Methodology for Translating Single-Tooth Bending Fatigue Data to be Comparable to Running Gear Data

D.R. McPherson and S.B. Rao

Introduction

The gear industry is under continual pressure to increase power density and reliability of geared transmissions while at the same time reducing costs. To meet these demands, new materials and manufacturing processes are being evaluated on a continuing basis. The first step in this evaluation is to conduct screening tests to compare the performance of gears fabricated using the new materials and processes with that of gears manufactured using the incumbent materials and processes. However, once promising new materials and processes have been identified, the issue rapidly becomes one of developing accurate design data to permit effective utilization of these new materials and processes. The accepted practice in the gear industry is that accurate design data be derived from running gear tests.

Running gears can fail via a number of modes, many of which are shown generically in Figure 1. Screening tests are conducted in a manner that allows evaluation of performance relative to one of these modes while avoiding damage via the others. A common approach is to evaluate bending strength using the single-tooth bending fatigue test (STF) and surface durability using the rolling/sliding contact fatigue test (RCF). From the bending strength point of view, this ensures that tests intended to evaluate bending strength will not have to be terminated due to surface durability (pitting, wear or scoring) failures. The design of running gear test specimens to evaluate bending strength (or surface durability) requires a good initial estimate of bending strength so that the specimens can be designed to fail by the target failure mode at about the desired life without

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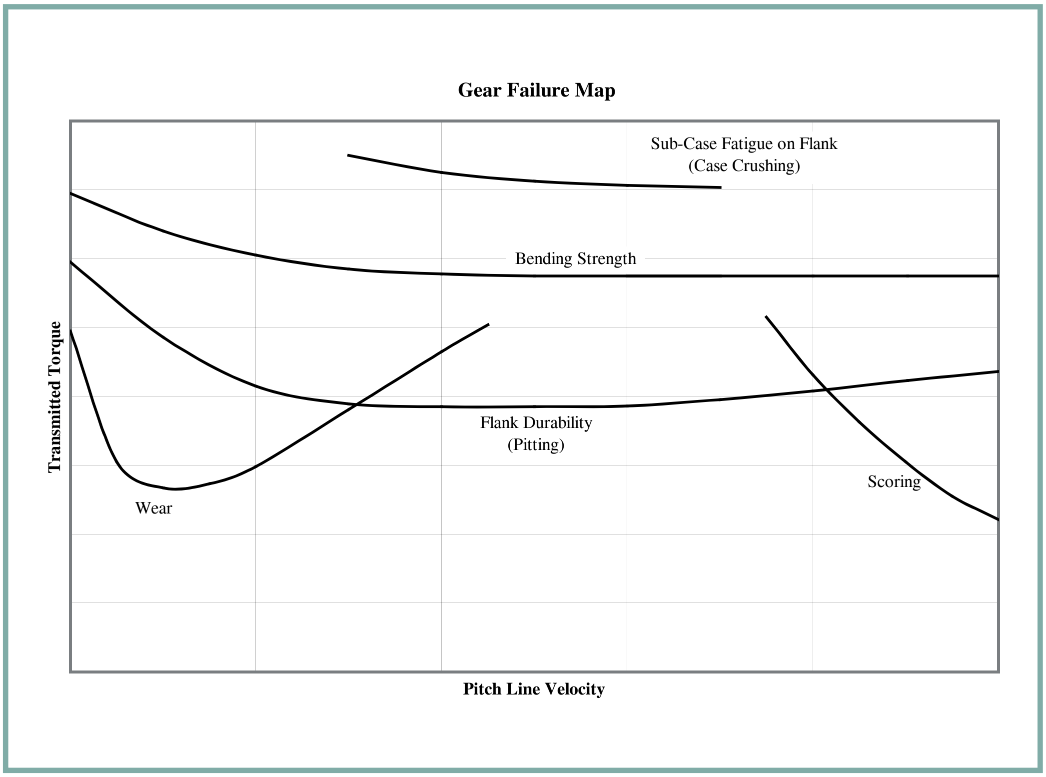

Figure 1—Gear failure map.

undue risk of failure by unwanted modes. This paper describes a method developed by the Gear Research Institute to extrapolate running gear bending strength data from STF results that is invaluable in comparing bending performance of different materials and processes. This methodology has also proven useful in the design of running gear bending strength test specimens. It is strongly recommended that data extrapolated by this or other similar methodologies should not be used as the basis for the design of gears for applications.

