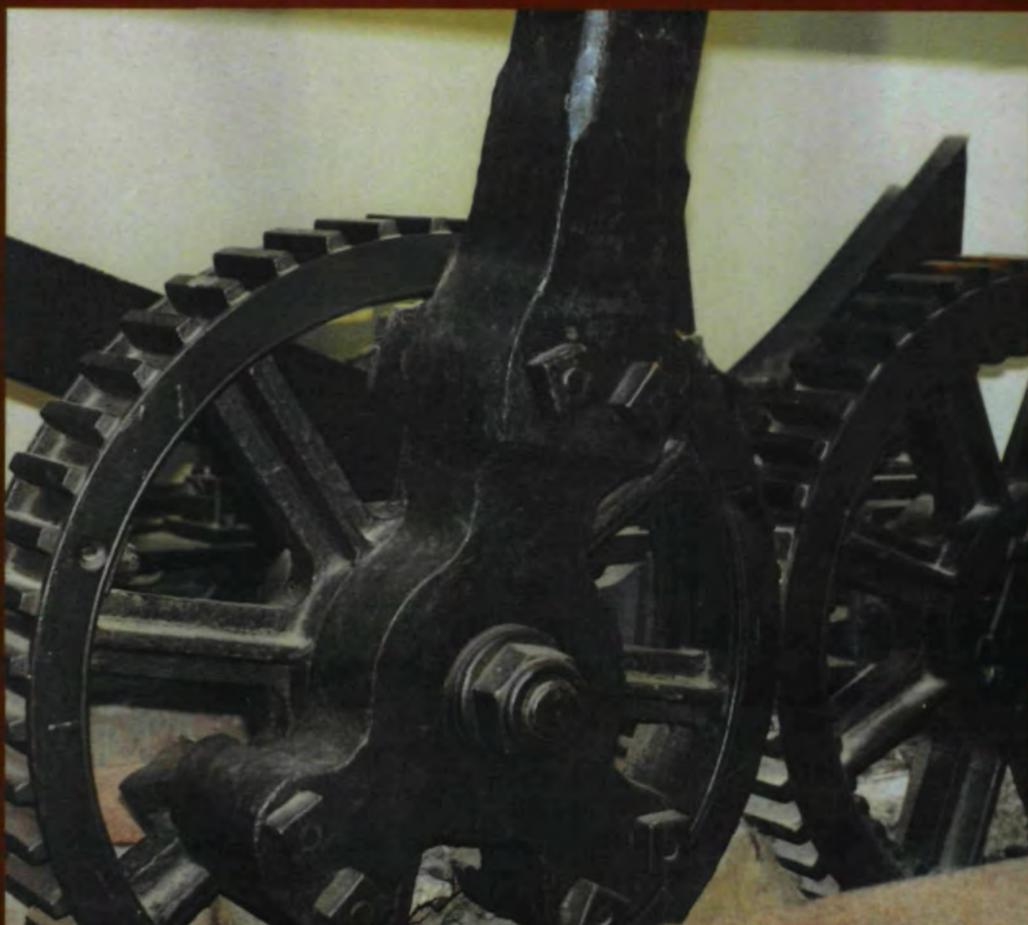
GEAR TECHNOLOGY

The Journal of Gear Manufacturing

NOVEMBER/DECEMBER 1990



Review of AGMA, ISO, and BS Gear Standards
Part I — Pitting

The Involute Helicoid & The Universal Gear

Limits of Worm Gear Surfaces to Prevent Undercutting

Fundamentals of Bevel Gear Hard Cutting

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P.O. Box 1426
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OUR COVER

"Sun and Planet" mechanism developed by James Watt to modify his steam engine to achieve rotary motion. In this system a cog wheel is attached to the connecting rod and then married to another, usually smaller, cog wheel joined to a flywheel. The machine shown was manufactured around 1787 by Boulton & Watt. It is at the Science Museum in Edinburgh, Scotland. Our thanks to Mr. Richard F. Beale of Brisbane, Australia, for use of the photos and information on this early steam engine.

GEAR TECHNOLOGY

The Journal of Gear Manufacturing

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November/December, 1990 Vol. 7, No. 6

GEAR TECHNOLOGY, The Journal of Gear Manufacturing (ISSN 0743-6858) is published bimonthly by Randall Publishing, Inc., 1425 Lunt Avenue, P. O. Box 1426, Elk Grove Village, IL 60007. Subscription rates are: $40.00 in the United States, $50.00 in Canada, $55.00 in all other countries. Second-Class postage paid at Arlington Heights, IL and at additional mailing office.

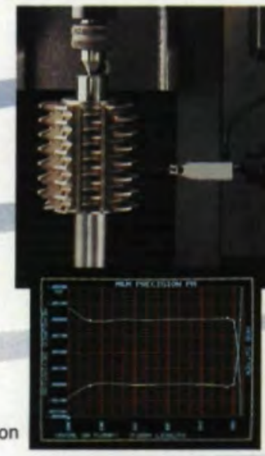
Postmaster: Send address changes to GEAR TECHNOLOGY, The Journal of Gear Manufacturing, 1425 Lunt Avenue, P. O. Box 1426, Elk Grove Village, IL 60007.

MANUSCRIPTS: We are requesting technical papers with an educational emphasis for anyone having anything to do with the design, manufacture, testing, or processing of gears. Subjects sought are solutions to specific problems, explanations of new technology, techniques, designs, processes, and alternative manufacturing methods. These can range from the "How to . . ." of gear cutting (BACK TO BASICS) to the most advanced technology. All manuscripts submitted will be carefully considered. However, the Publisher assumes no responsibility for the safety or return of manuscripts. Manuscripts must be accompanied by a self-addressed, self-stamped envelope, and be sent to GEAR TECHNOLOGY, The Journal of Gear Manufacturing, P.O. Box 1426, Elk Grove, IL 60009, (708) 437-6604.

M&M Precision Systems . . .

Total Quality in Gear Manufacturing

From Start



Part blank and gear cutting tools play critical roles in gear quality control. We offer optional software packages for inspecting gear blanks for circular geometry and gear cutting tools such as hobs, shaper/shaver cutters, and broaches.

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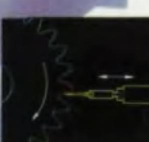
Gear Testing using Generative Metrology techniques is enhanced by computerized automation and analysis. True index, lead, and involute testing are performed as the test part and probe are engaged in inter-related continuous movement. Rotary and linear axis are synchronized as required by the test.



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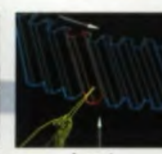
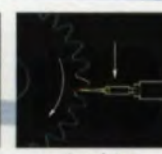
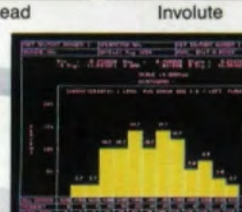


Lead



Involute

Process control can be implemented by analyzing data collected in gear testing. We offer SPC software to evaluate x-bar, R, histogram, and tooth surface topological studies.



Histogram

To Finish

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Editorial

LOOKING TO THE FUTURE – PART II

Beginning with our next issue, some of the promised changes in format for *Gear Technology* will begin showing up in these pages. As part of our commitment to provide you with important information about the gear and gear products industry, we are expanding our coverage. In addition to continuing to publish some of the best results of gear research and development throughout the world, we will be adding special columns covering vital aspects of the gearing business.

In our Shop Floor column, several well-known gearing professionals will discuss practical design and manufacturing problems that appear in the work place. We invite you to submit your questions to this panel of experts.

Management Matters will cover some of the challenges of running a gear design or manufacturing business in the 90s. We will cover such matters as doing business overseas, training, employee problems, product liability, marketing for your company, and other items of concern to gear shops, both large and small.

Along with these additions to our editorial line-up, we will continue to provide several articles on gear design, manufacturing, and research in every issue. This is one part of the magazine that will not change. While we are undergoing a facelift, we have not lost sight of the fact that providing the latest information about gear manufacturing, research, and development is our primary goal.

Along with these editorial improvements, we shall be making some cosmetic changes to *Gear Technology*. Look for some new type faces, headline styles, and design elements to appear beginning with next issue. Our goal with these changes is to make the magazine more contemporary, more readable, and more useful to our readers.

Perhaps the most readily apparent change to *Gear Technology* will be on our cover. With some regret, we have reached the end of our series of gear drawings by Leonardo da Vinci. In the course of nearly seven years of publishing, we have used most of the artist's gear-related drawings, and commissioning new ones is beyond the power of our editorial and art staff.

Instead, we will be featuring four-color art on our covers. If you or your company have photos of gears, gears in motion, or gear cutting that you think would make a good cover for *Gear Technology*, please send them to our art department for consideration. We will credit you or your company as the source, and the artwork will be returned to you after the magazine is printed.

Our goal in executing these changes to *Gear Technology* is to keep up with the changing needs and interests of you, our readers. As you strive to remain competitive and keep up with the changing business climate, we want to keep in step with you and continue to be a trusted and useful reference source for gear information of all sorts.

Michael Goldstein

Michael Goldstein, Editor/Publisher

IT'S YOUR MOVE

GEAR TECHNOLOGY always wants to be responsive to its readers. Please send us your reactions to the changes in our magazine. If you have ideas for additional or different columns, cover art, questions for our columnists, or just would like the opportunity to respond to something you've read in our pages, please let us know. A phone call or letter to our editorial offices is always welcome.

We also continue to remain on the lookout for articles on all aspects of gear manufacture and design. These articles remain the heart of our magazine. Please consider sharing any article you have written with us. Call or write for a copy of our editorial guidelines.

VIEWPOINT

Dear Editor:

Your article on the ITC's Report to the President on the condition of the U.S. gear industry (Sept./Oct. issue) was most interesting. I am wondering if the total report neglected to mention that some of our inability to export gears is due to our reluctance to provide metric countries with the metric module-based gears that overseas customers demand.

I also hope your readers are apprised of the fact that those involved in furnishing gears to the U.S. government as contractors will have to provide metric-dimensioned gears after 1992.

Sincerely,
Valerie Antoine
Executive Director
U.S. Metric Association, Inc.

EDITOR'S NOTE: The relevent portion of the Omnibus Trade and Competitiveness Act of 1988 reads as follows:

". . . It is therefore the declared policy of the United States (1) to designate the metric system of measurement as the preferred system of weights and measures for U.S. trade and commerce; (2) to require that each Federal agency, by a date certain and to the extent ecnomically feasible by the end of the fiscal year 1992, use the metric system of measurement in its procurements, grants, and other business-related activities, except to the extent that such use is impractical or is likely to cause significant inefficiences or loss of markets to U.S. firms . . ."

Readers should also note that in a July 30 letter to Secretary of Commerce Robert A. Mosbacher, NASA informed the secretary that "Effective with the start of the new fiscal year, October 1, 1990, all Requests for Proposals for new NASA flight programs will require use of the metric system of measurement." A memo dated July 20, 1990, which sets NASA policy on metric conversion states: "Ongoing programs may continue use of the conventional inch-pound system baseline for hardware design, but must plan to accommodate the metric hardware that will result from this transition."

Letters for this column should be addressed to Letters to the Editor, GEAR TECHNOLOGY, P.O. Box 1426, Elk Grove Village, IL 60009. Letters sent to this column become the property of GEAR TECHNOLOGY. Names will be withheld upon request, however, no anonymous letters will be published.

AGMA, ISO, and BS Gear Standards
Part I — Pitting Resistance Ratings

Doug Walton, Yuwen Shi, Stan Taylor,
Mechanical Engineering Department
University of Birmingham, Birmingham, U.K.

Summary:

A study of AGMA 218, the draft ISO standard 6336, and BS 436:1986 methods for rating gear tooth strength and surface durability for metallic spur and helical gears is presented. A comparison of the standards mainly focuses on fundamental formulae and influence factors, such as the load distribution factor, geometry factor, and others. No attempt is made to qualify or judge the standards other than to comment on the facilities or lack of them in each standard reviewed. In Part I a comparison of pitting resistance ratings is made, and in the subsequent issue, Part II will deal with bending stress ratings and comparisons of designs.

Introduction

Standard spur and helical gears are usually designed to specific standards to meet the requirements of proportions, manufacturing accuracy, and load rating. The load rating is the most important issue discussed in AGMA (American Gear Manufacturers Association), ISO (International Standards Organization), DIN Deutsche Industrie Normen) and BSI (British Standards Institution) gear standards. The standards written by these organizations are widely used for gear design throughout the world and also form the basis of "minority" gear standards. China, for example, issues a gear design standard based on ISO. European gear standards are now becoming very similar. The new BS and the draft ISO standard share much in common with DIN. This paper considers BS 436:1986, the draft ISO standard 6336, and AGMA 218.01. Since this review was written, AGMA introduced AGMA 2001-B88, although this new standard is not considered here. However, the trend toward a universal standard continues, with AGMA 2001 publishing rating factors, some of which are similar to the draft ISO standard.

This article is intended for designers who will appreciate a review of this complex subject. Many designers in the USA will still use AGMA 218 because they are familiar with it, and will, we suspect, continue to do so for some time. (This situation also exists in the UK with respect to the old and new British Standards on gear ratings.) It will take the authors some time before they have enough experience in the use of AGMA 2001 and have been able to validate it against real designs and other rating standards. While there are marked similarities between the old and new AGMA formulas for pitting resistance and bending strength ratings, we have mainly excluded any reference to AGMA 2001.

Although, theoretically, any standard will produce a gear pair which is satisfactory, it is no longer enough to accept any standard when other procedures might produce more competitive designs. On the other hand, if the design calls for special operating conditions, such as shock loads or flexible drives, it may be advantageous for the designer to address a standard which deals more closely with these conditions. In addition, the customer may specify the code to be used. A working knowledge of more than one standard is desirable, particularly if the product is aimed at international markets. An understanding of the differences between gear standards is, therefore, important. It should be pointed out, however, that in a review of gear standards it is impossible to cover every aspect of each code. ISO 6336 Parts 1 to 4, for example, contain over 90 figures and over 20 tables.

Standards for Spur and Helical Gears

The old British Standard, BS 436:1940,[1] in use for nearly fifty years, was a revision of the original Specification for Machine Cut Gears first issued in 1932. During its long existence, the rating method remained the same with only minor revisions. Though obsolescent, it is still used extensively throughout Britain and elsewhere, mainly because it is easy to use. The standard rates gears on the basis of bending strength and contact stresses, which are referred to as wear (meaning non-abrasive wear). The tooth root bending strength is based on the Lewis equation,[2] considering both tangential and radial loads. The effect of stress concentrations at the root is not taken into account directly, but allowances are made in the use of the allowable bending strength of the gear material, values for which are supplied in the standard. The bending strength is also factored for running speed and life. Contact stresses are based on a modified Hertz equation with allowances for speed, running time, and geometry. The latter item is taken into consideration by a zone factor, which accounts for the influence on the Hertzian stress of tooth flank curvature at the pitch point, and converts the tangential load to a normal force. No serious attempt was made to keep this standard up to date on newer gear materials and processes to predict the higher performances being achieved in practice. Another serious deficiency is that no account was made for surface finish or uneven load distribution.

NOMENCLATURE

BS 436:1940

d	Pitch diameters of pinion and wheel
E_e	Equivalent Young's modulus
F	Face width
k	Constant in Hertz contact stress formula
K	Pitch factor
n, N	Pinion and wheel running speed
P	Diametral pitch
R	Gear ratio
R_r	Relative radius of curvature
s	Maximum contact stress set in the surface layers of the gear cylinders
S_c	Surface stress factor
T	Number of teeth on wheel
X_c	Speed factor for contact stress
Z	Zone factor
α_t	Transverse pressure angle at reference cylinder
α_{tw}	Transverse pressure angle at pitch cylinder
β_b	Base helical angle

AGMA 218.01

C_a	Application factor for pitting resistance
C_c	Curvature factor at pitch line
C_f	Surface condition factor
C_H	Hardness ratio factor
C_L	Life factor for pitting resistance
C_m	Load distribution factor for pitting resistance
C_p	Elastic coefficient
C_R	Reliability factor for pitting resistance
C_s	Size factor for pitting resistance
C_T	Temperature factor for pitting resistance
C_v	Dynamic factor for pitting resistance
C_x	Contact height factor
C_ψ	Helical overlap factor
d	Operating pitch diameter of pinion
F	Net facewidth of the narrowest number
I	Geometry factor for pitting resistance
m_N	Load sharing ratio
n_p	Pinion running speed
P_n	Diametral pitch
S_{ac}	Allowable contact stress number

BS 436: 1986 and ISO/DIS 6336

b	Face width
d_1	Reference diameter of pinion
K_A	Application factor
$K_{H\alpha}$	Transverse load factor for contact stress
$K_{H\beta}$	Face load factor for contact stress
K_V	Dynamic factor
n_1	Pinion running speed
S_{Hmin}	Minimum demanded safety factor on contact stress
Z_M	(BS only material quality factor for contact stress)
Z_N	Life factor for contact stress
Z_X	Size factor for contact stress
Z_E	Elasticity factor for contact stress
Z_H	Zone factor for contact stress
Z_L, Z_R, Z_V	Lubricant influence, roughness, and speed factor for pitting
Z_W	Work-hardening factor for contact stress
Z_ϵ	Contact ratio factor for contact stress
Z_β	Helix angle factor
α_t	Transverse pressure angle at reference cylinder
α_{tw}	Transverse pressure angle at pitch cylinder
β_b	Base helix angle
σ_{Hlim}	Basic endurance limit for contact stress
σ_{HP}	Permissible contact stress

Smith[3] described BS 436 as "an average experience method, whereby gear manufacturers and users collaborate to provide extremely empirical rules of thumb based on operating experience. Permissible loads are specified for 'typical' manufacturing accuracies of a given class with 'typical' loading cycles and corrections for speed, etc." Although the standard did not take into account factors known to influence bending and contact stresses, such as application conditions (i.e., the load fluctuations caused by external sources), system dynamics and gear accuracy and the benefits of surface hardening, the standard served its users well.

