Laminated Gearing

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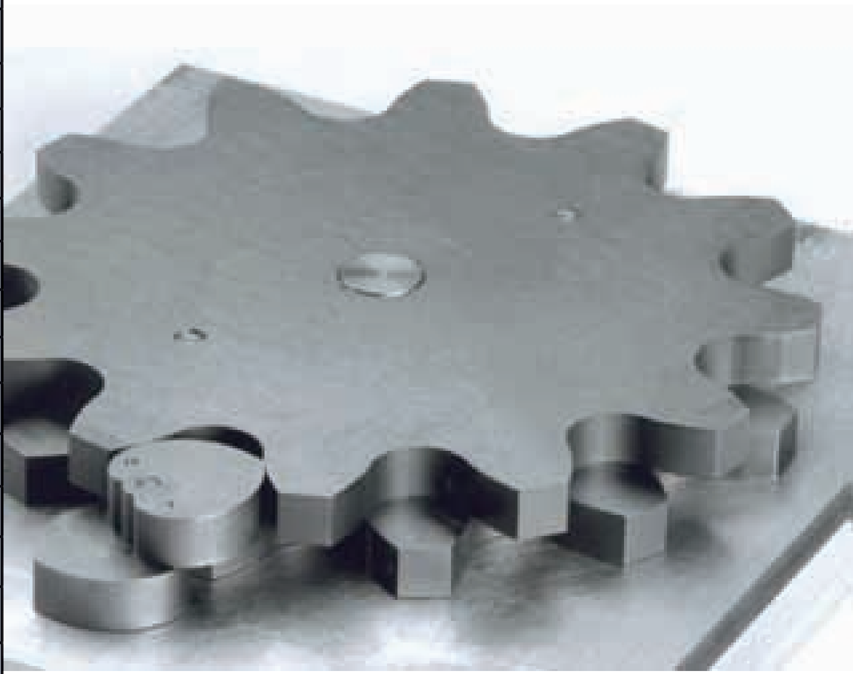
Nomenclature	
C	center distance
C_b	Basic center distance, $(N + n)/2P$ or $m(N + n)/2$
m	Module
m_p	Contact ratio
N	Number of teeth on gear
n	Number of teeth on pinion
P	Diametral pitch
p	Circular pitch on reference circles, π/P or πm
$p/2$	Basic tooth thickness
p_b	Base pitch
R	Radius of gear reference circle, $N/2P$ or $mN/2$
r	Radius of pinion reference circle, $n/2P$ or $mn/2$
R_b	Radius of gear base circle
r_b	Radius of pinion base circle
R_{if}	Inside form radius on gear
r_{if}	Inside form radius on pinion
R_{of}	Outside form radius on gear
r_{of}	Outside form radius on pinion
r_p	Radius where tooth becomes pointed
t	Tooth thickness on reference circle
ΔT	Deviation from $p/2$ on gear reference circle
Δt	Deviation from $p/2$ on pinion reference circle
Φ	Profile angle on basic rack
ϕ	Pressure angle
ϕ_p	Pressure angle at r_p

Management Summary

It is a common objective in geared product design that each stage of gearing provide the maximum gear ratio, or the number of teeth in the driven gear divided by the number in the driver. Further increase in the number of teeth in the driven gear is often restricted by an outside diameter limit and the selected diametral pitch or module. The alternate approach is to reduce the number of teeth in the driving gear down to a single tooth. This article points out that very low numbers of teeth lead to inadequate design values of contact ratio. This obstacle may be overcome by using helical gears instead of spur gears, but with noted manufacturing and other disadvantages. Spur gear designs may be maintained by the use of laminated gearing.



A one-tooth laminated pinion in mesh with a 12–tooth laminated gear. Photo courtesy of ACCU-Prompt Inc.

In this article, a laminated gear consists of two spur gears on the same axis, where the teeth on one gear are aligned with the tooth spaces on the other gear. The contact ratio for each mesh can be as low as 0.5. The basic geometry and circular fillet design are outlined.

Advantages

In some designs, it is advantageous to use laminated gearing instead of helical gearing. Specifically, in fine-pitch formed gearing (molded plastic, die cast, powder metal, stamped), tolerances are often so large relative to whole depth that the contact ratio is less than unity. In that case, it is necessary to use either laminated or helical gearing (Ref. 1). In choosing one or the other, note should be taken of the fact that tooling for laminated gearing is simpler and more accurate than that for helical gearing, since tooling for laminated gearing is made on a wire EDM machine, whereas that for helical gearing is made with a machined electrode guided by a rotating mechanism. Also, inspection equipment for laminated gearing is simpler than that for helical gearing (Ref. 2)

And it should be noted that a worm in mesh with a helical gear has only point contact between mating teeth, with the potential for high wear (Ref. 3), whereas laminated gearing has line contact, and no axial thrust on the bearings.