STF Tests Compared to Running Gears

In the STF test, a specimen gear is held in position and one tooth at a time is tested by applying a cyclic load normal to a fixed point on the flank. Care is taken in the selection of the load point and in the design of the loading appliance to ensure that the surface of the test tooth is not locally overloaded at the point of contact. Thus, the influences of all the surface durability aspects of testing running gears are eliminated, and tests may be continued as long as needed to achieve failure via bending. The cyclic load is varied between the selected maximum and some fixed percentage of that maximum (10% or 5%, depending on the compliance of the system) to maintain preload on the system. This limits the distance the hydraulic loading cylinder must travel and permits testing at comparatively high frequencies. A typical STF fixture, with specimen installed, is shown in Figure 2.

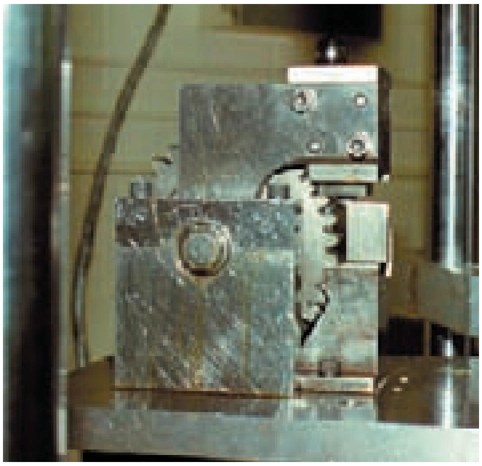

Figure 2—Gear Research Institute's standard STF specimen mounted in fixture.

These test conditions yield three categories of correlation issues that must be taken into account when translating STF test results to running gear test results. The easiest to explain is the correlation issue of the stress range experienced by running gear teeth compared to STF test teeth. As noted above, in the STF test, stress varies from 10% (or 5%) of the maximum up to the maximum. In running gears, the load is completely released as the tooth passes out of mesh. In some cases, depending on geometry and operating speed, the critical area in the root fillet is subjected to a small amount of compression as the next tooth is loaded. Thus stress varies from zero (or a small negative percentage of the maximum) to the maximum.

The next correlation issue is due to the fact that the teeth that will break on running gears represent only the weakest part of the statistical population defined by the teeth tested in the STF test. In the STF test, several (normally at least four, often eight, sometimes up to sixteen) teeth are tested on each gear, one at a time, and each represents a separate data point. In running gears all of the teeth are tested on each gear in the same time, and the failure of the first tooth represents failure of the gear. Much of this paper is dedicated to presenting a method to account for this difference.

The last correlation issue, and the most difficult to explain and quantify, is related to confidence. It is not economically feasible to conduct enough screening tests with new materials and manufacturing processes to be able to draw many statistically reliable inferences from the result. The object of most single-tooth fatigue testing is to determine the mean load resulting in failure at the run-out number of cycles. To make the best possible estimate of this load with a reasonable number of tests, recent test programs have been conducted using a two-load approach. Enough tests are conducted in an up-and-down sequence to find two loads that result in a non-zero and non-unity failure rate. In other words, at each load some of the tested teeth break by the run-out limit while others do not. Further tests are conducted until six have been completed at each of these loads. Ideally, one of the failure rates at these two loads will be above 0.5 and the other below 0.5. Analysis of the result will allow the load resulting in 50% failure at the run-out limit to be determined with some statistical reliability. The range within which the load to result in 50% failures could vary is illustrated by the confidence bands in Figures 5 and 9. In the test programs these figures are drawn from, tests were conducted following an up-and-down sequence and less than six tests

