Mirror Finishing of Tooth Surfaces Using A Trial Gear Grinder With Cubic-Boron-Nitride Wheel

by
A. Ishibashi, S. Ezoe, S. Tanaka
Saga National University, Japan

Fig. 1 – Gear mirror finished by the authors' grinder

AUTHORS:

DR. AKIRA ISHIBASHI *is a Professor of Mechanical Engineering at Saga National University, Japan. He is also director of the university's Laboratory of Machine Design and Manufacturing Technique. Since 1959, his primary research interest has been in the area of gear manufacturing techniques and load carrying capacities, and he has published the results of his work in the Transactions of the Japan Society of Mechanical Engineers. He is a member of JSME, the Japan Society of Precision Engineering, The Japan Society of Lubrication Engineers, and the Society of Automotive Engineers of Japan, as well as several other professional organizations.*

DR. SHIGETADA TANAKA *is Associate Professor of Mechanical Engineering at Saga National University, Japan. He has been involved in research on the load carrying capacities of gears since 1966 and has developed a new gear grinder with a cubic-boron-nitride wheel. He is a member of the JSME, the Japan Society of Precision Engineering and the Japan Society of Lubrication Engineers.*

MR. SATORU EZOE *has a Bachelor of Engineering degree and is a Research Associate in the Department of Mechanical Engineering at Saga National University, Japan. His work has been mainly in the area of gear manufacturing techniques and load carrying capacities of precision gears. He is a member of the JSME, the Japan Society of Precision Engineering and the Japan Society of Lubrication Engineers.*

Introduction

In conventional gear grinders, grinding wheels with Alundum grains and a hardness of about 2000 HV have been used for finishing steel gears with hardnesses up to about 1000 HV. In this case, the accuracy of the gears ground is greatly affected by wear of the grinding wheel because the difference in hardness is comparatively small when the gears are fully hardened.

It is generally accepted that the wear of material becomes smaller when its hardness is greater. Diamond is the hardest of all materials; however, grinding wheels with diamond grains (HK=6900 −9600)[1] are not suitable for the finishing of steel gears with hardnesses in the range of 100 to 1000 HV. Although they can efficiently finish the tungsten carbide tools with a higher hardness of about 1800 HV. Diamond is more reactive with steel than with tungsten carbide at the high temperatures and pressures which occur in the grinding process. Therefore, chemical reaction and mechanical adhesion are likely to occur at the interface of the cutting edges of diamond grains and the virgin surface of steel, causing larger losses in the diamond wheel.

Grinding wheels with cubic-boron-nitride (CBN) grains have been used to sharpen high speed steel cutters with a hardness of about 850 HV(2) and for finishing hardened steels. The hardness of CBN grains is about 4600 HV,(6) appreciably lower than that of diamond grains. However, wear of wheels with CBN grains is appreciably smaller than wheels with diamond grains in the grinding of steels.

Recently, gear grinders with a CBN wheel have been developed for finishing hardened steel gears, and a remarkable reduction in the wear of the grinding wheel has been achieved,(3,4) as suggested by the results obtained from grinding flat surfaces and circular cylinders.(5,7) However, the tooth surfaces finished by CBN wheels were appreciably rougher than those finished by Alundum wheels. It was believed that CBN wheels never produce tooth surfaces with a peak-to-valley roughness less than 1.0 μm R_{max} ($\pm$ 10μ in. Ra). This has been the most important problem to solve in the development of gear grinders with a CBN wheel.

In contrast to earlier results,(3,7) the authors succeeded in remarkably decreasing the surface roughness of gear teeth and finally achieved mirror-like finishing using a trial gear grinder with a CBN wheel. Fig. 1 shows a mirror finished spur-gear with a module of 5. Crossed lines on the floor are clearly seen on the tooth surfaces of the gear.

In this article, the mechanism of mirror finishing with a CBN wheel will be explained, and then wear characteristics of CBN wheels used in mirror finishing of steel gears will be investigated by changing the speeds of the wheel and work, the depth of cut, the amount of grinding fluid, etc.

Trial Gear Grinder for Mirror-like Finishing

Mechanism of Gear Grinding and the Shape of the Wheel

It is very important to examine the basic mechanism of gear grinding and the shape of the grinding wheel which is most suitable for obtaining very smooth mirror-like tooth surfaces. The most important factor in obtaining the mirror-finished surfaces is the minimization of both the vibration of the grinding wheel and the clearance in the guide ways for the saddle on a work gear being ground. Moreover, the mechanism of the grinder must be simple in order to obtain a high accuracy gear grinder which can bring about accurately finished teeth with a very small surface roughness of about 0.1 μm R_{max}.

After some investigation, it was found that the geometrical shape of the CBN wheels used in the earlier experiments (3,4) are not suitable for the mirror finishing of tooth surfaces. When the disk-type grinding wheel with a trapezoidal section or the forming-type grinding wheel with an involute profile is used, most parts of the tooth surfaces, (i.e., the different parts of a tooth profile) will be finished by different abrasive grains located at the different positions. In this case, the height of a large number of the abrasive grains on the wheel must be in the narrow range, less than 0.2 μm, in order to obtain mirror finished surfaces. This is nearly impossible in practice, and, therefore, mirror-finished tooth surfaces will never be obtained when the disk-type and forming-type grinding wheels are used.

The few protruding grains on the CBN wheel are considered the enemy which prevents production of smoother surfaces. This is because the protruding grains scarcely wear due to the high wear resistance of CBN grains. After some investigation, however, the authors succeeded in effectively utilizing a few protruding CBN grains for mirror finishing of tooth surfaces by using a trial gear grinder designed and made by the authors.

Trial Gear Grinder with CBN wheel

Fig. 2 shows a schematic drawing of the trial gear grinder used in the mirror finishing of spur gears. The wheel spindle was rotated through a flat belt with a width of 40 mm at a rotational speed of about 1800 or 3600 rpm using an induction motor with a maximum output of 1.5 KW. The rolling motion for tooth profile generation was given by two pairs of steel bands and a cylinder with a diameter approximately equal to that of the base circle of the gear being ground.

For guiding the saddle with a work-gear mounting shaft, two cylinders and linear ball guide bushings were used under preloading conditions to avoid even the slightest run out (meandering motion) which might occur during tooth profile generation. The two pairs of angular ball bearings for the CBN wheel spindle shaft were preloaded to about 1000 N using eight coil springs of the same size. Fig. 3 shows the sectional view of the wheel spindle head.

Table 1 shows specifications of grinding wheels used in the present experiments. The Alundum wheels were used for comparison tests. Truing of the CBN wheels was done by a multi-grain type diamond dresser using an attachment shown in Fig. 4. After truing, the CBN wheels were dressed by an abrasive tip made of Alundum grains with vitrified bonds. Grinding was done with and without grinding fluid. Nonsoluble grinding fluid was used and flooded at a rate of 0.05 to 6.0 L/min.

