

Development Of Conical Involute Gears (Beveloids) For Vehicle Transmissions

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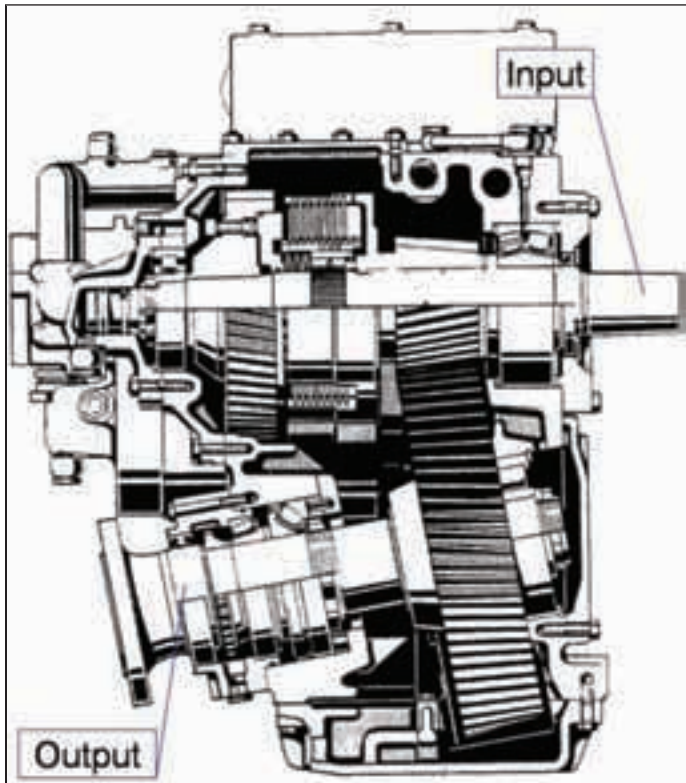


Figure 1—Reversing marine transmission with offset and down angle output shaft ($i = 1.2\text{--}2.9$; max. 1,350 kW).

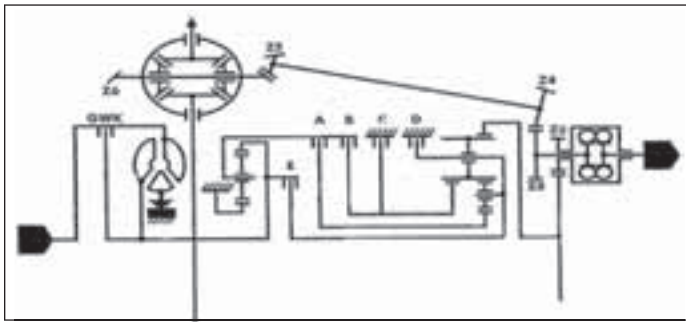


Figure 2—Schematic view of drive train of six-speed automatic gearbox 6HP32.

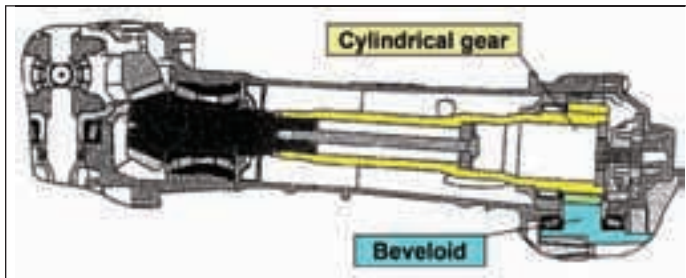


Figure 3—Drive shaft with beveloid gear pair.

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Management Summary

Conical involute gears (beveloids) are used in transmissions with intersecting or skewed axes and for backlash-free transmissions with parallel axes. The geometry of such gears is generally known, but applications in power transmissions are more or less exceptional.

ZF applications so far include marine transmissions with inclined output shafts and low-backlash robot transmissions. The latest development has been realized for the all-wheel drive (AWD) version of a six-speed automatic transmission. Here, the torque must be transmitted from a center differential to a front-axle bevel gear. By means of conical spur gears, it has been possible to realize an inclined drive shaft which offers considerable advantages in terms of installation space.

This article deals with the development of conical gears for vehicle transmissions: basic geometry, design, production and testing.

