

# Design of Oil-Lubricated Machine Components for Life and Reliability

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## Management Summary

In the post-World War II era, the major technology drivers for improving the life, reliability and performance of rolling-element bearings and gears have been the jet engine and the helicopter. By the late 1950s, most of the materials used for bearings and gears in the aerospace industry had been introduced into use. With improved manufacturing and processing, the potential improvement in bearing and gear life can be as much as 80 times that attainable in the early 1950s. This article summarizes the use of laboratory fatigue data for bearings and gears coupled with probabilistic life prediction and EHD theories to predict the life and reliability of a commercial turboprop gearbox. The resulting predictions are compared with field data.

## Introduction

By the close of the 19th century, the bearing industry began to focus on sizing bearings for specific applications and determining bearing life and reliability. In 1896, R. Stribeck (Ref. 1) in Germany began fatigue testing

full-scale bearings. J. Goodman (Ref. 2) in 1912 in Great Britain published formulas based on fatigue data that would compute safe loads on ball and cylindrical roller bearings. In 1914, the *American Machinists Handbook and Dictionary of Shop Terms* (Ref. 3) devoted six pages to rolling-element bearings, discussing bearing sizes and dimensions and recommending (maximum) loading and specified speeds. However, this publication did not address the issue of bearing life. During this time, it would appear that rolling-element bearing fatigue testing was the only way to determine or predict the minimum or average life of ball and roller bearings.

In 1924, A. Palmgren (Ref. 4) in Sweden published a paper in German outlining his approach to bearing life prediction and presented an empirical formula based upon the concept of an  $L_{10}$  life, or life at which 90 percent of a population survives. During the next 20 years, he empirically refined his approach to bearing life prediction and matched his predictions to test data (Ref. 5). However, his formula lacked a theoretical basis or an analytical proof.

In 1939, W. Weibull (Refs. 6 and 7) in Sweden published his theory of failure. He was a contemporary of Palmgren and shared the results of his work with him. In 1947, Palmgren, in concert with G. Lundberg, also of Sweden, incorporated his previous work along with that of Weibull and what appears to be the work of H. Thomas and V. Hoersch (Ref. 8) in a probabilistic analysis to calculate rolling-element (ball and roller) life. This has become known as the Lundberg-Palmgren theory (Refs. 9 and 10). (In 1930, H. Thomas and V. Hoersch (Ref. 8) at the University of Illinois, Urbana, developed an analysis for determining subsurface principal stresses under Hertzian contact (Ref. 11). Lundberg and Palmgren (Refs. 9 and 10) do not reference the work of Thomas and Hoersch (Ref. 8) in their papers.)

The Lundberg and Palmgren life equations have been incorporated in both the International Organization for Standardization (ISO) and the American National Standards Institute (ANSI)/American Bearing Manufacturers Association (ABMA) standards for the load ratings and life of rolling-element bearings (Refs. 12–14), as well as in current bearing codes to predict life.

As mentioned, in the post-World War II era, the major technology drivers for improving the life, reliability and performance of rolling-element bearings and gears have been the jet engine and the helicopter. By the late 1950s, most of

