

Hypoloid Gears with Small Shaft Angles and Zero-to-Large Offsets

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Management Summary

Beveloid gears are used to accommodate a small shaft angle. The manufacturing technology used for beveloid gearing is a special setup of cylindrical gear cutting and grinding machines. A new development, the so-called Hypoloid gearing, addresses the desire of gear manufacturers for more freedoms. Hypoloid gear sets can realize shaft angles between zero and 20° and at the same time, allow a second shaft angle (or an offset) in space that provides the freedom to connect two points in space. The first application of Hypoloids is found in all-wheel-drive vehicles. Because of their unique geometry, Hypoloids can be used to connect the transfer case to the front axle without the use of CV joints, allowing for greater efficiency, reduced NVH characteristics and tighter packaging. The Hypoloid technology does not apply only to automotive drive trains. Its advantages apply also to aircraft as well as general gearbox manufacturing.

Introduction

If two shafts are neither parallel nor perpendicular, but include a small angle in the plane that is defined by the axis of rotation, then two possible gearing solutions are known to accomplish

a motion transmission.

One possible solution is called beveloids. Beveloids are manufactured like cylindrical gears using, for example, a hobbing process for soft manufacturing and a threaded-wheel grinding for

hard finishing. Shaft angles between 0° and 15° can be realized according to the beveloid method—which results, depending on the ratio—in gear pitch angles between 0° and 7.5° . Or, in case of the combination of one conical gear with one conventional cylindrical gear, the maximal required pitch angle might be as high as 15° (Refs. 1–2).

The second possibility is the application of angular spiral bevel gears. The ratio in most real applications is close to miter, which results in pitch angles between 0° and 7.5° .

The described gearsets are generally used in automotive transfer cases to transmit rotation and torque from the output shaft of a transmission to the front axle of an all-wheel-drive vehicle.

The mechanical function of both tapered cylindrical gears (beveloids) and spiral bevel gears is to provide an angle between the shafts in the plane that their two axes define. In most cases concerning all-wheel-drive vehicles, this will still require two constant

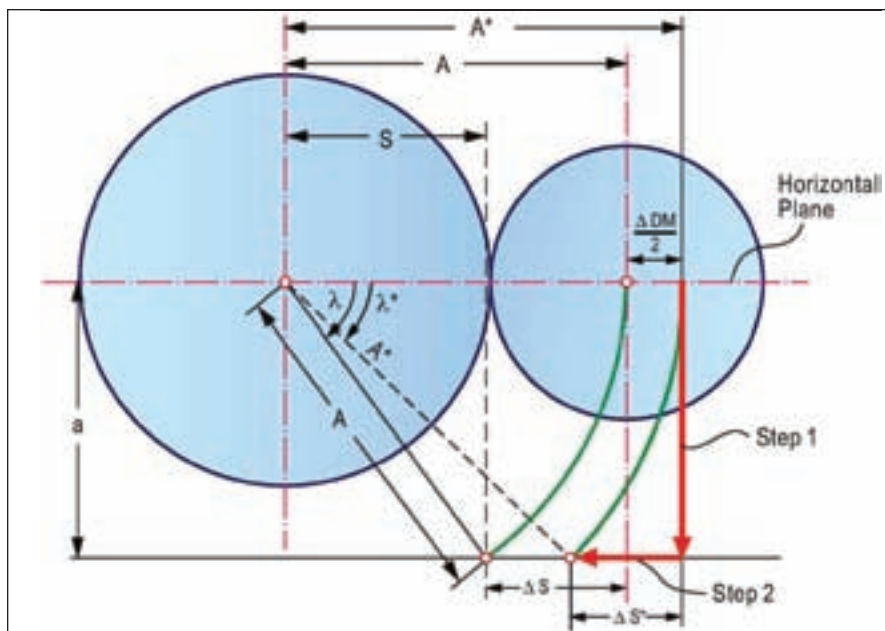


Figure 1—Pinion offset position (see ANSI/AGMA/ISO 23509-A08).

velocity joints or two universal joints (one on each end of the drive shaft) in order to connect the output shaft of the gearbox with the input shaft of the front axle, which commonly have different vertical locations.

In order to connect two points in space, as in the case of a propeller shaft between the output of a transfer case and a front axle input, it is necessary to provide one angle and a linear offset or two angles in perpendicular planes. Hypoid gears represent such a general valid solution of input/output shaft orientation in three-dimensional space. However, the features of today's hypoid gear designs do not cover the case of low shaft angle and high offset. The different hypoid theories applied today do not even allow gear engineers to design low shaft angle gears with any offset. The common hypoid theories rely on a flat or conical generating gear as the basis for basic setting and tool parameter calculation (Ref. 3). Shaft angles close to and including 90° —combined with ratios of 2.5 and higher—lead to gear pitch cone angles of 68° and higher and pinion pitch cone angles of 22° and lower. This leads to a typical ring gear whose cone is close to a plane, with a tangent plane to the pitch cone, which is close enough to the pitch cone in the neighborhood of the contacting line. This allows application of certain amounts of hypoid offset, derived in the pitch cone tangent plane in the traditional hypoid theory. But the traditional theory fails in cases of high hypoid offsets (close or equal to half the ring gear diameter). The traditional hypoid theory also fails in cases of low ratio (close to 1.0). In cases of high ratios, worm gear drives can be used to realize a 90° shaft angle and an offset of half the gear diameter, plus half the worm diameter (like center distance in cylindrical gearing). In the case of low ratios, crossed helical gears can be used to achieve any desired shaft angle combined with an offset equal to the center distance of those crossed helical gears.

