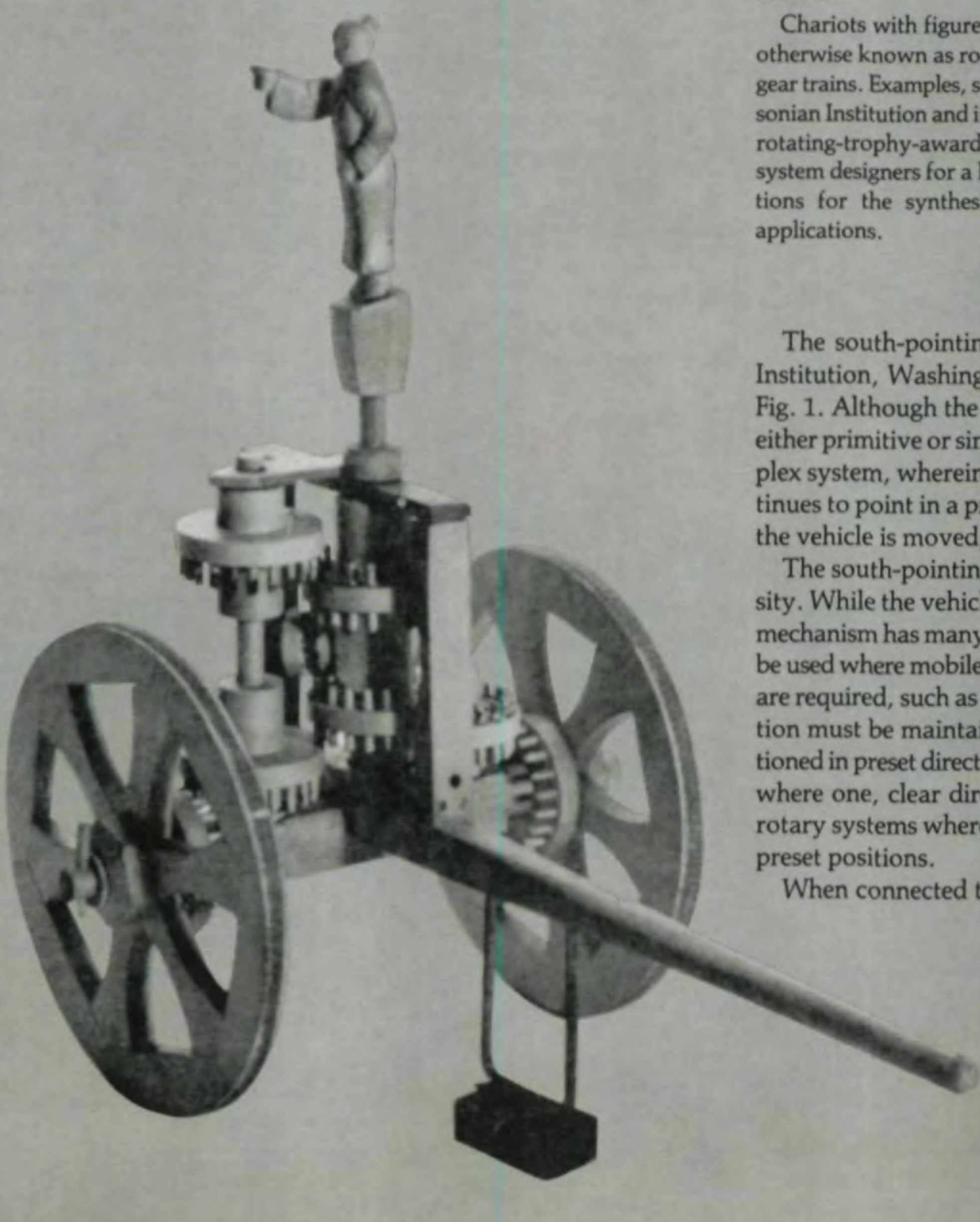
# The Elementary Theory for the Synthesis of Constant Direction Pointing Chariots (or Rotation Neutralizers)

Cemil Bagci,
Tennessee Technological University
Cookeville, TN



**Abstract:**

Chariots with figures pointing in a fixed direction as they move, otherwise known as rotation neutralizers, are built with differential gear trains. Examples, such as the south-pointing chariot at the Smithsonian Institution and its replica, the Ohio State University-pointing-rotating-trophy-award-chariot, have been fascinating mechanical system designers for a long time. This article offers the design equations for the synthesis of such chariots and their engineering applications.