Mechanism of Mirror Finishing by CBN Wheel

As shown in Fig. 2, a dish-type CBN wheel was used in the authors' grinder. Using this grinding wheel, plunge grinding in the direction of the wheel axis was done on a test

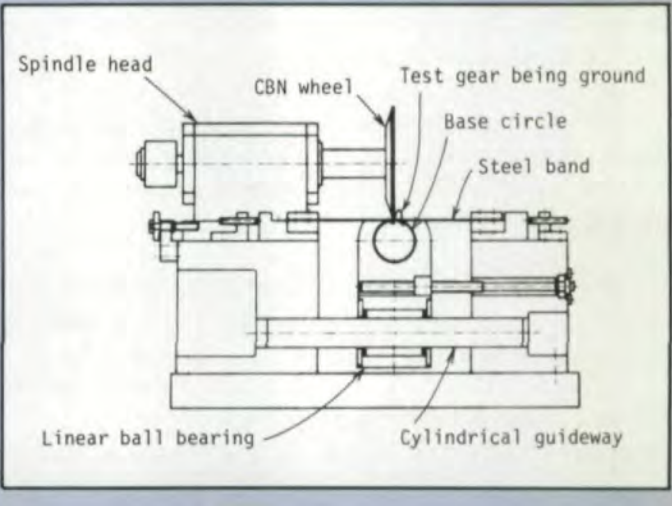


Fig. 2 – Schematic drawing of the grinder used for mirror finishing of tooth surface

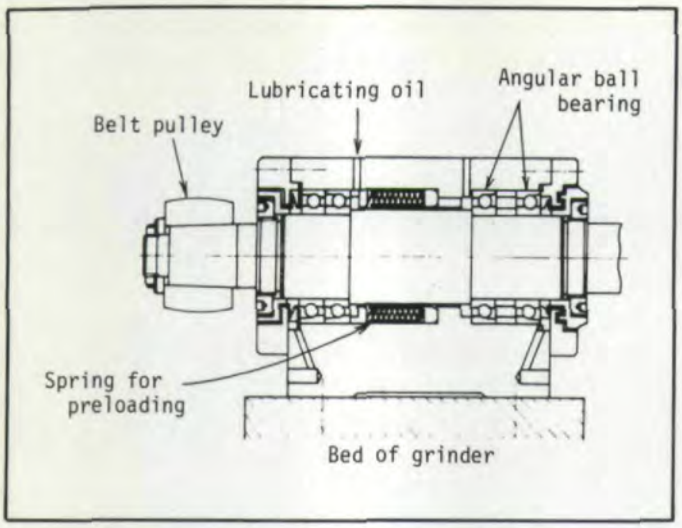


Fig. 3 – Sectional view of wheel spindle head

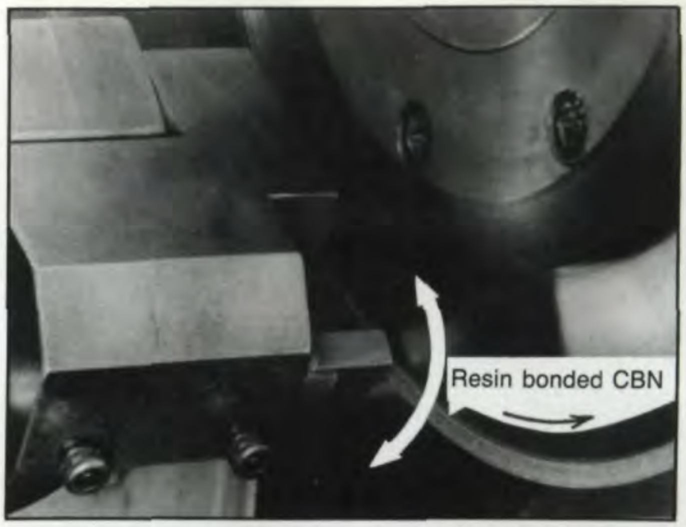


Fig. 4 – Attachment for truing of CBN wheel

TABLE 1 – TYPES AND SPECIFICATIONS OF GRINDING WHEELS

Abrasive grain	Grain size	Grade	Concentration	Bond	Diameter
CBN	50	N	100	B	200 mm
	100	N	100	B	200 mm
	100	N	75	M	200 mm
	200	N	100	B	200 mm
Abrasive grain	Grain size	Grade	Structure	Bond	Diameter
WA	60	K	6	V	200 mm
	220	K	6	V	200 mm

specimen with a width of 5 mm, and then the ground surface was measured by a roughness meter to determine the effects of truing and dressing upon the ground surface. As supposed, it was impossible to obtain surface roughnesses less than 1.0 μm R_{max} even when careful truing was conducted using the multi-grain diamond tip. An example of the roughness of the plunge ground surface is shown in Fig. 5(a). From this figure we see that the surface roughness is about 1.5 μm R_{max} and agrees with the one generally supposed from earlier investigations.

However, surprisingly, the mirror finished surface with a roughness of about 0.1 μm R_{max} is obtained. Fig. 5(b) shows the same wheel used for generating tooth surfaces just after the plunge grinding.

Mirror Finishing by CBN Wheel With No Wear.

A roughness curve obtained from the plunge grinding is shown on the right side of Fig. 6. This curve indicates the envelope of the effective grains which participate in the final finishing of the plunge grinding. One of the valleys in the roughness curve may not be finished by the top of a single grain, but for the sake of simplicity, it is assumed that one valley is produced by one grain. In Fig. 6, Numbers 2′, 1′, 0, 1, 2, and 3 are given to the valleys of the roughness curve to indicate the tops of the effective grains in the plunge grinding.

In the case of generating grinding, only a few grains participate in final finishing and can bring about mirror finished surfaces if the gear grinder is properly designed, accurately made, and if the work feeding speed (rolling speed of base cylinder) is properly selected.

For example, when a CBN wheel which has produced a plunge ground surface with a roughness of about 1.5 μm R_{max} is used for generating tooth profiles of the test gear as shown Table 2, only grains 0, 1, and 2 participate in the final finishing of teeth as shown in Fig. 6. Only the grain "0" participates in the final finishing at the section (A - A), while the grain "1" does at the sections (B - B) and (B′ - B′). The three limited areas finished by the corresponding three grains 0, 1, and 2 are indicated in Fig. 6(a).

Mirror Finishing by CBN Wheel With Wear

When wear of a CBN wheel has occurred after grinding many teeth, the effective surface of the wheel deviates from a straight line, which is schematically shown on the right side of Fig. 7. The maximum wear at the edge of the wheel is 6 μm, and the height of effective grains is 2 μm, and their pitch is 0.1 mm. The calculated results indicate that the effective grains participating in final finishing are three in number as shown in Fig. 7(a).

In the case of the wheel with actual shape of wear pattern, similar results were obtained when the maximum wear, the height of effective grains and the pitch of the grains were nearly equal to those of the schematical shape of wear pattern. This result was supported by experiments in which mirror finished surfaces were obtained using a CBN wheel with wear.

Mirror Finished and Conventionally Finished Gears

Specifications of test gears used in the comparison tests are shown in Table 2. The hardness of test gears used for the following experiments was the same (800 HV) although some gears with different hardnesses were used for obtaining the grinding ratio.