The original AGMA standard was issued in 1926, and the first draft of AGMA 218.01,[4] used in this review, was drafted in 1973 and approved for publication in 1982. AGMA 218 also rates gears on the basis of bending strength and surface contact stresses, (referred to as surface durability or pitting) but also introduces a number of other factors in the rating equations. For example, influence factors are used to take into account load distribution across the face width, quality of the transmission drive, and transmission accuracy relating to manufacture. Considerable knowledge and judgment is required to determine values for these factors.

Compared to the old British Standard, AGMA 218 is considerably more comprehensive. Ratings for pitting resistance are based on the Hertzian equation for contact pressure between curved surfaces, which is modified for the effect of load sharing between adjacent teeth. The Lewis equation has been modified to account for effects, such as stress concentrations at the tooth root, compressive stresses resulting from the radial component of the gear load, load distribution due to misalignment between meshing teeth, and load sharing. The critical bending stress is assumed to occur at the tooth fillet but, as in the old British Standard, the effect of blank geometry (e.g., rim and web size and how the relative size of these effects stresses at the tooth root) is not considered.

The ISO standard, ISO 6336,[5] was issued in 1980, though it is still a draft. The standard covers a wide range of designs and applications, and is the most detailed document among the gear rating standards considered here. It contains a vast amount of collected knowledge and the options to calculate factors at various levels of complexity. It gives procedures for determining gear capacity as limited by pitting and tooth breakage, as in other standards, and also considers

AUTHORS:

DR. DOUG WALTON *is a Senior Lecturer in the Mechanical Engineering Department of the University of Birmingham. He holds BTech and PhD degrees in Aeornautical Engineering and is Chairman of the department's Design Group. He is a Member of the Institution of Mechanical Engineers and the British Gear Association.*

YUWEN SHI *is a visting research student from the Harbin Institute of Technology, People's Republic of China. He has a BS degree in Mechanical Engineering and is currently working in the area of the CAD of geared drives at the University of Birmingham.*

DR. STAN TAYLOR *is a Lecturer in the Mechanical Engineering Department of the University of Birmingham. He holds a PhD in Mechanical Engineering and is a Member of the Institution of Mechanical Engineers. He has been a pioneer in the use of computers in solving engineering problems.*

scuffing. The basic equations are modified by applying influence factors as in AGMA. These procedures demand a realistic appraisal of all influence factors, particularly those for the allowable stress, the probability of failure, and the appropriate safety factor. ISO also offers three different methods for determining bending and contact stresses and their influence factors, depending on the application and accuracy required. Since the latest German standard, DIN 3990: 1987,[6] is substantially similar to ISO 6336, a detailed discussion of DIN is excluded.

The new British standard, BS 436:1986,[7] is similar to ISO/DIS 6336 and is a complete revision of the old standard. It is, however, much more user friendly than ISO. Like the other standards, the new British standard uses modified Lewis and Hertz equations, using correction factors, such as the dynamic factor to account for load fluctuations arising from manufacturing errors, and a load distribution factor to take into account the increase in local load due to non-uniform loading arising from conditions such as shaft stiffness and, in the case of helical gears, helix error. The correction factors are the same or are similar to those used by ISO. Geometry factors in the new BS 436 are similar to those in ISO (method B) while other methods in ISO use different approaches for geometry factors. The new British standard, however, draws on a considerable amount of previously published research and uses additional parameters, like material quality factors, not allowed for by ISO or AGMA. This standard does not work on "typical" figures for each rating factor as in the old standard, but uses researched data to predict load increases caused by deflections, alignment tolerances, and helix modifications. Throughout this review, the term BS refers to the new standard, except where stated.

In the stress analysis procedures of BS and ISO, bending and contact stresses are classified into three groups: 1) Nominal or basic stresses, which are calculated for geometrically perfect gears meshing with perfect load distribution, 2) Actual stresses, calculated from the nominal stresses, but allowing for manufacturing and mounting errors, and 3) Permissible stresses, calculated for the gear material taking into account the required life, gear finish, lubrication, and the minimum specified factors of safety. The BS differs significantly from ISO and DIN in the determination of permissible stresses.

Literature Survey

Comparisons have been made between AGMA and ISO covering basic theories and results for applications. Those comparisons that were published were mainly based on old versions of AGMA (AGMA 215.01 and AGMA 225.01)[8,9] and the older, approved version of ISO/DP 6336. No detailed comparisons have been made between BS 436 and other standards. More recently some comparisons have been made between the latest British and German standards by Hofmann,[10] who described the theoretical basis of the latest BS and DIN and the differences in determining permissible stresses.

Imwalle and Labath[11] made a design survey of different gear sets for the purpose of comparing AGMA (AGMA 215 and AGMA 225) and ISO/DP 6336. The results were summarized for a comparison of dynamic load distribution and geometry factors and allowable stress. In the comparison of geometry factors, all the factors which are linked with gear tooth geometry were combined to form a "total geometry factor". Comparisons showed that ISO usually gives a higher factor of safety and higher calculated bending and contact stresses for case-hardened gears compared to AGMA, but lower values for through-hardened examples. It may be noted that ISO provides data on the latest and most advanced gear materials and treatments. In another paper[12] by the same authors the concept of a basic stress was used, defined as the stress which is calculated if all the modifying factors are set at unity. The results showed that ISO usually gives a higher basic bending stress, but a lower basic contact stress compared to AGMA. Again, comparisons were made of geometry, dynamic load distribution factors, life factors, and allowable stresses.

Mathematical means were provided by Castellani and Castelli[13] to compute the parameters for calculating the tooth form factor and the stress correction factor (allowing for stress concentrations at the tooth fillet) which are used in AGMA and ISO. Comparisons made between these two factors in the gear strength ratings gave the following two differences:

1) A different choice of reference points on the tooth root profile for the tooth form and stress correction factors is made. ISO chooses the same critical point for both tooth form and stress correction factors relating to the point of the fillet whose tangent forms a 30° angle with the tooth axis. The critical point for the tooth form factor depends on the gear type (spur or helical) and its accuracy. AGMA considers the minimum curvature radius for the stress correction factor relating to the point where the fillet connects to the root circle.

2) Both standards assume that the load application side of the tooth flank is critical with respect to bending failure. AGMA takes this assumption into consideration by subtracting the radial, compressive stress component from the bending stress, while ISO only considers the tangential bending stress. ISO compensates for this by making allowances on the values of the stress correction factor.

Comparison of Pitting Resistance Ratings

Comparing gear standards can present difficulties for the following reasons:

1) There are a number of influence factors included in each standard, but the number and the numerical values of these factors differ. Taking the power rating formula for pitting resistance as an example, BS 436:1940 has four influence factors, while there are twelve in AGMA 218, compared with sixteen in both ISO and the BS. These are discussed later.

2) The determination of influence factors usually requires a knowledge of additional parameters, some of which are not always readily available. For example, in order to use the analytical method to calculate the AGMA geometry factor for pitting resistance, four additional data items (a curvature factor at the pitch line to determine the radius of curvature between the two contact surfaces; a contact height factor to adjust the location of the height of the tooth profile where the stress is calculated; a helical factor to account for the effect of helix angle on contact stresses; and a load sharing ratio) have to be employed. To determine these four factors more infor-

mation is required and some, like generating the tooth layout to give the necessary geometry data, may be difficult to obtain.

3) Each standard has its own definitions for the influence factors, but factors bearing the same names do not necessarily have the same effects. This makes direct comparisons of influence factors difficult. For example, although AGMA and ISO both introduce a service factor, the values cannot be compared directly.

The terms used in each standard are listed in the nomenclature.

The power rating for contact stresses given by BS436:1940 is

$$\frac{X_c\, S_c\, Z\, F\, N\, T}{126\,000\, K\, P} \tag{1}$$

Equation 1 can be rewritten as

$$\frac{n\, F\, d^2}{126\,000}\; \frac{R_r^{0.8}}{d}\, X_c\; \frac{s^2}{k\, E_e} \tag{2}$$

For AGMA 218.01 the transmissible power based on pitting is

$$\frac{n_p\, F\, d^2}{126\,000}\; \frac{I\, C_v}{C_s\, C_m\, C_f\, C_a}\; \left(\frac{S_{ac}\, C_L\, C_H}{C_p\, C_T\, C_R}\right)^2 \tag{3}$$

The BS 436:1986 power rating is

$$\frac{b\, d_1{}^2\, n_1}{126\,000}\; \frac{u}{u+1}\; \frac{1}{Z_H{}^2\, Z_\epsilon^2}\; \frac{1}{K_A\, K_v\, K_{H\alpha}\, K_{H\beta}}\; \left(\frac{\sigma_{HP}}{Z_E}\right)^2 \tag{4}$$

$$\text{where } \sigma_{HP} = \sigma_{Hlim}\, Z_L\, Z_V\, Z_R\, Z_M\, Z_W\, Z_X\, Z_N / S_{Hmin} \tag{5}$$

and for ISO/DIS 6336 the power is

$$\frac{b\, d_1{}^2\, n_1}{126\,000}\; \frac{u}{u+1}\; \frac{1}{Z_H{}^2\, Z_\epsilon^2\, Z_\beta^2}\; \frac{1}{K_A\, K_v\, K_{H\alpha}\, K_{H\beta}}\; \left(\frac{\sigma_{HP}}{Z_E}\right)^2 \tag{6}$$

where

$$\sigma_{HP} = \left(\frac{\sigma_{Hlim}\, Z_N}{S_{Hmin}}\right) Z_L\, Z_R\, Z_V\, Z_W\, Z_X \tag{7}$$

From Equations 2-7 it may be seen that for a given gear ratio the transmissible power is proportional to the square of the pinion pitch circle diameter and permissible contact stresses. Therefore, to increase gear power capacity in terms of surface durabilty, it is more effective to increase the pinion PCD or permissible surface stresses than to increase the gear pair facewidth.

The pitting resistance related factors above may be grouped into common and non-common factors. Common factors are those which have the same meanings in all the standards, (not all these factors appear in the standards) and their values can be compared directly. For example, the dynamic factor which allows for internally generated gear tooth loads induced by non-conjugate meshing action of the gear teeth, appears in all the standards except the old BS, and values can be compared directly. Non-common factors, such as the geometry factor, are those which are only equivalent to each other in the sense of having the same effects on the rating results, although specific values cannot be compared.

Table 1 classifies all the contact stress influence factors into either common or non-common groups where it can be seen that the old British Standard had few parameters for comparison with other standards. The similarity between BS and ISO *is also apparent. A comparison between the influence* factors given in each standard is made in the following paragraphs.

1) Application factors. An application factor is used in AGMA, ISO, and BS to evaluate external influences tending to apply a greater load to the gear teeth than that based on steady running conditions. Typical external influences are the drive characteristics (e.g., smoothness and load fluctuations) of the prime mover and of the driven machine. Values suggested in ISO correspond to those for the overload factor in AGMA 215. While no data appears in AGMA 218, these factors may be found in related AGMA application standards. BS gives more detailed conditions than ISO, although values are similar.

2) Dynamic factors. As discussed above, the application factor is used to handle dynamic loads unrelated to tooth accuracy. The effect of dynamic load related gear tooth accuracy is then evaluated by the inclusion of a dynamic factor which accounts for effects of gear set mass elastic effects and transmission errors. AGMA modified the experimental work of Wellauer[14] to obtain dynamic factors as a function of transmission error. These accuracy levels can under certain manufacturing conditions be the same as the gear quality numbers given in AGMA 390.[15]

ISO dynamic factors are based on Buckingham's incremental load method[16] and work by Weber and Banaschek.[17] ISO (analytical) method B presents a calculation procedure for the main resonance speed and divides the running speed into three sectors. The dynamic factor corresponds to each of these speed sectors and may help designers to adjust the operating speed or alter the design to avoid critical speeds. ISO method C is only applicable to gears with accuracy numbers of 3 to 10 and cannot be used for gears operating at or near the main resonance speed. The dynamic factor in the BS is very close to ISO method C. In the old BS, dynamic effects were not considered. The speed factor used in the old BS is not to be confused with dynamic loads, but was intended to allow for load reversals and their effect on fatigue during the life of the gear.

It has been customary for AGMA to put the dynamic factor in the denominator of the rating formula, whereas ISO and BS apply it to the numerator. Nevertheless, the dynamic factor is defined as a multiplier of the transmitted load in all the standards, although some believe that the effect should be additive.[18]

3) Load distribution factors. A load distribution factor is used in the rating equations to reflect the non-uniform load distribution along the contact lines caused by deflections, alignment, and helix modifications (including crowning and end relief), and profile and pitch deviations. The evaluation procedure for this factor is rather complex, since many variables are involved, and some of them, such as the component of the gear system alignment and manufacturing errors, are difficult to determine.

TABLE 1 COMPARISON OF PITTING RESISTANCE INFLUENCE FACTORS

	BS 436:1940	AGMA 218	BS 436:1986	ISO/DIS 6336
Geometry Factors*	$\frac{R_r^{0.8}}{d}$	I	$\frac{u}{u+1}\ \frac{1}{Z_H^2\ Z_\epsilon^2}$	$\frac{u}{u+1}\ \frac{1}{Z_H^2\ Z_\epsilon^2\ Z_\beta^2}$
Elasticity Factors*	$(kE_e)^{0.5}$	C_p	Z_E	Z_E
Size Factors*	–	C_s	$\frac{1}{Z_x^2}$	$\frac{1}{Z_x^2}$
Lubrication Film Factors*	–	$C_f^{0.5}\ C_T$	$\frac{1}{Z_L Z_V Z_R}$	$\frac{1}{Z_L Z_V Z_R}$
Application Factors†	–	C_a	K_A	K_A
Dynamic Factors†	–	C_V	$\frac{1}{K_V}$	$\frac{1}{K_V}$
Load Distribution Factors†	–	C_m	$\frac{1}{K_{H\alpha}K_{H\beta}}$	$\frac{1}{K_{H\alpha}K_{H\beta}}$
Work Hardening Factors†	–	C_H	Z_W	Z_W
Life Factors†	–	C_L	Z_N	Z_N
Reliability Factors†	–	C_R	S_{Hlim}	S_{Hmin}
Material Quality Factor†	–	–	Z_M	–
Speed Factor†	X_C	–	–	–

† denotes common and * non-common factors

In AGMA the load distribution factor is the product of the face and transverse load distribution factors. The face or longitudinal (as described in the ISO and BS) load distribution factor accounts for the non-uniform load across the face of the gear, while the transverse load factor reflects the effect of non-uniform distribution of load down the tooth flank due to profile, pitch deviations, and tooth modifications. Although AGMA uses this factor to allow for the effect of the non-uniform distribution of load among the teeth which share the total load, no specific information is given in the standard. The AGMA standard assumes that if the gears are accurately manufactured, the value of the transverse load distribution factor can be taken as unity. AGMA provides both empirical and analytical methods to determine the face load distribution factor. The empirical method is recommended for normal, relatively stiff gear assemblies, and only a minimum amount of information is required. The second method is based on elastic and non-elastic lead mismatch and needs information about design, manufacture, and mounting and is, theoretically, suitable for any gear design.

The ISO load distribution factor is also the product of the transverse and longitudinal load factors. Three different approaches have been made by ISO to determine the longitudinal load factor. Method B is a final proof rating calculation method based on known manufacturing errors. Method C is a preliminary rating method and uses assumed values of manufacturing errors within limits of prescribed tolerances. Method D is even more simplified than method C. The transverse load factor is a function of longitudinal load factor, contact ratio, pitch tolerance, and mean load intensity. Procedures for calculating the load distribution factors in ISO are the most complex and are still under revision. Load distribution factors in the BS employ virtually the same procedure as method C in ISO, except for a difference in determining total misalignment. ISO gives five approximation methods for this, while the BS only gives one.