The freedom of any small shaft angle (e.g., 0° to 20°) combined with any offset between zero and the sum

of half the mean pitch diameters of the two members will, for the first time, be possible by applying the Hypoloid system.

The Basics of Hypoloids

The generating principle is applied between the driving pinion and the driven gear. Although, in most cases, pinion and gear might have the same number of teeth, the gear is used as a generating gear. The new method even goes one step further and uses a non-generated gear with straight or curved tooth profile as generating gear for the pinion. In the bevel and hypoid gears, the non-generated principle is generally used only in cases when the ring gear pitch cone angle is 68° and higher. The Hypoloid method derives, in a first step, bevel gear machine basic settings for a non-generated gear member. Those settings are used to derive, in a second step, the basic settings for a bevel gear generator in order to manufacture the pinion. The cutter head for the pinion cutting (positioned by the basic settings and rotated around the cradle axis) represents one tooth of the non-generated gear member on which the pinion rolls during the generating roll process.

The principle of applying a non-generated gear member in order to generate the mating pinion is the only technique that delivers a precise conjugate relationship between pinion and gear, even if the axes of the two members are not in one plane. A conjugate basic geometry also requires the pitch cone to be parallel to the root cone. It has been observed that in the case of low shaft angle spiral bevel gear sets, the tooth depth was calculated to be taller at the toe (reverse tooth taper) in order to fulfill the requirements of completing (matching tooth thickness and opposite member slot width). One element of the Hypoloid geometry is a parallel-depth tooth design, which will lead to more optimal tooth proportions than a reverse taper, and also fulfill the requirement of parallelism between pitch line and root line. If the axes of the two members are in two parallel planes, the distance between the planes

is defined as offset. In the case of conical pitch elements of the two members, this offset is commonly called hypoid offset.

One member is defined as a pinion (generated member) and one member is defined as a gear (non-generated member). In spite of the traditional definition, Hypoloid gears and pinions can have a similar number of teeth. It is even possible that the pinion has a higher number of teeth than the gear.

The conjugacy between the two members is only the basis for the generating principle. In order to make the gearset insensitive to tolerances in manufacturing and assembly, a located contact is achieved using flank surface crowning in the direction of the tooth profile, the lead and the path of contact.

If the non-generating process of the gear member is performed with straight cutting blades, then the generation of a pinion tooth will cause additional profile curvature versus an involute (or more precisely defined as spherical involute or octoid). The additional pinion profile curvature can cause undercut in the pinion root area and a pointed top land. To reduce the additional profile curvature in the generated pinion, it is possible to manufacture the non-generated gear teeth with curved blades. If the gear cutter blades are formed like the involute of a similar generated gear, then the pinion tooth profiles will be regular involutes without additional profile curvature, without additional undercut in the root area and with no pointed top land versus a standard profile. It is also possible to approximate the involute function of the gear blade profile with circular or parabolic shape functions. This will achieve a similar effect and reduce the complexity of blade grinding or grinding wheel dressing kinematics.

The adjustment of the offset is done in two steps starting from the spiral bevel non-generated gear design. The axis of the spiral bevel gear set lies in a horizontal plane (Fig. 1). The first step to offset the pinion relative to the gear is to move the pinion axis in a

continued

where

$\delta_{2\text{-hypoid}}$ is pitch angle of hypoid gear set member.

Establishing the gear blank dimensions is explained in a four-step process:

1. The spiral bevel gear version has a pitch angle of $\delta_{2\text{-spiral}}$ and a distance crossing point to pitch cone (ZTKR2) of zero (Fig. 2).
2. In case of a pinion diameter increase ΔDM , the crossing point moves in negative z_2 axis direction and establishes a new coordinate system origin (Fig. 3):

$$ZTKR2^* = \frac{\Delta DM}{2} \quad (10)$$

$$ZTKR2^* = \frac{\Delta DM}{2 \sin \Sigma} \quad (11)$$

where

ZTKR2* is crossing point to pitch apex.

3. The changed pitch angle of the gear for a hypoid pair shifts the location of the pitch apex (while the pitch point P at the center of the face width remains unchanged) as shown in Figure 4. The value of ZTKR2** is negative, because the gear cone apex is now shifted in negative z_2 direction and lies left of the R_2 axis.

$$ZTKR2^{**} = \frac{\Delta DM}{2 \sin \Sigma} + \frac{RPO}{\tan \delta_{2\text{-spiral}}} - \frac{RPO}{\tan \delta_{2\text{-hypoid}}} \quad (12)$$

$$\delta_{2\text{-hypoid}} = \delta_{2\text{-spiral}} \cos \lambda^* \quad (13)$$

4. The horizontal movement (Fig. 1) of ΔS (or ΔS^*) will move the pinion axis in case of positive ΔS values toward the gear axis, and move the crossing point in positive Z -axis direction by $\Delta S / \sin(\Sigma)$, which will establish the final distance crossing point to pitch apex (Fig. 5). The value of ZTKR2*** is negative because the gear cone apex is now shifted in negative z_2 direction and lies left of the R_2 axis.