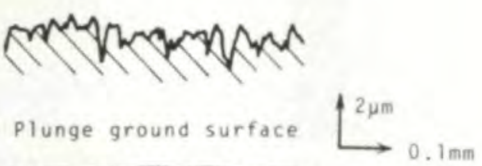
Tooth Surface Roughness

Fig. 8 shows surface roughnesses of test gears finished by three different methods. The roughnesses were measured in the direction of tooth profile using a Talysurf roughness meter. Fig. 8(a) indicates the surface roughnesses of the test gear which was mirror finished by a CBN wheel using the trial gear grinder designed and made by the authors. Fig. 8(b) shows the surface roughnesses of the gear finished by a conventional precision gear grinder with an Alundum wheel. Fig. 8(c) shows the roughnesses of the gear finished on a precision hobbing machine with a carbide skiving-hob. It is evident from Fig. 8 that the surface roughness of the test gear finished by the authors' grinder is extremely small and is about 0.1 μm R_{max} (about 1.0 μ in. Ra).

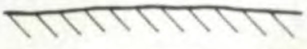
Tooth Profile and Tooth Trace

Surface durability of gears cannot be increased sufficiently by a reduction in roughness of teeth alone. Accuracies of the tooth profile and the tooth trace must be increased at the same time. Figs. 9 and 10 show the tooth profiles and traces of test gears finished by three different methods. From these figures it may be seen that both the tooth profile and the tooth trace of the gear mirror finished by the authors' grinder are the best. This is because the mechanism of the grinder is very simple, resulting in an accurate generating motion.

Fig. 5 – Roughnesses of surface produced by CBN wheel



(a) Plunge ground surface



(b) Generated tooth surface

Conditions for Mirror Finishing

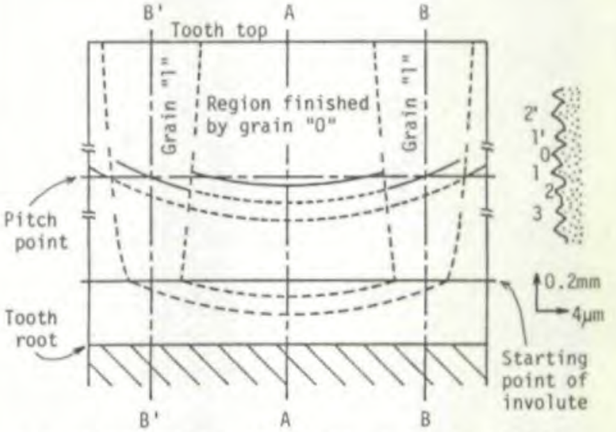
Effects of Grain Size and Wheel Speed

It is generally supposed that grinding wheels with a very small grain size are indispensable for obtaining mirror-finished surfaces. However, this supposition does not apply to the authors' grinder as can be understood from the mechanism

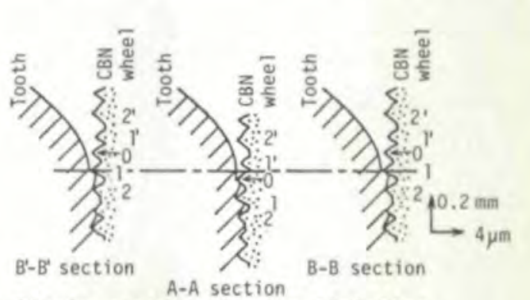
TABLE 2 – SPECIFICATIONS OF TEST GEARS

Module	m	3
Pressure Angle	α	20°
Number of teeth	Z	25
Face width	b	15 mm
Pitch circle dia.	d	75 mm
Outside dia.	d_k	81 mm
Helix angle	β	0°
Hardnesses (Materials)	800 Hv (SCM415) 290 HB (SCM435)	

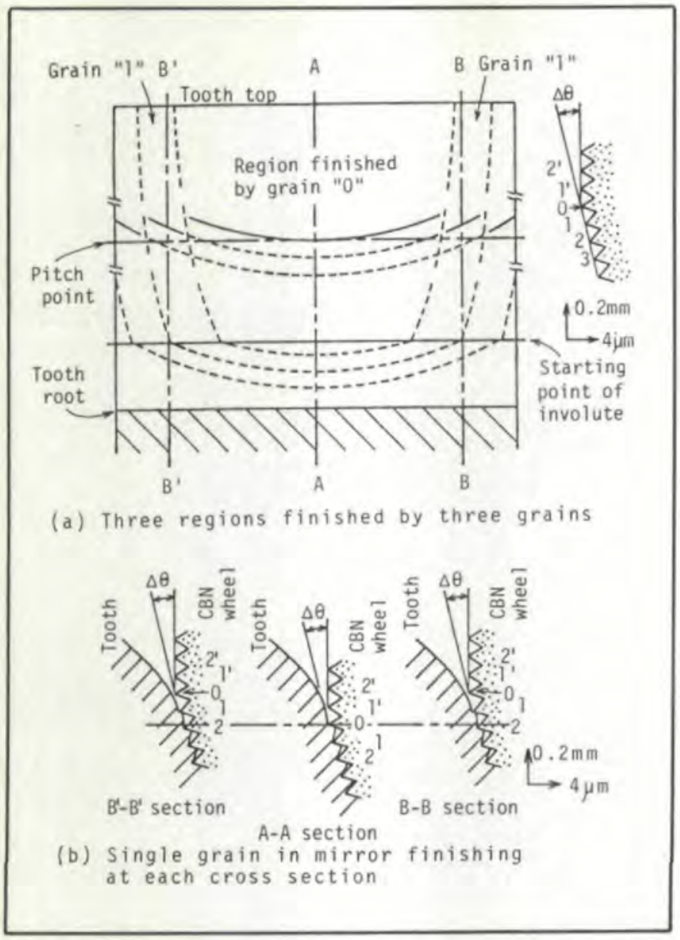
Fig. 6 – Mechanism of mirror finishing by wheel with no wear



(a) Three regions finished by three grains



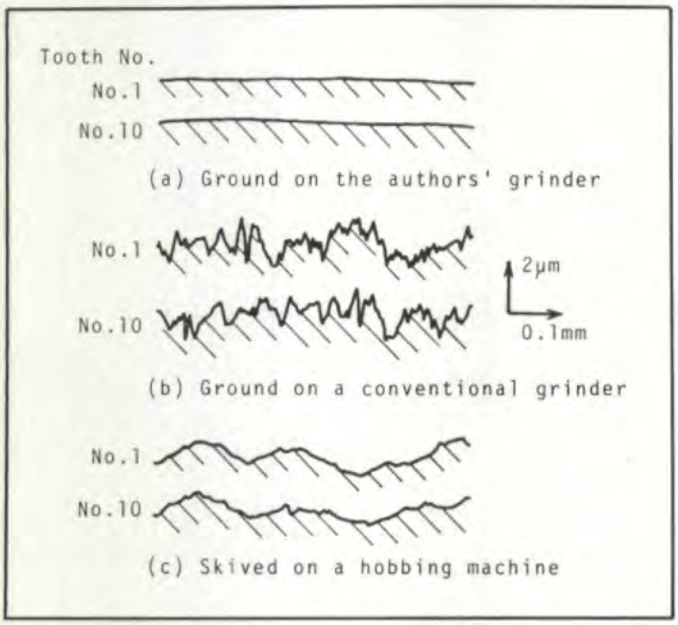
(b) Single grain in mirror finishing at each cross section