## Introduction

The south-pointing chariot exhibited at the Smithsonian Institution, Washington, D.C., (circa 2600 BC) is shown in Fig. 1. Although the mechanism is ancient, it is by no means either primitive or simplistic. The pin-tooth gears drive a complex system, wherein the monk on the top of the chariot continues to point in a preset direction, no matter what direction the vehicle is moved, without a slip of the wheels.[1]

The south-pointing chariot is more than a historical curiosity. While the vehicle in Fig. 1 is a demonstration model, the mechanism has many practical engineering applications. It can be used where mobile or adjustable reference or tracking planes are required, such as in instrumentation where constant direction must be maintained; in light or signal beams to be positioned in preset directions; in rotary gravitational test chambers where one, clear directional view must be provided; and in rotary systems where reference planes must be maintained in preset positions.

When connected to the base or revolute joints of a robot,

**Fig. 1** – The south pointing chariot exhibited at the Smithsonian Institution, Washington, D.C.

such a system will orient a coordinate reference plane in its originally set position to form a base for output of position sensors. It can move the reference plane with a sine function, retaining it parallel to itself, an application useful for tracing and machining circular paths and cylindrical and toroidal surfaces, welding and the laser machining of shell surfaces by the incorporation of a parallelogram linkage loop as illustrated in Fig. 9.

### Design Equations

The constant-direction-pointing chariot is driven by a planetary gear train differential whose train value is $(-1)$.[2] Fig. 2 shows a bevel gear planetary gear train differential having the train value of $(-1)$. It is used in the rear end differentials of vehicles. Figs. 3 and 4 show two other planetary gear differentials generating the train value of $(-1)$, but using spur gears. In these systems, let n designate the speed of a shaft and $n_a$ the speed of the arm. A train value of $(-1)$ means that when the planet arm is held stationary, $n_a = 0$, and when the first gear (gear 2) is rotated one revolution in one direction, $n_2 = 1$, the last gear (gears 5, 6 and 8 in Figs. 2, 3 and 4, respectively), rotates in the opposite direction one revolution, $n_5 \equiv n_6 \equiv n_8 = -1$ in these figures.

The equations of motion for a planetary gear train in general form are[2]

$$e = \frac{n_L - n_a}{n_F - n_a} \tag{1}$$

e being the train value defined by

$$e = (-1)^q \frac{\Sigma\, \mathrm{PDVER}}{\Sigma\, \mathrm{PDVEN}} \tag{2}$$

where Σ PDVER is the product of all the driver gear tooth numbers starting with the first gear considered, and Σ PDVEN is the product of all the driven gear tooth numbers, including the last gear. They are formed by keeping the planet arm stationary. The number of external contacts of the gears in the train is q; $n_F$, $n_a$ and $n_L$ are the speeds of the first gear, planet arm and the last gear in the train. See detailed applications of Equations 1 and 2 for the analysis and synthesis of simple and compound planetary gear trains and automatic vehicle transmissions in Reference 2. Let $N_i$ be the tooth number of the ith gear. In Fig. 2 tooth numbers of gears satisfy $N_2 = N_5$; $N_4$ is of any practical number greater than 18 for efficient operation, and

$$e = -\frac{N_2 N_4}{N_4 N_5} = -1 \tag{3}$$

**AUTHOR:**

**DR. CEMIL BAGCI** *is professor of mechanical engineering at Tennessee Technological University, Cookville, TN, where he teaches courses and conducts research in machine design, kinematics, and robotics. Dr. Bagci studied at Technical Teachers College, Anakara, Turkey, and received his undergraduate and graduate degrees from Oklahoma State University, Stillwater, OK. He is author of over 150 technical papers and the recipient of numerous research awards. He is a member of AGMA, ASME and several other technical societies.*