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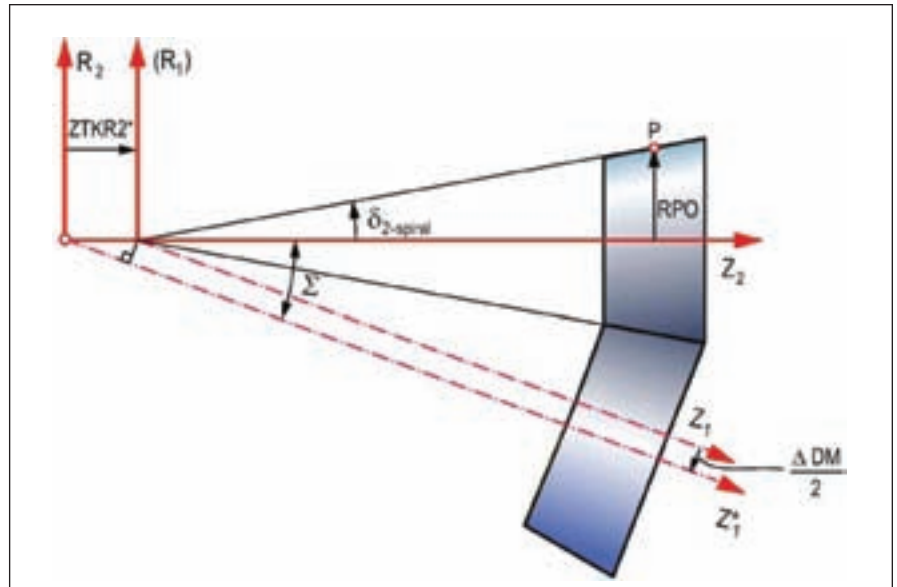


Figure 3—Pinion diameter increase.

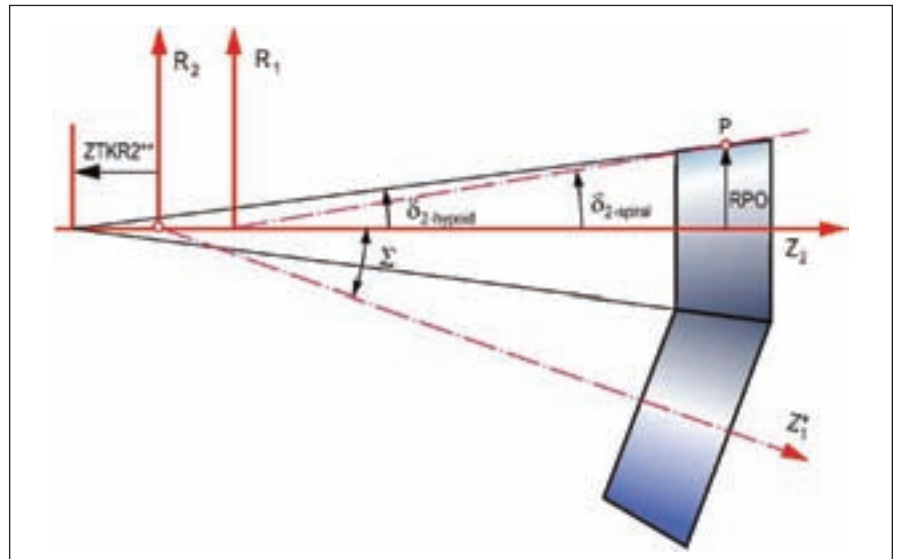


Figure 4—Gear hypoid pitch angle.

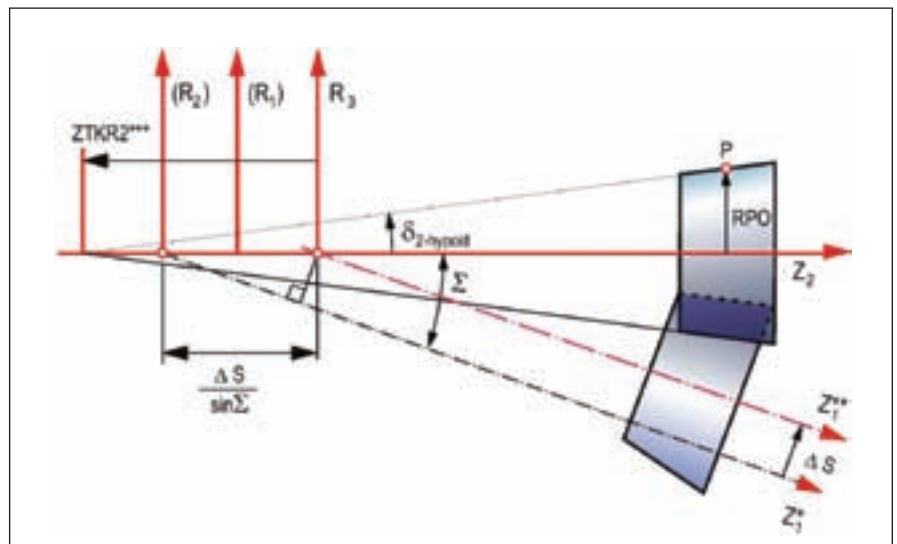


Figure 5—Hypoid crossing point.

is now shifted in negative z_2 direction and lies left of the R_3 axis.

$$ZTKR2 * * * = \frac{\Delta DM}{2 \sin \Sigma} + RPO \left(\frac{l}{\tan \delta_{2\text{-spiral}}} - \frac{l}{\tan \delta_{2\text{-hypoid}}} \right) - \frac{\Delta S}{\sin \Sigma} \quad (14)$$

The pinion cone angles are calculated using the three-dimensional point generating principle between two axes in a general three-dimensional arrangement in space (Fig. 6). This principle is

based on a given gear cone and a given pinion axis. A pinion cone—or, more correctly a hyperboloid—around the pinion axis is calculated such that it eases perfectly onto the gear-enveloping surface.

