30.000	-30.000	
Tolerance	0.0864	0.0864	0.0193	0.0193	0.076	0.076	0.152	60	60	60	60	
Toler Type	Uni	Uni	Uni	Uni	Uni	Uni	Bi	Bi	Bi	Bi	Bi	
Pp Spec							1.67	1.67	1.67	1.67	1.67	
Ppk Spec	1.33	1.33	1.33	1.33	1.33	1.33	1.67	1.67	1.67	1.67	1.67	
Bilat Pp							3.71	3.76	4.46	4.08	2.50	
Bilat Ppk							2.58	3.16	3.98	3.57	2.27	
Unilat Ppk	3.62	2.67	1.36	0.45	3.84	4.03						
Bilat Pp							PASS	PASS	PASS	PASS	PASS	
Bilat Ppk							PASS	PASS	PASS	PASS	PASS	



**Appendix 3—Short study GR&R, range method.**

Machine S/N 951104				DATE: March 12, 2002			
Short Study GR&R Coast Side				5 parts		10 parts	
Results in Arc Seconds				d2		1.19    1.16	
Reading	Operator A	Operator B	Delta	Tolerance: 5 Arc Seconds			
1	1.01000	1.09000	0.08000	GRR 0.20784 (5.15 x Avg. R / 1.19)			
2	2.63000	2.65000	0.02000	GRR% 4.2 (100 x GRR / 5)			
3	3.29000	3.24000	0.05000				
4	2.62000	2.64000	0.02000				
5	2.10000	2.17000	0.07000				
		Sum	0.24000				
		Avg. R	0.04800				
				GRR Sigma = 0.04034 (GRR/(1.19 x 4.33))			



## Appendix 4—Long study GR&R, average and range method.

Phoenix 500HCT GR&R Study—8.8–3.73

Customer:

Machine Serial Number: 951104

Study Date: March 6, 2002

Study Conducted By: Andrew DeSantis

Operator 1: S.G.

Operator 2: S.Z.

Operator 3: J.Z.

Side: Coast

Units: arc sec.

Apprais.:3

Trials: 3

Sets: 10

Operator #1						Operator #2				
Set #	1/A	1/B	1/C	Avg.	Range	2/A	2/B	2/C	Avg.	Range
1	1.06	1.17	1.24	1.16	0.18	1.18	1.19	1.30	1.22	0.12
2	3.02	3.08	3.05	3.05	0.06	3.05	3.12	3.01	3.06	0.11
3	3.54	3.62	3.62	3.59	0.08	3.59	3.65	3.67	3.64	0.08
4	3.07	3.16	3.17	3.13	0.10	3.14	3.35	3.20	3.23	0.21
5	2.56	2.64	2.68	2.63	0.12	2.73	2.71	2.64	2.69	0.09
6	2.50	2.63	2.70	2.61	0.20	2.60	2.59	2.71	2.63	0.12
7	4.34	4.37	4.39	4.37	0.05	4.36	4.43	4.35	4.38	0.08
8	1.66	1.89	1.93	1.83	0.27	1.72	1.93	1.91	1.85	0.21
9	3.00	3.24	3.23	3.16	0.24	3.11	3.05	3.21	3.12	0.16
10	3.03	3.03	3.21	3.09	0.18	2.93	3.10	3.09	3.04	0.17
Totals	27.78	28.83	29.22	28.61	1.48	28.41	29.12	29.09	28.87	1.35
Avg.	2.78	2.88	2.92	2.86	0.15	2.84	2.91	2.91	2.89	0.14

Operator #3					Sample Avg.	Sample Max.	Sample Min.	Sample Range	
Set #	3/A	3/B	3/C	Avg.	Range				
1	1.17	1.15	1.26	1.19	0.11	1.19	1.30	1.06	0.24
2	2.96	3.12	3.20	3.09	0.24	3.07	3.20	2.96	0.24
3	3.54	3.63	3.70	3.62	0.16	3.62	3.70	3.54	0.16
4	3.17	3.31	3.39	3.29	0.22	3.22	3.39	3.07	0.32
5	2.59	2.66	2.80	2.68	0.21	2.67	2.80	2.56	0.24
6	2.62	2.57	2.64	2.61	0.07	2.62	2.71	2.50	0.21
7	4.39	4.42	4.43	4.41	0.04	4.39	4.43	4.34	0.09
8	1.81	1.88	1.92	1.87	0.11	1.85	1.93	1.66	0.27
9	3.11	3.14	3.00	3.08	0.14	3.12	3.24	3.00	0.24
10	3.06	3.28	3.09	3.14	0.22	3.09	3.28	2.93	0.35
Totals	28.42	29.16	29.43	29.00	1.52	Rsubp			
Avg.	2.84	2.92	2.94	2.90	0.15	3.20			