(a) Three regions finished by three grains

(b) Single grain in mirror finishing at each cross section

Fig. 7—Mechanism of mirror finishing by wheel with wear



(a) Ground on the authors' grinder

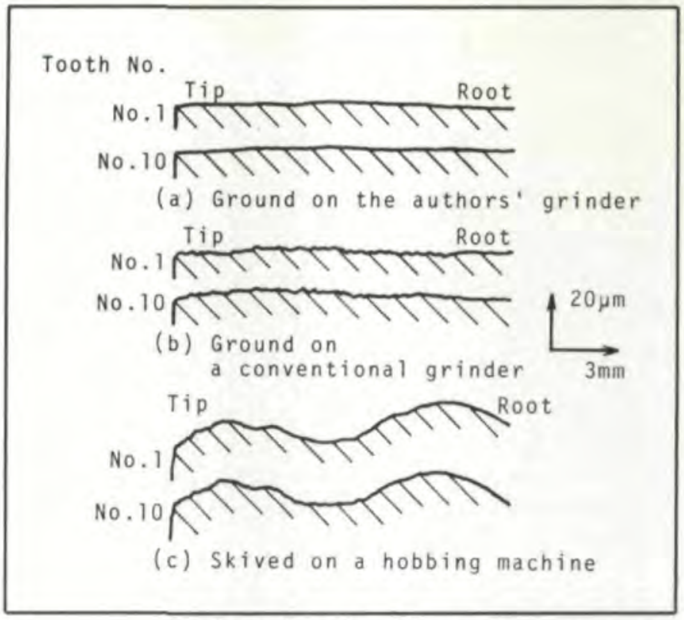
(b) Ground on a conventional grinder

(c) Skived on a hobbing machine

Fig. 8—Roughnesses of tooth surfaces

previously mentioned.

Fig. 11 shows surfaces of two CBN wheels with which mirror finished tooth surfaces were easily obtained at a work rolling speed less than 20 mm/min and a depth of cut less than 30 μm. For clarification purposes, Fig. 12 shows surface roughnesses of teeth finished under grinding conditions outside the best for mirror finishing. When the CBN wheels with grain sizes of #100 and #200 were used, surface roughnesses in the range of 0.2 to 0.4 μm R_{max} were obtained at a work rolling speed of $V_g = 60$ mm/min and a depth of cut $\sigma = 30$ μm. When the CBN wheel with a grain size of #50 was used, mirror finished tooth surfaces were obtained by reducing the work rolling speed to 10 mm/min. However, some waviness was observed at the surfaces, and therefore, the surface quality was not as good as that finished by the wheels with a grain size of #100 or #200.



(a) Ground on the authors' grinder

(b) Ground on a conventional grinder

(c) Skived on a hobbing machine

Fig. 9—Tooth profiles of three gears

Increases in the wheel speed are beneficial for obtaining a smoother surface if the vibration of the wheel can be avoided at higher speeds. See Figs. 12(b) and (b′).

Effects of Depth of Cut

As estimated from the mechanism of mirror finishing, tooth surface roughnesses hardly increase when the depth of cut is increased at a constant work-rolling speed. Fig. 13 shows surface roughnesses of teeth finished at a work rolling speed of $V_{2g} = 20$ mm/min. The surface roughness of teeth finished at a depth of cut of 3 μm was about 0.1 μm R_{max} ($\doteq 1.0$ μ in. Ra). When the depth of cut was increased by a factor of about 30, the surface roughness increased to about 0.5 μm R_{max}.

Effects of Grinding Fluid

Application of grinding fluid is very effective for improving both the surface finish and accuracy of the tooth trace. Fig. 14 shows surface roughnesses and tooth traces of gears finished with and without grinding fluid. In dry grinding, insufficient cleaning of removed chips sticking to the edges of the abrasive grains and insufficient removal of the grinding heat bring about reduction in the surface quality of teeth ground. Thermal expansion of gear teeth is greater at and near the center of the face width. Due to this effect, the tooth traces become appreciably concave when the depth of cut and/or the work rolling speed exceed a certain limit.

Application of a small amount of grinding fluid improves surface roughness appreciably as seen in Fig. 14(b). In order

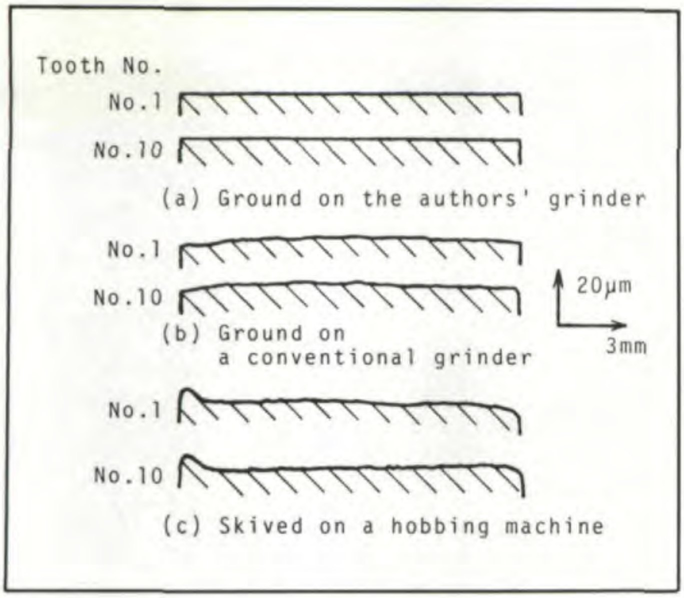


Fig. 10 – Tooth traces of three gears

to prevent the thermal effect which brings about concave tooth traces, a larger amount of grinding fluid must be applied to the tooth being ground.

Wear and Grinding Ratio of CBN Wheels

Using the trial gear grinder, the wear and the grinding ratio (volume of removed metal / worn volume of grinding wheel) of CBN wheels were investigated under different grinding conditions.

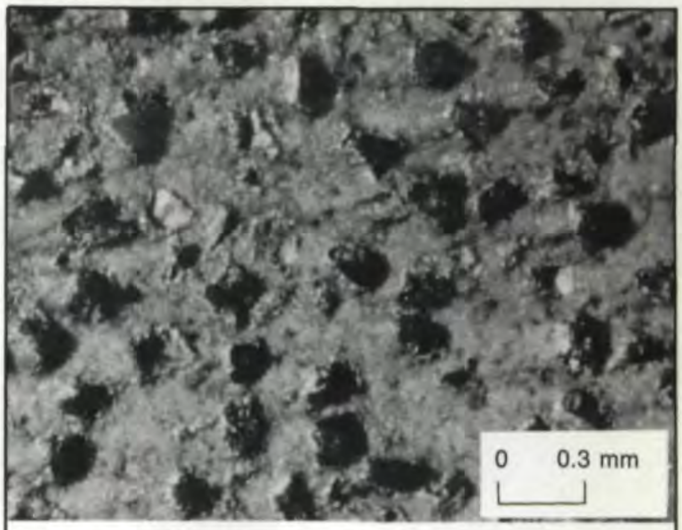
Effects of Grinding Fluid

Fig. 15 shows an example of the grinding ratios obtained in finishing of hardened gears with a hardness of about 800 HV using a resinoid CBN wheel with a grain size of #200. Wheel and work speeds were 1130 m / min and 20 mm / min, respectively. Effects of grinding fluid upon the grinding ratio were very small under these moderate grinding conditions. In some cases, grinding fluid can prevent abnormal wear under severe grinding conditions.