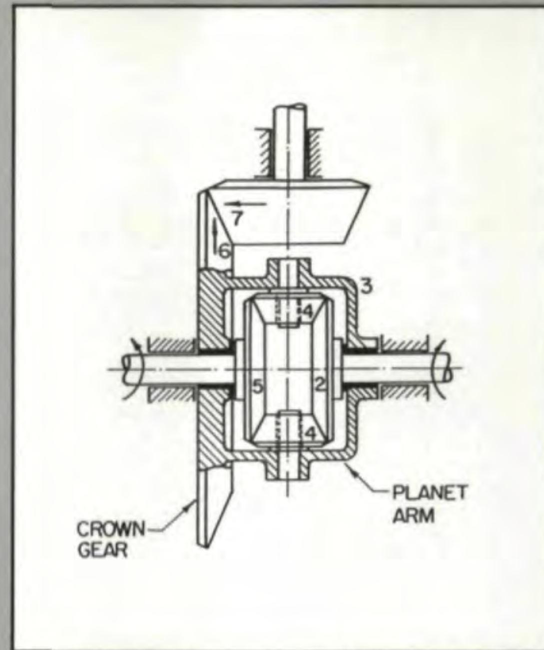



Fig. 2 – Bevel gear planetary gear train differential of train value $(-1)$.

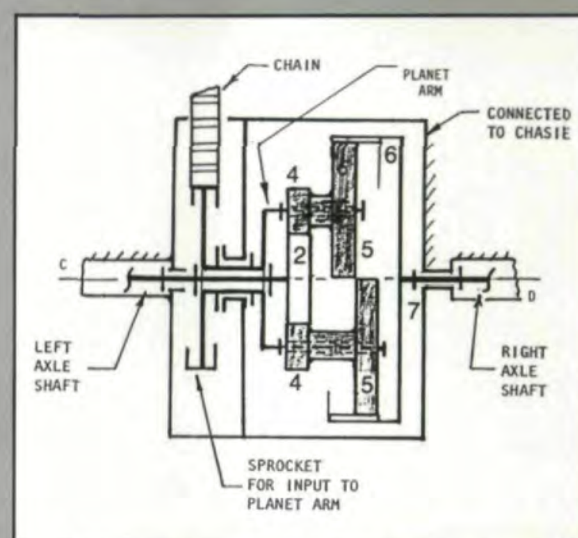



Fig. 3 – Spur gear planetary train differential with an internal gear and train value of $(-1)$.

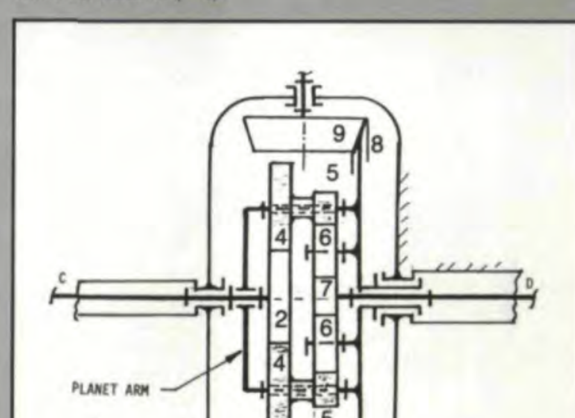



Fig. 4 – Spur gear planetary gear train differential with all external gears and train value of $(-1)$.

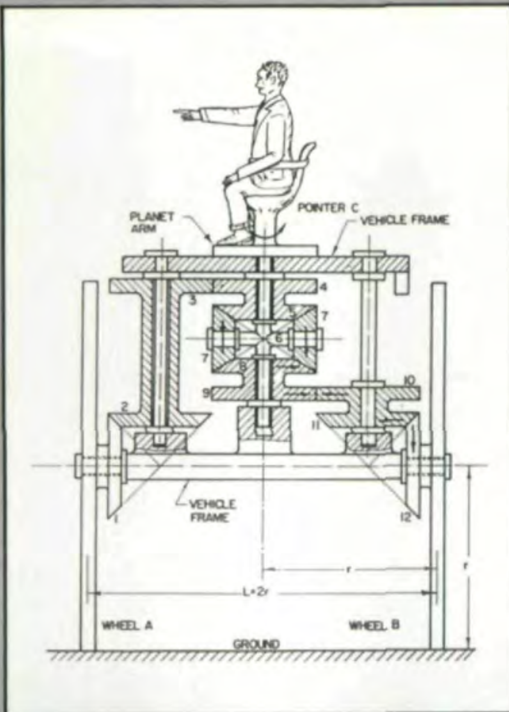



Fig. 5 – The form of the constant direction pointing chariot.