4) Life factors. Life factors take into account the effects of increments in permissible stresses if a limited number of load cycles is demanded. Among AGMA, ISO, and BS, the most distinct difference lies in the definition of endurance limits.

AGMA 218 sets 10^7 load cycles as the endurance limit for both bending and pitting, while ISO and BS define limits of $2x10^6$, $5x10^7$, and 10^9 cycles for contact stresses. Although there is no life factor in the old BS, a procedure to calculate variable duty cycles by determining an equivalent running time was provided. BS 436:1986 also has a procedure to deal with variable duty cycles, while this aspect of gear running is not considered by ISO.

5) Material quality factor. Among the four standards, only the BS introduces material factors in its bending and contact stress ratings in an attempt to allow for the higher permissible stresses to be obtained from using higher quality materials.

6) Size factors. Size factors are used in all, except the old BS, to take into account the influence of tooth size on surface fatigue strength. Values are usually taken as unity because no further information is provided in any of the standards.

7) Work-hardening factors. When the pinion material is substantially harder than the wheel, the effects of cold work hardening and internal stress changes in the softer wheel material may occur, in which case the surface contact stresses will be reduced. These effects have been considered by AGMA, ISO, and BS by introducing a hardness ratio or work-hardening factor. In AGMA, the hardness ratio factor is a function of the gear ratio and pinion and wheel hardness, but AGMA only applies this factor to the wheel rating. A guidance diagram is given by BS for determining the work-hardening factor, based on surface roughness and hardness. The ISO work-hardening factor is only related to wheel hardness.

8) Permissible stresses. Permissible bending and contact stresses are given in the old BS for a limited number of materials listed in the standard. It is usually agreed that the values are generally too pessimistic for surface-hardened gears. Allowable bending and contact stresses, based on laboratory and field experience for each material and heat treatment condition, are provided in AGMA. For most of the steels the allowable bending and contact stress numbers are functions only of material hardness.

In both BS and ISO the permissible bending/contact stress is based on the bending/surface fatigue endurance limit for the material, taking into account the required life and running conditions. According to the BS, for most gear materials the bending/contact endurance limit depends only on hardness without differentiating between materials and heat treatments. In ISO, bending/contact endurance limit is determined either based on experimental data for test gears of the same material or on prepared, polished specimens. Values are provided in the ISO standard for a wide range of steels and heat treatments.

For surface-hardened gears, the BS bending endurance limit is based on residual stresses and the ultimate tensile strength of the gear material. Determining the residual stresses and tensile strength of surface-hardened gears is, however, difficult, casting some doubt as to the ease with which this method can be used.

9) Factors of safety and reliability. So far there is no accepted method of relating gear reliability to safety factors considering the effect of material quality and gear accuracy. The AGMA reliability factor accounts for the effect of the normal statistical distribution of failures from the allowable stresses and can be chosen according to the reliability required. BS and ISO leave the user to specify a value for this factor. Minimum demanded safety factors for bending strength and contact stress are recommended by both ISO and BS to reflect the confidence in the actual operating conditions and material properties, but the values for these factors are different. The safety factor for bending strength in the old BS is defined as the ratio of ultimate tensile strength to the product of the speed factor and bending stress factor.

10) Non-common geometry factors. Geometry factors account for the influence of the helix angle, contact ratio, and tooth flank curvature at the pitch point on gear load capacity. Ignoring the experimental exponent of 0.8, the geometry factor for the old BS can be written

$$\frac{R}{R+1}\;\frac{\cos\alpha_t \cos\alpha_{tw}}{2\cos\beta_b} \qquad (8)$$

For the BS and ISO the geometry factor is

$$\frac{1}{Z_H^{\,2}} = \frac{\cos\alpha_t \sin\alpha_{tw}}{2\cos\beta_b \cos\alpha_{tw}} \qquad (9)$$

The similarity between Equations 8 and 9 is not apparent when expressed in the way given in the standards. (See Geometry Factors, Table 1.)

The AGMA geometry factor, I, for contact stress is

$$\frac{C_c\, C_x\, C_\psi^2}{m_N} \qquad (10)$$

where C_c is the curvature factor at the pitch line and is a function of the gear ratio and pressure angle. C_x is a contact height factor adjusting the location of the tooth profile where the stress is calculated. The helical factor C_ψ accounts for the helical effect in low contact ratio helical gears, and m_N is the load sharing ratio which depends on the transverse and face contact ratios. Similarly, ISO uses a helix angle factor to account for the helix effect on contact stresses. Both ISO and BS include a contact ratio factor to allow for the influence of transverse contact ratio and overlap ratio on contact stress based ratings.

11) Non-common elasticity factors. Elasticity factors account for the influence of material mechanical properties on the Hertzian stress. Those used in AGMA, ISO, and BS are identical. The only difference between the old BS and the others is that the equation for calculating this factor has been simplified by assuming that Poisson's ratios for the pinion and wheel are the same.

12) Non-common lubrication film factors. BS and ISO account for minimal film thickness between contacting teeth on surface load capacity. In their rating procedures, oil viscosity, surface hardness, and pitch line velocity are considered to be the main factors influencing film thickness. There are some differences between the calculation methods used by BS and ISO. BS gives two diagrams: one for roughness and the other for the product of a lubricant and speed factor. ISO provides three equations and corresponding diagrams to determine these factors. Although AGMA

218 does not consider lubrication, it does take tooth surface roughness and temperature effects into account by introducing a surface condition and a temperature factor. In the old BS, lubrication was ignored altogether. Tooth scuffing, *which is covered by ISO and DIN in separate parts, attempts* to predict the temperature at which scuffing will occur. This is not dealt with by any of the other standards and, therefore, no comparisons can be made, although scuffing does appear in the new AGMA standard.

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The Involute Helicoid and The Universal Gear

Leonard J. Smith
Invincible Gear Co., Livonia, MI.

Introduction

A universal gear is one generated by a common rack on a cylindrical, conical, or planar surface, and whose teeth can be oriented parallel or skewed, centered, or offset, with respect to its axes. Mating gear axes can be parallel or crossed, non-intersecting or intersecting, skewed or parallel, and can have any angular orientation. (See Fig. 1.) The taper gear is a universal gear. It provides unique geometric properties and a range of applications unmatched by any other motion transmission element. (See Fig. 2.) The taper gear can be produced by any rack-type tool generator or hobbing machine which has a means of tilting the cutter or work axis and/or coordinating simultaneous traverse and infeed motions.

Traditionally this has entailed the use of proprietary or special machines – however, with the advent of numerical control for axis synchronization, conventional machines can be employed. These are the same machines used for spur and helical gear generation.

The taper gear provides features not attainable with any other type of gear. It merits consideration for what it can do, and it may well be the answer to a problem which heretofore has eluded satisfactory resolution.

Application

The taper gear has many familiar applications; for example, the gear shaper cutter, where the taper is employed to provide a relieved cutting edge. (See Fig 3.) Another familiar application is the rack-and-pinion automotive steering mechanism where a taper is used to eliminate backlash by axial adjustment. In marine engine prop drives a tapered gear is meshed with a cylindrical spur or helical pinion to provide an angular takeoff and/or to enable an optimum placement for the engine. The taper gear also allows several unusual gear meshes in the mechanism of a well-known aircraft gun and provides a lightweight reliable design in a minimum envelope.

Taper gears have found a niche in many commercial and military applications, but have not been widely embraced by the general gear industry, because of a requirement for special machines, and because of lack of information in the literature.

The taper gear concept provides a powerful tool to the geometer, and it is hoped this article will encourage the expansion of fundamental gear theory to include this versatile machine element in the basic gearing literature for widespread evaluation.

The Involute Helicoid

The involute helicoid which is conjugate to a straight-sided rack, when converted to a complex involute helicoid by the addition of a taper, provides the basis of a universal gear system.

The spur gear is the simplest form embodying involute tooth surfaces. (See Fig. 4.) The helical gear adds a helical twist to the surface which results in a simple involute helicoid. (See Fig. 5.)

The involute helicoid has three major characteristics: the involute in any transverse section, a helix in any cylindrical section, and an axvolute in any axial section.

Applying a taper to cylindrical spur involute gears provides an additional degree of freedom and results in a complex, involute helicoid surface on the tooth flanks. Opposite flanks will have equal, but opposite hands of helix and a common lead. (See Fig. 6.) The cylindrical spur gear thus may be considered a special case of the involute helicoid with zero taper, just as the cylindrical spur gear may be considered a special case of the involute helicoid with zero helix; i.e., infinite lead.

Applying a taper to a cylindrical helical gear also provides an additional degree of freedom to a gear which is initially a simple involute helicoid with equal and parallel helices of the same hand and common lead, and results in a complex involute helicoid of compound structure.

The helix resultant of the taper is additive to the original helix on one flank and is reductive to the helix on the opposite flank. There are thus two entirely different helix angles and differing leads on opposite flanks. Relative magnitudes of helix and taper determine whether

AUTHOR:

LEONARD J. SMITH *is vice-president of the Invincible Gear Co. With over a half-century of experience in precision metrology and metalworking, he has pioneered developments in machine design, numerical control, adaptive control, servomechanisms, electrical discharge machining, abrasive machining, engineering reprographics and archiving, and computer integration. He has been active in AGMA, ASME, SME, ASME-GRI, and other technical associations.*

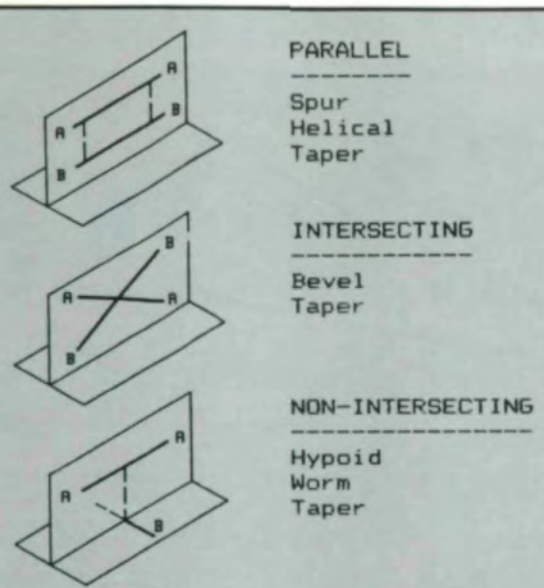

Fig. 1 – Axes Orientation

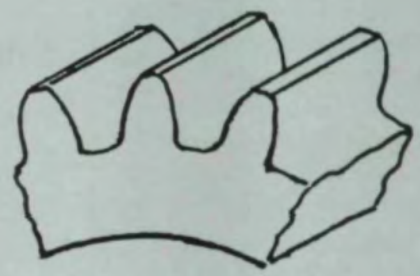

Fig. 4 – Spur Gear Tooth – Zero Helicoid

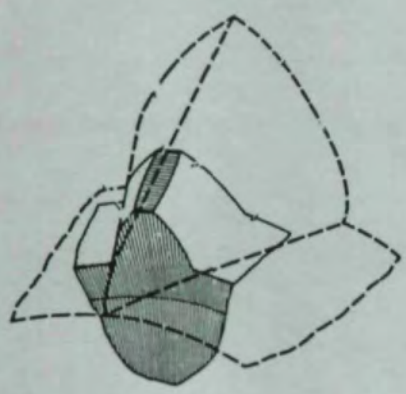

Fig. 2 – Taper Gear Tooth

Fig. 5 – Helical Gear Tooth – Simple Helicoid

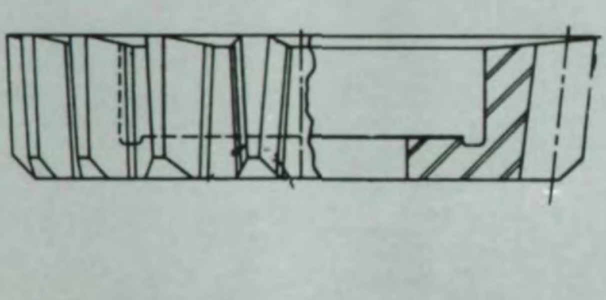

Fig. 3 – Shaper Cutter

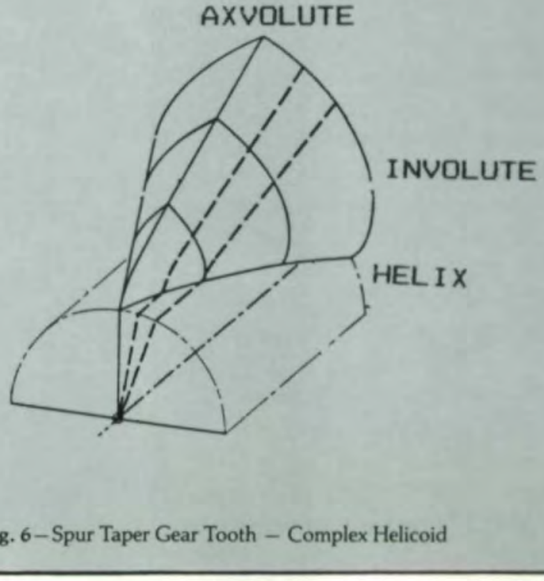

Fig. 6 – Spur Taper Gear Tooth – Complex Helicoid

the flank hands are the same or opposite.

Myriad possibilities are available for unlike profiles and leads for each flank, including providing a spur flank on one side and a helix on the other. Buttress profiles and one way ratcheting as well as back stopping are possible.

The axvolute is the key to the universality of the taper gear, since it provides a three-dimensional cam or crowned surface allowing complete freedom of mesh conditions.

Comparison

The superficial resemblance of the taper gear to a bevel gear is misleading. They are two distinct entities.

BEVEL GEAR. The bevel gear is generated from a conical surface. Its tooth surfaces converge to a common apex. Each transverse section represents a geometric reduction in a progression from back to front. Each section represents a different diametral pitch, and by custom is referenced at the back cone. (See Fig. 7.) The face width is restricted by the parameters of number of teeth and cone angle, since the width of the cutting tool tip at the front face becomes a limit factor. Conjugate bevel gears must have the same diametral pitch at their back cones, must be flush matched, have complementary cone angles equal to the sum of the shaft angle, and have a common apex. Tooth elements in all sections have a common angular dimension. (See Fig. 8.)

TAPER GEAR. The taper gear is generated from a cylindrical surface, the base cylinder. All straight line generatrices converge to a common origin on a base plane tangent to this cylinder. (See Fig. 9.) Angular symmetry of the tooth does not exist, as each cross section is a different angular value, since each tooth section is smaller than its predecessor, and its tooth space is correspondingly larger. The taper gear is controlled by a tool traveling a constant path parallel to the cone and produces a pitch point at the center of equal velocity which corresponds to the pitch of the cutting tool. This is generally referenced at the center of the face width. (See Fig. 10.)

Like all involute gears, the pitch and pressure angle vary according to the diameter ratio to the base circle. Each cross section may be considered as a profile shift or addendum correction,

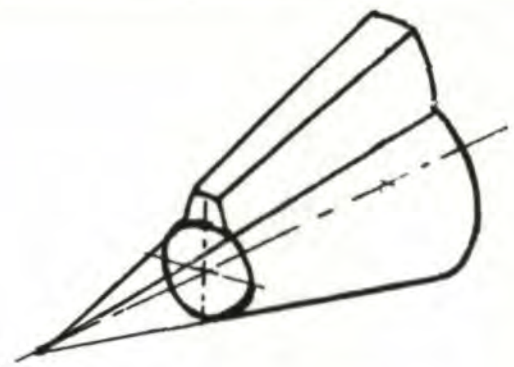

Fig. 7 – Base Cone

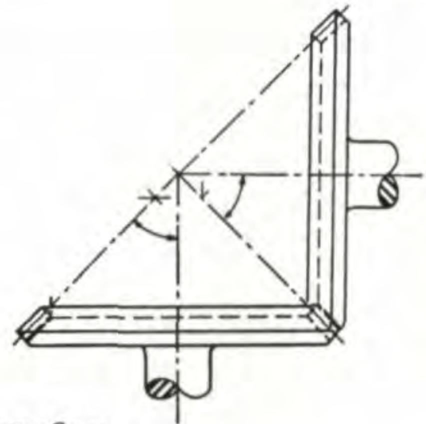

Fig. 8 – Complementary Cones

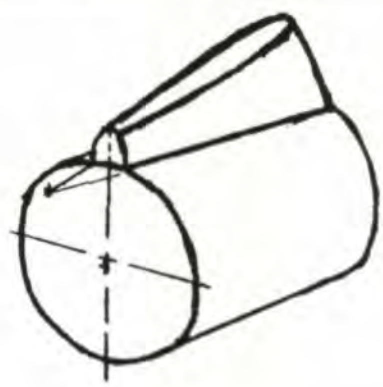

Fig. 9 – Base Cylinder

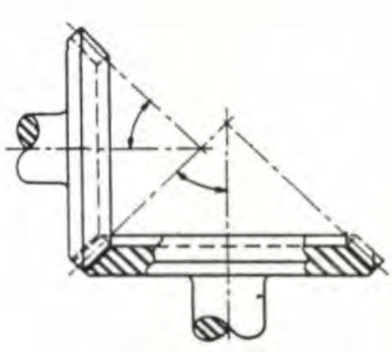

Fig. 10 – Independent Cones

since each section employs a different portion of the same involute.