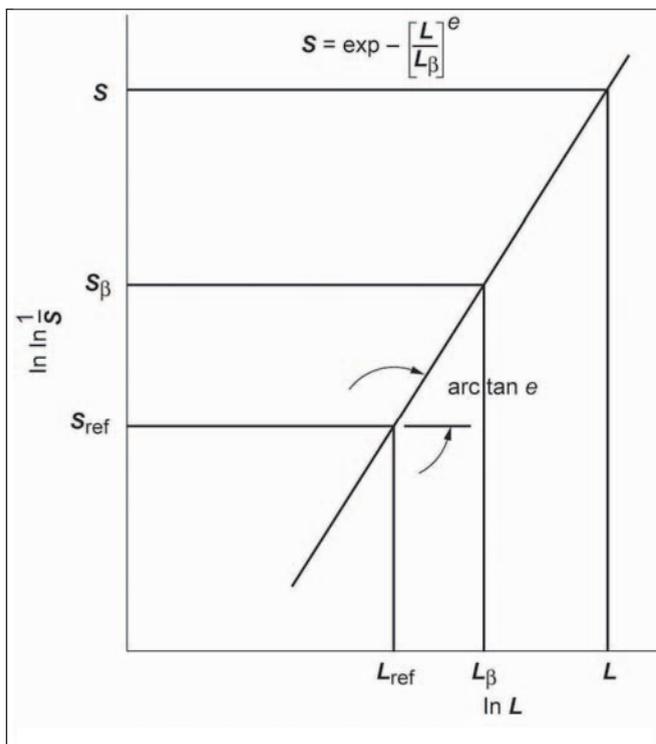


Figure 1—Weibull plot where (Weibull) slope of tangent or line is  $e$ . Probability of survival  $S_\beta$  of 36.8 percent, at which  $L = L_\beta$  or  $L/L_\beta = 1$ .

the materials used for bearings and gears in the aerospace industry were introduced into use. By the early 1960s, the life of most steels was increased over that experienced in the early 1940s, primarily by the introduction of vacuum degassing and vacuum melting processes in the late 1950s (Ref. 15).

The development of elastohydrodynamic (EHD) lubrication theory in 1939 by A. Ertel (Ref. 16), and later by A. Grubin (Ref. 17) in 1949, showed that most rolling bearings and gears have a thin EHD film separating the contacting components. The life of these bearings and gears is a function of the thickness of the EHD film (Ref. 15).

Computer programs modeling bearing and gear dynamics that incorporate probabilistic life prediction methods and EHD theory enable the optimization of rotating machinery based on life and reliability. With improved manufacturing and material processing, the potential improvement in bearing and gear life can be as much as 80 times that attainable in the early 1950s (Ref. 15).

Between 1975 and 1981, Coy, Townsend and Zaretsky (Refs. 18–21) published a series of papers developing a methodology for calculating the life of spur and helical gears based upon the Lundberg-Palmgren theory and methodology for rolling-element bearings.

A probabilistic life model for planetary gear trains has been developed (Refs. 22–27). This model is based on the individual reliabilities of the gearbox bearings and gears based on classical rolling-element fatigue. The reliability of the gearbox system is treated as a strict series probability combination of the reliabilities of the gearbox components based on the Lundberg-Palmgren theory (Refs. 9 and 10). Each bearing and gear life was calculated, and the results were statistically combined to produce a system life for the total gearbox. The method was applied to a turboprop gearbox by Lewicki, et al. (Ref. 28).

The work presented in this report summarizes the use of laboratory fatigue data for bearings and gears coupled with probabilistic life prediction and EHD theories to: (1) predict the life and reliability of a commercial turboprop gearbox, and (2) compare the resulting prediction with field data.

### Enabling Equations and Analysis

**Weibull Analysis.** In 1939, Weibull (Refs. 6 and 7) developed a method and equation for statistically evaluating the fracture strength of materials. He also applied the method and equation to fatigue data based upon small sample (population) sizes, where the two-parameter expression relating life and probability of survival is

$$\ln \ln \left( \frac{1}{S} \right) = e \ln \left( \frac{L}{L_B} \right) \quad \text{where } 0 < L < \infty; 0 < S < 1 \quad (1)$$

When plotting the  $\ln \ln [1/S]$  as the ordinate against the  $\ln L$  as the abscissa, fatigue data are assumed to plot as a straight line (Fig. 1). The ordinate  $\ln \ln [1/S]$  is graduated in statistical percent of components failed or removed for cause