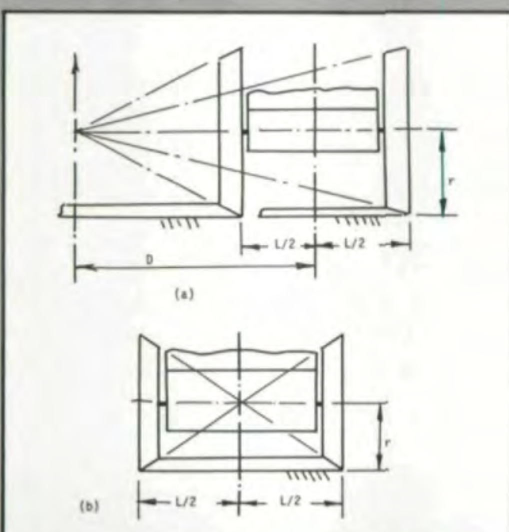



Fig. 6 – Wheels rolling on crown gear platforms.

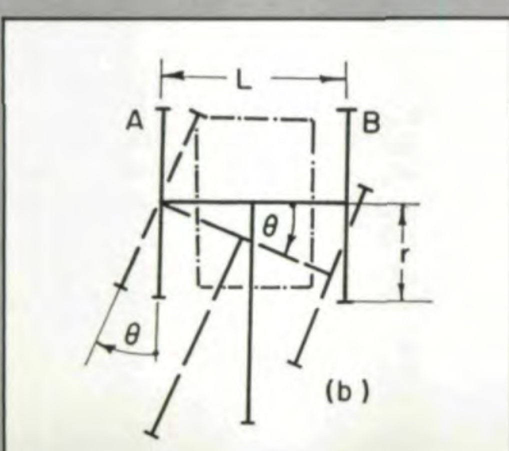



Fig. 7 – Chariot rotated through $\theta$ about the point of contact of wheel A.

In Fig. 3, for example, with tooth numbers $N_2 = 40$, $N_4 = 20$, $N_5 = 60$ and $N_6 = 120$,

$$e = \left(-\frac{N_2}{N_4}\right) \left(\frac{N_5}{N_6}\right) = -1 \tag{4}$$

In Fig. 4 with $N_2 = N_4 = 60$, $N_5 = N_7 = 20$, and $N_6 = 40$

$$e = \left(-\frac{N_2}{N_4}\right) \left(-\frac{N_5}{N_6}\right) \left(-\frac{N_6}{N_7}\right) = -1 \tag{5}$$

Fig. 5 shows the skeleton of the constant direction pointing chariot, where the bevel gear differential of Fig. 2 is used. Bevel gears 8, 7 and 5 correspond to the bevel gears 2, 4 and 5 in Fig. 2. $N_3 = N_4 = N_9 = N_{10}$ for the spur gears, and $N_1 = N_2 = N_{11} = N_{12}$ for the bevel gears that transform the motions of the wheels to the first and last gears of the bevel gear differential gear train of gears, 5, 7 and 8. In precision machines and instrumentation systems for low torque applications, when the chariot is moving on a platform, wheels at A and B may have point contact on the platform, and they are compressed against the platform surface to cause proper level of traction as seen in Fig. 8. For large torque applications, when the chariot rotates about a fixed vertical axis, wheels are bevel gears rolling on a stationary crown gear to eliminate slip as shown in Figs. 6 a and b.

The function of the two-wheel, 12-gear mechanism is to keep the pointer or the planet arm of the differential gear train stationary with respect to the platform (ground) regardless of the direction in which the vehicle travels. The constant direction pointer is connected to the planet arm with an adjustable coupling, permitting repositioning of the pointer as desired. Let the radii of the wheels be r. They are positioned from the center of the chariot at equal distance L/2. Consider the rotation of the chariot through angle $\theta$ about the vertical axis passing through the contact point of wheel A. (See Fig. 7.) Gears 1, 2, 3, 4 and 5 remain stationary, and $n_5 = 0$. The vehicle frame has rotated through $\theta$ (with the pointer if wheel B did not rotate), but wheel B rolls on a circle of radius L causing gear 12 to rotate through