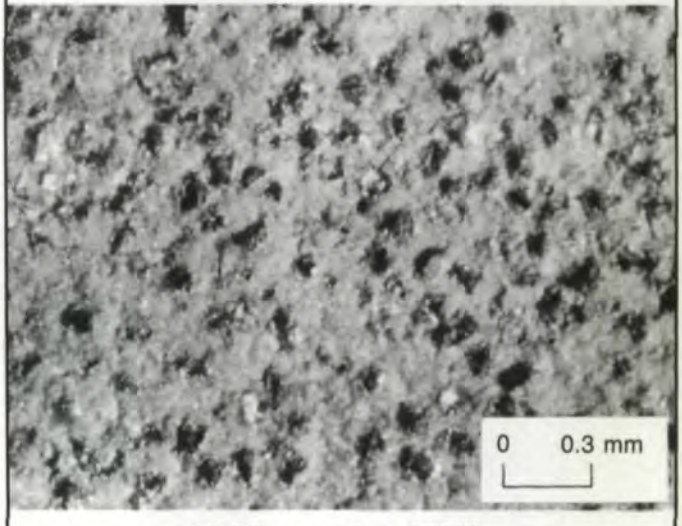
Effects of Wheel Speed and Bonding Material

The effects of the CBN wheel speed upon the wear of the wheel were comparatively small. Fig. 16 shows changes in the worn volume when the resin bonded (resinoid) and metal bonded wheels were used at speeds of 1800 and 3600 rpm. It should be noted that the worn volume of the CBN wheel with metal bonded grains was appreciably smaller than that of the resin bonded wheel. Increase in the wheel speed caused an increase in the wear of the wheel. This is contrary to the general expectation based on the results obtained in the high speed grinding with Alundum wheels.[8]

Fig. 17 shows grinding ratios obtained when grinding hardened gears using the resin bonded and metal bonded wheels with the same grain size (#100). Grinding conditions (V_g = 60 mm / min, σ = 30 μm) were comparatively severe, and therefore, grinding fluid was applied at a flow rate of 6 L / min. When the resinoid wheel was replaced by the metal bonded wheel, the grinding ratio increased by a factor of about 6. However, this replacement of wheels brought about



(a) With grain size of #100



(b) With grain size of #200

Fig. 11 – Surfaces of resinoid CBN wheel

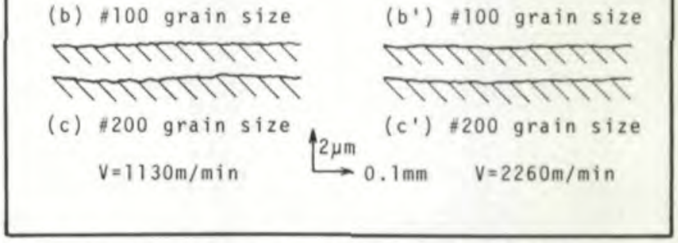


Fig. 12 – Effects of grain size and wheel speed upon roughnesses

(continued on page 22)

an increase in surface roughnesses. Of course, mirror finished tooth surfaces with a roughness of about 0.2 μm R_{max} can be achieved using the metal bonded CBN wheel when the grinding conditions are properly selected.

Effects of Work Speed

The grinding ratio changes with the work rolling speed at the same wheel speed. Fig. 18 shows grinding ratios obtained under different work rolling speeds using a resinoid CBN wheel with a grain size of #200. The grinding ratio increased with the work rolling speed up to a certain limit (about 800 at V_g = 60 mm/min and σ = 30 μm), and thereafter, decreased due to the effect of abnormal wear. The abnormal wear is caused by the detachment of effective grains with strong cutting ability. This detachment is due to excessive force acting at the cutting edges of the grains.

Effects of Work Hardness

It is generally believed that the ground surfaces become a little rougher when the hardness of work materials is made lower. However, mirror finishing of low hardness gears was achieved when the authors' grinder was used. For example, Fig. 19 shows surface roughnesses of a gear with a hardness of 290 HB. Mirror finished surfaces with a roughness of about 0.1 μm were obtained at a comparatively large depth of cut (30 μm) but a low work-rolling speed of 6 mm/min.

Note that the wear of CBN wheels becomes appreciably larger in the grinding of low hardness steel gears. Fig. 20 shows grinding ratios obtained in grinding of a high hardness gear with 800 HV and a lower hardness gear with 290 HB. In grinding the lower hardness gear, the lower grinding ratios were obtained when the work rolling speed was increased from 20 to 60 mm/min. This result is contrary to the one obtained when grinding high hardness gears.

The appreciably lower grinding ratios obtained in the grinding of low hardness steel may be ascribed to the higher plastic deformability which brings about a difficulty in production of grinding chips.

Discussion

Errors Caused by Wear of Grinding Wheel

Wear of CBN wheels is very small under normal grinding conditions, and its effect is negligible in grinding one test gear.

Comparison with Alundum Wheel

In order to compare their cutting ability, two Alundum wheels with much the same grain sizes as those of the CBN wheels were used on the trial gear grinder. Fig. 21 shows changes in the tooth profiles and tooth traces of a hardened gear of up to 150 teeth, ground by an Alundum wheel without re-dressing. The tooth trace error (concavity) was clearly observed after grinding 150 teeth when the ground surface began to burn. In the case of the CBN wheel, a sufficient grinding ability had been retained under the same grinding conditions and no traces of burning were seen on the tooth surfaces even after grinding 150 teeth or more. Accuracies of the gear ground by the CBN wheel were better than those of the gear ground by the Alundum wheel as shown in Fig. 22.

Changes in Hardness Due to Grinding

It is generally accepted that CBN wheels can grind cooler than Alundum wheels and hardly bring about tempering effects at and below ground surfaces. However, it is better to use a grinding fluid when the grinding conditions are comparatively severe. Fig. 23 shows changes in hardnesses of four gears ground with and without grinding fluid. In the case of fully hardened gears, an appreciable decrease in hardness was

Fig. 13 – Effects of depth of cut upon surface roughness

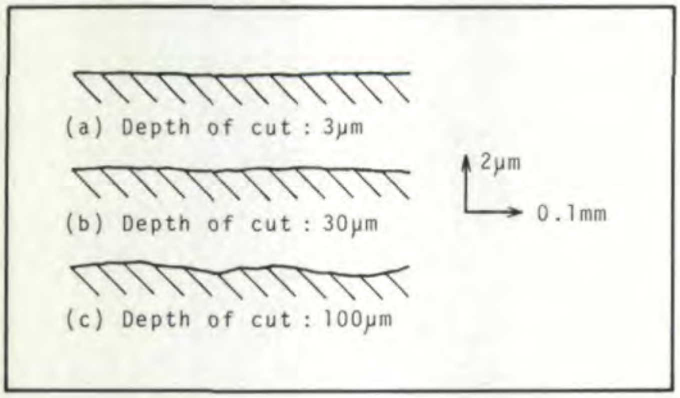


Fig. 14 – Surface roughnesses and tooth traces (Effect of grinding fluid)