### Nomenclature

$a$	major semiaxis of contact ellipse, m (in.)
$a_1$	life adjustment factor for reliability
$a_2$	life adjustment factor for materials and processing
$a_3$	life adjustment factor for operating conditions including lubrication
$B$	gear material constant, N/m <sup>1.979</sup> (lbf/in. <sup>1.979</sup> )
$C_D$	basic dynamic capacity of a ball or roller bearing, N (lbf)
$C_t$	basic dynamic capacity of gear tooth, N (lbf)
$c$	stress-life exponent
$d$	diameter of rolling element, m (in.)
$e$	Weibull slope; exponent
$F_t$	normal tooth load, N (lbf)
$f'$	tooth face width, m (in.)
$f_{cm}$	bearing geometry and material coefficient
$h$	elastohydrodynamic (EHD) lubricant film thickness, m (in.); exponent
$i$	number of rows of rolling elements
$k$	gear tooth stress cycles per input shaft revolutions
$L$	life, hr, stress cycles, or revolutions
$L_B$	characteristic life or life at which 63.2 percent of population fails, hr, stress cycles, or revolutions
$L_{10}$	10-percent life or life at which 90 percent of a population survives, hr, stress cycles, or revolutions
$\ell$	length of stressed track, m (in.)
$L$	roller length, m (in.)
$N$	number of gear teeth
$n$	life, stress cycles
$P_{eq}$	equivalent bearing load, m (in.)
$p$	load-life exponent
$r$	pitch circle radius of gear, m (in.)
$S$	probability of survival, fractional percent
$V$	stressed volume, m <sup>3</sup>
$X_n$	fraction of time spent at load-speed condition $n$
$Z$	number of rolling elements per row
$z$	depth beneath the surface of maximum orthogonal or maximum shear stress, m (in.)
$\alpha$	contact angle, deg
$\eta_{10r}$	$L_{10}$ life of single gear tooth, stress cycles
$\Lambda$	lubrication film parameter $h/\ell$ (Eq. 14)
$\rho$	curvature sum, m <sup>-1</sup> (in. <sup>-1</sup> )
$\sigma$	composite surface roughness, rms, m (in.)
$\sigma_1, \sigma_2$	surface roughness of bodies 1 and 2, rms, m (in.)
$\tau$	maximum orthogonal or maximum shear stress, Pa (psi)
$\varphi$	gear pressure angle, deg

### Subscripts

1, 2	bodies 1 or 2; load-life condition 1, 2, etc.
$B$	bearing
$G$	gear
$n$	body $n$ or load-life condition $n$
$ref$	define
$s, sys$	system
$t$	tooth

as a function of  $\ln L$ , the log of the time or cycles to failure. The tangent of the line is designated the Weibull slope  $e$ , which is indicative of the shape of the cumulative distribution or the amount of scatter of the data.

The method of using the Weibull distribution function for data analysis for determining component life and reliability was later developed and refined by Johnson (Ref. 29).

**Bearing Life Analysis.** Lundberg and Palmgren (Refs. 9 and 10) extended the theoretical work of Weibull (Refs. 6 and 7) and showed that the probability of survival  $S$  could be expressed as a power function of shear stress  $\tau$ , life  $n$ , depth of maximum shear stress  $z$ , and stressed volume  $V$ :

$$\ln \frac{1}{S} \sim \frac{\tau^c n^e}{z^h} V \quad (2)$$

$$\ln \frac{1}{S} \sim \frac{\tau^c n^e a l}{z^{h-1}} \quad (3)$$

By substituting the bearing geometry and the Hertzian contact stresses for a given load into Equation 3, the bearing basic dynamic load capacity  $C_D$  can be calculated (Ref. 9). The basic dynamic load capacity  $C_D$  is defined as the load that a bearing can carry for a life of one-million inner-race revolutions with a 90-percent probability of survival ( $L_{10}$  life). Lundberg and Palmgren (Ref. 9) obtained the following additional relation:

$$L_{10} = \left( \frac{C_D}{P_{eq}} \right)^p \quad (4)$$

where  $P_{eq}$  is the equivalent bearing load and  $p$  is the load-life exponent.

Formulas for the basic dynamic load ratings derived by Lundberg and Palmgren (Refs. 9 and 10) and incorporated in the ANSI/ABMA and ISO standards (Refs. 12–14) are as follows:

Radial ball bearings with  $d \leq 25$  mm:

$$C_D = f_{cm} (i \cos \alpha)^{0.7} Z^{2/3} d^{1.8} \quad (5)$$

Radial ball bearings  $> 25$  mm:

$$C_D = f_{cm} (i \cos \alpha)^{0.7} Z^{2/3} d^{1.4} \quad (6)$$

Radial roller bearings:

$$C_D = f_{cm} (i l \cos \alpha)^{7/9} Z^{3/4} d^{29/27} \quad (7)$$

Equation 4 can be modified using life factors based on reliability  $a_1$ , materials and processing  $a_2$ , and operating conditions such as lubrication  $a_3$  (Refs. 15 and 30) where