BarA	2.86100	Trials	3
RBarA	0.14800	Dsub4	2.58
XBarB	2.88733	UCLr	0.374100
RBarB	0.13500	LCLr	0
XBarC	2.90033	Ksub1	3.0419
RBarC	0.15200	Operators	3
Rp	3.19556	Ksub2	2.7
RBarBar	0.14500	Ksub3	1.62
XBarDiff	0.03933	Parts	10

EV	0.44108	Repeatability - Equipment Variation (EV)
AV	0.06924	Reproducibility - Appraiser Variation (AV)
R&R	0.44648	Repeatability & Reproducibility (R&R)
PV	5.17680	Part Variation (PV)
TV	5.19602	Total Variation (TV)
%EV	8.49%	
%AV	1.33%	
%R&R	8.59%	
%PV	99.63%	

$EV = RBarBar * Ksub1$   
 $AV = \text{SQRT}((XBarDiff * Ksub2)^2 - (EV^2/nr))$   
 $R\&R = \text{SQRT}(EV^2 + AV^2)$   
 $PV = Rp * Ksub3$   
 $TV = \text{SQRT}(R\&R^2 + PV^2)$   
 $\%EV = 100 * (EV/TV)$   
 $\%AV = 100 * (AV/TV)$   
 $\%R\&R = 100 * (R\&R/TV)$   
 $\%PV = 100 * (PV/TV)$

## Appendix 5—Useful definitions from the SPC reference manual and Delphi specification SD-002.

**analysis of variance (ANOVA)**—statistical method to evaluate the data from a designed experiment.

**apparent resolution**—the size of the least increment on the measurement instrument, this value is typically used in literature as advertisement to classify the measurement instrument; the number of data categories can be determined by dividing the size into the expected process distribution spread ( $6\sigma$ ).

**appraiser variation**—variation due to difference in appraiser method, calculated as variation due to the inability of one appraiser to reproduce the measurements of another appraiser; appraiser variation is referred to as “reproducibility” in the calculation worksheets.

**assignable cause**—sometimes referred to as a special cause, a source of variation that is intermittent, often unpredictable, and unstable.

**average (x)**—the sum of the numerical values

in a sample divided by the number of observations.

**bias**—difference between the observed average of measurements and the master average of the same parts using precision instruments.

**bilateral specification**—bilateral tolerances are those that define a nominal dimension along with a  $\pm$  allowance.

**center line**—the horizontal line in the middle of a control chart that shows the average value of the items being plotted.

**common cause**—a source of variation that affects all the individual values of the process variation.

**control chart**—a chart that shows the plotted values, a central line and one or two control limits that are used to monitor a process over time; the types of control charts used are:

**X chart**—a control chart where the average of a subgroup of data is monitored over a period

of time.

**R chart**—a chart used to monitor the range of a subgroup of data over a period of time.

**p chart**—used for data that consists of the ratio of the number of occurrences of an event to total occurrences, generally used to report the fraction non-conforming or defective; p charts can have a variable sample size.

**control limit**—a dashed line or lines on a control chart used as a basis for judging the significance of variation from subgroup to subgroup. Variation beyond a control limit shows that special causes may be affecting the process. Control limits are calculated from process data and are not to be confused with engineering specifications.

**Cpk**—the capability index for a stable process, typically defined as the minimum of CpkU or CpkL.

**CR**—capability ratio.

**data**—variable data: measurements of a sampled part. attribute data: qualities and pass/fail test results of a sampled part.

**designed experiment**—a plan to conduct tests that involves all of the prework that must be accomplished before any tests are conducted. Prework requirements are: questions be written; data collection sheets be prepared; analysis of data be laid out; and the limitations of the test be known.

**discrimination**—discrimination is the larger of the apparent and effective resolutions for single reading systems. The number of data categories is often referred to as the discrimination ratio since it describes how many classifications can be reliably distinguished given the observed process variation.

**distribution**—a way of describing the output of a natural cause system of variation, in which individual values are not predictable but in which the outcomes as a group form a pattern that can be described in terms of its location, spread and shape. Location is commonly expressed by the mean or average, or by the median; spread is expressed in terms of the standard deviation or the range of a sample; shape involves many characteristics such as symmetry and peakedness, but these are often summarized by using the name of a common distribution, such as the normal, binomial, or Poisson.

**effective resolution**—the size of the data category when the total measurement system variation is considered is the effective resolution. This size is determined by the length of the confidence interval based on the measurement system variation. The number of data categories can be determined by dividing the size into the expected process distribution spread. For the effective resolution, a standard estimate of this (at the 97% confidence level) is 1.41 [PV/R&R].

**finish tool**—any tool that generates a part feature being evaluated (final or in-process).

**gage**—any device used to obtain measurements, frequently used to refer specifically to the devices used on the shop floor, includes go/no-go devices.

**gage repeatability & reproducibility (GR&R)**—a statistical method of determining the accuracy, repeatability, and the relative ease of use of a gaging system.

**histogram**—a bar chart that represents data in cells of equal width. The height of each cell is determined by the number of observations that occur in each cell.