Basic Geometry

From Figure 1, it is seen that the contact ratio for each mesh is $m_p = z/p_b$, where $z = \sqrt{r_{of}^2 - r_b^2} - u_p$ so that

$$m_p = \frac{\sqrt{r_{of}^2 - r_b^2} - u_p}{p_b} \tag{1}$$

where

$$u_P = \sqrt{C^2 - (r_b + R_b)^2} - \sqrt{R_{of}^2 - R_b^2} \tag{2}$$

For maximum contact ratio, the $u_P = 0$ in Equation 2, i.e.,

$$C^2 - (r_b + R_b)^2 = R_{of}^2 - R_b^2$$

which reduces to

$$C = \sqrt{R_{of}^2 + r_b(r_b + 2R_b)} \tag{3}$$

For a given center distance, the sum of the deviations from basic tooth thickness is (Ref. 4)

$$\Delta t + \Delta T = 2C_b(\mathrm{inv}\phi - \mathrm{inv}\Phi) \tag{4}$$

where

$$\cos\phi = \frac{C_b}{C}\cos\Phi \tag{5}$$

Usually, the R_{of} in Equation 3 and ΔT in Equation 4 can be basic, namely, $R_{of} = (N + 2)/2P$ and $\Delta T = 0$.

If the contact ratio of Equation 1 is somewhat greater than 0.5, then the sliding can be reduced by reducing the outside form radius of the gear (R_{of}) (Ref. 5). For a reduced R_{of}, the

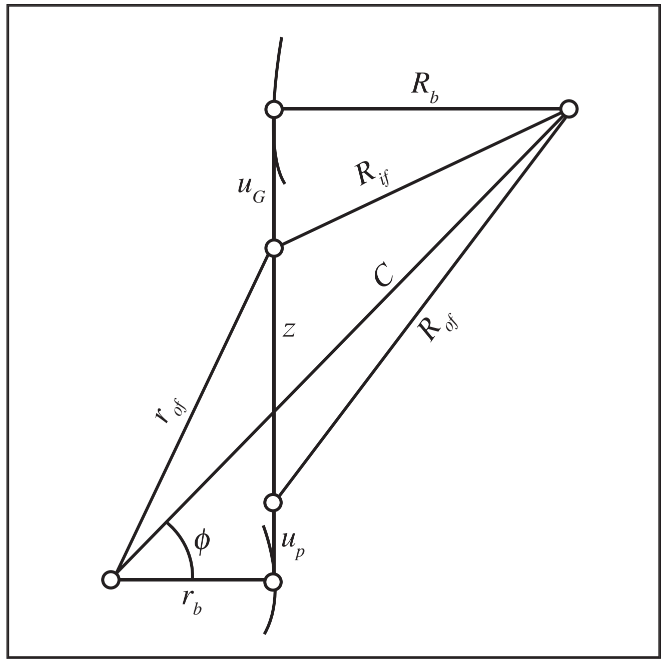


Figure 1—Calculation of contact ratio.

inside form radius of the pinion is

$$r_{if} = \sqrt{r_b^2 + u_P^2} \tag{6}$$

where u_p is that in Equation 2.

Also from Figure 1, it is apparent that the inside form radius of the gear is

$$R_{if} = \sqrt{R_b^2 + u_G^2} \tag{7}$$

where

$$u_G = \sqrt{C^2 + (r_b + R_b)^2} - \sqrt{r_{of}^2 - r_b^2} \tag{8}$$

Circular Fillets

With the advent of wire EDM, it has become common practice to replace the fillet generated by a basic rack with a circular arc of specified radius. Consequently, the designer now has to determine the location and radius of a circular arc that does not intersect the fillet generated by the tip of the mating tooth. If a computer program is not accessible (some programs do not determine the fillet generated by the tip of the mating tooth and some programs do not accept one and two-tooth pinions), then the fillet can be determined graphically—the old-fashioned way (Ref. 6).

To plot the fillets, the mating gears are rolled together in tight mesh (Ref. 7). Rather than draw the rolling circles (also known as operating pitch circles), only the tooth profiles are plotted (which permits greater magnification) on 8½ x 11 vellum, as shown in Figure 2.