Basic Setting Calculation for the Non-Generated Gear

A basic machine setup is the method used to define the manufacturing machine setup and kinematics. This will indirectly also define the flank surfaces as well as the tooth root geometry of both members. Figure 7 shows the non-generated gear with its axis Z_3 in

the horizontal basic machine plane $Y_4\text{-}Z_4$. The crossing point on the gear axis is located at the machine center (origin of the coordinate system $Y_4\text{-}Z_4$). The axis Y_4 is the cradle axis. The gear is positioned with the angle $\gamma_{\text{gear-hypoid}}$ such that the pitch line and root line are parallel to the axis Z_4 . The cutter radius vector is adjusted to achieve the desired spiral angle at the center face with mean cone distance. All relevant formulas for non-generated gear basic machine setup can be derived from the graphical representation in Figure 7.

$$\vec{RMW} = \begin{Bmatrix} 0 \\ 0 \\ RM_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ ZTKR2 \sin \delta_{2\text{-hypoid}} \\ ZTKR2 \cos \delta_{2\text{-hypoid}} \end{Bmatrix} \quad (15)$$

$$\vec{RM}_2 = \vec{RMW} + ZTKR2 \quad (16)$$

$$\vec{RW}_2 = \begin{Bmatrix} -RW_2 + \cos \beta_2 \\ 0 \\ RW_2 + \sin \beta_2 \end{Bmatrix} \quad (17)$$

$$\vec{HF}_2 = \begin{Bmatrix} 0 \\ HF2 \\ 0 \end{Bmatrix} \quad (18)$$

$$\vec{EX}_2 = \vec{RM}_2 + \vec{HF}_2 - \vec{RW}_2 \quad (19)$$

Basic Setting Calculation for the Generated Pinion

In order to establish the generated pinion basic machine setup, the triangular Vector RM_2 , RW_2 and EX_2 has to be rotated by $\gamma_{\text{gear-hypoid}}$ about the X_4 -axis. Then the three vectors are rotated by 180° about the Y_4 -axis and in a third rotation about the X_4 -axis by 90° in order to line up the pinion axis with the Y_4 -axis (as shown in Figure 8). The cutter axis vector $Y_{\text{cut}2}$ also has to perform all those rotations and then be reversed in its direction to define the correct cutter or grinding wheel orientation ($Y_{\text{cut}1}$).

$$Y_{\text{cut}1} = (\text{ROT}2)(\text{ROT}0)(\text{ROT}1)(\text{ROT}0) Y_{\text{cut}2} \quad (20)$$

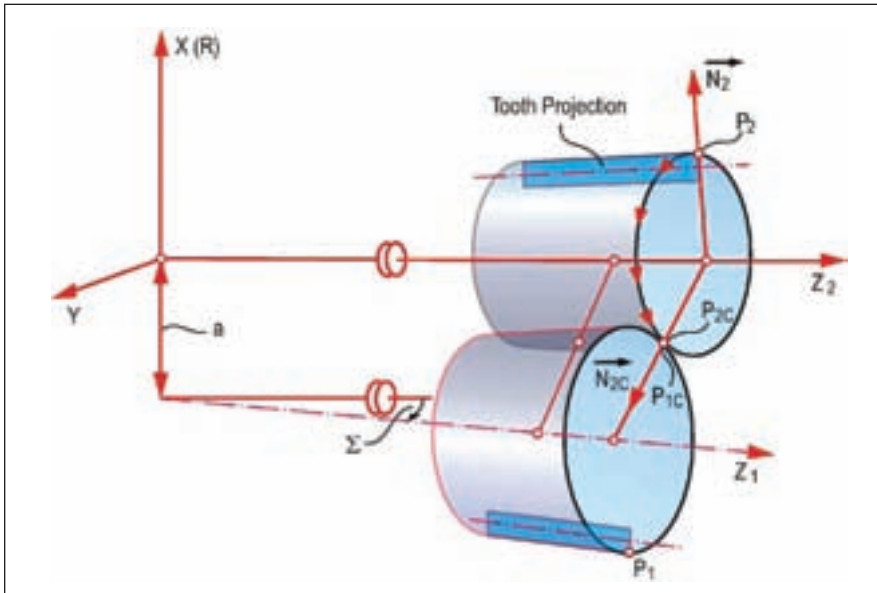


Figure 6—Principal of pinion blank dimension calculation.