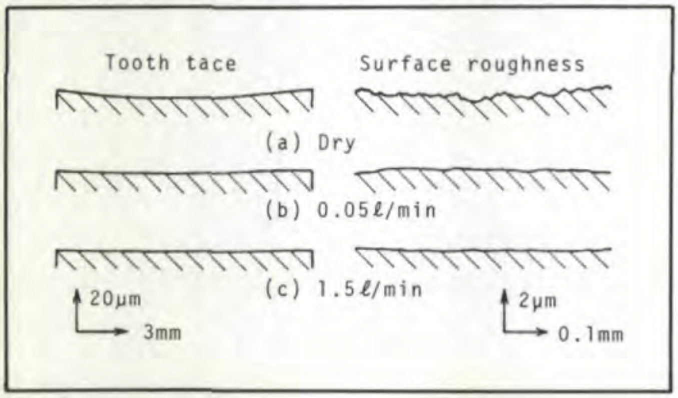
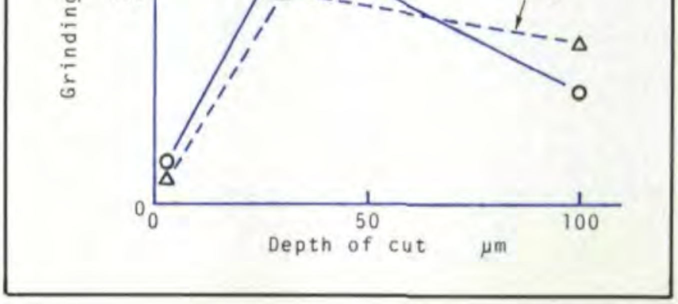


Fig. 15 – Grinding ratio in dry and wet grinding



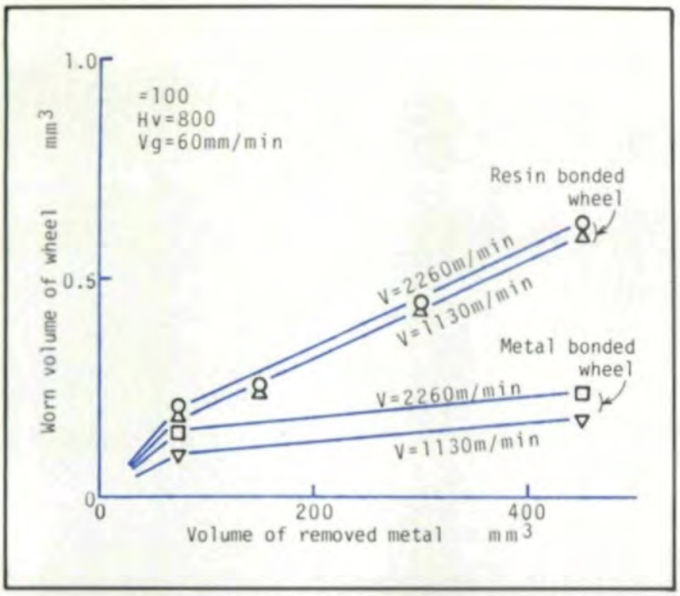


Fig. 16 – Effects of grain bonding material upon wheel wear

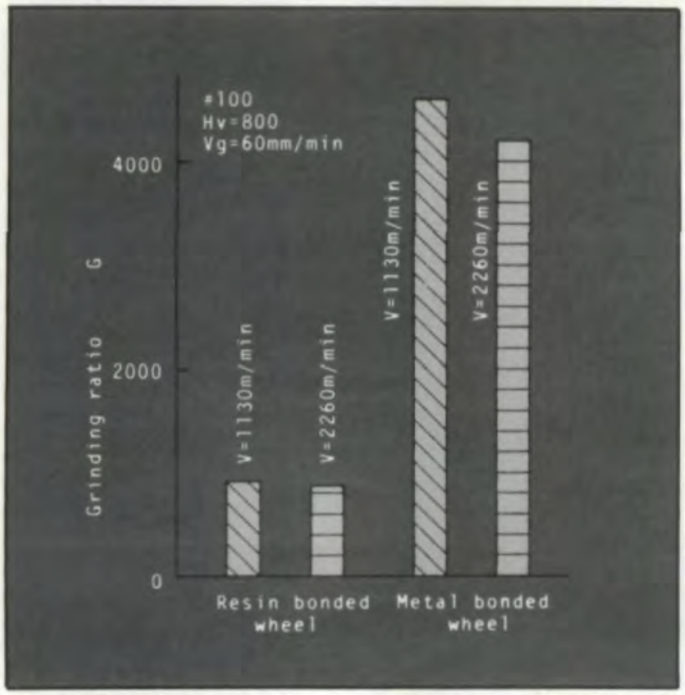


Fig. 17 – Effects of grain bonding material upon grinding ratio

observed in dry grinding, while an increase in hardness was observed in the case of a lower hardness gear with 290 HB. In wet grinding with a non-soluble grinding fluid flooded at a rate of 1.5 L/min, no decrease in the hardness was observed in the fully hardened gear.

Application to Other Grinders

The results shown in this article can be applied effectively to improve a shaving-cutter grinder because its grinding process is almost the same as that in the trial gear grinder.

Mirror finishing of circular-arc tooth-trace cylindrical gears will be possible when a cup-type grinding wheel with an effective surface at and near the edge is used. Note, in this case that the face milling cutters[9] cannot be used for rough and semi-finishing of gears.