$$n_{12} = \frac{L\theta}{r} \tag{6}$$

This rotation causes gear 8 to rotate in relation to the vehicle frame through

$$n_8 = -\frac{L\theta}{r} \tag{7}$$

Applying Equation 1 using $n_L = n_5 = 0$, and $n_F = n_8$ we have

$$-1 = \frac{0 - n_a}{-\dfrac{L\theta}{r} - n_a} \tag{8}$$

and

$$2n_a = -\frac{L}{r}\,\theta \tag{9}$$

Since the planet arm must rotate through $(-\theta)$ about the vertical axis to maintain the fixed position of the pointer, Equation 9 suggests that

$$r = L/2 \tag{10}$$

must be satisfied in the design of the chariot. Then,

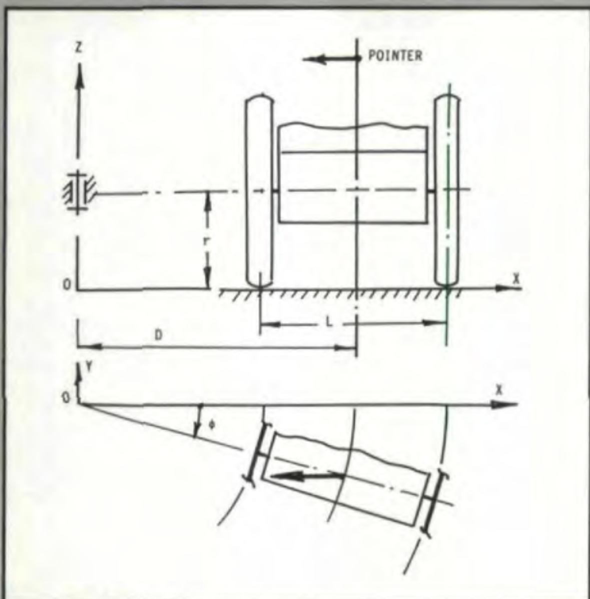



Fig. 8 – Chariot rotated through $\phi$ about a pole axis OZ.



$$n_a = -\theta \tag{11}$$

neutralizing the pointer rotation.

Let us observe more general motion of the chariot. Let it be rotated through $\phi$ about a vertical pole axis OZ at some distance D from the center of the chariot. (See Fig. 8.) The wheel A experiences rotation

$$n_1 = n_5 = \frac{D - \frac{L}{2}}{r}\,\phi \tag{12}$$

On the other hand, wheel B experiences rotation

$$n_{12} = -n_8 = \frac{D + \frac{L}{2}}{r}\,\phi \tag{13}$$

Substituting $n_5$ and $n_8$ into Equation 1 we again find

$$-1 = \frac{\frac{D - \frac{L}{2}}{r}\,\phi - n_a}{-\frac{D + \frac{L}{2}}{r}\,\phi - n_a} \tag{14}$$

and

$$2n_a = -\frac{L}{r}\,\phi \tag{15}$$

with

$$r = L/2 \tag{10}$$

$$n_a = -\phi \tag{16}$$

Therefore, the pointer is undisturbed wherever the chariot goes without the slip of wheels.

In addition to positioning planes or directing light or signal beams in preset directions, the chariot can be used with a parallelogram linkage loop and extended links to function as a tracer of exact circles of both large and small radii, where the use of wormgear or ordinary gear drives may be considered very costly. In that form, it can carry cutter to machine, grind, cut with a laser beam or weld inner surfaces of shells and large bearings. (See Fig. 9, where 0 is the center of the planet arm, OE is the constant direction pointer, EP is the task performing link such as an end-effector link of a robot, OG=EF and GF=OE.) In this form EP is normal to the shell surface, and it can be extended to the desired size of the machined surface, (D+EP) being its radius. Tools are mounted on vertical extensions to retain the chariot outside the shell surface. The platform (or crown gear) on which the wheels rotate is moved in the vertical direction for machining or task feed. If full rotation of the chariot about the vertical axis is required, a second parallelogram loop (OHIE) is added with $60° \le \beta \le 120°$ so that each loop moves the other parallelogram loop from its dead center position. Forming EP'

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to give presidential access to the national treasury to this man? One has the nagging feeling, on studying Mr. Bentsen's record, that he could put himself first, Texas second and the country third — not exactly the priorities we'd like to see in a potential president.

Mr. Bush's running mate, Dan Quayle, seems much more a liability than an asset to the campaign and has a long way to go to achieve presidential stature. It's not his youth that's a problem; both Theodore Roosevelt and John Kennedy were within a year of his age when they assumed the presidency. It's the sense that he is untried, unaccomplished, inexperienced and, yes, immature.