Introduction

In transmissions with shafts that are not arranged parallel to the axis, torque transmission is possible by means of various designs, such as bevel or crown gears, universal shafts or conical involute gears (beveloids). The use of conical involute gears is particularly ideal for small shaft angles (up to 15°), as they offer benefits with regard to ease of production, design features and overall input. Conical involute gears can be used in transmissions with intersecting or skewed axes or in transmissions with parallel axes for backlash-free operation. Conical gears are spur or helical gears with variable addendum correction (tooth thickness) across the face width. They can mesh with all gears made with a tool with the same basic rack.

ZF has developed various applications with conical gears:

- Steering transmissions (Ref. 1);
- Low-backlash planetary gears (shaft angle $1\text{--}3^\circ$) for robots (Ref. 2);
- Transfer gears for commercial vehicles;
- Marine transmissions with down-angle output shafts (Refs. 1 and 3, Fig. 1); and
- Automatic transmissions for AWD vehicles (Fig. 2).

ZF has developed a six-speed AWD automatic transmission for high-end passenger cars with 750 Nm of torque. For the first time, AWD has been realized for a six-speed automatic transmission. For this project, the AWD concept of VW/AUDI with Torsen differential (Ref. 4) was optimized, functions were improved, and the number of components and tooth engagements was reduced.

Originating from the planetary gear, the power flows via a spur gear to the Torsen differential, where the torque is distributed to the cardan shaft (50%) and via an intermediate stage to the front-axle bevel gear (50%), as shown in Figure 2. The skew drive shaft is an indispensable benefit of the package. With a parallel drive shaft arrangement, the tunnel dimensions would have been too big to permit realization of a right-hand drive vehicle. The skew drive shaft was made possible by a beveloid gear and an adapted bevel gear (85° shaft angle). Transfer gear and bevel gear share a common oil circuit. Lubrication is effected via a separate oil pump, which is driven via the drive shaft, as shown in Figure 3. Development of this beveloid gear is described below.

Basics of Conical Gears

Macrogeometry. In simple words, a beveloid gear is a spur gear with variable addendum correction across the face width. In the process, the tool is tilted by the cone angle δ relative to the gear axis. This results in the following basic gear dimensions:

| Nomenclature | |
|-----------------|------------------------------|
| a | center distance |
| a_s | substitute center distance |
| d_b | base diameter |
| m_n | normal module |
| r | radius |
| r_w | operating pitch radius |
| z | number of teeth |
| P | point |
| P_a | active point |
| α_n | normal pressure angle |
| α_t | transverse pressure angle |
| β | helix angle |
| γ | polar angle |
| δ | cone angle |
| ω | angular velocity |
| Φ | local angle |
| Σ | shaft angle |
| \underline{a} | center distance vector |
| \underline{n} | normal vector |
| \underline{r} | radius vector |
| \underline{v} | velocity vector |
| c_a | tip relief |
| c_f | root relief |
| f_s | profile lean (profile twist) |
| Index | |
| 0 | tool |
| 1 | pinion |
| 2 | wheel |
| L | left |
| R | right |
| b | base |
| t | tangential |
| ' | coordinate system 1 (pinion) |
| " | coordinate systsem 2 (wheel) |

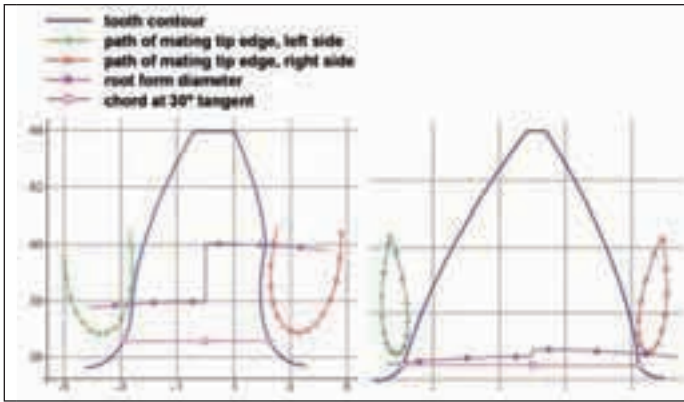


Figure 4—Tooth profile at the inner (toe) and outer (heel) face for helical beveloids.