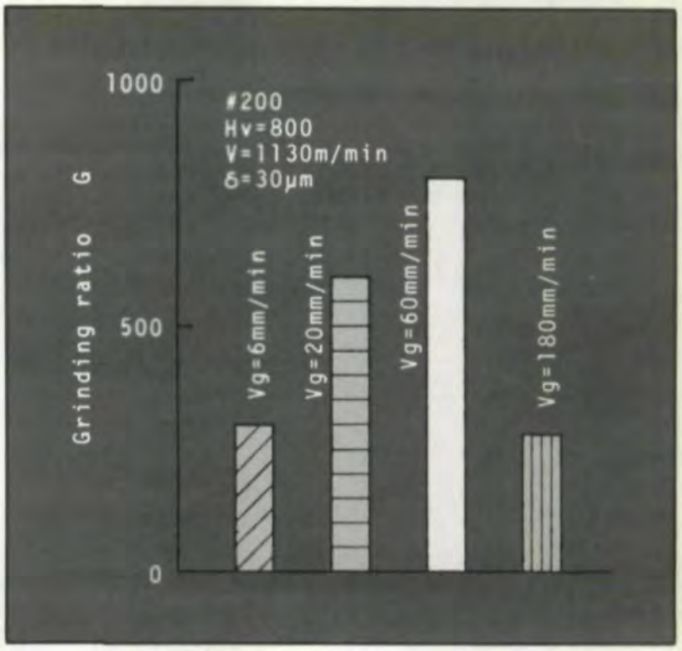


Fig. 18 – Effects of work speed upon grinding ratio

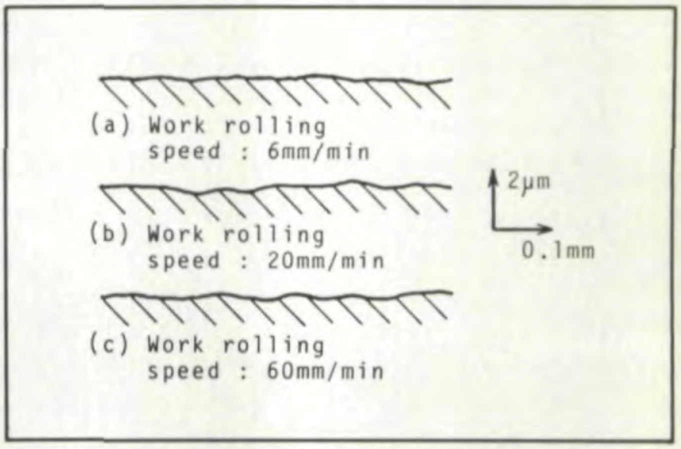


Fig. 19 – Surface roughnesses of lower hardness gear

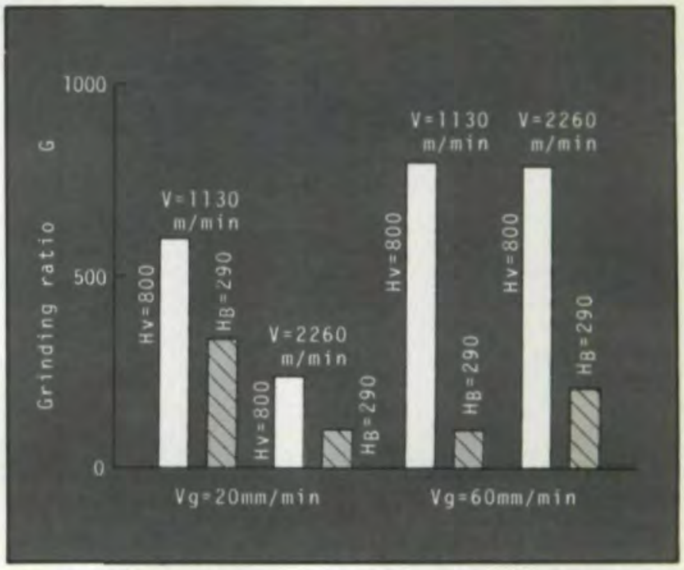


Fig. 20 – Effects of gear hardness upon grinding ratio

(continued on page 26)

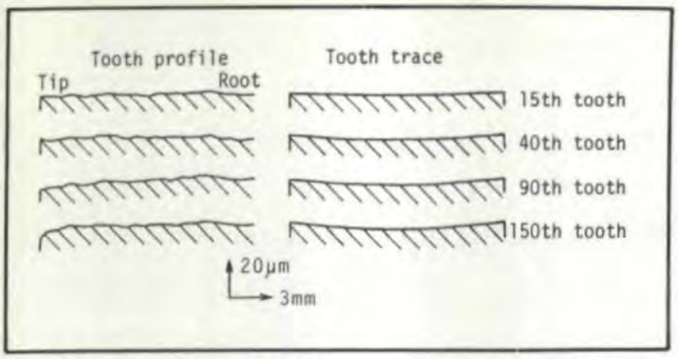


Fig. 21 – Tooth profiles and tooth traces with Alundum wheel V−1240 m/min, V_g − 60 m m/min, σ −30 μm

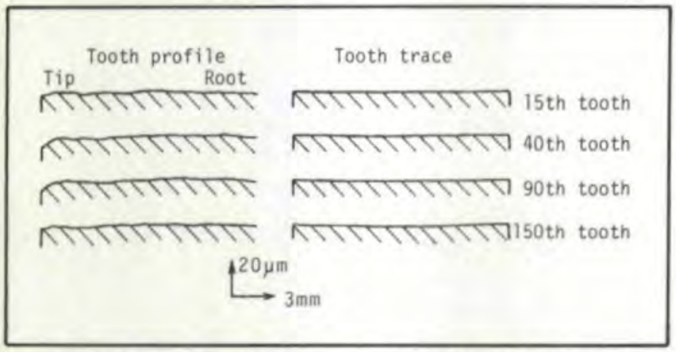


Fig. 22 – Tooth profiles and tooth traces with CBN wheel V − 1130 m/min, V_g − 60 mm/min, σ −30 μm

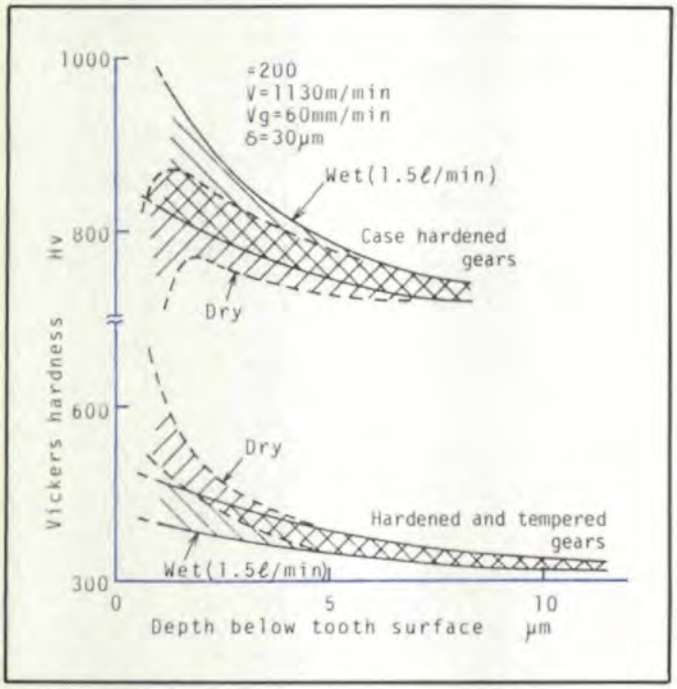


Fig. 23 – Changes in hardness of ground gear

Production Rate in Mirror Finishing of Gears

As described in the mechanism of mirror finishing and the experimental results, mirror finishing of tooth surfaces can be achieved at a lower work rolling speed less than 100 mm/min. However, a comparatively large depth of cut (30-50 μm) is allowable in the mirror finishing when sufficient grinding fluid is applied to the grinding region. Notice that mirror finished tooth surfaces can be obtained without the use of the spark-out grinding process. This saves much time in the final grinding process. Rough grinding is possible, leaving a finishing stock of 10-30 μm, and then mirror finishing can be done by a single pass of the wheel. For example, mirror finishing of the test gears with 25 teeth could be achieved within 20-30 minutes when the trial gear grinder was used. Production rates in mirror finishing of tooth surfaces will be improved in the future. At the present time, it may be estimated that mirror finishing of tooth surfaces may be performed at a production rate acceptable in practice.

Conclusions

Using a gear grinder designed and made by the authors, the following results were obtained:

(1) In contrast to the general expectation that CBN wheels produce rougher surfaces than conventional Alundum wheels, very smooth mirror-like surfaces have been obtained using a CBN wheel (Fig. 1).

(2) The mechanism of mirror finishing with CBN wheels has been clarified and supported by experiments.