$$L = a_1 a_2 a_3 L_{10} \quad (8)$$

**Gear Life Analysis.** Between 1975 and 1981, Coy, Townsend and Zaretsky (Refs. 18–21) published a series of papers developing a methodology for calculating the life of spur and helical gears based upon the Lundberg-Palmgren theory and methodology for rolling-element bearings. Townsend, Coy and Zaretsky (Ref. 31) reported that for AISI 9310 spur gears, the Weibull slope  $e$  is 2.5. Based on Equation 2, for all gears except a planet gear, the gear life can be written as

$$L_{10G} = \frac{N^{-1/e_G} \eta_{10t}}{k} \quad (9)$$

For a planet gear, the life is

$$L_{10G} = \frac{N^{-1/e_G} (\eta_{10t1}^{-e_G} + \eta_{10t2}^{-e_G})^{-1/e_G}}{k} \quad (10)$$

The  $L_{10}$  life of a single gear tooth can be written as

$$\eta_{10t} = a_2 a_3 \left( \frac{C_t}{F_t} \right)^{p_G} \quad (11)$$

where

$$C_t = B f^{0.907} \rho^{-1.165} l^{-0.093} \quad (12)$$

and

$$\rho = \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \frac{1}{\sin \phi} \quad (13)$$

and  $\eta_{10t}$  is the  $L_{10}$  life in millions of stress cycles for one particular gear tooth. This number can be determined by using Equation 11, where  $C_t$  is the basic load capacity of the gear tooth;  $F_t$  is the normal tooth load;  $p_G$  is the load-life exponent usually taken as 4.3 for gears based on experimental data for AISI 9310 steel; and  $a_2$  and  $a_3$  are life adjustment factors similar to that for rolling-element bearings (Table 1). The value for  $C_t$  can be determined by using Equation 12, where  $B$  is a material constant that is based on experimental data and is approximately equal to  $1.39 \times 10^8$  when calculating  $C_t$  in SI units (newtons and meters), and  $21,800$  in English units (pounds and inches) for AISI 9310 steel spur gears;  $f$  is the tooth width; and  $\rho$  is the curvature sum at the start of single-tooth contact.

Life factors  $a_2$  for materials and processing are determined experimentally. Table 2 shows representative life factors obtained from surface fatigue testing of spur gears by NASA (Refs. 15 and 30).

The  $L_{10G}$  life of the gear (all teeth) in millions of input shaft revolutions at which 90 percent will survive can be determined from Equation 9 or Equation 10 where  $N$  is the total number of teeth on the gear;  $e_G$  is the Weibull slope for the gear and is assumed to be 2.5 (from Ref. 31); and  $k$  is the number of load (stress) cycles on a gear tooth per input shaft

revolution.

For all gears except the planet gears, each tooth will see load on only one side of its face for a given direction of input shaft rotation. However, each tooth on a planet gear will see contact on both sides of its face for a given direction of input shaft rotation. One side of its face will contact a tooth on the sun gear, and the other side of its face will contact a tooth on the ring gear. Equation 10 takes this into account, where  $\eta_{10r1}$  is the  $L_{10}$  life in millions of stress cycles of a planet tooth meshing with the sun gear, and  $\eta_{10r2}$  is the  $L_{10}$  life in millions of stress cycles of a planet tooth meshing with the ring gear.

**Elastohydrodynamic Lubrication.** An important parameter to consider when designing and operating rolling bearings and gears is the elastohydrodynamic (EHD) lubricant film thickness that forms between heavily loaded contacting bodies. Ertel (Ref. 16) and Grubin (Ref. 17) are credited with the first useful solution. A summary of EHD film thickness calculations can be found in Reference 15.

The life of a rolling bearing or gear is a function of a lubrication film parameter  $\Lambda$  where

$$\Lambda = \frac{h}{\sigma} \quad (14)$$

and

$$\sigma = (\sigma_1^2 + \sigma_2^2)^{1/2} \quad (15)$$

The lubricant film parameter  $\Lambda$  can be used as an indicator of bearing and gear performance and life. For  $\Lambda < 1$ , surface smearing or deformation accompanied by wear will occur on the rolling surfaces. For  $1 < \Lambda < 1.5$ , surface distress may be accompanied by superficial surface pitting. For  $1.5 < \Lambda < 3$ , some surface glazing can occur with eventual failure caused by classical subsurface-origin, rolling-element fatigue. At  $\Lambda \geq 3$ , minimal wear can be expected with extremely long life, and failure will eventually be by classical subsurface-origin, rolling-element fatigue.