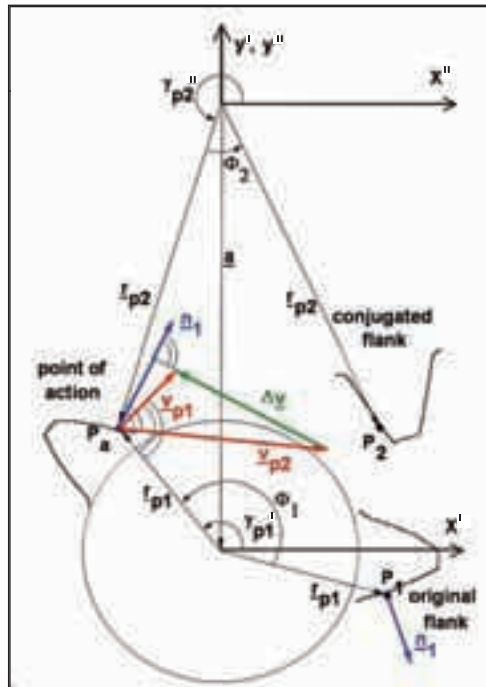


Figure 5—Generating of the conjugate flank point, simplified by showing in one plane.

Helix angle right/left side:

$$\tan \beta_{R,L} = \tan \beta \cdot \cos \delta \mp \frac{\tan \alpha_{n0} \cdot \sin \delta}{\cos \beta} \quad (1)$$

Transverse pressure angle right/left:

$$\tan \alpha_{t,R,L} = \frac{\tan \alpha_{n0} \cdot \cos \delta}{\cos \beta} \pm \tan \beta \cdot \sin \delta \quad (2)$$

Base circle diameter right/left:

$$d_{b,R,L} = \frac{z \cdot m_n \cdot \cos \alpha_{t,R,L}}{\cos \beta_{R,L}} \quad (3)$$

The differing left- and right-flank base circles result in unsymmetrical tooth profiles in helical gears (see Fig. 4).

Manufacturing by means of a rack-type cutter results in a tooth root cone angle equal to the cone angle δ . The addendum angle is designed in

a manner that avoids tip edge interference with the meshing gear and to achieve the maximum possible transverse contact ratio. As a result, we have a different tooth height across the face width.

Due to the geometric design limits for undercut and pointed teeth, the face width decreases with increasing cone angle. Sufficiently well-proportioned gears are possible up to a cone angle of about 15° .

Microgeometry. The pairing of two conical gears generally results in a point-like tooth contact. Outside this contact, spacing (separation) occurs between the tooth flanks. The object of gear modification is to reduce spacing (gaping) in order to obtain a planar and even contact.

Exact calculation of the tooth flank is possible through applying the rule of the common normal, as shown in Figure 5 (Ref. 5). To this end, a flank point P_1 with the radius r_{p1} and corresponding normal vector \underline{n}_1 is generated on the original flank. We get the speed vector \underline{v}_{p1}' through:

$$\underline{v}_{p1}' = \underline{\omega}_1 \cdot \begin{pmatrix} r_{p1} \cdot \sin \gamma_1' \\ r_{p1} \cdot \cos \gamma_1' \\ 0 \end{pmatrix} \quad (4)$$

For the point generated on the mating flank, the radial vector \underline{r}_{p2}

$$\underline{r}_{p2} = \underline{a} - \underline{r}_{p1} \quad (5)$$

and the velocity vector \underline{v}_{p2}'' apply.

$$\underline{v}_{p2}'' = \underline{\omega}_2 \cdot \begin{pmatrix} r_{p2} \cdot \sin \gamma_2'' \\ r_{p2} \cdot \cos \gamma_2'' \\ 0 \end{pmatrix} \quad (6)$$

The angular velocities result from the gear ratio:

$$\frac{\underline{\omega}_1}{\underline{\omega}_2} = -\frac{z_2}{z_1} \quad (7)$$

The angle γ is iterated until the rule of the common normal

$$\underline{n}_1 \times (\underline{v}_{p2} - \underline{v}_{p1}) = \underline{0} \quad (8)$$

is fulfilled. The point of action P_a found is rotated through the angle Φ_2

$$\Phi_2 = -\Phi_1 \cdot \frac{z_1}{z_2} \quad (9)$$

around the gear axis and thus the conjugate flank point P_2 is obtained.

Gear set arrangements. Beveloids can be installed in the following axis configurations:

- parallel axes,
- intersecting axes and
- skewed axes.