(3) Mirror finished tooth surfaces with a roughness of about 0.1 μm R_{max} (≐ 1.0 μ in. R_a) are easily obtained using CBN wheels with grain sizes of #100 and #200.

(4) Mirror finishing of low hardness gears is possible, but the grinding ratio in the mirror finishing becomes appreciably smaller than that in the grinding of fully hardnened gears.

(5) Grinding ratios obtained using a metal-bonded CBN wheel are appreciably greater than those obtained with a resin-bonded CBN wheel,but the tooth surface quality with the metal-bonded wheel is a little lower under the same grinding conditions.

(6) It is better to use grinding fluid for improving the surface finish and also for avoiding the reduction in hardness at and near the ground surface of fully hardened gears.

References

1. FIELD, J.E., "The Properties of Diamond", Academic Press, London, 1979, p. 396.
2. ISHIBASHI, A., and NAKAE, M., "Wear of HSS Hob Ground by Boron-Nitride Wheel and Accuracy and Surface Durability of Gears Hobbed", *Bull. Japan Soc. Pre. Eng.*, Vol. 15, No. 2, June 1981, p. 137-138.
3. KÖNIG, et al., "Teilwälzschleifen von Einsatzgehärteten Zylinderräderen mit CBN-Doppelkegelscheiben", antantriebstechnik, Bd. 20, Nr. 7-8, 1981, s. 314-317.
4. DAIMON, M., et al., "Performance of CBN Wheel; 4th Report", Preprint of Japan Soc. Pre. Eng., March 1983, p. 51-54, (in Japanese).
5. For example, more than a hundred papers are shown in the following paper: Yokokawa, K., "CBN Wheels and Future of Grinder", *Tool Engineer,* Vol. 26, No. 12, Dec. 1982, p. 95-101, (in Japanese).
6. YOKOKAWA, K., "Performance of CBN Wheel; 14th Report," Preprint of Japan Soc. Pre. Eng., Oct. 1983, p. 533-534, (in Japanese).
7. KISHI, K., et al., "Wear Characteristics of Resin Bonded CBN Wheel", Preprint of Japan Soc. Pre. Eng., Oct. 1983, p. 527-528, (in Japanese).

(continued on page 48)

KINEMATIC ANALYSIS OF ROBOTICS . . .

(continued from page 13)

References

1. ALLEN, R. R., "Multiport Models for the Kinematic and Dynamic Analysis of Gear Power Transmission," ASME *Journal of Mechanical Design*, Vol. 101, No. 2, Apr. 1979, pp. 258-267.
2. ANONYMOUS, "Bevel Gears Make Robot's 'Wrist' More Flexible," *Machine Design*, Vol. 54, No. 18, Aug. 12, 1982, p. 55.
3. BUCHSBAUM, F., and FREUDENSTEIN, F., "Synthesis of Kinematic Structure of Geared Kinematic Chains and Other Mechanisms," *J. Mechanisms and Machine Theory*, Vol. 5, 1970, pp. 357-392.
4. DAY, C. P., AKEEL, H. A., and GUTKOWSKI, L. J., "Kinematic Design and Analysis of Coupled Planetary Bevel-Gear Trains," ASME *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 105, No. 3, Sept. 1983, pp. 441-445.
5. DIMENTBERG, F. M., "Determination of the Positions of Spatial Mechanisms," (Russian), *Izdat. Akad. Nauk*, Moscow, 1950.
6. FREUDENSTEIN, F., "An Application of Boolean Algebra to the Motion of Epicyclic Drives," ASME *Journal of Engineering for Industry*, Vol. 93, 1971, pp. 176-182.
7. FREUDENSTEIN, F., and YANG, A. T., "Kinematics and Statics of a Coupled Epicyclic Spur-Gear Train," *J. Mechanisms and Machine Theory*, Vol. 7, 1972, pp. 263-275.
8. MERRITT, H. E., *Gear Trains*, Pitman and Sons, London, 1947.
9. POLDER, J. W., *A Network Theory of Variable Epicyclic Gear Trains*, Eindhoven, Greve Offset, 1969.
10. YANG, A. T., and FREUDENSTEIN, F., "Mechanics of Epicyclic Bevel-Gear Trains," *ASME Journal of Engineering for Industry*, Vol. 95, 1973, pp. 497-502.

The authors are grateful to the General Motors Research Laboratories for the support of this research through a grant to Columbia University.

This article was previously presented at the ASME Design Engineering Technical Conference, October 1984. Paper No. 84-Det-22.

CURVIC COUPLING DESIGN . . .

(continued from page 46)

to keep the clutch teeth in engagement or to move them out of engagement. Higher pressure angles are often used for shift clutches to obtain a proportionately wider space between the toplands of teeth for easy engagement.

The tooth contact of non-generated clutch teeth with positive pressure angle will move very quickly to the edge of the tooth at the heel as the clutch is disengaged under load. To obtain proper tooth contact at all depths of engagement, a generated helical surface should be used. For the great majority of small clutches which shift under load, however, it is entirely satisfactory to design both members with identical convex teeth. When both members are convex, the localized tooth contact remains safely positioned on the surface of the teeth at all depths of engagement thus approximating the action of a helical surface.

Since this localized tooth contact travels from toe to heel as the teeth are disengaged, the amount of this bearing shift should be calculated.

$$\Delta S_L = \frac{h_o}{2} \tan \phi \frac{r_c}{A}$$

where ΔS_L = bearing shift lengthwise on the tooth
h_o = contact depth
ϕ = pressure angle
r_c = cutter radius
A = mean radius of coupling

This calculated amount of bearing shift should be compared with the available face width as follows:

$$\Delta S_L = F - \tfrac{1}{2}\sqrt{\frac{r_c}{1000}}$$

where F = face width

The shift clutch diameter which has been determined in a previous section should be checked according to the formula below. This applies to case-hardened teeth which shift under load and the calculated stress should not exceed 150,000 psi. maximum at operating temperatures.

$$s_c = \frac{0.9T}{AF\ h_o}$$

where s_c = surface stress, psi.
T = torque, lbs. inches
A = mean radius of clutch, inches
F = face width, inches
h_o = contact depth

For clutches which shift under stationary no-load conditions, the surface stress should not exceed 40,000 psi. for case-hardened steel, as given by the following formula:

$$s_c = \frac{T}{AFN\ h_o}$$

The standard tooth proportions given in an earlier section are suggested for initial use in designing shift and overload clutches.

* * *

MIRROR FINISHING OF TOOTH SURFACES . . .

(continued from page 26)

8. OPITZ, H., and GÜHRING, K., "High Speed Grinding", *Annals of CIRP*, Vol. 16, 1968, p. 61-73.
9. ISHIBASHI, A., "The Characteristic of Circular-Arc-Toothed Cylindrical Gears", *Bull. Japan Soc. Mech. Engrs.*, Vol. 9, No. 33, Feb. 1966, p. 200-208.

The authors express their thanks to Emeritus Profs. A. Wakuri and T. Ueno, Kyushu University, for their encouragement. They are also indebted to the staff of the Machine Shop of the Faculty of Science and Engineering, Saga University, for making the gear grinder used in this investigation.

This article was previously presented during the November, 1984 ASME Technical Conference. Paper no. 84-DT-153.