The most expedient way of attaining a higher  $\Lambda$  ratio is to reduce the bearing or gear operating temperature and thus increase the lubricant viscosity. Another way is to select a lubricant with a higher viscosity at operating temperature, a larger pressure-viscosity coefficient, or both. The most expensive way of attaining a higher  $\Lambda$  ratio is to select a high-quality surface finish on bearings and gears (Refs. 15 and 30). The effect of film thickness on bearing life is shown in Figure 2. The life factor (LF) obtained from this figure is used to modify or adjust the calculated lives of bearings and gears (Refs. 15 and 30). This constitutes the life factor  $a_3$  in Equations 8 and 11.

**System Life Prediction.** The  $L_{10}$  lives of the individual bearings and gears that make up a rotating machine are calculated for each condition of their operating profiles. For each component, the resulting lives from each of the operating conditions are combined using the linear damage

Life adjustment factor	Variable
Reliability, $a_1$	Probability of failure
Materials and processing, $a_2$	Bearing steel Material hardness Residual stress Melting process Metal working
Operating conditions, $a_3$	Load Misalignment Housing clearance Axially loaded cylindrical bearings Rotordynamics Hoop stresses Speed Temperature Steel Lubrication Lubricant film thickness Surface finish Water Oil Filtration

Steel <sup>a</sup>	10-percent relative life, $L_{10}$
VAR AISI 9310	1.0
VAR AISI 9310 (shot peened)	1.6
VIM-VAR AISI 9310	2.5
VAR Carpenter EX-53	2.1
CEVM CBS 600	1.4
VAR CBS 1000	2.1
CEVM Vasco X-2	2.0
CEVM Super Nitra	1.3
VIM-VAR AISI M-50 (forged)	3.2
VIM-VAR AISI M-50 (ausformed)	2.4
VIM-VAR M50 NiInloy (5Ni-2A1)	11.5

<sup>a</sup> VAR, vacuum arc remelting; CEVM, consumable-electrode vacuum remelting; VIM-VAR, vacuum induction melting - vacuum arc remelting.

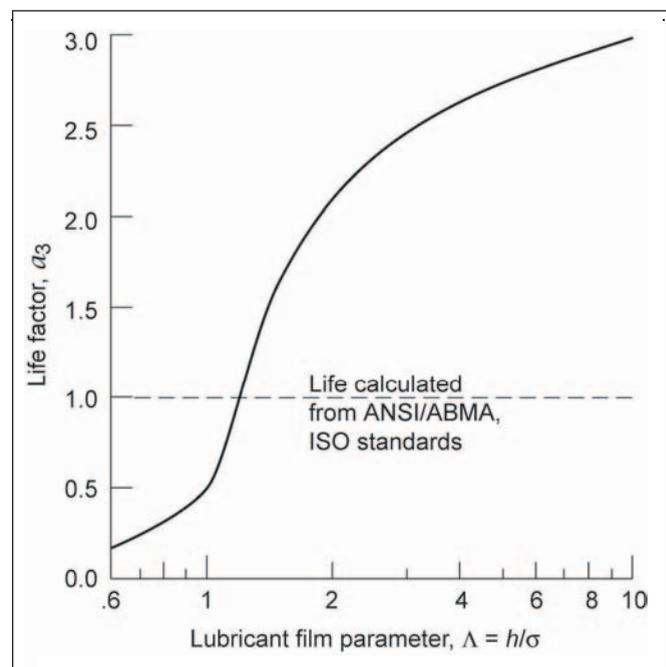


Figure 2—Life factor  $a_3$  as function of lubricant film parameter  $\Lambda$  (Ref. 15).