**in control**—state of a process when it exhibits only random variations (as opposed to systematic variations and/or variations with assignable sources).

**in-process dimensions**—dimensions that occur on the process routings but usually not on the part print. Normally they are intermediate dimensions of a complex process.

**interaction**—found in GR&R. Non-additivity between appraiser and part. Appraiser differences depend on the part being measured.

**key control characteristic (KCC)**—a process parameter for which variation must be controlled around some target value to ensure that variation in a KPC is maintained around its

target value during manufacturing and assembly. A method for adjusting the KPC to its target value is required.

**key product characteristic (KPC)**—a product characteristic for which reasonably anticipated variation could significantly affect the product's safety or compliance with government standards or regulations, or is likely to significantly affect customer satisfaction with a product.

**linearity**—difference in the bias values of a gage through the expected operating range of the gage.

**long term capability**—statistical measure of the within-subgroup variation exhibited by a process over a long period of time. This differs from performance because it does not include the between-subgroup variation.

**mean**—the average of values in a group of measurements.

**median**—the middle value of a group of measurements, when arranged from lowest to highest, if the number of values is odd. By convention, if the number of values is even, the average of the middle two values is the median.

**measurement system**—the collection of operations, procedures, gages and other equipment, software and personnel used to assign a number to the characteristic being measured; the complete process used to obtain measurements. The actual gages or measurement devices utilized to monitor a process.

**measurement system error**—the combination of gage bias, repeatability, reproducibility, stability and linearity.

**normal distribution**—a continuous, symmetrical, Gaussian-bell-shaped frequency distribution for variable data that underlies the control charts for variables. When measurements have a normal distribution, about 68.26%, 95.44%, 99.73% of all individuals lie within plus and minus one, two, and three standard deviations from the mean, respectively. These percentages are the basis for control limits and control charts analysis.

**out-of-control**—condition describing a process from which all special causes of variation have not been eliminated. This condition is evident on a control chart by the presence of points beyond the control limits or by patterns that are not random within the control limits.

**Ppk**—the performance index for a stable process, typically defined as the minimum of PpkU or PpkL.

**PR**—performance ratio.

**probability**—set of conditions or causes working together to produce an outcome.

**process**—the combination of people, machines and equipment, raw materials, methods and environment that produces a given product or service.

**process capability**—the total range of a stable process's inherent variation ( $6\hat{\sigma}_{R\&D}$ )

**process performance**—the total range of a stable process's total variation ( $6\sigma_s$ ).

**process routings**—the documents that describe the processes required to produce a product.

**range (R)**—the difference between the highest and lowest values in a subgroup.

**reference value**—1. a value that serves as an agreed upon reference for comparison. It may be a theoretical or established value based on scientific principles; an assigned value based on some national or international organization; a consensus value based on collaborative experimental work under the auspices of a scientific or engi-

neering group; or for a specific application, an agreed upon value obtained using an accepted reference method. 2. a value attributed to a specific quantity and accepted, sometimes by convention, as appropriate for a given purpose. 3. a value consistent with the definition of a specific quantity and accepted, sometimes by convention, as appropriate for a given purpose.

**regression analysis**—a calculation to define the mathematical relationship between two or more variables.

**repeatability**—variation in measurements obtained with one gage when used several times by one appraiser while measuring a characteristic on one part.

**reproducibility**—variation in the average of the measurements made by different appraisers using the same gage when measuring a characteristic on one part.

**resolution**—the capability of the measurement system to detect and faithfully indicate even small changes of the measured characteristic; see also discrimination.

**scatter diagram**—a plot of two variables, one against the other, to display trends.

**sigma ( $\sigma$ )**—the measure of variability, or dispersion, that indicates how data spreads out from the mean. It gives information about the variation in a process.

**stability**—the condition describing a process from which all special causes of variation have been eliminated, and only common causes remain; evidenced by the absence of points beyond the control limits and by the absence of non-random patterns or trends within the control limits.

**standard deviation**—see sigma.

**statistical process control**—the use of statistical methods and techniques, such as control charts, to analyze a process or its output so as to take appropriate actions to achieve and maintain a state of statistical control and continue improvement of process variability.

**subgroup**—one or more events or measures used to analyze the performance of a process. Rational subgroups are chosen so that the variation represented within each subgroup is as small as feasible.

**tolerance**—allowable deviation from standard. That is, the permitted range of variation about a nominal value. The permitted tolerance is the difference between the upper and lower specification limits. Specification limits should not be confused with control limits.

**unilateral specification**—unilateral tolerances are those which have only a single limit, e.g. must not exceed 1,000 lbs. or hardness to be 60 Rockwell or less.

**variation**—the inevitable differences among individual outputs of a process; the source of variation can be grouped into two major classes: common causes and assignable (special) causes.

**$\bar{X}$  and R chart**—see control chart.

**zero based dimensions**—these dimensions have a value of zero as their inherent target value, e.g. roundness, concentricity, and surface finish. They usually generate distributions that have a visible amount of skewness or non-normality.