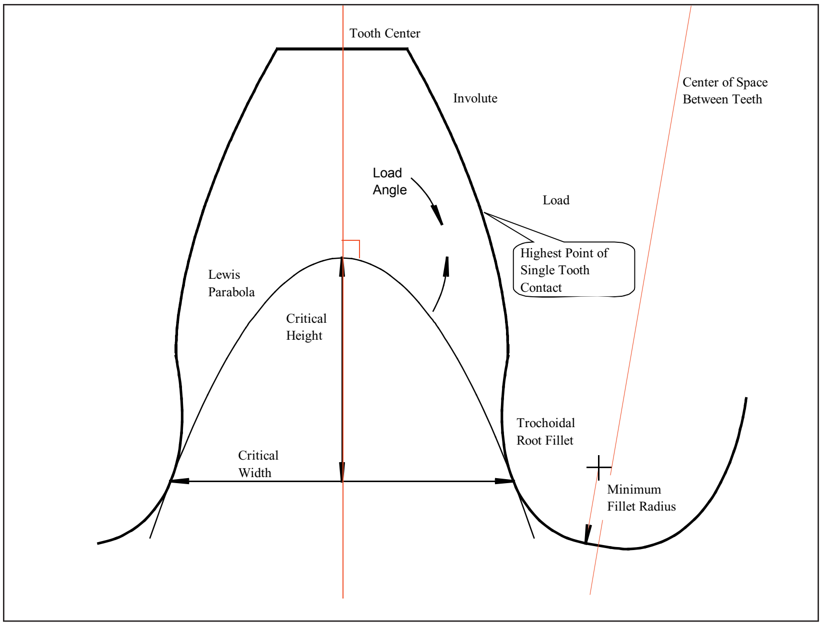

Figure 3—Layout of spur gear showing loading and Lewis parabola.

$$\text{Bending Stress} = \frac{\text{Load} \times \text{Cos}\left(\text{Load Angle}\right)}{\text{Face Width}}\left[\frac{6 \times h}{s^2} - \frac{\text{Tan}\left(\text{Load Angle}\right)}{s}\right]K_f$$

Equation 1.

$$K_f = \text{Stress Concentration Factor} = H + \left(\frac{s}{r}\right)^L\left(\frac{s}{h}\right)^M$$

Equation 2.

were conducted at most of the loads. If there were two loads at which six tests had been conducted, the range would correspond to an approximate 90% confidence interval.

What the result of this limited number of tests does not provide with any reliability is the standard deviation of the load that results in failure at the run-out limit. Methods are presented in the literature to determine standard deviation (Refs. 1, 2); however, to accomplish this with statistical reliability requires that two, four or more times as many tests be conducted. A good knowledge of the standard deviation is needed to adjust loads to account for different failure rates, such as are encountered with running gears where all of the teeth are subjected to loading compared to one tooth at a time with single-tooth fatigue testing. The empirical method presented here avoids this need for an accurate value of standard deviation.

Conversion of Load to Stress

Before comparisons can be made between tests with specimens having differing geometries—such as those used in STF and running gear tests—the applied test loads must be converted to bending stress. Figure 3 shows a spur gear tooth with a point load applied at the highest point of single-tooth contact. This point of loading corresponds to the highest bending stress when there is effective load sharing between gear teeth. Specimen gears used in rig tests should have effective load sharing, so this is the appropriate point of loading for determining bending stress for running gear specimens used in rig tests. For gears tested in single-tooth bending fatigue, the actual point of loading established by the test fixture is used in calculating bending stresses.

The Lewis parabola is drawn from the point the load line intersects the center of the gear tooth and is tangent to the root fillet. The methods used to lay out this parabola vary, depending on how the root form is generated, and the full particulars are lengthy and presented in detail elsewhere (Ref. 3). The critical height and width are determined from the Lewis parabola as shown in Figure 3. The angle between the load line and a normal to the tooth center is termed the load angle (it differs from the pressure angle at the point of loading because of the thickness of the tooth). The bending stress is thus:

See pg 44 for Equation 1

Where
s = Critical Width from Lewis Parabola
h = Critical Height from Lewis Parabola

See pg 44 for Equation 2

r = Minimum Fillet Radius
H = 0.331–0.436 x (Nominal Pressure Angle –Radians)
L = 0.334–0.492 x (Nominal Pressure Angle –Radians)
M = 0.261–0.545 x (Nominal Pressure Angle –Radians)

This equation for bending stress can be derived from first principles or from AGMA standards (Refs. 3, 4) by taking the forms of relevant formulas pertinent to spur gears and setting all design factors at unity. A similar formula could be developed for helical gears.