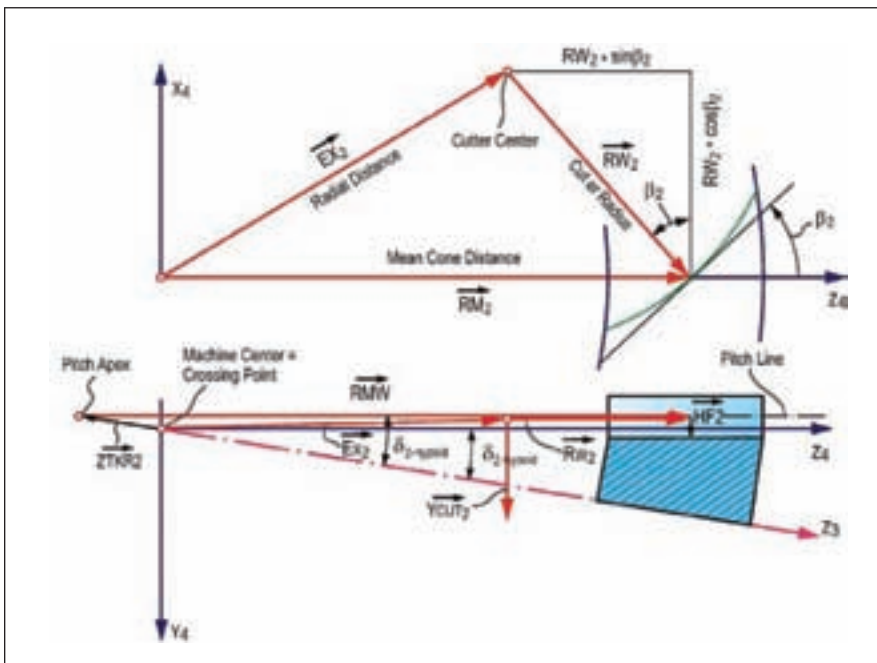


Figure 7—Basic settings for non-generated gears.

$$\vec{EX}_1 = (\text{ROT2})(\text{ROT0})(\text{ROT1}) \cdot \begin{Bmatrix} EX_{2x} \\ EX_{2y} - HF2 + HF1 \\ EX_{2z} \end{Bmatrix} \quad (21)$$

$$\vec{TT}_1 = \begin{Bmatrix} -a \\ 0 \\ 0 \end{Bmatrix} \quad (22)$$

The tool definitions, such as point radius, blade angle, etc., of inside and outside cutting blades for pinion and gear, are calculated using the common rules for bevel and hypoid gear sets manufactured in a completing process. After defining the pressure angles of the gear tool first, the pinion tool pressure angles have to be calculated differently than normally done for bevel and hypoid gears (derived from a flat generating gear).

Non-Generated Pressure Angle Calculation

In order to achieve correct pressure angles between pinion and gear flanks, it had been observed that flank points or surface elements are generated in surface lines while they are passing through the generating plane (or more correctly, the generating surface). The trapezoidal profile of the gear teeth is oriented around the gear root cone (not bent around). The gear tooth profile, not the slot profile, has to be used to define the blade pressure angles for the blades and the basic setting arrangement to duplicate the non-generated gear while generating the pinion slots.

Hypoloid presents a new solution to eliminate the mismatch problem between pinion and gear teeth cut with identical corresponding blade angles. The effect of trapezoidal slots wound around a cylinder or a slim cone requires a correction or adjustment of the pinion blade pressure angle (Fig. 9) of:

$$\Delta\alpha = \frac{-360^\circ}{(2z_2)} \quad (23)$$

where

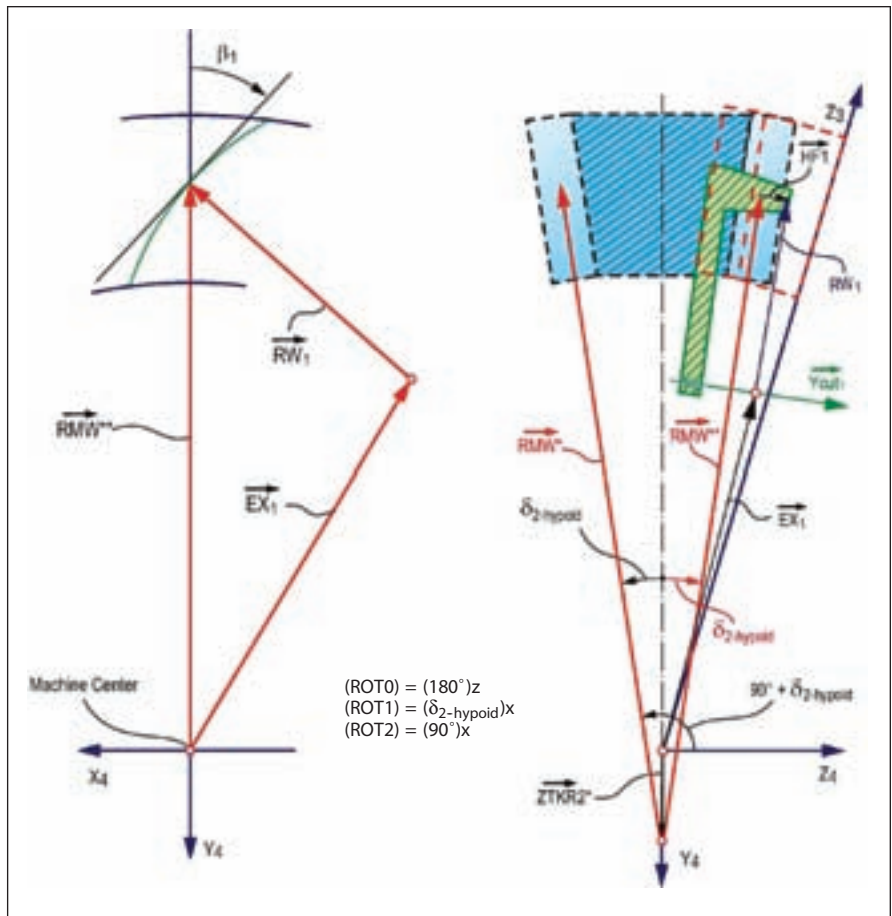
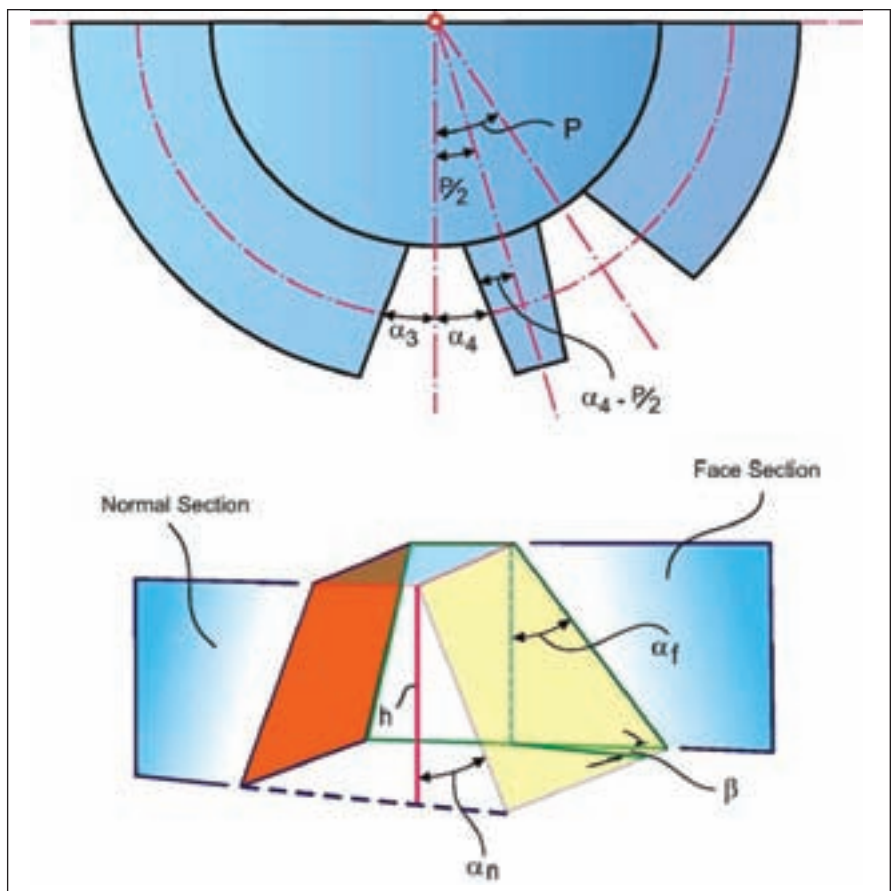


Figure 8—Basic settings for generated pinion.



continued Figure 9—Pressure angle calculation.

$\Delta\alpha$ is change in pinion pressure angle due to non-generated gear member.

Where teeth are oriented under a spiral angle to an axial plane (in addition to the curved shape in longitudinal direction), the angle $\Delta\alpha$ is calculated as:

$$\Delta\alpha_F = \frac{-360^\circ}{(2z_2)} \quad (24)$$

$$\Delta\alpha_N = \arctan \left\{ \tan \left[\arctan \left(\frac{\tan \alpha_N}{\cos \beta} \right) - \frac{360^\circ}{(2z_2)} \right] \cos \beta \right\} \quad (25)$$

where

$\Delta\alpha_F$ is a change in pressure angle in face section;

$\Delta\alpha_N$ is a change in pressure angle in normal section;

β spiral angle at center face width of gear

The choice of completing tools will generate a certain amount of length crowning between the mating pinion and gear flanks. This length crowning might be reduced or increased by a cutter tilt and subsequent blade angle adjustment (as known in the art).

Other elements of crowning, such as profile crowning, flank twist or, in general terms, first, second and higher-order flank modifications, can be applied to Hypoloid gearsets in order to optimize their physical properties. For those modifications, the existing Gleason bevel and hypoid gear correction software can be applied.

Example. The input data for an example Hypoloid development are listed in the Table in Figure 10. The

ratio is 1.056, the shaft angle is 12° and the pinion offset is 50 mm.

Optimal values for tooth depth, profile shift (addendum modification) and pinion diameter increase are found as in a standard bevel gear design calculation in an iteration process. At first, the default values given by the program are used in order to conduct the first-tooth contact analysis and undercut check. Further optimization steps are done to avoid undercut and maximize the active working profile of pinion and gear flanks. The resulting tooth contact analysis is shown in Figure 11.

The left vertical sequence in Figure 11 displays the coast side analysis (pinion concave flanks in mesh with the gear convex flanks). The right vertical sequence in Figure 11 displays the results of the drive side (pinion convex flanks in mesh with the gear concave flanks). The top section of the graphic shows the ease-offs, which represent