As in all involute gears, this provides the relationship of a whole family of racks capable of generating the profile or of operational mesh at any diameter.

Machining Methods

CONVENTIONAL HOBBING. In the conventional hobbing process, the basic rack, represented by the hob, traverses the gear blank in a plane parallel to the gear axis and at a fixed center distance from the gear axis, and generates a spur gear in the simplest embodiment. (See Fig. 11.)

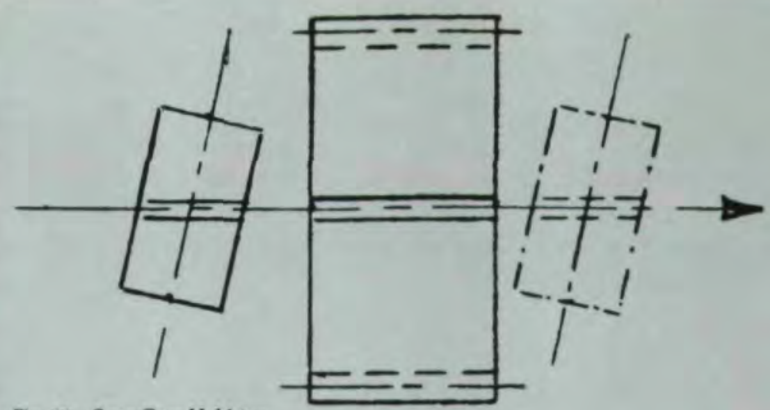
Fig. 11 – Spur Gear Hobbing

A helical gear can be generated by skewing the work to the helix angle and traversing along the axis of the rack tooth. This is frequently termed oblique hobbing and has the unique characteristic of shifting the contact across the rack. (See Fig. 12.)

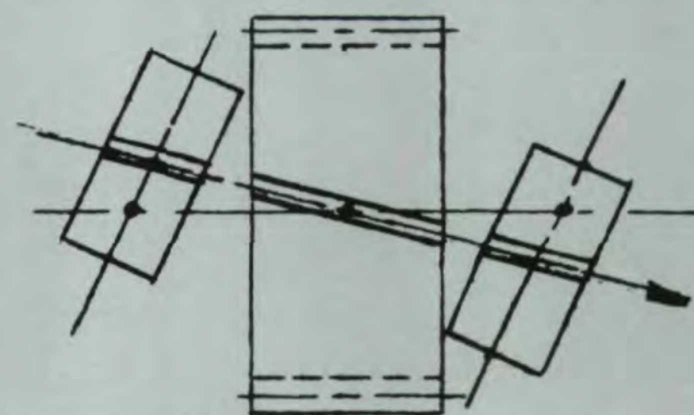
Fig. 12 – Helical Oblique Hobbing

The more common approach skews the rack to the helix angle and requires an additive rotational timing to produce the helix, while traversing along the gear axis. This method employs a fixed portion of the rack for full generation. (See Fig. 13.)

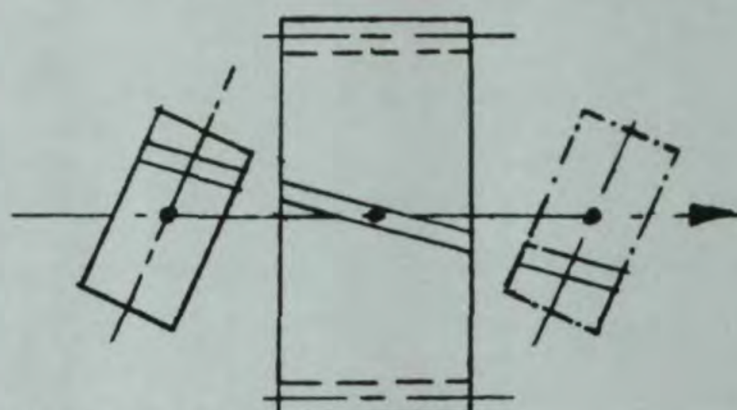
Fig. 13 – Helical Skew Hobbing

These methods provide a constant tooth thickness in any transverse plane. Tooth thickness increase or decrease is obtained through radial infeed of the rack or hob; i.e., a change in their center distance. Additional compensatory devices could be employed to impart nonuniform helix control.

TAPER HOBBING. In tapered gearing an additional degree of freedom is required: an angular relationship between the axis of the rack and work, which provides a uniform rate of change of center distance in relation to the traverse of the face width. The radial distance of the rack from the center line of the work is not constant, but diminishes from the back face to the front face. As a consequence, the tooth thickness gradually decreases. (See Figs. 14-15.)

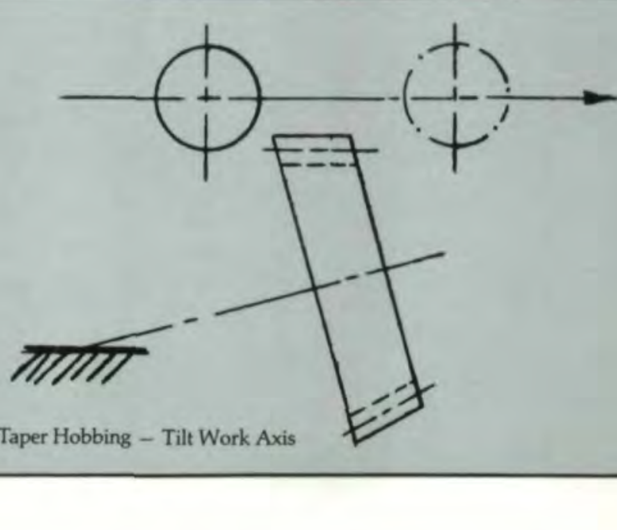
Fig. 14 – Taper Hobbing – Tilt Work Axis

A tapered gear which is generated in this manner has the superficial appearance of a bevel gear, which it is not. Each transverse section represents a spur gear of differing tooth thickness. In digitally controlled machines it is possible to synchronize the traverse and infeeds as a step function to produce the angular effect without requiring the

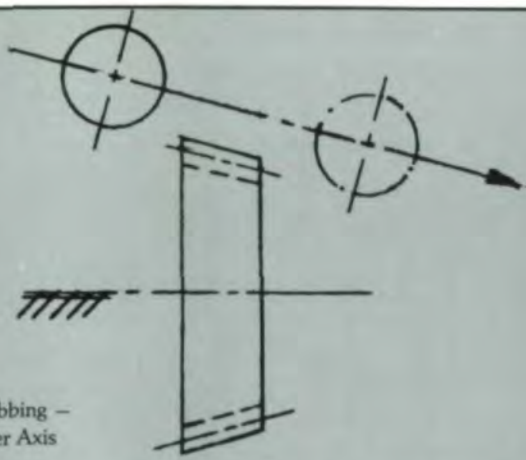

Fig. 15 – Taper Hobbing – Tilt Cutter Axis

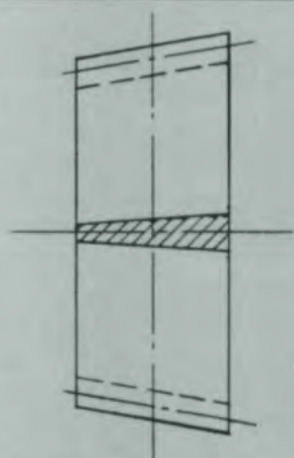

Fig. 16 – Spur Taper Gear

Fig. 17 – Helical Taper Gear

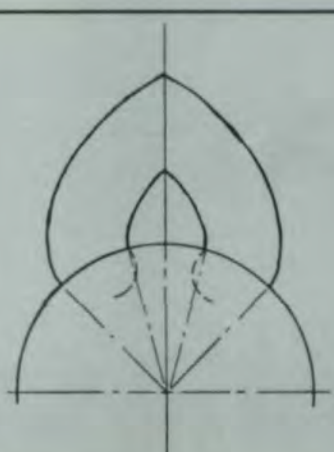

Fig. 18 – Limit Geometry

added degree of freedom in the machine tool. The helix may be obtained by oblique orientation or by supplemental timing.

TAPER SHAPING. By employing a circular gear type cutter in place of the rack for generation, the same requirements and relations as in hobbing apply. However, the resultant taper gear will be substantially, but not exactly, the same as its hobbed equivalent.

Taper Gear Geometry

BASIC (SPUR) GEOMETRY. The basic geometry of the spur taper gear results in a complex involute helicoid. The tilt of the cutting tool path produces a reduced transverse pressure angle symmetrical on both sides of the tooth and a symmetrical base circle for both flanks. The tool traverse provides reduced tooth thickness in each cross section.

This uniform reduction is along a constant helix and results in a constant lead of the helicoid surface. It is evident that equal and opposite hand helix angles are produced. (See Fig. 16.)

BASIC (HELICAL) GEOMETRY. The basic geometry of the helical taper gear results in a compound involute helicoid. The tilt of the cutting tool path in addition to the helix generation produces non-symmetrical flanks on the teeth and results in different base circles for each side.

The opposing geometric influences, the conventional helix generation with symmetrical parallel flanks and parallel leads, and the action resulting from the taper produces non-symmetry, the result of which is the compound helicoid.

On one flank the action of the taper produces an increased helix angle and reduced lead, and on the other flank it decreases the helix angle and increases the lead. (See Fig. 17.)

LIMIT GEOMETRY. The limit of a taper gear is identical to that of any involute of a circle constrained by an opposing involute of opposite directional orientation. The involute becomes pointed where the profile paths cross. (See Fig. 18.)

For the spur taper gear this crossover is equiangular from the center line of the tooth. In the case of a helical taper gear, there is no tooth symmetry, and the center of the tooth apex is the intersection of two opposing involutes struck from two different base circles. The involute angles are obviously different for each flank.

The other limit occurs at the base circle of the gear where generation originates. If the generating tool operates in a zone not defined by the involute, it produces a degeneration of the desired profile. This is the familiar undercut of involute gears with low tooth number and standard tooth proportions.

TAPER ANGLE. For intersecting drives, the taper angle may or may not be related to the ratio of the mesh. They operate as tapered cylindrical gears and are independent of cone angles. (See Fig. 19.)

For example, a 2:1 ratio set could consist of both gears with 45° cone angles, or one could be 30° and its mate 60°, or any other combination deemed suitable. There are, of course, some preferred approaches, but anything is possible. There is no requirement that cone angles intersect at a common apex. This allows multiple takeoffs from a common gear at various angles. (See Fig. 20.)

Taper gears operate on pitch cylinders not pitch cones. It is obvious that as cone angles increase, the relative face width usable must decrease for a given number of teeth, since the limit conditions of apex and undercut are met at a faster rate of change.

HELIX ANGLE. Infinite selection of helix angles is also permissible in cross axes orientation so long as the sum is correct. For parallel axis operation the taper provides a third variable for maximizing contact ratio and allows reduction in face width for equivalent loading to a conventional helical gear.

The combination of high cone angle and high helix angle provides a unique design opportunity, since the high helix increases the virtual number of teeth and allows increased cone angle without exceeding limits of apex and undercut.

CENTER DISTANCE MATCH. The taper gear has the conventional advantage of employing slight changes in helix angles to provide a given center distance while employing standard tools and tooth proportions.

Taper gears provide even greater advantage by allowing axial change of position to accomodate variations in center distance or for adjustment of backlash in over- or undersize centers.

It would not take a great deal of imagination to envision automatic means of takeup from thermal variations or even adjustment based on the load environment.

The minimum secondary benefit of the taper gear is that it provides for manufacturing variation without compromising the mesh or, conversely, allows greater latitude in tolerancing both gears and housings.

CONTACT. Each spur section of the taper gear is conjugate to the generating rack and contacts the rack continuously during its rotation. Hence, the taper tooth is conjugate to the generating rack. Contact between the taper gear tooth and the basic rack occurs along a straight line common to the rack and the taper tooth, and this contact line is inclined against the pitch plane of the rack. (See Figs. 21-22.)

If two taper gears are meshed at a shaft angle equal to the sum of the generating angles, a hypothetical rack surface of zero thickness may be assumed as existing between the meshing gears. This hypothetical rack surface meshes with both component parts which are contacted along two straight, non-parallel lines on opposite sides of the rack surface. At the point of intersection of the two contact lines, simultaneous contact exists between each taper gear and the rack, and, therefore, also between the two taper gears.

If the rack surface is ignored, it may be concluded that mating gears of this character which mesh at non-parallel axes are conjugate to each other, but contact only at a point which travels, as the gears rotate, on the tooth surfaces and through space. If the cone angle is small, the tapered gears approach spur gears, and the contact approaches line contact. (See Fig. 23.)

Contact may range from line contact with a rack or parallel axis mounting to point contact on cross axes similar to so-called spiral gears. Separation of pitch planes is possible, providing all the leeway for matching centers and ratios inherent in those gears, with the additional feature of backlash takeup.

CROWNING. In common with all involute helicoids, the line of contact is inclined across the face of the rack. Full face contact is obtained by parallel mounting in an anti-backlash mode. Angular mesh provides a meshing angle equal to the sum of the taper angles, and the contact lines are inclined to each other. These lines are straight line elements representing contact with the rack, but provide theoretical limited contact at their intersection. In effect the tooth profile is crowned in both the profile and lead directions.

Judicious use of mismatch in crowning can provide all the desirable characteristics of controlled crowning for deflection, mismatch, or load compensation, enabling smooth transition from no-load to load and avoiding the harmful effects of heavy end bearing.

Taper Gear Features

COMMONALITY. All gears generated from the same basic rack have a common normal base pitch and are, therefore, conjugate to each other no matter what the taper inclination or helix angle of an individual gear.

UNIVERSALITY. With unlimited angle selection for providing motion control between any two places in space at any ratio, these gears have the most universal application of any motion transmission device extant. In parallel applications optimized involute length and helical overlap provide for maximized power in a given face width.

INTERCHANGEABILITY. Taper gears are interchangeable without requirement for matching or provision for pairs or sets. Because of variation insensitivity, the only results of mismatch are slight backlash differences which can be compensated for by axial shift. Off-the-shelf gear replacement is possible even in the most demanding application.

Taper gears are subject to the same inspection procedures used for spur and helical gears. They can be inspected for all elements, such as involute, lead, spacing, runout, and pitch, as well as for composite operation with single or double flank inspection.

NOISE REDUCIBILITY. In parallel gears all the parameters for successful reduction of dynamic variations are available for optimizing. High profile contact ratio, helical overlap, and variable addendum with progression from all-recess to all-approach action, provide the tools from pursuing minimum noise design. Cross-axis application tends to be naturally quieter as a consequence of less dynamic variation due to the natural crowning effect.