For a given gear design and loading condition, such as the Gear Research Institute's Standard STF specimen, loaded in its standard fixture, Equation 1 can be simplified to the following form:

$$\text{Bending Stress} = \text{Load x Stress Factor} \quad (2)$$

Where the stress factor comprises all the items on the right side of Equation 1 except load. For the Gear Research Institute's standard STF test, this becomes:

Face Width	1.000 inch
Load Angle	24.8 degree
h	0.286 inch
s	0.335 inch
K_f	1.53
Stress Factor	19.3 psi bending stress per pound load

Correlation Issues

The three correlation issues discussed earlier are now treated in detail.

Correction for Allowable Stress Range. In the Gear Research Institute's standard STF test, the load is varied from 10% to 100% of the maximum load. These R = 0.1 stresses are converted to the required R ratio stresses via ASR diagrams. The ASR diagrams are constructed to be representative of brittle materials following the method described in Reference 5. The pertinent equations are as follows:

See pg 45 for Equation 3

See pg 45 for Equation 4

(Ultimate stress is taken as the bending stress corresponding to the linear deviation point load from the fast bend single overload test.)

$$Y = \frac{1 - \dfrac{\sigma_M}{\sigma_U}}{1 + \dfrac{\sigma_M}{\sigma_U}} \quad (5)$$

$$\sigma_R = \text{Fully Reversed Stress} = \frac{\sigma_A}{Y} \quad (6)$$

These equations can be algebraically manipulated to yield an expression for any desired R ratio stress. Strain gage calibration with the Gear Research Institute's standard running gear bending test specimens show that the stress varies from negative 20% to positive 100% when the gears are tested at standard operating speed (Ref. 6). Thus, R loading for the examples shown here is equal to negative 0.2. By way of example, an allowable

$$\sigma_A = \text{Alternating Stress} = \frac{\text{Maximum Stress} - \text{Minimum Stress}}{2}$$

Equation 3.

$$\sigma_M = \text{Alternating Stress} = \frac{\text{Maximum Stress} - \text{Minimum Stress}}{2}$$

$$\sigma_U = \text{Ultimate Stress}$$

Equation 4.

stress range diagram constructed in the manner described above is shown for the running gear G50 stress for the first example set of data.

Statistical Analysis—Accounting for Differing Populations. The statistical step from failures of individual teeth to failures of gears is made using a probability diagram comparing maximum applied test load (abscissa) and failure rate in terms of the variate of a probability distribution (ordinate). The exact nature of the scales to be used on this diagram is not intuitively obvious. It is customary to make the scale on the life axis of stress-life diagrams logarithmic; however, the scale on the stress axis may be either linear or logarithmic. Based on this precedent, the scale on the load axis for the diagram to be constructed here could be either linear or logarithmic. The Weibull distribution is frequently used to characterize fatigue; however, it is customary to use the normal probability distribution to analyze failure rates at the fatigue endurance limit. Thus, there are four reasonable sets of scales that could be used on this diagram.

ANSI/AGMA 2001-C95 gives a table of reliability factors to relate allowable stress and various failure rates (Ref. 4). The values in this table represent experience with gears and show the magnitude of difference in applied stress to result in progressively lower failure rates. The reliability factor appears in the denominator of the rating equations. Thus the reciprocal of the table values is proportional to the stress difference associated with the difference in failure rate. These are failure rates for gears each having a definite number of teeth. The STF results are for tests of individual teeth. To be helpful in determining the scales to be used on the diagrams constructed here, this information must be converted to failures rates of teeth. The Gear Research Institute's standard specimen that has been used in running gear tests has 18 teeth, making this a convenient number of teeth-per-gear for the calculations presented here. In this case, one failure in two gears tested corresponds to one failure in 36 teeth tested