(Palmgren-Langer-Miner) rule (Refs. 4, 32 and 33) where

$$\frac{1}{L} = \frac{X_1}{L_1} + \frac{X_2}{L_2} + \dots + \frac{X_n}{L_n} \quad (16)$$

The cumulative lives of each of the machine components are combined to determine the calculated machine system  $L_{10}$  life using the Lundberg-Palmgren formula (Ref. 9):

$$\frac{1}{L_{sys}^e} = \left( \frac{1}{L_{B_1}^{e_B}} + \frac{1}{L_{B_2}^{e_B}} + \dots + \frac{1}{L_{B_n}^{e_B}} \right) + \left( \frac{1}{L_{G_1}^{e_G}} + \frac{1}{L_{G_2}^{e_G}} + \dots + \frac{1}{L_{G_n}^{e_G}} \right) \quad (17)$$

The calculated system life is dependent on the resultant value of the system Weibull slope  $e$ . This value is normally not known with absolute certainty and is usually assumed to be the same as that of the shortest-lived component in the system.

### Results and Discussion

**Predicted Life of a Turboprop Gearbox.** A commercial turboprop gearbox used for this analysis is shown in Figure 3. It consists of two stages with a single-mesh spur reduction followed by a 5-planet planetary gearbox comprising 11 rolling-element bearings and 9 spur gears (Ref. 28). The first stage consists of the input pinion gear meshing with the main drive gear. The second stage is provided by the fixed-ring planetary driven by a floating sun gear as input with a five-planet carrier as output. The input pinion speed is constant at 13,820 rpm, producing a carrier output speed of 1,021 rpm.

The operational profile includes loads for takeoff, climb, cruise and descent. The cruise segment of the profile consumes 68 percent of the flight time with a little less than half of the power required for the takeoff, which lasts for less than 3 percent of the flight time.

The cause for removal can be assumed to be that one or more bearings or gears had fatigue or damage resulting in wear and/or vibration detected by magnetic chip detectors and/or vibration pickups. The gearboxes are removed from service before secondary damage occurs. The removed gearbox is inspected and the failed part or parts are replaced. The

gearbox is then put back into service. Individual occurrences are not predictable but are probabilistic. No two gearboxes run under the same conditions fail necessarily from the same cause and/or at the same time. At a given probability of survival, the life of the gearbox will always be less than the shortest-lived element in the gearbox.

Using Equations 4–8 for bearings and Equations 9–13 for gears and appropriate computer programs incorporating these equations, the lives of each of the bearings and gears making up the gearbox were calculated for each of their operating conditions. These lives are shown as the Weibull plots in Figure 4.

The  $L_{10}$  life of a single double-row, spherical bearing is 3,529 hr. From Equation 14, the system  $L_{10}$  life for the five-bearing planetary set is 774 hr. For all the bearings in the gearbox, the bearing system  $L_{10}$  life is also predicted to be 774 hr.

Using Equation 17 for the individual gears, the gear system predicted  $L_{10}$  life is 16,680 hr. Combining the bearing and gear lives to obtain a gearbox  $L_{10}$  life, again using Equation 17, the predicted  $L_{10}$  life for the gearbox is 774 hr. The lives of the individual bearings, and more specifically, those of the planet double-row spherical bearings, determine the life of the gearbox in this example. The system lives of the bearings, gears and gearbox are summarized in Figure 4c.

**Gearbox Field Data.** The application of the Lundberg-Palmgren theory (Ref. 9) to predict gearbox life and reliability needs to be benchmarked and verified under a varied load and operating profile. The cost and time to laboratory test a statistically significant number of gearboxes to determine their life and reliability is prohibitive. A practical solution to this problem is to benchmark the analysis to field data. Fortunately, these data were available for the commercial turboprop gearbox used in this study.

No two gearboxes are expected to operate in exactly the same manner. Flight variables include operating temperature and load. Small variations in operational load can result in significant changes in life. Hence, the accuracy of our calculations is dependent on how close the defined mission profile is to actual flight operation.

The gearboxes are condition-monitored and are removed from service on the detection of a perceived component failure. At the time of removal, the gearboxes are functional. The removal precludes secondary damage. That is, the damage is limited to the failed component.