Due to the fact that selection of the cone angle does not depend on the shaft angle, pairing is also

possible with cylindrical gears. Beveloids can be produced as external or internal gears, as shown in Table 1, so a whole matrix of pairing options results, and the designer is provided with a high degree of flexibility (Ref. 6).

Load capacity calculation. Certain conditions provided, analytical calculation methods for involute spur gears, such as ISO 6336, DIN 3990 or AGMA C95, can be used for sufficiently accurate determination of the load capacity of beveloids in intersecting axial arrangement. This is, however, only possible if the tooth contact is linear. Since the tooth contact of most beveloid sets without flank modifications is point contact, flank modifications must bring about approximate linear contact.

The tooth profile changes across the face width, and consequently load capacity in the individual face sections across the face width changes. The main influences on tooth root capacity change across the face width in opposite directions, which are shown in Table 2.

Homogenous tooth root capacity across the face width is to be aimed at. The influences on flank capacity also vary across the face width, as shown in Table 3.

The load capacity of the tooth center section can thus be seen as representative for the entire gearing. All geometric gear parameters are determined in the tooth center. Only the center distance must be converted into a substitute center distance a_s and is obtained by adding up the pitch radii.

$$a_s = r_{w1} + r_{w2} \quad (10)$$

Contact pattern quality can be assessed on the basis of the width factors (e.g., $K_{H\beta}$ and $K_{F\beta}$ in DIN/ISO), which are to be determined as precisely as possible on the basis of the contact conditions. Load capacity calculations according to the finite element method are of a higher quality and to be preferred over analytical methods.

Beveloid Gear Sets in Automatic Car Transmissions

Macrogeometry. Based on experience with the design of marine and robot gears, beveloid gear sets for a new range of 6-speed automatic transmissions have been designed. But in car transmissions, macrogeometry is more restricted, and the demands regarding low-noise design are much higher. For the smallest possible transmission error to be attained, the gear was designed with a pressure angle $\alpha_n = 17.5^\circ$ and a helix angle $\beta = 29.5^\circ$. This gives a transverse contact ratio of $\epsilon_\alpha = 1.7$ on the coast side and 1.8 on the drive side. The face contact ratio ϵ_β is 1.89 on the coast side and 1.63 on the drive side.

Due to the relatively wide helical angle, the

| Gear | Type | Axes | | |
|----------|-------------------|----------|---------|--------|
| | | parallel | crossed | skewed |
| External | Beveloid Beveloid | | | |
| | Cylindr. Beveloid | | | |
| Internal | Beveloid Beveloid | | | |
| | Cylindr. Beveloid | | | |

| | Toe | Heel |
|--|-----|------|
| Root chord | S ☹ | L ☺ |
| Root fillet radius | L ☺ | S ☹ |
| Tangential force at operating diameter | L ☹ | S ☺ |
| Profile contact ratio | L ☺ | S ☹ |

| | Toe | Heel |
|-----------------------|-----|------|
| Radius of curvature | S ☹ | L ☺ |
| Profile contact ratio | L ☺ | S ☹ |

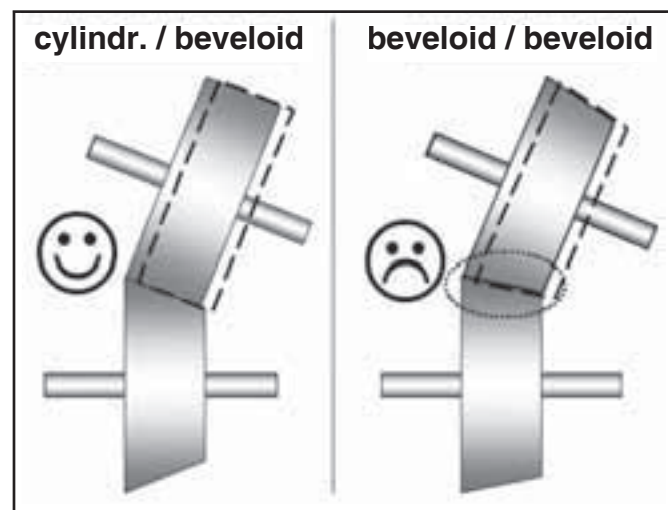


Figure 6—Backlash reduction due to axial offset of the driven gear. Left: No Influence, Right: Jammed.