MESH INSENSITIVITY. The three-dimensional curvature of the taper gear

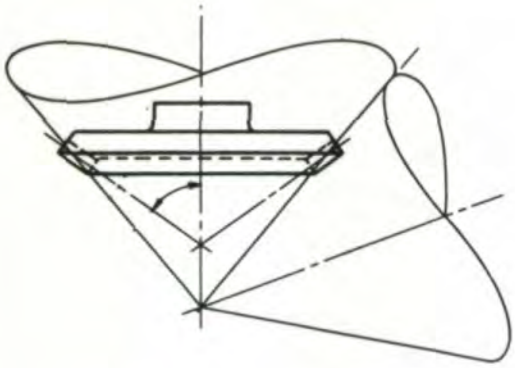

Fig. 19 – Cone/Taper Independence

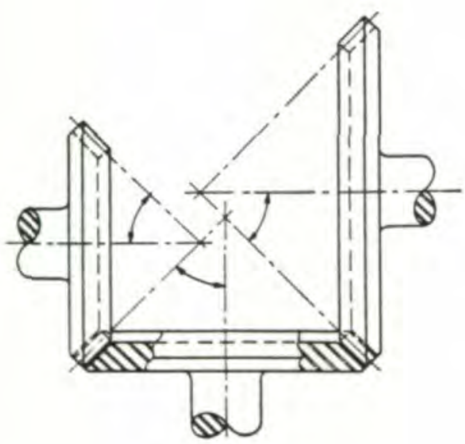

Fig. 20 – Angle Independence

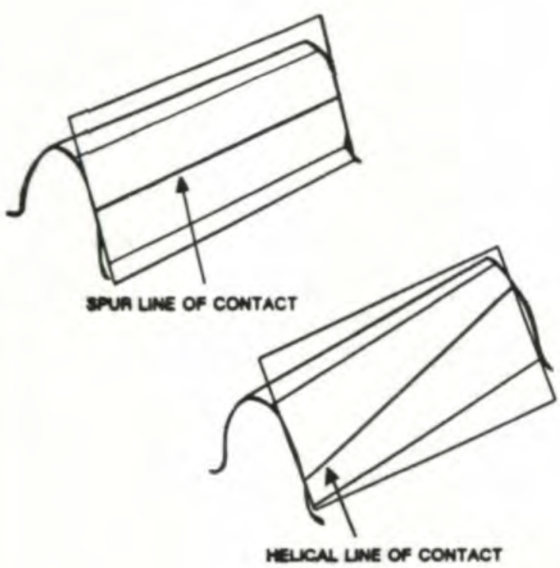

Fig. 21 – Line of Contact – Spur/Helical

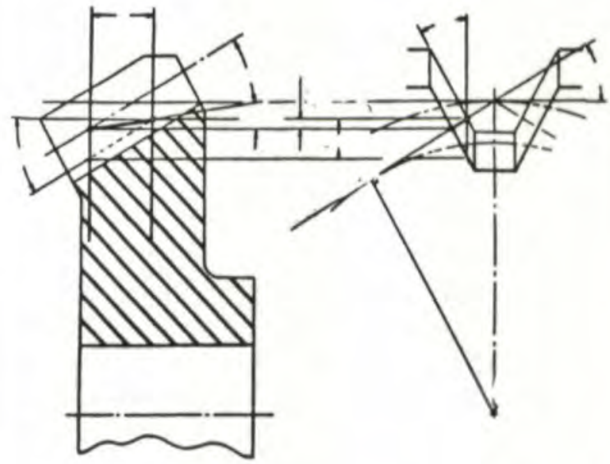

Fig. 22 – Line of Contact – Taper Gear

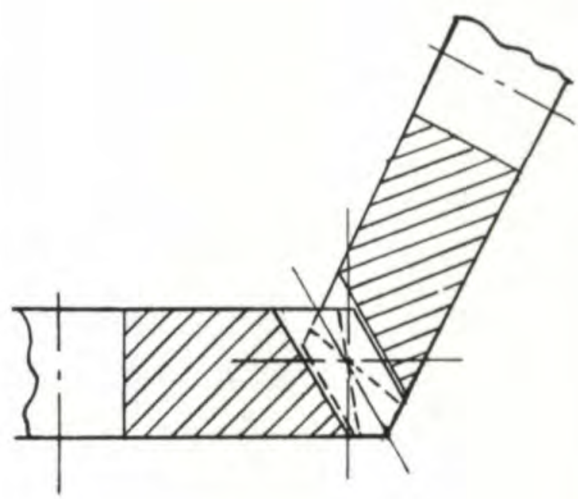

Fig. 23 – Line of Contact – Non-parallel Axes

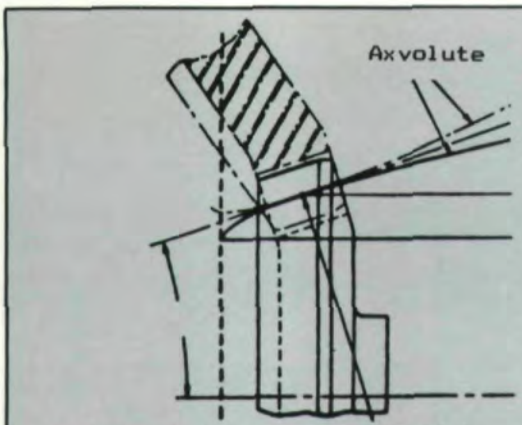

Fig. 24 – Angular Insensitivity – Axvolute Mesh

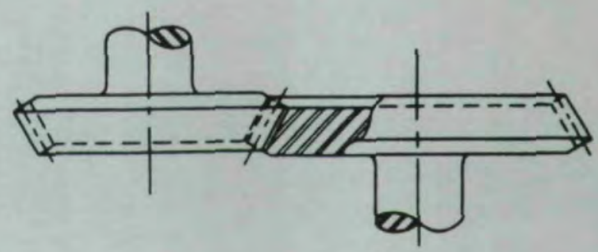

Fig. 28 – Parallel

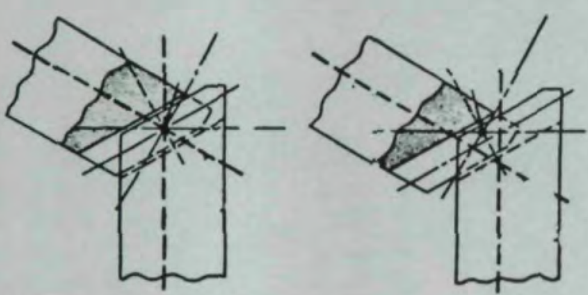

Fig. 25 – Position Insensitivity

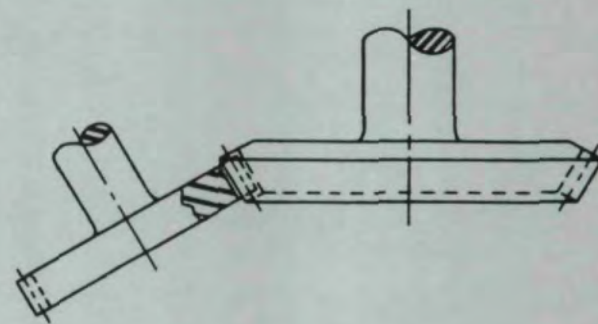

Fig. 29 – Spur

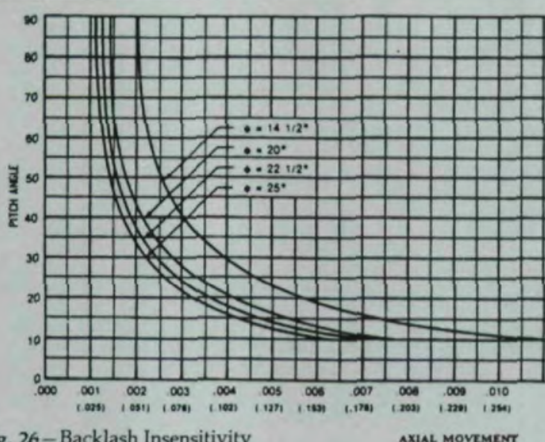

Fig. 26 – Backlash Insensitivity

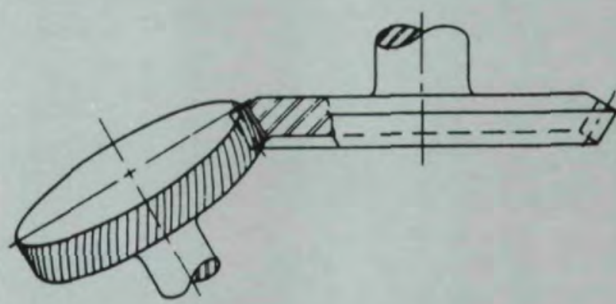

Fig. 30 – Helical

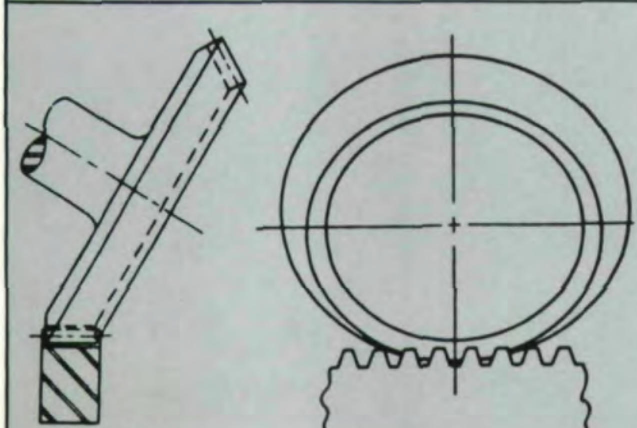

Fig. 27 – Rack

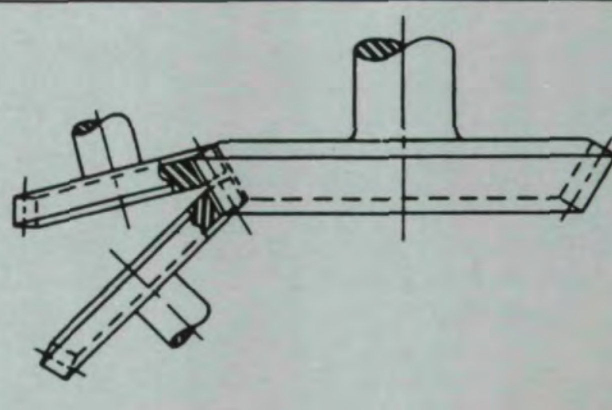

Fig. 31 – Multiple

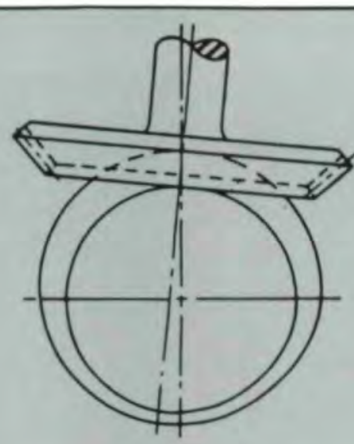

Fig. 32 – Skew

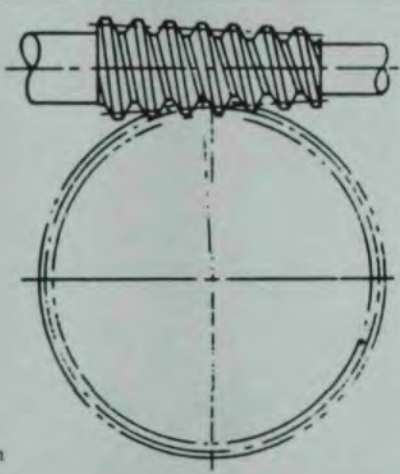

Fig. 33 – Taper Worm

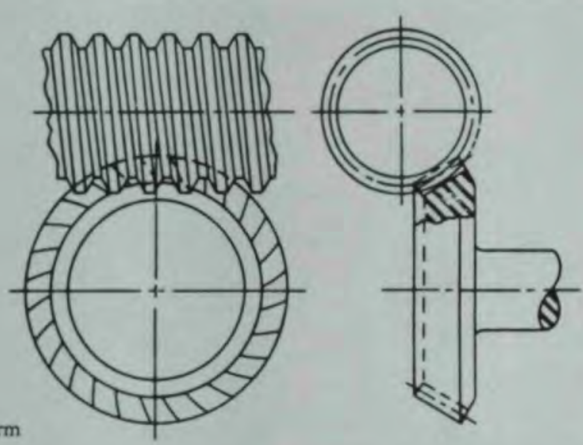

Fig. 34 – Worm

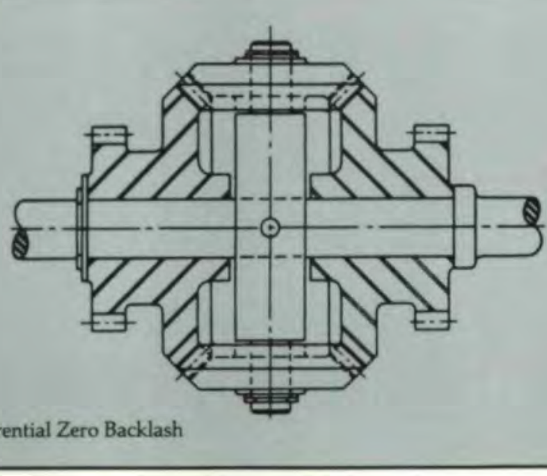

Fig. 35 – Differential Zero Backlash

tooth results in a remarkable ability to resolve angular misalignment, axis skew, deflection, twist, and positional mismatch without affecting conjugate action. The only requirement for mesh is a common base pitch. (See Fig. 24.)

Positional mismatch is limited only by the tight mesh condition, which can be relieved by a simple axial shift of either member. (See Fig. 25).

BACKLASH CONTROL. An outstanding feature of taper gears is their ability to be set for minimum backlash in any mode by axial adjustment of one member to take up play, without affecting center distance or mesh integrity. For parellel-axis mode, the taper angle can be selected to provide any degree of sensitivity. (See Fig. 26.)

Precision differentials have been constructed to provide zero backlash and essentially zero lost motion transfer between input and output shafts. (See Fig. 34.)

UNLIMITED ORIENTATION. Taper gears can be employed on intersecting or non-intersecting axes, parallel or non-parallel, and any angle of orientation. (See Figs. 27-35.)

Conclusion

Given the remarkable geometric properties accruing from this simple conceptual change in basic gearing fundamentals, combined with the availability of axis-synchronized machine tools, the taper gear provides a new tool to the general gearing industry.

Note: Taper gears are generally referred to as "Beveloids" in the literature, however, this a registered trademark of Invincible Gear.

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1. BEAM, A.S. "Beveloid Gearing." *Machine Design*, Dec, 1954.
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Our thanks to MR. WILLIAM L. JANNINCK for assistance with the technical editing of this article.

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Limitations of Worm and Worm Gear Surfaces in Order to Avoid Undercutting

Dr. Vadim Kin
Purdue University
Hammond, IN

Abstract:

The dimensions of the worm and worm gear tooth surfaces and some of the worm gear drive parameters must be limited in order to avoid gear undercutting and the appearance of the envelope of lines of contact on the worm surface. The author proposes a method for the solution of this problem. The relations between the developed concept and Wildhaber's concept of the limit contact normal are investigated. The results of computations are illustrated with computer graphics.

Basic Kinematic Equations

Investigation of Undercutting of Spatial Gears. The investigation is based on the following equations and theorems that have been proposed by Litvin.[1,2]

$$\underline{v}_r^{(1)} + \underline{v}^{(12)} = \underline{0} \tag{1}$$

$$\frac{d}{dt}(\underline{n} \cdot \underline{v}^{(12)}) = \frac{d}{dt}[f(u,\theta,\phi)] = f_u\frac{du}{dt} + f_\theta\frac{d\theta}{dt} + f_\phi\frac{d\phi}{dt} = 0 \tag{2}$$

where: $\underline{v}_r^{(1)}$ is the velocity of motion of the contact point over the worm surface, $\underline{v}^{(12)}$ is the sliding velocity, $\underline{n}$ is the worm surface unit normal, u and θ are the worm-surface curvilinear coordinates, and ϕ is the generalized parameter of motion. Equations 1 and 2 yield the following equations

$$\begin{vmatrix} \frac{\partial x_1}{\partial u} & \frac{\partial x_1}{\partial \theta} & v_{x_1}^{(12)} \\ \frac{\partial y_1}{\partial u} & \frac{\partial y_1}{\partial \theta} & v_{y_1}^{(12)} \\ \frac{\partial f}{\partial u} & \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial \phi}\omega \end{vmatrix} = \begin{vmatrix} \frac{\partial x_1}{\partial u} & \frac{\partial x_1}{\partial \theta} & v_{x_1}^{(12)} \\ \frac{\partial z_1}{\partial u} & \frac{\partial z_1}{\partial \theta} & v_{z_1}^{(12)} \\ \frac{\partial f}{\partial u} & \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial \phi}\omega \end{vmatrix} = \begin{vmatrix} \frac{\partial y_1}{\partial u} & \frac{\partial y_1}{\partial \theta} & v_{y_1}^{(12)} \\ \frac{\partial z_1}{\partial u} & \frac{\partial z_1}{\partial \theta} & v_{z_1}^{(12)} \\ \frac{\partial f}{\partial u} & \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial \phi}\omega \end{vmatrix} = 0 \tag{3}$$