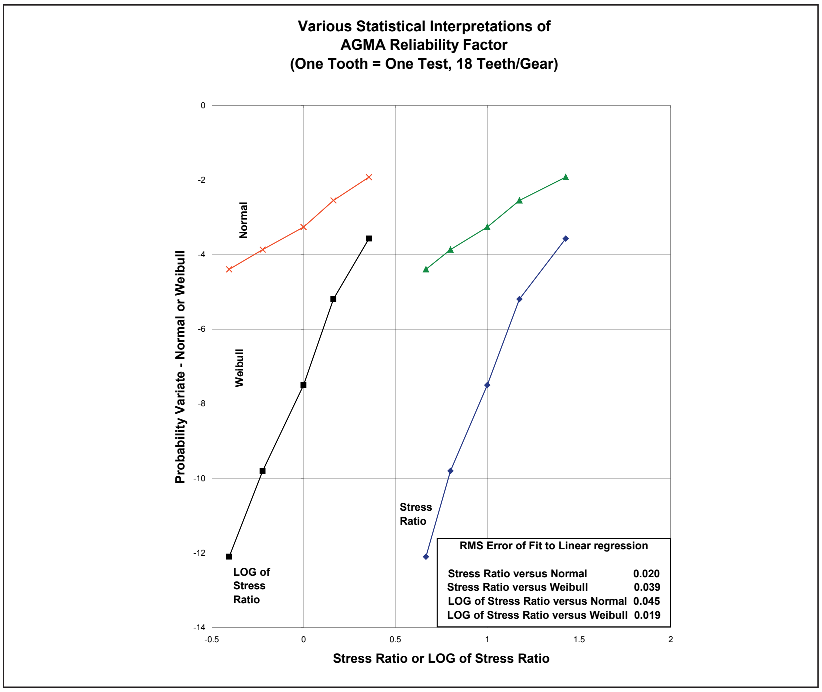

Figure 4—Various statistical interpretations of AGMA reliability factor (1 tooth = 1 test, 18 teeth/gear).

and the corresponding normal probability variate is negative 1.916 (contrasted to considering failures of gears where one failure in two gears tested results in a failure rate of 0.5 and a normal probability variate of 0.000).

Figure 4 shows the reciprocals of the values from Table 11 in Reference 4 scaled four ways. The abscissa shows these reciprocals plotted on a linear scale and a logarithmic scale. The ordinate shows the failure rate values divided by 18 to represent failures of individual teeth, plotted against normal probability variate and Weibull probability variate. Normal probability variate is x from Equation 7, and is shown as "x" in normal probability tables.

$$\text{Failure Rate} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt \qquad (7)$$

Weibull probability variate is given by Equation 8.

$$w = Ln\left(Ln\left(\frac{1}{1-\text{Failure Rate}}\right)\right) \qquad (8)$$

The relationships in Figure 4 that are closest to linear are the ones with the logarithmic stress scale versus Weibull probability and the linear stress scale versus normal probability. Both have roughly equally good fit to a linear regression. Since normal probability has been used frequently to characterize failure rates at the fatigue endurance limit, the charts presented here were based on normal probability. The best fit with reliability factors and normal probability was with the linear stress (or load, since it is proportional to stress) scale; thus, the diagrams presented here use these scales.

Maximum test load is used in constructing the diagrams presented here. This is an artifact of the manner in which the method was developed; load could just as easily be converted to stress before the statistical analysis as after (as is done here). Test results are sorted by load and the failure rate is determined at each load. These results are plotted in terms of normal probability variate value corresponding to the failure rate at each load tested. These points are plotted as hollow diamonds in the sample diagrams presented later in this paper. The normal probability variate corresponding to 100% failures is positive infinity, that corresponding to 0% failures is negative infinity. In order to keep the scale of the diagram reasonable, these values are plotted as positive three and negative three, respectively, and not used in fitting a line to the data representing a mixture of failures and no failures. Because of this, it is necessary to have data from at least two loads that resulted in a mixture of failures and no failures.

The step from failures of individual teeth to failures of gears is based on fitting a line through these data points. When a line is fit to data using a least squares fitting technique, it can be shown that one point on the line will always be the average point of the data. The average point of the data is noted mean failure point on the charts presented here, and is plotted as a solid diamond. The abscissa for this point is defined as the average maximum load in tests at loads that resulted in a mixture of failures and no-failures. The ordinate is defined as the NPV value corresponding to the average failure rate in these tests. The tests are summed individually to account for different numbers of tests at each load. A line is fit through the mean failure point and is extrapolated down to normal probability variate equal to negative 1.916, which corresponds to one failure in 36 teeth or one failure in two 18-tooth gears to find the 50% failure load corresponding to running gears. The exact slope of this line cannot reasonably be determined with the limited data typically available; the method used to fit it is discussed in the following section.