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*****
***      INPUT DATASET FOR THE PROGRAM H Y P O L O I D      ***
***      CALCULATION OF BASIC MACHINE SETTINGS FOR HYPOLOID GEARS      ***
*****
*[SAMPLE ]*
*
*****
***      GENERAL DATA BLOCK      ***
*****
I
[ 36.000]      Z1      NUMBER OF TEETH PINION      [-] 1
[ 38.000]      Z2      NUMBER OF TEETH GEAR      [-] 2
[ 10.000]      BETA2    SPIRAL ANGLE OF GEAR      [DEG] 3
[LH]          HOSP     HAND OF SPIRAL PINION (LH OR RH)  [-] 4
[ 20.000]      ALF3    PRESSURE ANGLE I.B.      [DEG] 5
[ 20.000]      ALF4    PRESSURE ANGLE O.B.      [DEG] 6
[ 35.000]      F        GEAR FACE WIDTH      [MM] 7
[ 122.000]     DUMR2   OUTER DIAMETER GEAR      [MM] 8
[ 12.000]      AWI     SHAFT ANGLE      [DEG] 9
[ 50.000]      TTX     OFFSET      [MM] 10
I
*****
***      MISCELLANEOUS DATA BLOCK      ***
*****
I
[ 152.000] 12°      RCP     CUTTER RADIUS NOMINAL VALUE      [MM] 11
[ 0.000]      KZW     NUMBER OF CUTTER STARTS      [-] 12
[ 1.000]      WROW1   BLADE EDGE RADIUS PINION      [MM] 13
[ 1.000]      WROW3   BLADE EDGE RADIUS GEAR      [MM] 14
[ .000]      PCR     PROFILE CROWN (0.0=STRAIGHT EDGE) [MM] 15
[ .000]      LCR     LENGTH CROWNING      [MM] 16
[ 0.075]      SPLF    BACKLASH      [MM] 17
[ -0.600]      X      V0-PROFILE SHIFT +/- BASED ON PIN [-] 18
[ -0.600]      Y      PROP SIDE SHIFT +=>THICKER PIN TOP[-] 19
[ 15.000]      DELDM  INCREASE OF PINION MEAN DIAMETER [MM] 20
[ 1.000]      DPTHF  (1.0->STD, 0.8->STUB, 1.2->HIGH) [-] 21
I
*****

```

Figure 10—Input data for Hypoloid calculation.

length and profile crowning as well as flank twist in the interaction between pinion and gear. The bottom section shows the calculated tooth contact between pinion and gear flanks, drawn within the boundaries of the gear tooth. The center section shows the motion transmission error resulting from the tooth contact and ease-off development. After this development step, the contact size and motion transmission error, as well as the contact position, can be changed without changing the basic parameters of Figure 10. Tooth contact optimization programs identical to those used for regular hypoid gears can be employed.

After geometrical- and strength-related optimizations are finished, the settings for the manufacturing machines (for blade sharpening, cutting and grinding) can be directly retrieved from the basic setting data file. Figure 12 shows the basic settings for pinion

and gear cutting, as they can be directly entered into the control of a Phoenix bevel and hypoid gear cutting or grinding machine.

The finished gearset of this example calculation is shown after heat treatment and tooth grinding in Figure 13. The testing performed with the manufactured sample Hypoloid gear sets confirmed the high degree of insensitivity to deflections of the gear box housing, bearings and gears under load.

Summary

A new development—Hypoloid gearing—addresses the desire of gear manufacturers for more versatility. Hypoloid gearsets can realize shaft angles between zero and 20° and at the same time, allow a second shaft angle (or an offset) in space, which provides the freedom to connect two points in space.

The first application of Hypoloids

is found in all-wheel-drive vehicles that typically use a transfer case with a pinion/idler/gear arrangement or a chain. In those cases, the exit of the transfer case needs to be connected with the front axle. The obstacle here is the fact that the propeller shaft between the transfer case and the front axle will require two CV joints because the front axle input point has a vertical offset and is shifted sideways with respect to the transfer case exit. The tight packaging of modern vehicles requires the possibility to offset the two connecting points of a front propeller shaft. The axle housing in vehicles with independent front axle is in most cases directly mounted to the bottom of the engine, such that no variations in shaft angle is required. However, the penalty for such a design is the cost of two CV joints as well as the reduced efficiency of the front drive train, which could range