It is important to remember that whenever the outside form

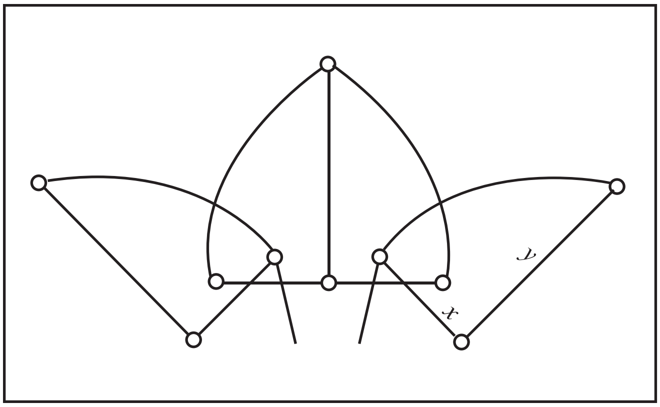


Figure 2—Plot for generation of fillets.

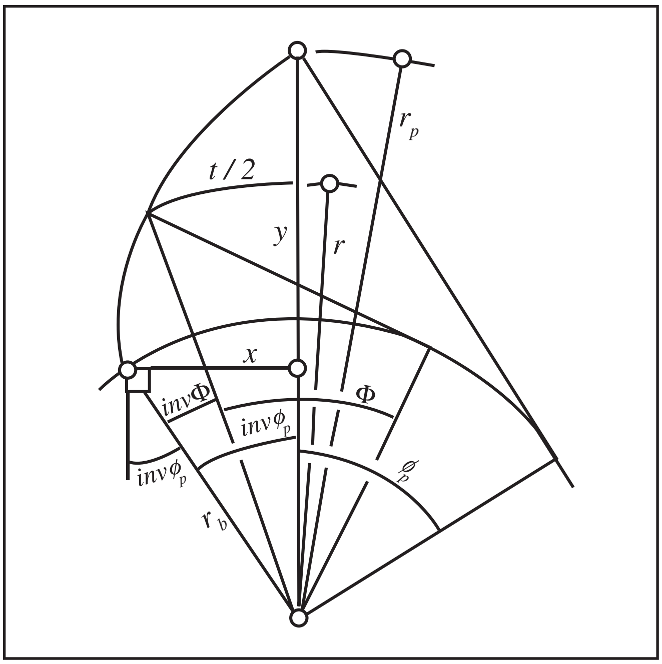


Figure 3—Coordinates of involute endpoints.

radius of the gear (R_{of}) contacts the base point of the pinion profile (as in Equation 3), then the specified circular arc falls on the radial line to the base point, not on the involute.

The four involute profiles in Figure 2 are traced from a single involute plotted on 8 ½ x 11 vellum (Ref. 8). From Figure 3, it is seen that the angle between the base point of the involute and the centerline of the tooth is

$$\text{inv}\phi_p = \text{inv}\Phi + \frac{t/2}{r}$$

where

$$\frac{t}{2r} = \frac{1}{d}\left(\frac{p}{2} + \Delta t\right) = \frac{1}{d}\left(\frac{\pi d}{2n} + \frac{n}{n}\Delta t\right)$$

so that

$$\text{inv}\phi_p = \text{inv}\Phi + \frac{1}{n}\left(\frac{\pi}{2} + \frac{n}{d}\Delta t\right) \tag{9}$$

where

$$\frac{n}{d} = P \quad \text{or} \quad \frac{1}{m}$$

Thus, the endpoints of the involute are at

$$x = r_b \sin(\text{inv}\phi_p) \tag{10}$$

and

$$y = r_p - r_b \cos(\text{inv}\phi_p) \tag{11}$$

where

$$r_p = \frac{r_b}{\cos\phi_p} \tag{12}$$

The locations of the *y* endpoints on the centerline of each tooth in Figure 2 are calculated from right triangles.

Possible Disadvantages

It should be noted that this method does have some limitations. For instance, even though the illustration show both the driver and driven laminated gears with the laminations flat in form and closely side by side, practical manufacturing and assembly conditions may dictate the need for a spacer of some form on at least one of the gears. The benefits of laminated gearing are only available if, in each set of laminations, there is a very accurate rotational orientation of one outline to the other and that the two outlines themselves are very closely matched. Finally, another limit on reducing the number of teeth in the driver is the need for a root diameter of adequate size, either to permit mounting on a shaft or simply of adequate torsional cross-section to transmit torque to the gear teeth.

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