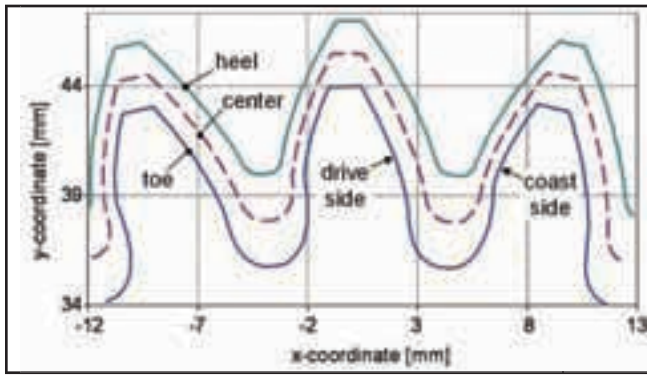


Figure 7—Profiles of the beveloid gear at heel, center, and toe.

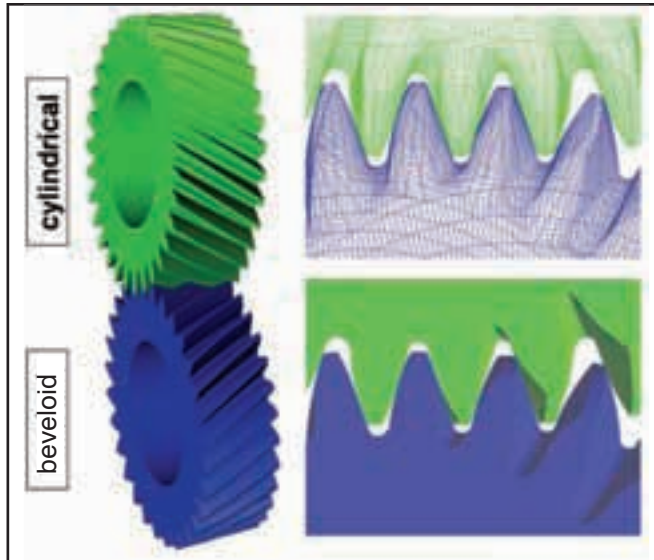


Figure 8—FE model of the beveloid gear set.

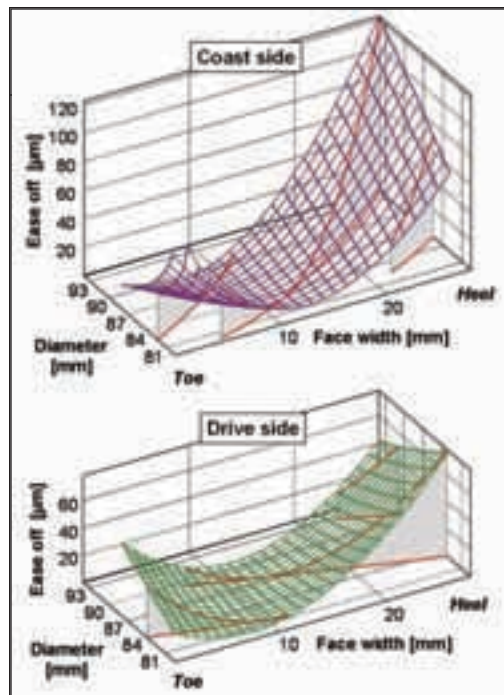


Figure 9—Ease-off for the unmodified gear set on coast and drive sides with emphasized spacing along the line of contact in one mesh position.

transverse pressure angle α_t on the drive side is 15.3° and on the coast side 23.9° . For pairing with a shaft angle of 8° , combination of a beveloid gear (input) with a cylindrical gear (output) was chosen. This resulted in a root angle of 8.6° on the beveloid gear. This combination was necessary because the output gear must be axially displaceable over a relatively wide range for adjustment of the bevel drive pinion on the output shaft. If two beveloid gears were used, axial displacement would result in the tooth backlash being too heavily affected, as is shown by Figure 6. Selectable face width is limited by the limits for pointed teeth on the heel and the undercut on the coast side on the toe of the beveloid gear.

For maximum load capacity to be achieved on the drive side, a relatively large undercut was accepted on the toe of the coast side, with such an undercut being unusual for cylindrical gears. The profiles of the beveloid gear for the inner edge (toe), the center and the outer edge (heel) are shown in Figure 7. Figure 8 shows the FE model of the gear set. This model was used in the FE program PERMAS for calculating precisely the stresses resulting from the contact conditions and the load capacity of the gear set (Ref. 7).