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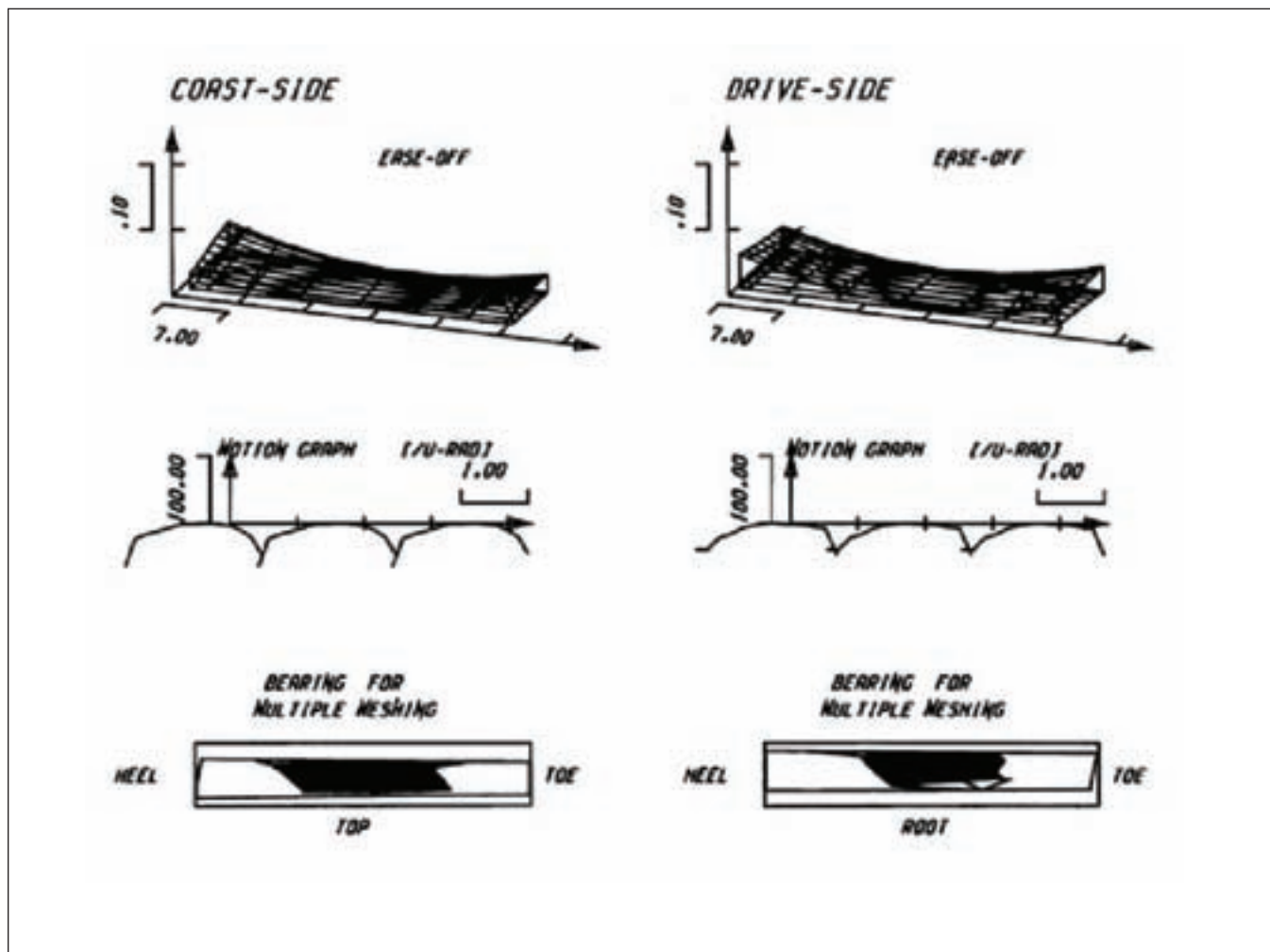


Figure 11—Ease-off, motion transmission and tooth contact analysis.

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 FORMATE HYPOID SUMMARY No. SAMPLE

P I N I O N - Finishing - C U T T I N G S P E C I F I C A T I O N S

26. RADIAL DISTANCE	165.705 mm
27. TILT ANGLE	82.912 deg
28. SWIVEL ANGLE	-21.313 deg
29. WORK OFFSET	49.999 mm
30. MACHINE ROOT ANGLE	-77.809 deg
31. MACHINE CENTER TO CROSSING PT.	1.724 mm
32. SLIDING BASE	477.712 mm
33. RATIO OF ROLL	1.089291
34. CENTER ROLL POSITION	93.067 deg

G E A R - Finishing - C U T T I N G S P E C I F I C A T I O N S

41. VERTICAL	149.691 mm
42. HORIZONTAL	533.180 mm
43. MACHINE CENTER TO CROSS PT.	53.678 mm
44. MACHINE ROOT ANGLE	5.560 deg

Figure 12—Basic settings for Hypoloid pinion and gear cutting.



Figure 13—Ground automotive Hypoloid gear set.

between 0.5% and 2% of the power that flows through the front axle.

Beveloids can realize an angle in one plane, which in most cases is not sufficient to connect the two points in question without the additional requirement of two CV joints. Only the newly developed Hypoloids can connect those two points due to the shaft angle and the additional offset and thereby make the use of CV joints obsolete.

The Hypoloid technology not only reduces cost and increases efficiency, but also has an enhanced performance compared with beveloids with straight teeth. The curved Hypoloid teeth enhance the NVH characteristic and show less contact displacement under load. Flank form generation, tooth contact analysis, ease-off calculation and coordinate measurement with corrective feedback are already possible with today's cutting and grinding machines.

The Hypoloid technology is not restricted to automotive drive trains; all of the mentioned advantages apply also to aircraft as well as general gearbox manufacturing. ⚙

(Editors' note: Hypoloid is a trademark of the Gleason Corporation.)

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Dr. Hermann Stadtfeld received a bachelor's degree in 1978 and in 1982 a master's degree in mechanical engineering at the Technical University in Aachen, Germany. He then worked as a scientist at the Machine Tool Laboratory of the Technical University of Aachen. In 1987, he received his Ph.D. and accepted the position as head of engineering and R&D of the Bevel Gear Machine Tool Division of Oerlikon Buehrle AG in Zurich, Switzerland. In 1992, Dr. Stadtfeld accepted a position as visiting professor at the Rochester Institute of Technology. From 1994 until 2002, he worked for The Gleason Works in Rochester, New York—first as director of R&D and then as vice president of R&D. After an absence from Gleason between 2002 to 2005, when Dr. Stadtfeld established a gear research company in Germany and taught gear technology as a professor at the University of Ilmenau, he returned to the Gleason Corporation, where he holds today the position of vice president-bevel gear technology and R&D. Dr. Stadtfeld has published more than 200 technical papers and eight books on bevel gear technology. He holds more than 40 international patents on gear design and gear process, as well as tools and machines.