Field data were collected for 64 new commercial turboprop gearboxes. From these field data, the resultant time to removal of each gearbox is plotted in the Weibull plot of Figure 5. For these data, there was not a breakdown of the cause for removal or the percent of each component that had failed. The resultant  $L_{10}$  life from the field data was 5,627 hr., and the Weibull slope  $e$  was 2.189. Using the Lundberg-Palmgren method (above), the predicted  $L_{10}$  life was 774 hr and the Weibull slope  $e$  was 1.125. The field data suggest

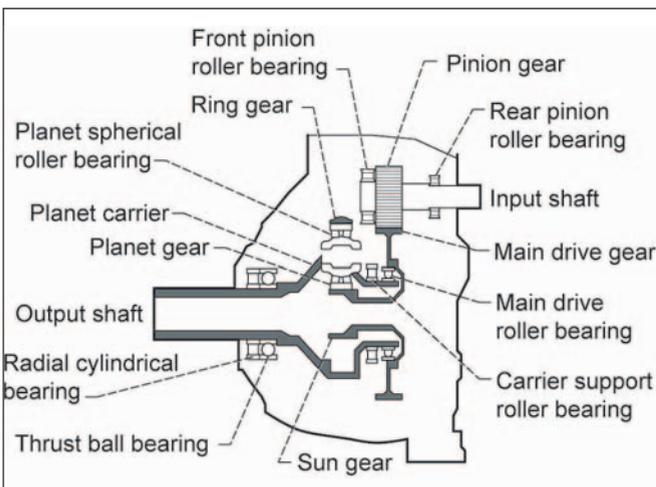


Figure 3—Commercial turboprop gearbox.

that the life of the gearbox was underpredicted by a factor of 7.56.

Although errors in the assumed operating profile of the gearbox may account for the difference between actual and predicted life, it is suggested that using the Lundberg-Palmgren equations results in a life prediction that is too low for the bearings.

Referring to Equation 4, in their 1952 publication (Ref. 10), Lundberg and Palmgren calculate a load-life exponent  $p$  equal to  $10/3$  for roller bearings, where one raceway has point contact and the other raceway has line contact. The  $10/3$  load-life exponent has been incorporated in the ANSI/ABMA/ISO standards first published in 1953 (Refs. 12–14). Their assumption of point and line contact may have been correct for many types of roller bearings then in use. However, it is no longer the case for most roller bearings manufactured today and, most certainly, for cylindrical roller bearings. Experience and the analysis suggest that the  $10/3$  load-life exponent  $p$  for roller bearings is incorrect and underpredicts roller bearing life (Ref. 34).

The work of Poplawski, Peters and Zaretsky (Ref. 34) suggests that  $p$  for roller bearings is equal to or greater than 4 but is less than 5. This premise can be easily tested based on the data for the turboprop gearbox.

From Equation 17, assuming that the bearing system has the same Weibull slope as that of the gearbox ( $e = 2.189$ ),

$$\frac{1}{L_{sys}^{e_s}} = \frac{1}{L_B^{e_B}} + \frac{1}{L_G^{e_G}} \quad (18a)$$

$$\frac{1}{(5627)^{2.189}} = \frac{1}{L_B^{2.189}} + \frac{1}{(16680)^{2.5}} \quad (18b)$$

From equation (18b), the actual bearing system life is

$$L_B = 5627 \text{ hr} \quad (18c)$$

From Lundberg-Palmgren (Ref. 9), the predicted bearing system life is

$$L_B \sim \left( \frac{C_D}{P_{eq}} \right)^4 \sim 774 \text{ hr} \quad (19a)$$

Then,

$$\left( \frac{C_D}{P_{eq}} \right) \sim 5.27 \quad (19b)$$

Calculating a revised value for the load-life exponent  $p$  for the gearbox bearings based on the actual bearing system life of 5,627 hr. (Eq. 18c),

$$\left( \frac{C_D}{P_{eq}} \right)^p \sim (5.27)^p \sim 5627 \text{ hr} \quad (20a)$$