Microgeometry. The point contact of the beveloid gear set is a major problem when it comes to realizing design targets. This unfavorable contact behavior negatively affects load capacity and at low load results in high travel deviations, which cause an inordinately high noise level. The mismatch dimensions (ease-off) determined by means of a tooth contact analysis are shown in Figure 9. On the unmodified gear, we have both crowning and profile twist, which clearly differ for the drive flank and coast flank. Figure 9 also shows the spacing (gap) on the potential contact lines of a mesh position; they illustrate the crowning and tilt caused along the contact line. The effects of spacing (contact gap) on flank pressure were analyzed by a load distribution calculation with the program LVR (Ref. 8). In the computation, the gear set was approximately modeled as a parallel axis drive, and contact gap was considered as flank topology on one gear of the gear set. This permitted adequate representation of the contact conditions, which were verified by a comparison of calculated load distributions with contact patterns measured.

Figure 10 shows LVR-computed load distributions and the contact patterns for the same loads. To significantly improve contact conditions through enhanced contact width, topological flank modifications were computed for the cylindrical gear of the gear set. The object was to compensate as far as possible the existing spacing (gap) without completely eliminating the crowning. Particular attention was

paid to the compensation of the profile twist, which causes the diagonal contact pattern and, like the crowning, prevents a full contact. The topologies shown in Figure 11 were generated on test gears on a 5-axis KAPP grinding machine and tested for comparison with the unmodified gears.

The contact pattern was significantly improved, as shown in Figure 12. Gear noise was also clearly reduced with the topologically modified gears. In the following sequence, the topologies were converted into conjugate topologies for the beveloid gear and defined as the specification for batch-production methods. Optimum topology has been very closely realized in the batch production launched in the meantime. In fine adjustment of the topology, great attention was paid to the resulting noise excitation, which needed to be improved—especially for partial loads.

With the present fine geometry, it has been possible to achieve the low noise level aimed at, which corresponds with that of a well designed set of helical gears. Compared to the unmodified variant, a noise reduction of more than 10 dB was achieved in critical speed ranges, especially under partial-load conditions. Assessment of the excitation level was also carried out with the program LVR, which was used in serial computations to determine transmission error for various loads and modifications (Ref. 8). Using the diagrams of the excitation levels calculated on the basis of the transmission error, it was possible to assess the efficiency of individual corrections. Figure 13 shows an example of the diagrams. The transmission error curves for the unmodified gears and the topologically modified gear set are shown in Figure 14 for low partial load. Reduction of the amplitude of transmission error for the topologically modified gears is evident. In tests with the transmission on the test bench and in the vehicle, the low noise level and the improvement compared to the original variant were proven. The gearing also fulfilled all requirements regarding load capacity.

Manufacture and testing. Beveloids are machined on commercial cylindrical gear production machines, which generate the tooth profile via a hobbing process. Hobbing and shaping are the cutting methods used for carburized gears; worm grinding and honing are the methods used for hard/fine machining. The beveloid gears of a car transmission are cut with hobs, and following hardening, they are fine-machined on generating grinding machines (Reishauer RZA and Kapp KX1). It is of particular importance for tool design that hobbing creates different topologies for the drive and coast sides. In order not to lose too much root thickness due to protuberance undercut, the protuberance chosen was equivalent to the machining allowance. Notwithstanding the protuberance, a tip

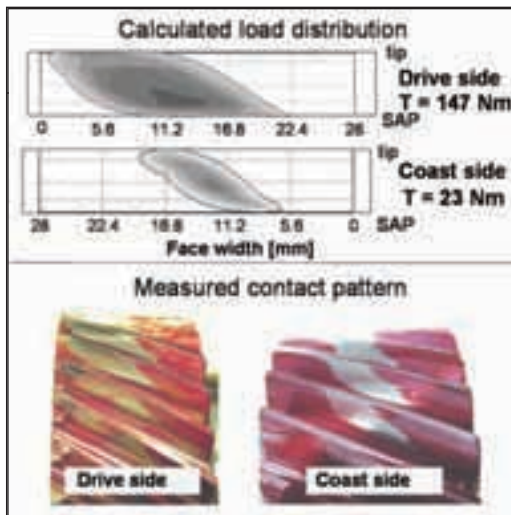


Figure 10—Calculated load distributions in comparison to loaded contact patterns from measurement runs.