Here:

$$\underline{r}_1(u,\theta) = x_1(u,\theta)\,\underline{i}_1 + y_1(u,\theta)\,\underline{j}_1 + z_1(u,\theta)\,\underline{k}_1 \tag{4}$$

are the equations of the tool surface Σ_1 and (u,θ) are the

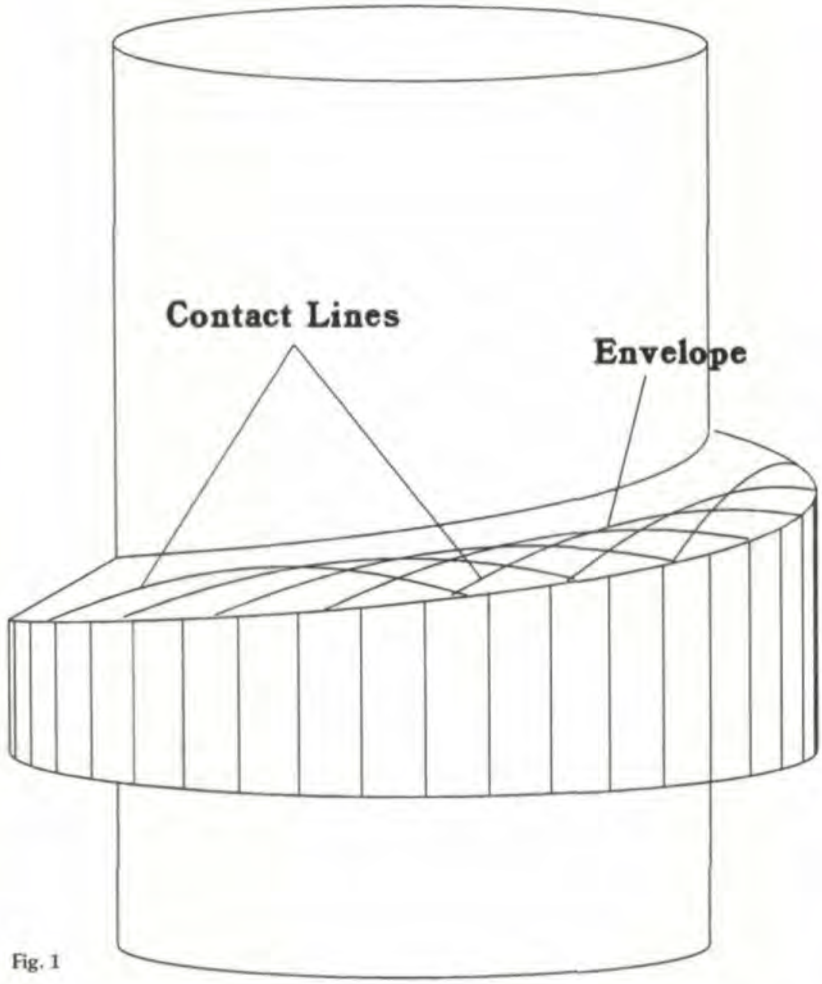

Fig. 1

curvilinear surface Σ_1 coordinates. Surface Σ_1 is a regular surface, and

$$\underline{n} \cdot \underline{v}^{(12)} = f(u,\theta,\phi) = 0 \quad (5)$$

is the equation of meshing with ϕ as the generalized parameter of motion. (One may chose that $\phi \equiv \phi_1$ and $\frac{d\phi}{dt} \equiv \omega^{(1)}$ where ϕ_1 is the angle of rotation of the tool.) The sliding velocity $\underline{v}^{(12)}$ is represented by

$$\underline{v}^{(12)} = (\underline{\omega}^{(12)} \times \underline{r}) - (\underline{R} \times \underline{\omega}^{(2)}) \quad (6)$$

where $\underline{\omega}^{(12)} = \underline{\omega}^{(1)} - \underline{\omega}^{(2)}$; $\underline{r}$ *is the position vector of the* instantaneous contact point M that is drawn from the line of action of the sliding vector $\underline{\omega}^{(1)}$ to M; $\underline{R}$ is the position vector that is drawn from the origin of $\underline{r}$ to any point of the sliding vector $\underline{\omega}$.[2]

Equations 3 yield the relation

$$F(u,\theta,\phi) = 0 \quad (7)$$

Equations 5, 6, 4, and 7 determine a line L on surface Σ_1 that generates singular points on surface Σ_2. We call L the limiting line because if Σ_1 is limited with L, singular points on Σ_2 do not appear.

Envelope of Contact Lines on the Worm Surface. The envelope of lines of contact on surface Σ_1, if it exists, is determined by the following equations:

$$\underline{r}_1 = \underline{r}_1(u,\theta),\ \underline{n} \cdot \underline{v}^{(12)} = f(u,\theta,\phi) = 0$$

$$q(u,\theta,\phi) = \frac{\partial f}{\partial \phi}(u,\theta,\phi) = 0 \quad (8)$$

Fig. 1 shows an envelope of contact lines on the surface of an involute worm. The existence of an envelope on Σ_1 is not desirable because a part of the worm surface without contact lines is without meshing, and the conditions of heat transfer and lubrication in the area close to the envelope are not favorable. For these reasons, the existence of the envelope of contact lines must be avoided. This can be done by choosing the appropriate design parameters for the gear drive.

Instead of the envelope E on surface Σ_1, an envelope of contact lines on the plane P of surface curvilinear coordinates (u, θ) might be considered (Fig. 2). Both envelopes,

AUTHOR:

DR. VADIM KIN *completed his graduate studies in mechanical engineering at the University of Illinois at Chicago in the summer of this year. He is presently a lecturer at the Purdue University – Hammond, IN, campus.*

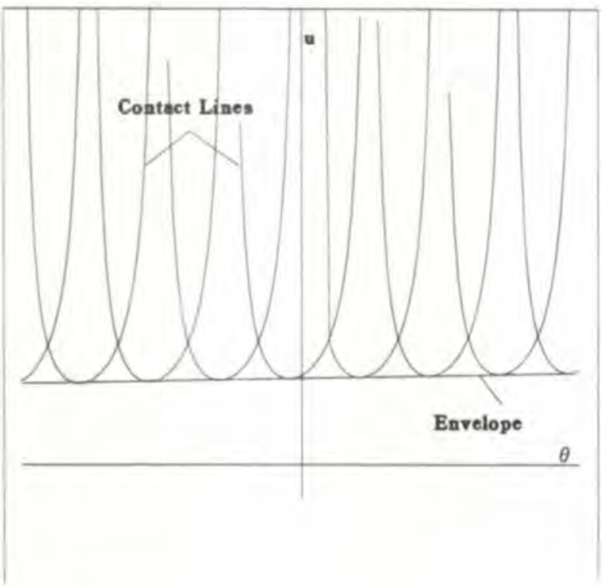

Fig. 2

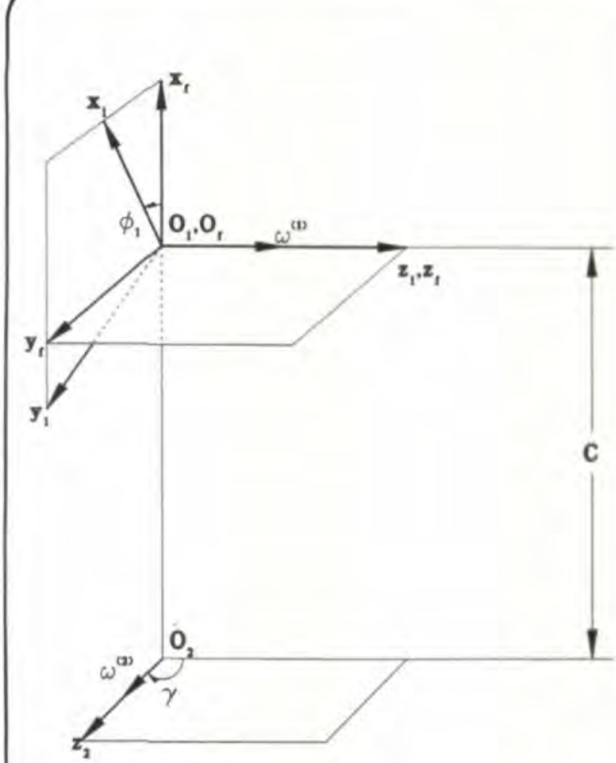

Fig. 3

if they exist, appear simultaneously on Σ_1 and on P. It is easy to verify that at the point of the envelope, the direction of the velocity of contact point in its relative motion over surface Σ_1, $\underset{\sim}{v}_r^{(1)}$ cannot differ from the tangent to the envelope. This means that $\underset{\sim}{v}_r^{(1)}$ is equal to zero in any direction that differs from the common tangent to the contact line and the envelope.

Applications to the Involute Worm Gear Drive

The proposed approach is applied to the case of an involute worm gear drive. The goal is to determine the design parameters with which the appearance of the envelope of contact lines on the worm surface Σ_1 and the appearance of singular points on Σ_2 can be avoided. Worm tooth surface Σ_1 is a screw involute surface represented in coordinate system S_1 rigidly connected to the worm by the following equations:[1]

$$\underset{\sim}{r}_1 = \begin{bmatrix} r_b\cos\theta + u\cos\lambda_b\sin\theta \\ r_b\sin\theta - u\cos\lambda_b\cos\theta \\ p\theta - u\sin\lambda_b \end{bmatrix} \tag{9}$$

where u and θ are the surface curvilinear coordinates, r_b and λ_b are the base cylinder radius and the lead angle on this cylinder. The screw parameter ($p>0$ for a right-hand thread) is $p = r_b\tan\lambda_b$. Equation 9 works for both side surfaces if u is considered as an algebraic value. The surface Σ_1 unit normal is represented by the equations

$$\underset{\sim}{n}_1 = \frac{\underset{\sim}{N}_1}{|\underset{\sim}{N}_1|},\quad \underset{\sim}{N}_1 = \frac{\partial \underset{\sim}{r}_1}{\partial u} \times \frac{\partial \underset{\sim}{r}_1}{\partial \theta}$$

that yield

$$\underset{\sim}{n}_1 = \sin\lambda_b\sin\theta\ \underset{\sim}{i}_1 - \sin\lambda_b\cos\theta\ \underset{\sim}{j}_1 + \cos\lambda_b\ \underset{\sim}{k}_1 \tag{10}$$

Equation of Meshing for Worm Gear Surfaces. A hob that is identical to the worm generates the worm gear tooth surface. The meshing by cutting of the hob with the to-be generated worm gear simulates the meshing with the worm gear in the drive. Coordinate systems S_1, S_2, and S_f are rigidly connected to the worm, the worm gear, and the frame, respectively. (Fig. 3) The equation of meshing is represented as follows:

$$\underset{\sim}{n}_1 \cdot \underset{\sim}{v}_1^{(12)} = f(u,\theta,\phi_1)$$

$$= u - p\theta\sin\lambda_b + \cot(\theta+\phi_1)\,(r_b\cos\lambda_b + E\cot\gamma\sin\lambda_b)$$

$$- \left(p\,\frac{1 - m_{21}\cos\lambda}{m_{21}\sin\gamma} - E\right)\frac{\cos\lambda_b}{\sin(\theta+\phi_1)} \tag{11}$$

where ϕ is the angle of rotation of the worm, γ is the twist angle of the worm gear axes (Fig. 3), and $m_{12} = \frac{\omega^{(2)}}{\omega^{(1)}}$ is the gear ratio. The worm gear tooth surface is represented by

$$[r_2] = [M_{21}]\,[r_1],\ f(u,\theta,\phi_1) = 0 \tag{12}$$

where the 4x4 matrix $[M_{21}]$ describes the coordinate transformation in transition from S_1 to S_2.

Envelope of Contact Lines on Σ_1. The envelope of contact lines on Σ_1 is determined by the equations

$$\underset{\sim}{r}_1(u,\theta),\ f(u,\theta,\phi_1) = 0,\ \frac{\partial f}{\partial \phi_1} = 0 \qquad (13)$$

The envelope of contact lines on the plane of parameters (u,θ) is represented by the equations

$$f(u,\theta,\phi_1) = 0,\ \frac{\partial f}{\partial \phi_1} = 0$$

that yield

$$\cos(\theta + \phi_1) = \frac{r_b + E\cot\gamma\tan\lambda_b}{p\,\dfrac{1 - m_{21}\cos\gamma}{m_{21}\sin\gamma} - E} \qquad (14)$$

It is easy to verify that the envelope exists if $|\cos(\theta+\phi_1)| \leqslant 1$. The appearance of an envelope of contact lines may be avoided by appropriately chosen design parameters. For a one-thread worm the parameter is the twist angle γ, and for an orthogonal ($\gamma=90°$) worm gear drive it is the number of threads, i.e., the lead angle λ_b. Fig. 4 shows that the contact lines on Σ_1 do not have an envelope in the working space in the case of a two-thread worm with the lead angle $\lambda_b=21.68°$, $\gamma=90°$. We emphasize that the pattern of contact lines favors the conditions of lubrication and efficiency of the worm gear drive.

Singular Points on Σ_2. The investigation of the singularity of Σ_2 is based on application of Equations 3. Fig. 5 shows the limiting line L on the plane of parameters (u,θ). The working space of the worm must be limited with $L(u,\theta)$ to avoid the appearance of singular points on Σ_2.

In the case of the worm gear drive, the envelope of contact lines and the limiting line usually do not appear simultaneously. However, in some particular cases these two lines may have a common point as shown in Fig. 6. The computations and drawings correspond to the case of a three-threaded worm gear drive with the following parameters:

$$m_{21} = \frac{3}{25},\ \gamma = \frac{\pi}{2},\ E = 150,\ r_b = 40.29,\ \lambda_b = 16.59°$$

The common point of both lines appears in the non-working space of the discussed example.

Relations Between Concepts of Line Contact Envelope, Singularity of Σ_1, and Wildhaber's Concept of Limit Pressure Angle

Wildhaber's concept of the limit pressure angle has been developed on the basis of scientific conditions of force transmission by gear tooth surfaces.[4,5,6] However, Wildhaber's equations may be and should be interpreted geometrically, and this can be done on the basis of the concept of the envelope of contact lines and the concept of

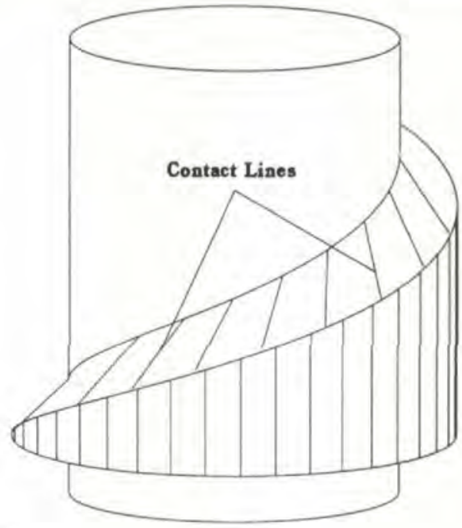

Fig. 4

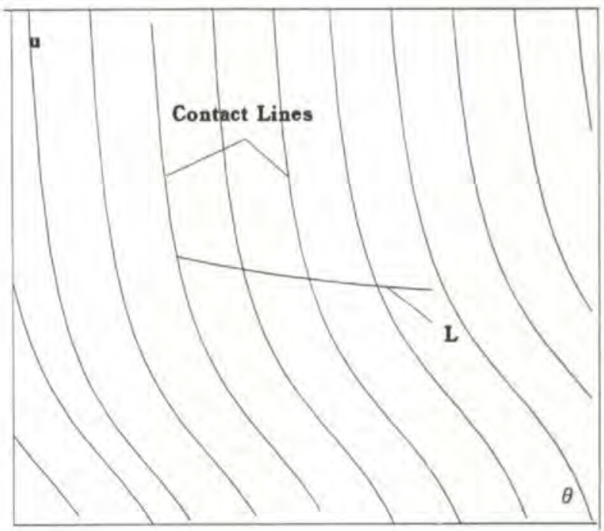

Fig. 5

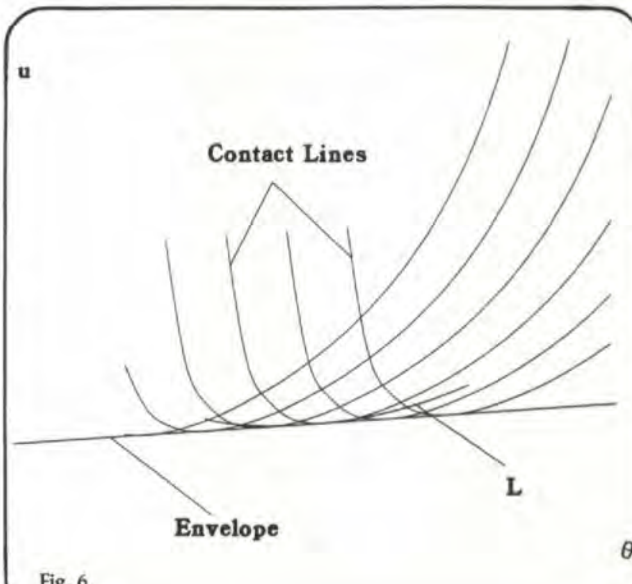

Fig. 6

singularity of generated surface Σ_2. Consider the equation of meshing that is represented by

$$\underline{n} \cdot \underline{v}^{(12)} = \underline{n} \cdot (\underline{\omega}^{(12)} \times \underline{r} - \underline{R} \times \underline{\omega}^{(2)}) = 0 \qquad (15)$$

The equation of meshing is observed in the neighborhood of the contact point, and, therefore, we have

$$\frac{d}{dt}(\underline{n} \cdot \underline{v}^{(12)}) = 0 \qquad (16)$$

Let us differentiate Equation 15, assuming first that $\underline{v}_r^{(2)}=0$ and singular points on Σ_2 appear, and then $\underline{v}_r^{(1)}=0$, and an envelope of contact lines exists. We assume by differentiation that vectors $\underline{R}$, $\underline{\omega}^{(1)}$, $\underline{\omega}^{(2)}$, and $\underline{\omega}^{(12)} = \underline{\omega}^{(1)} - \underline{\omega}^{(2)}$ are constant. Generally, the differentiation of Equation 15 yields the following equation:

$$\dot{\underline{n}} \cdot \underline{v}^{(12)} + \underline{n}^{(i)} \cdot (\underline{\omega}^{(12)} \times \dot{\underline{r}}^{(i)}) = \qquad (17)$$

$$(\underline{n}_r^{(i)} + \dot{\underline{n}}_{tr}^{(i)}) \cdot \underline{v}^{(12)} + \underline{n}^{(i)} \cdot [\underline{\omega}^{(12)} \times (\underline{v}_r^{(i)} + \underline{v}_{tr}^{(i)})] = 0$$

Here:

$$\dot{\underline{n}}_{tr}^{(i)} = \underline{\omega}^{(i)} \times \underline{n}^{(i)}, \quad \underline{v}^{(12)} = \underline{v}_{tr}^{(1)} - \underline{v}_{tr}^{(2)}, \text{ and}$$

$$\underline{n}^{(i)} = \underline{n}^{(1)} = \underline{n}^{(2)} = \underline{n}$$

is the common contact normal. Considering the particular cases where $\underline{v}_r^{(2)}=0$ and singular points on Σ_2 appear; $\underline{v}_r^{(1)}=0$, and an envelope of contact lines exists, we receive from Equation 17 that

$$\underline{n} \cdot [(\underline{\omega}^{(1)} \times \underline{v}_{tr}^{(2)}) - (\underline{\omega}^{(2)} \times \underline{v}_{tr}^{(1)})] = 0 \qquad (18)$$

In addition, we have to consider that the contact point satisfies the equation of meshing (15). Equations 18 and 15, if satisfied, provide the conditions when Σ_2 has singularities or the envelope of contact lines on Σ_1 exists, or both singularities on Σ_2 and the envelope on Σ_1 exist simultaneously. The disadvantage of application of Equation 18 is that it is impossible to recognize which of the three above mentioned cases is observed. The direction of the contact normal $\underline{n}$ depends on two design parameters – the helix angle on the worm and the pressure angle. The application of Equations 18 and 15 may provide information about the limit pressure angle if the helix angle is considered as given.