Solving for load-life exponent  $p$ ,

$$p = 5.2 \quad (20b)$$

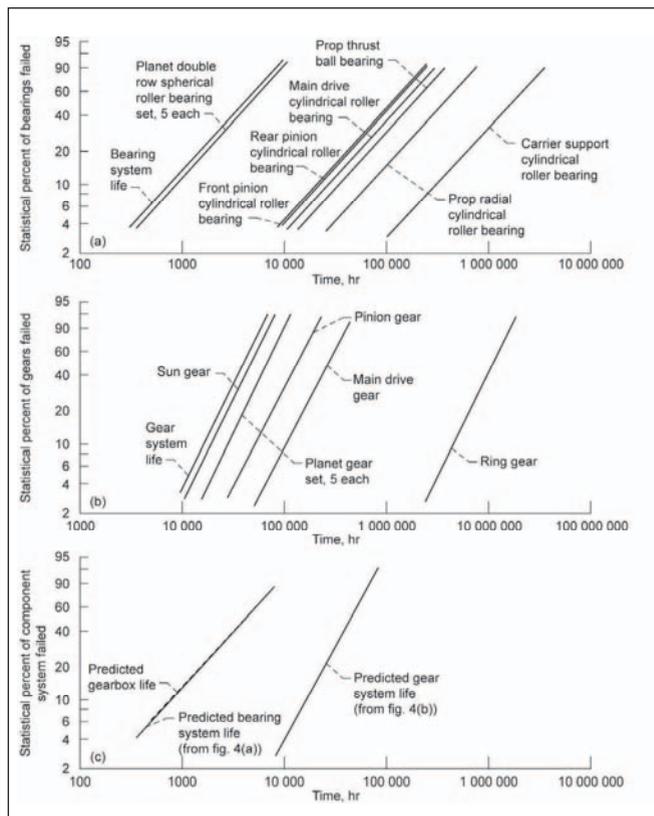


Figure 4—Predicted lives for commercial turboprop gearbox and its respective bearing and gear components using Lundberg-Palmgren life theory: (a) bearing component lives; (b) gear component lives; (c) gearbox life and component lives.

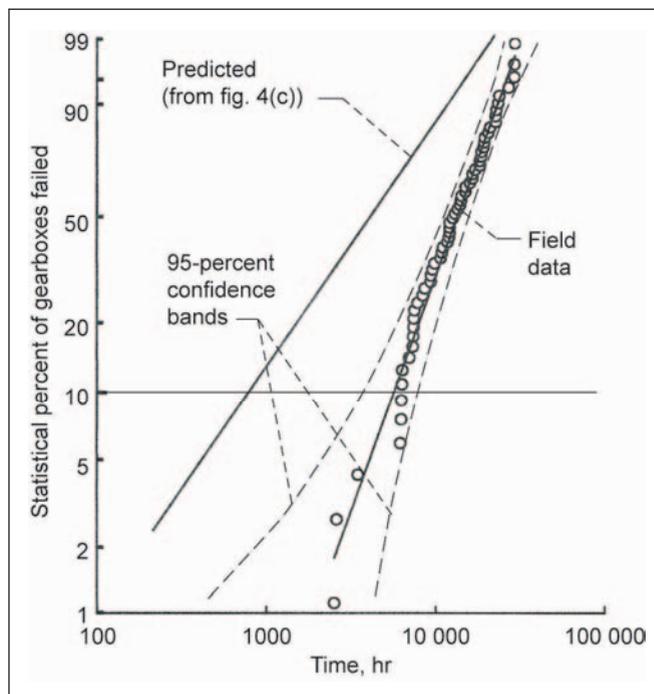


Figure 5—Field data for lives of turboprop gearboxes compared with predicted lives using Lundberg-Palmgren life theory. Failure index: 59 out of 64.

where according to Poplawski, Peters, and Zaretsky (Ref. 34),

$$4 \leq p \leq 5 \quad (20c)$$

Were the bearing lives to be recalculated with a load-life exponent  $p$  equal to 5.2, the predicted  $L_{10}$  life of the gearbox would be equal to the actual life obtained in the field.

### Summary of Results

Laboratory fatigue data for rolling-element bearings and gears coupled with probabilistic life prediction and elasto-hydrodynamic analysis were used to predict the life and reliability of a commercial turboprop gearbox. These data were compared with field data. The following results were obtained:

1. Using Lundberg-Palmgren theory, the predicted gearbox  $L_{10}$  life was less than that obtained in the field. The predicted gearbox  $L_{10}$  life was 774 hr. whereas the actual  $L_{10}$  life was 5,627 hr.
2. The gearbox life was dependent on the system life of the rolling-element bearings in the gearbox. Changing the load-life exponent  $p$  for the roller bearings from 4 to 5.2 would give a predicted gearbox  $L_{10}$  life of 5,627 hr.
3. The ANSI/ABMA/ISO standards using the unmodified Lundberg-Palmgren theory underpredict rolling-element bearing life, resulting in conservative values for the gearbox life prediction. ⚙️

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