Statistical Analysis—Accounting for Confidence. In most instances, three to six teeth were tested at each load. Thus the confidence in the ordinate values is limited. An approximate confidence interval is constructed based on the limiting normal probability variate values if one additional test were to be conducted at each load. The upper bound represents what the failure rate would have been if the additional test failed, the lower bound that if it did not fail. When all teeth tested at a given load fail, the corresponding normal probability variate value would be positive infinity. In most cases, no more than six tests are conducted at loads that result in 100% failure. Thus, if one more tooth were tested that did not break by the designated run-out limit, the failure rate would be six of seven and the corresponding limiting (minimum) normal probability variate value would be on the order of positive one. As noted previously, infinite values of normal probability variate are not useful for fitting lines to the data; however, the limiting values at the bottom of the range (or top of the range if several teeth are tested at a common load and all do not fail) are useful, as described below.

For each set of STF test results, two statistical diagrams are constructed. The first (labeled Step One) is focused on the load range used in STF tests. A line is fit by eye judgment through the mean failure point with a slope that seems to fit the data, and the 50% failure load is determined from this line. Standard deviation of the mean test load for the STF condition (sigma) is the reciprocal of the slope of this line. Given the size of the confidence intervals at each point (see Figures 5 and 9), it is clear that standard deviation cannot be estimated within a factor of two with any statistical reliability, given the number of tests conducted. Rather than attempt to extract standard deviation from too little data, it is assumed that sigma (for the STF test condition) is a fixed percentage of the 50% failure load. Based on examination of as many STF data sets as possible, with tests conducted over a span in excess of 10 years, this value for sigma is taken as 10% of the 50% failure load.

A second statistical diagram (labeled Step Two) is then constructed for each set of STF test results. A wider range of maximum loads is included in this diagram, and it is used to find the 1% failure and/or minus three-sigma load. A line is drawn through the mean failure point at the slope determined in Step One. This line is labeled Mean Fit—Load versus Failure Rate. A second line is drawn parallel to the first located to encompass all (or most) of the confidence intervals for each data point. (In some cases, the confidence intervals diverge further below the mean fit line than they do above. In these cases, the second line is drawn as far above the first as one would have to be drawn below it to encompass the confidence intervals.) This second line is labeled Conservative Fit—Load versus Failure Rate. The idea behind using two lines is to attempt to untangle scatter inherent in fatigue test results from change in failure rate with changing load, and to ultimately make a consistently conservative estimate of 1% failure and/or minus three-sigma bending strength.

The 50% failure load for running gears is selected from the Mean Fit—Load versus Failure Rate line. As noted previously, with the Gear Research Institute's eighteen-tooth specimen, 50% failure corresponds to one failure in 36 teeth tested, and the NPV is negative 1.916. Thus, the 50% failure load for running gears is the point on the Mean Fit—Load versus Failure Rate line at NPV equal to negative 1.916. The 10% failure load for running gears is selected from the Conservative Fit–Load versus Failure Rate line. With the 18-tooth specimen, 10% failure corresponds to one failure in 180 teeth tested, and the NPV is negative 2.54. Thus, the 10% failure load for running gears is the point on the Conservative Fit—Load versus Failure Rate line at NPV equal to negative 2.54.

Many industries consider the design condition to be 1% failure. With 18-tooth specimens, this is one failure in 1,800 teeth tested, and the corresponding NPV value is negative 3.26. The load corresponding to 1% failure is found by drawing a line through the loads selected for 10% and 50% failure with running gears and picking off the value at x equal negative 3.26. The aerospace industry considers the design condition to be minus three-sigma (i.e., one failure in 740 odd parts tested). With 18-tooth gears this is one failure in 13,333 teeth tested, and the corresponding NPV value is negative 3.79. The load corresponding to minus three sigma is found by drawing a line through the loads selected for 10% and 50% failure, with PC gears and picking off the value at x equal to negative 3.79.