Conclusion

Methods for determination of an envelope of contact lines on the generating surface and singular points on the generated surface have been developed and applied to the case of involute worm gear drives. A bridge between the developed theory and Wildhaber's concept of the limit contact normal has been established.

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Fundamentals of Bevel Gear Hard Cutting

Yogi Sharma,
Philadelphia Gear Corporation
King of Prussia, PA

Introduction

Some years back, most spiral bevel gear sets were produced as cut, case hardened, and lapped. The case hardening process most frequently used was and is case carburizing. Many large gears were flame hardened, nitrided, or through hardened (hardness around 300 BHN) using medium carbon alloy steels, such as 4140, to avoid higher distortions related to the carburizing and hardening process.

The use of a quench press can control, but not eliminate distortions. A lapping operation cannot remove runout, pitch error, profile error, and other errors caused by heat treatment distortions. It can only improve active tooth profile finish and tooth contact location, provided that gears did not have excessive errors during soft cutting or high distortions in heat treatment. As a matter of fact, overlapping sometimes does more harm than good on a bevel set.

Bevel tooth grinding was very limited due to cost and size.

Advancement in bevel generators, carbide technology, and many other factors allowed introduction of hard cutting in bevel gears. Initially, the process was limited to special requirements because of low carbide tool life, the need for frequent sharpening, and limited experience. But the picture changed dramatically after some time. The experience gained and the development of the CBN (Cubic Boron Nitride) tool made bevel gear hard cutting very effective from cost and quality viewpoints.

As with any process, hard cutting has its limitations and problems. A properly controlled process, starting from the design concept, good bevel generators, hard cutting tools, including sharpening fixtures, special machines, and trained work force, are a must for successful bevel hard cutting.

This article describes the process and steps required for spiral bevel hard cutting on small batches or in a jobbing atmosphere.

Process Description

Bevel hard cutting can be defined as an operation in which gear teeth flanks are finished by removing the stock allowance left during the soft or rough teeth cutting. The process is similar to gear grinding, with almost all the operations remaining the same, with the exception of tooth grinding, which is replaced by hard cutting.

A simplified manufacturing process sheet for a hard cut bevel gear will contain the following:

- Complete machining of gear blanks for teeth cutting – including various operations, such as turning, milling, drilling and tapping, etc.
- Bevel teeth cutting – consisting of teeth cutting, testing, and any tooth contact development with master or mate, teeth deburring, etc.
- Heat treatment – mostly case carburizing and hardening (nitriding, flame hardening, and induction hardening are rarely used for hard cut bevel gears).
- Finish machining – all machining operations required before teeth hard cutting, such as turning O.D./I.D. grinding, special machining, etc.
- Hard cutting – bevel gear arrives at hard cutting with most or all operations done. It is important that the mating part or master is available for testing purposes. Normally, the gear (member with higher number of

teeth) is finished first and pinion teeth are modified to get correct tooth contact along profile and length of tooth. The modification for length of contact is normally made by change of radius of curvature of the cutting blades. The location of tooth contact along the length is usually controlled by machine settings. The profile correction or modification can be made by different means depending on the type of machine or system used to cut bevel gears, size of gears, and pitch. On fine pitch gears, high pitch line bearing can be obtained by using cutter blades with modified profiles. On coarse pitch gears, profile modification can be made using taper shims. Once the tooth contact requirements are met, the gear teeth are finally checked for spacing and mounted on a gear checker.

- Final inspection – includes dimensional checks, magnaflux, and any other special requirements.

Advantages of Hard Cutting

• Higher power transmission by spiral bevel gears, as both members are case carburized and hardened and finished by hard cutting.

• Higher and predictable quality levels in gear teeth.

• A surface finish of 16 RMS or better.

• Lower noise level, lower internal dynamic forces due to higher geometric accuracy, and better load distribution. A hard cut bevel set significantly reduces the gear box vibration problem caused by higher internal dynamic forces due to poor quality gear teeth, which can cause premature gear box failure.

On the other hand the gear grinding operation is always very sensitive to many factors, such as rate of material removal, grinding wheel, coolant, etc. Any compromise or loose control can cause surface tempering or cracks or both. Surface tempering or cracks are practically unknown problems in hard cutting.

• Hard cutting is performed on the same type of machines as soft-cutting, making the process much more economical. Of course, special tooling is required.

• The same person or group of persons are involved in soft and hard cutting. It has been found that control in roughing operation (soft cutting in bevel gears) is quite important for a successful finishing operation. Proper stock allowance, tooth depth, tooth contact, etc. are necessary for bevel hard cutting. Too much stock allowance can cause loss of case depth and longer cutting times, while too little stock allowance also can cause a variety of problems. Hard cutting time cycles can be reduced by making some adjustments at soft cutting for heat treatment distortions, and it can be done very simply, as both operations are performed by the same person or group of persons.

• Consistency in bevel hard cutting can eliminate the need for matched bevel sets by careful planning. Some of the requirements for elimination of matched sets are as follows:

– Manufacturing and storing of case hardened and hard cut master gear and pinion for checking the gears and pinions in all future setups.

– Optimization of design and cutting data so that it does not have to change for the period unmatched sets are required.

– Tighter control of critical dimensions on gear/pinion blanks. Hard cut unmatched sets are not only useful in assemblies, but they can eliminate many production problems, as each member can be processed independent of others. The unmatched set approach must be used very selectively, as it needs careful planning, customized fixtures, optimized design, cutting summaries, and long term commitment. The unmatched set approach combined with standardized bevel sets can cut down delivery times and cost very effectively in spiral bevel gear boxes.

Preparation

Following are some items which should be considered in detail for successful and economical bevel hard cutting.

Practical Tooth Design. A balanced tooth geometry is a must for a good hard cut set. All new tooth geometry must be reviewed carefully from a manufacturing point of view. In a jobbing or low batch environment, the new tooth geometry should try to use existing tools, as new tool requirements can cause cost and delivery problems. Even in high batch production where tools can be designed around gears, poor tooth geometry can cause multiple problems, such as low tool life, smaller fillet radius, etc.

Gear Blank Design. Fig. 1 to Fig. 4 show a bored pinion, a stem spiral bevel pinion, a solid gear, and a ring type gear. As shown in Fig. 2, the bevel pinion can be indicated in both planes by means of an extra extension in front of the teeth. Both gears show proof bands for indicating the gear blank at hard cutting. Special attention must be paid in blank design so that revalidated proof surfaces at final machining can be used for indication purposes at hard cutting. In high batch production, blanks, truing, and use of proof surfaces are not required because of special customized fixtures. Still, it is good practice to create proof bands at final machining for inspection or assembly purposes. In the

AUTHOR:

YOGI SHARMA *is employed in gear manufacturing and design at Philadelphia Gear Corp. He holds an M.S. degree in mechanical engineering from Villanova University and one in industrial engineering from Penn State. Mr. Sharma is a licensed mechanical engineer in the state of Pennsylvania and a senior member of SME.*

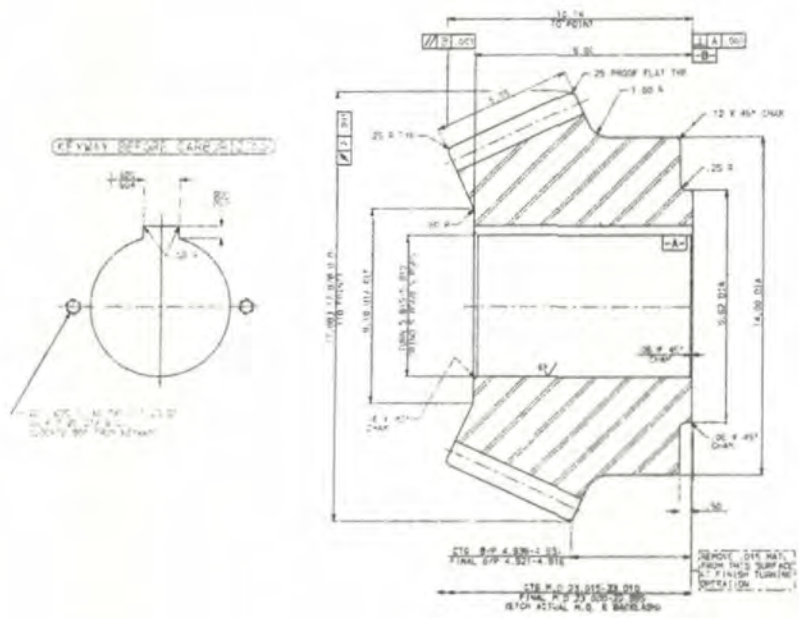

KEYWAY BEFORE CARBURIZING

Fig. 1

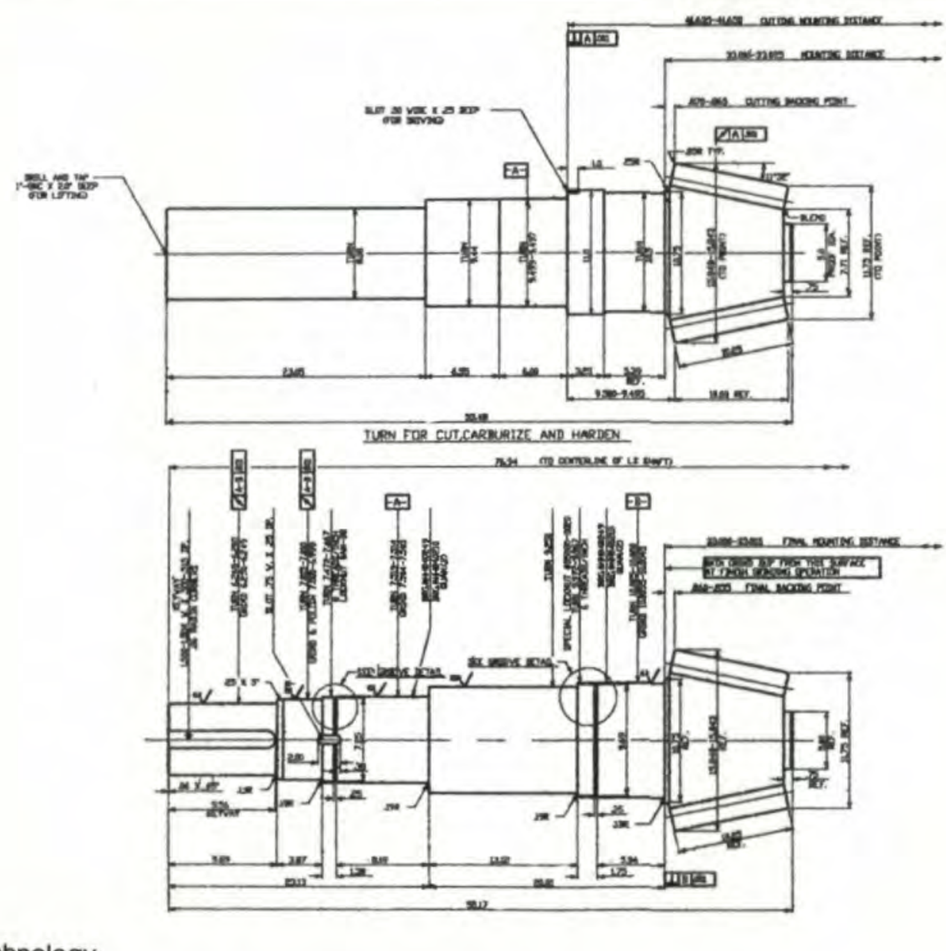

TURN FOR CUT,CARBURIZE AND HARDEN

Fig. 2

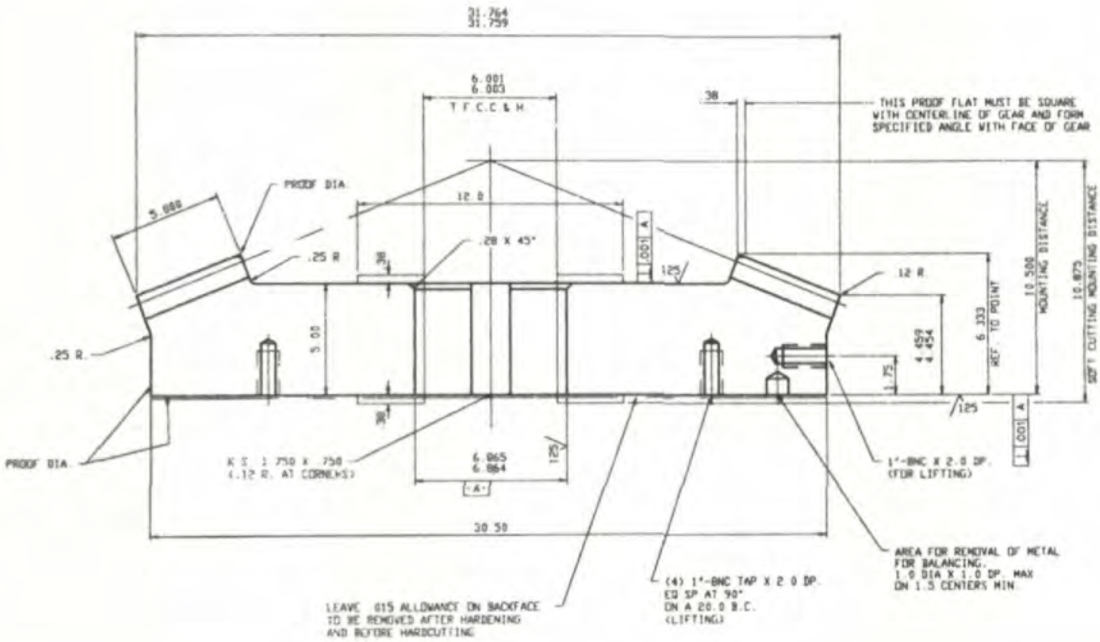

31.764
31.759
6.001
6.003
T.F.C.C.& H.
.38
THIS PROOF FLAT MUST BE SQUARE
WITH CENTERLINE OF GEAR AND FORM
SPECIFIED ANGLE WITH FACE OF GEAR
PROOF DIA.
5.000
12.0
.28 X 45°
.25 R
.12 R.
.25 R.
5.00
4.459
4.454
1.75
6.333
REF. TO POINT
10.500
MOUNTING DISTANCE
10.875
CUTTING MOUNTING DISTANCE
PROOF DIA.
K.S. 1.750 X .750
(.12 R. AT CORNERS)
6.865
6.864
-A-
30.50
1"-8NC X 2.0 DP.
(FOR LIFTING)
AREA FOR REMOVAL OF METAL
FOR BALANCING.
1.0 DIA X 1.0 DP. MAX
ON 1.5 CENTERS MIN.
(4) 1"-8NC TAP X 2.0 DP.
EQ SP AT 90°
ON A 20.0 B.C.
(LIFTING)
LEAVE .015 ALLOWANCE ON BACKFACE
TO BE REMOVED AFTER HARDENING
AND BEFORE HARDCUTTING