Mr. Quayle is a charming and attractive man, apparently one of Nature's darlings. Thanks to a cushion of family wealth and influence, he has coasted easily to the right places at the right times to get into college and law school; to get the good jobs and make the right acquaintances; and to have the best chances without having to have the backup credentials demanded of others. He has also benefitted from the generosity of lobbyists, being ranked 15th in the Senate in the amount of money earned from honorariums for speeches, articles, travel expenses and lobbyists' golf outings. This puts him well above the amount earned by numerous more experienced, better known politicians — including his running mate and his two opponents, who take no such fees at all. For a man to whom much in life has come easily, the remark that he should not be judged harshly for a decision made when he was young and under pressure is both disturbing and revealing. He needs to be reminded of the sign on the President's desk that says, "The buck stops here."

Fairness demands that we remind ourselves that an unpromising candidate does not necessarily make a bad president. Popular wisdom in 1860 was that the man from Illinois was a rube and an amateur, a good choice for the bosses because he could be easily manipulated. Abraham Lincoln proved popular wisdom wrong, and he is not the only man of less-than-obvious presidential stature to grow into the job. Maybe the country will be as fortunate again.

But the disturbing question remains, why is it that in the last twenty years, our national search for presidential timber too often seems to yield nothing but twigs? The people with real character, leadership ability and vision for the future don't seem to want to run. Those who do want to run seem less than the best.

I don't know what the answer is. Maybe there isn't one— or at least not a simple one. I do know that it is demoralizing for individual citizens and bad for a country to have election after election where the best candidate is "None-of-the-Above."

Michael Goldstein
Editor/Publisher

on the reverse side and mounting cutters on vertical extensions, the chariot-linkage system performs operations on the outside surfaces of shells, wheels and shafts. If the platform is tilted like a swash plate about a horizontal axis, JK for example, P can machine a toroidal surface.

The chariot rotating about a pole as in Figs 8 and 9 can be used in rotating roller coasters in which the passengers always look in one desired direction.

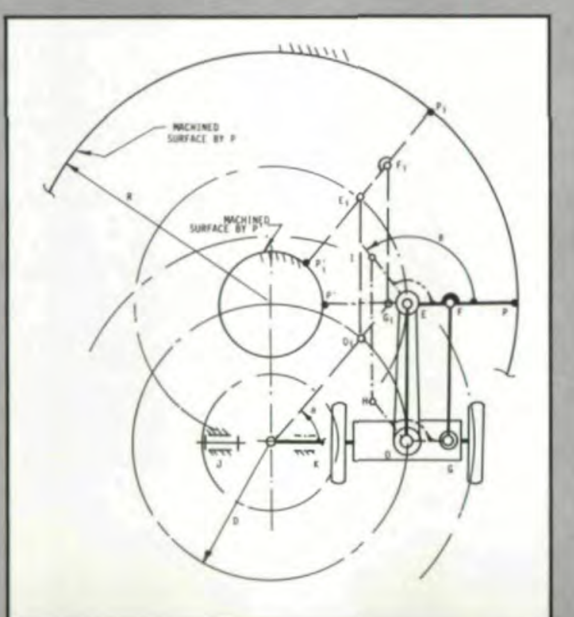


Fig. 9 – Chariot with parallelogram linkage loops performing tasks on cylindrical and toroidal surfaces.

### Conclusions

In the foregoing discussion, the simple equations of motion for the ever-puzzling constant-direction-pointing chariots are given. They can easily be designed for machining, robotics, roller coaster and instrumentation applications. The parameters that must be observed are as follows: $r = L/2$, $N_3 = N_4 = N_9 = N_{10}$, $N_1 = N_2 = N_{11} = N_{12}$. The differential gear train driven with gears 4 and 9 has train value of $(-1)$. Although the bevel gear planetary gear train differential is shown in Fig. 5, one can replace it with the spur gear planetary gear train differentials shown in Figs. 3 and 4, where gears 4 and 9 in Fig. 5 are connected to the C and D shafts of the first and last gears of the spur gear differentials. Chain and crown gear driven planet arms are connected to the pointer. Chariots with parallelogram linkage loops and tilting platforms offer precision task performing systems on cylindrical and toroidal inner and outer surfaces. Many other industrial applications of the constant direction pointing chariots are limited only by the ingenuity of the designer.

**References**


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