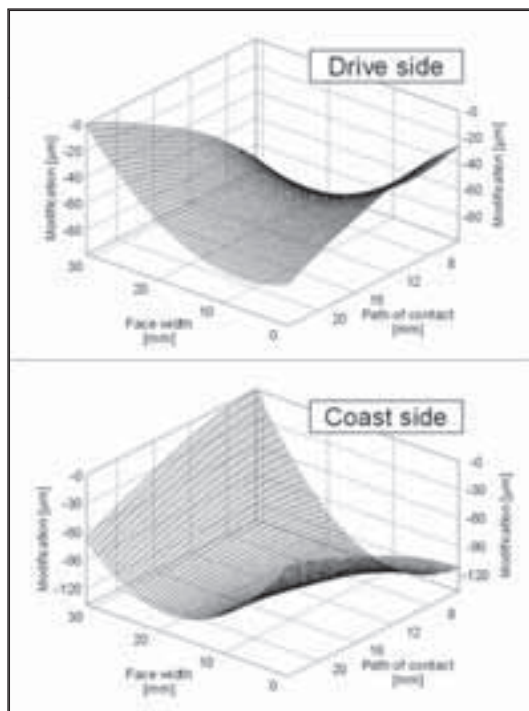


Figure 11—Desired topologies for the driven gear of the beveloid gear set.

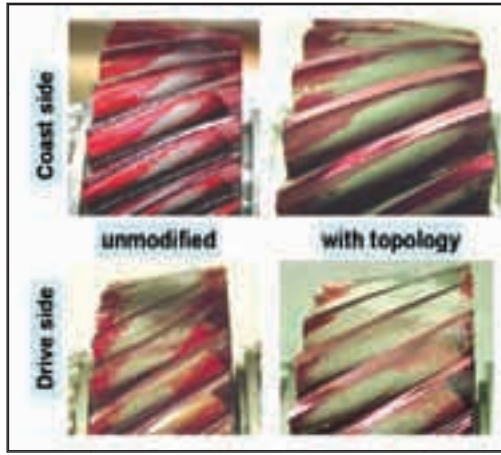


Figure 12—Improvements in the loaded contact pattern achieved with topological modifications.

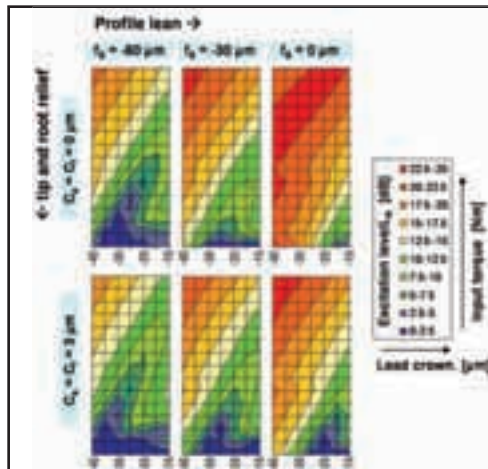


Figure 13—Excitation level for various loads and modifications, calculated for evaluating the noise excitation.

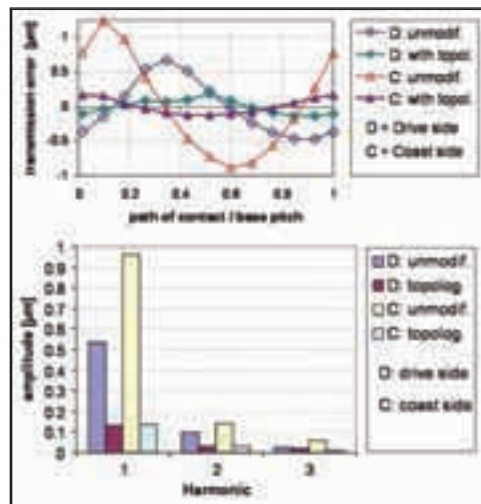


Figure 14—Transmission error, calculated with torque $T = 100 \text{ Nm}$ (15% nominal load) for unmodified and topologically modified gears.

radius on the grinding tool was required to avoid sharp grinding shoulders in the area of the heel. In this area, an addendum modification coefficient of about 1.3 occurs. If there is no tool tip radius, this addendum modification results in a small radius of curvature of the trochoid in the root fillet due to the generating process.