Fig. 3

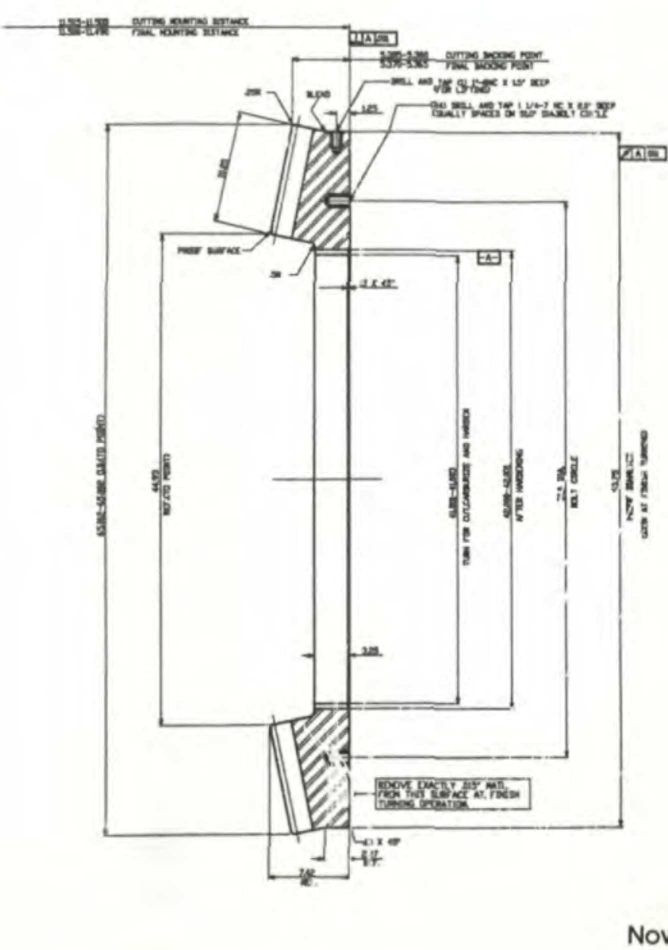


Fig. 4

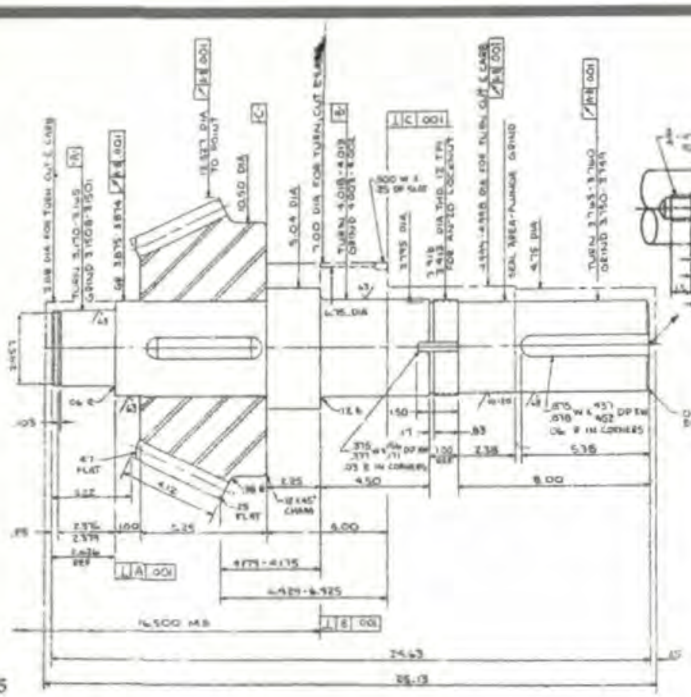


Fig. 5

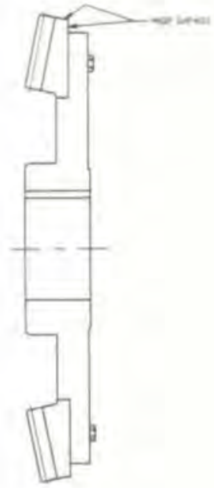


Fig. 6

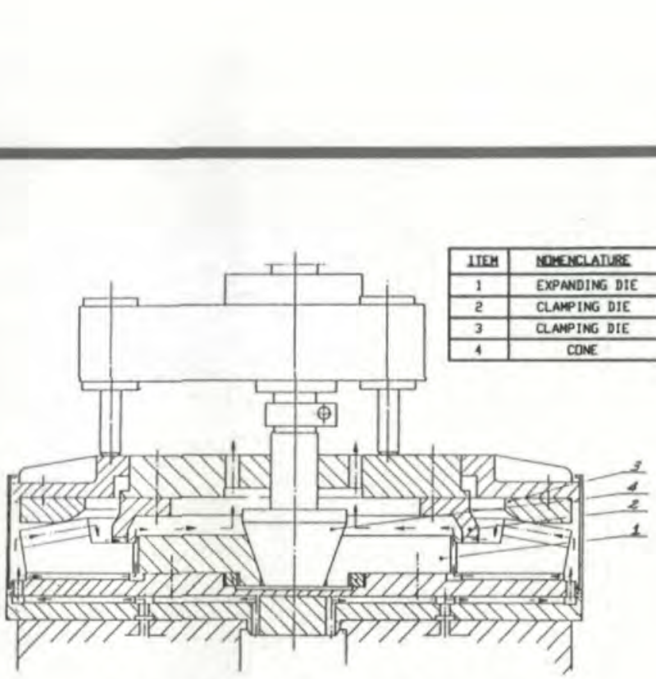
case of a bored pinion/gear, the subassembly with shaft should be done wherever possible to avoid concentricity and squareness problems (Fig. 5). Fig. 6 shows the subassembly of a ring gear and spider. In pinions, the one-piece design does offer some manufacturing advantages over the two-piece design. Also, small to medium size gears should be investigated for one-piece design, as there are some manufacturing advantages in the one-piece configuration.

Previously, gears were mostly designed as bored rings so that they could be die quenched, but hard cutting has changed the picture somewhat. Distortions in free quenching of small to medium size solid gears can be kept low with proper control. Hard cutting will remove the distortion from teeth. Caution must be used for any switch over from two-piece design to one-piece. There are many factors which affect distortions in heat treatment, such as material, blank configuration, fixturing in heat treatment, etc. Wherever possible, some experimental pieces should be manufactured to evaluate the situation before making the final decision, as excessive distortion is highly undesirable and can cause various problems at hard cutting.

Soft and Hard Cutting Summaries. A tooth cutting summary for a bevel generator provides machine set up data, cutter data, tooth measurements, and other tooth geometry information related to a specific spiral bevel set. Bevel hard cutting blades are very sensitive to proper feed and speed. Selection of correct amount of crowning stock allowance affects the overall quality and cutting times. Normally, in high batch production, a sample set or dummy set is always useful to fine tune the summary. In a jobbing or low batch atmosphere, a dummy set or experimental set may or may not be possible for various reasons; therefore, a jobbing atmosphere requires initial selection of all variables affecting tooth contact. The length and location of tooth contact should be based on factors, such as assembly conditions (overhung or straddle mounted members), tooth stiffeners, and loading, application, and past experience. Wherever possible, fine tuning of tooth contact can be achieved through load testing of bevel gears.

Heat Treatment Process Control. Excessive heat treatment distortions are highly undesirable for hard cutting of bevel gears for a variety of reasons, such as loss of case thickness, longer cutting times, tooth thickness problems, and mounting distance problems. Ring gears must be die quenched utilizing proper quenching dies. Fig. 7 shows a quench press. A pre-quench and temper operation of rough-turned blanks for large gears and pinions can assist in stabilizing the pieces during heat treatment. In the jobbing atmosphere, gears and pinions should be checked for distortion before releasing for final machining.

Finish Machining Operation. The finish machining of gears and pinions for hard cut bevel gears is very important. The runout and squareness and all other geometric tolerances of bearing diameters, proof diameters, etc. must be kept to less than 40% of the values allowed on the gear teeth. Any loose control in finish machining operations can add to distortion, leading to longer cutting cycles and various problems. It is vital that proof surfaces in both planes are indicated within specified values before proceeding with final machining. The big difference in the final machining of a lapped and a hard cut gear is that, in the event of a lapped set, the back face or mounting face is usually left as it is, while in hard cutting, it is ground or turned square to bore. In addition, it is critical that some extra material is left in the back of a hard cut



CROSS SECTION OF KLINGELNBERG QUENCH PRESS

Fig. 7

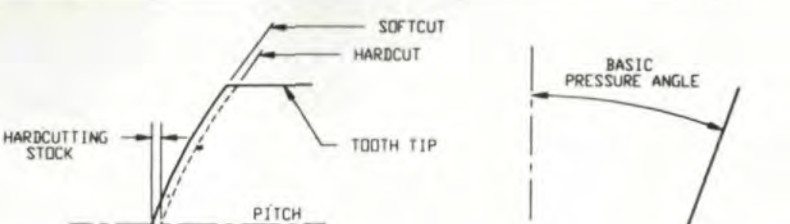


Fig. 8

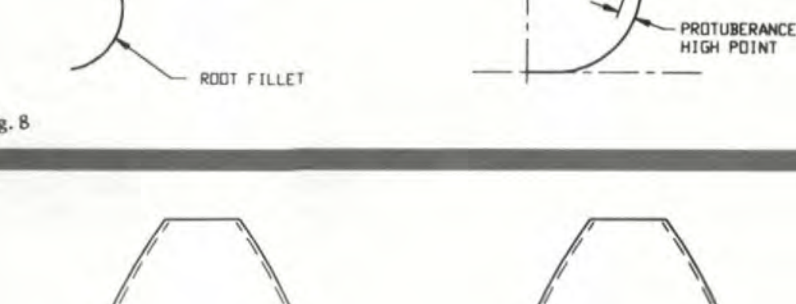


Fig. 9

gear at blank turning, which will then be removed at final machining. This will reduce the variations in mounting distances. In multiple pieces, consistency of the crown to back dimension is important for hard cutting cycles. Variation will cause adjustments for every piece on the machine, resulting in longer cutting cycles. Proper proof bands and surfaces must be created at final machining, with concern given to their location as well, so that they can be easily used for indication purposes at hard cutting.

Soft Cutting Blades. Fig. 8 depicts a blade with protuberance. Where ever possible, protuberance blades should be used. They will cut down hard cutting time, extend tool life, and reduce the possibility of steps in fillet. The use of customized protuberance blades is no problem in high batch production, but in jobbing or low batch production the picture is quite opposite. Amount and height of protuberance selection becomes difficult as blades are stretched to cover maximum range. Blades can still be obtained with protuberance selected very carefully, which will help hard cutting. Setting middle blades slightly deeper than side blades also helps in hard cutting and is a common practice in bevel cutting. Fig. 9 shows a fillet created with a deeper middle blade. Fig. 10 shows a blade for soft cutting.

Hard Cutting Blades. Bevel hard cutting began with the use of carbide inserts. Carbide inserts are clamped to steel bodies for coarse pitches and brazed for medium to fine pitch gears. In the beginning, low tool life with carbide inserts, frequent tool sharpening, insufficient experience, high cost, and many other factors kept hard cutting's use quite low. As hard cut bevel gears reached into the field, their performance in all aspects was the single most reason to increase the use of bevel hard cutting.

In the meantime, CBN inserts were introduced for hard cutting. The BZN (BZN is a trade mark of General Electric Company) compact blank is a combination of a layer of Borozon CBN (Cubic

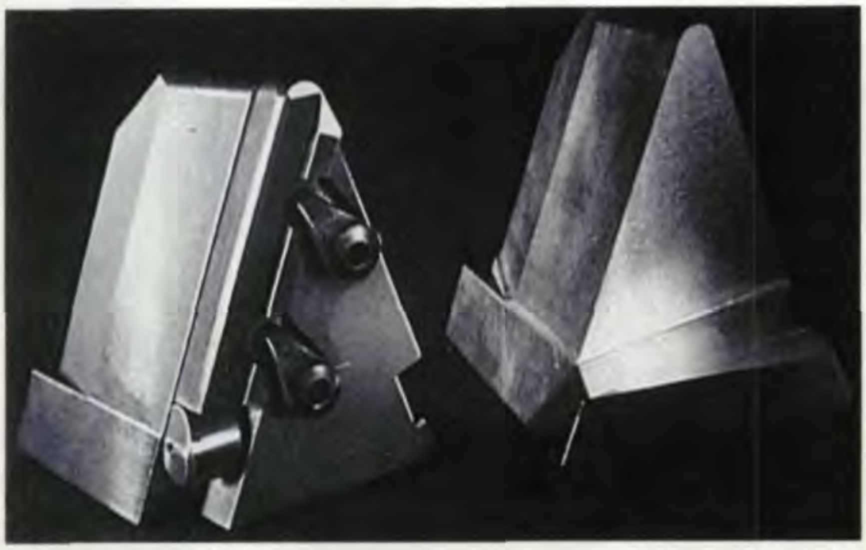


Fig. 10 — Courtesy of Klingelnberg Corp.

Boron Nitride) and a cemented tungsten carbide substrate produced as an integral blank using an advanced high pressure, high temperature process. The use of CBN inserts increased the tool life many times. Cutting times decrease appreciably as less sharpening is required. From a performance standpoint, the CBN inserts excelled considerably over the carbide inserts in every aspect, even though the price of the CBN insert is much higher than that of the carbide insert, tool price per gear did not rise because of improved tool life.

The CBN tools need much more careful sharpening, including the use of special machines, special fixtures, and diamond grinding wheels. The blades are sharpened on the cutting face. Extra care should be taken in storing, handling, and the use of CBN inserts. The correct feed, speed, and depth of cut is also very critical in the usage of CBN inserts.

Limitation of Bevel Hard Cutting

• Machine Accuracy. Bevel hard cutting and soft or rough teeth cutting are performed on the same or similar types of machines. The accuracy of the bevel generator reflects directly on the quality of gear teeth. Where more than one similar machine is available, it may be beneficial in the long run to use the better machine for hard cutting all the time.

• Single Indexing Versus Continuous Indexing System. Bevel generators with single indexing directly affect tooth spacing; whereas, continuous indexing offers significant accuracy in tooth spacing due to the natural hunting action between gear teeth and cutter blades.

• Multi-Start Cutter System. In cutting systems which utilize multi-start cutter heads, attention must be paid, so that the number of teeth is not divisible by the number of starts in the cutter head. Otherwise, the cutter head blade spacing can affect tooth spacing in hard cutting. In high batch production, this is never a constraint, as special cutter heads can be obtained for a set, but it can become a limiting factor in jobbing, where the exact ratio is a necessity. A multi-start cutter head can also be used as a single start cutter head, which will eliminate this problem; however, cutting times will increase.

• Fillet Finishing. In bevel grinding, the tooth fillets are completely ground, and the desired fillet radius is obtained by dressing the grinding wheel. In the bevel hard cutting process, the fillets are normally finished at soft cutting. Sometimes step problems may appear at hard cutting due to low or no protuberance, higher distortions, etc. The hard cutting blades are manufactured with a certain fillet radius, which should closely match with the roughing blade radius. During hard cutting, the fillet corners can be slightly skimmed with hard cutting blades, but any excessive fillet material removal compromises the tool life.

Summary

The hard cut bevel gears have performed very well in all applications. Hard cutting has improved load carrying capacity and has made quality bevel gears available to the industry at justifiable costs.

The improvement in large gears is even more noticeable as tooth grinding was not possible because of size. The introduction of CBN tooling has brought the tooling cost and cutting times to reasonable and economical values. Furthermore, consistency of hard cutting has made unmatched sets a reality under a controlled atmosphere and for certain applications. Hard cutting also provides gear teeth with the quality and surface finish of grinding without any possibility of any metallurgical damage. Finally, the utilization of CBN tools for hard cutting is being applied more frequently to parallel axis gears using shaper type machines.

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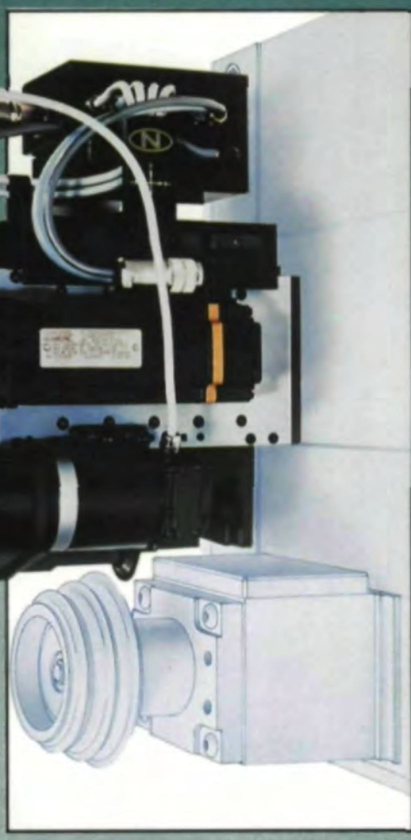

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