To test the topology required, the traditional measurements on four teeth in the center of tooth width and height are no longer sufficient. Grid measurements need to be carried out, with additional profile records on the heel and the toe and additional line records on tooth tip and tooth root. Permissible tolerance zones have been specified for these measuring records (K diagrams). Measurements can be conducted on any measuring machine used for cylindrical gears. For line measurements, modern measuring machines permitting a conical course of the measuring line should be used. If the measurement device doesn't allow a conical measurement, the face width has to be divided in sections. In each section, the measuring feeler touches the tooth on a certain diameter. The measuring result will then, however, be influenced by profile deviations in the flank record.

Test bench. In addition to theoretical capacity computations, a test bench conforming to the installation conditions of a car transmission was built for basic tests. Figure 15 shows the back-to-back test bench used. Two shafts with their axes intersecting at an angle of $\Sigma = 8^\circ$ are linked via two beveloid gear sets and braced against each other by means of a coupling. The test gear has a center distance of $a = 87 \text{ mm}$; the transmitting gear set has a center distance of $a = 200 \text{ mm}$.

The significant center distance difference guarantees that the transmitting gearing is not damaged—not even during overload testing. Compared to the test gearing, the number of teeth of the transmitting gearing has been doubled for the same ratio to be achieved. This test bench is used to determine S/N curves for tooth root and tooth flank capacity.

Summary

For a six-speed automatic transmission with integrated AWD, a beveloid gear set with 8° shaft angle was developed for a skew drive shaft linked to the front axle, as shown in Figure 16. This permitted compact tunnel dimensions. Since mid-2002, the transmission has been in production in a high-end passenger car. Development work included the following tasks:

- Development of specific software for the design and recalculation of beveloids.
- Because of topological corrections, the capacity and noise levels reached are comparable with those of helical gears.
- Beveloids are cut on NC hobbing machines, and after hardening, they are finish-machined on con-

tinuous worm grinding machines.

- Beveloids require comprehensive quality specifications (toe, center, root). Measurements are carried out on the usual measuring machines, with specific setting values being considered.
- A back-to-back test bench with suitable shaft angle was set up for basic testing. Allowable stresses have to be determined under different conditions. ☉

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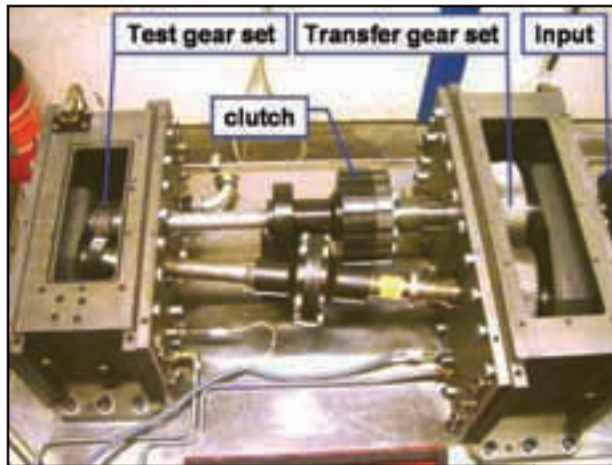


Figure 15—Back-to-back test rig for beveloid gear sets.

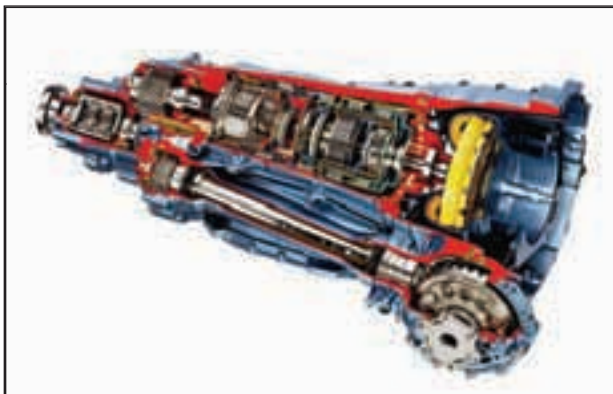


Figure 16—Six-speed automatic transmission for all-wheel drive with side shaft driven by a beveloid gear set.