Analysis Method—Step by Step

The first task in the analysis is to sort the data by load and find the failure rate at each

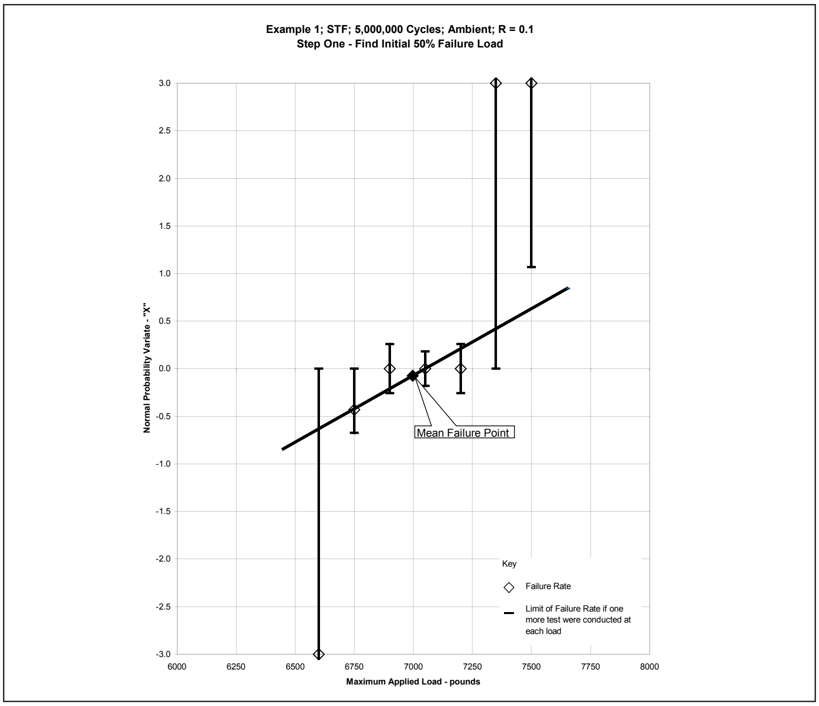

Figure 5—Example 1: STF; 5,000,000 cycles; ambient; R = 0.1. Step 1—Find initial 50% failure

Table 1 — Example 1 (Carburized) Data			
Maximum Load (Pounds)	Number of Tests	Number of Failures	Failure Rate
7,500	6	6	1.000
7,350	1	1	1.000
7,200	4	2	0.500
7,050	6	3	0.500
6,900	4	2	0.500
6,750	3	1	0.333
6,600	2	0	0.000

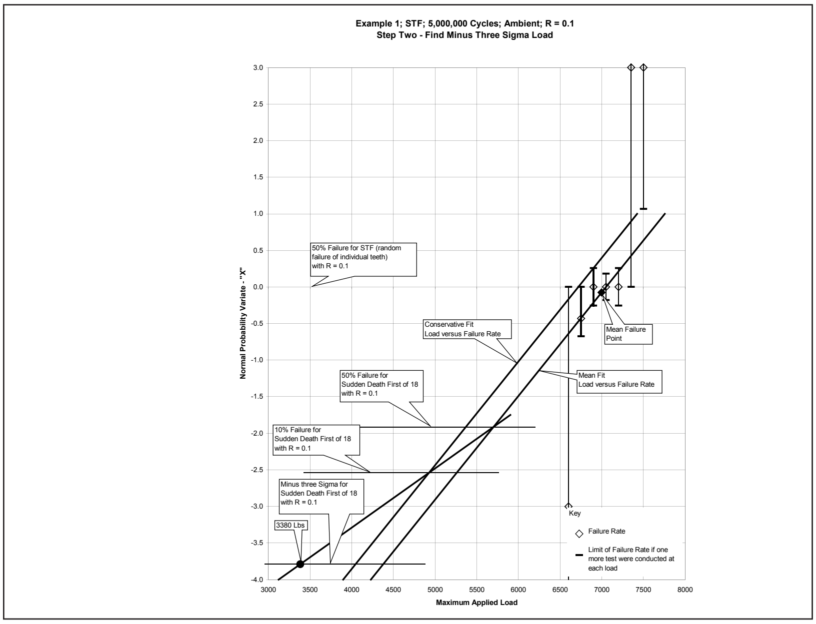

Figure 6—Example 1: STF; 5,000,000 cycles; ambient; R = 0.1. Step 2—Find minus 3 sigma load.

load. These failure rates are plotted in terms of normal probability variate versus load as hollow diamonds on the chart previously described. The mean failure point (average of all the tests at loads that resulted in a mixture of failures and no failures) is plotted as a solid diamond. The confidence ranges are calculated for each load and plotted as lines for each load. A line is fit through the mean failure point to suit the data and stay within the limits for each load. This line is used to determine the load corresponding to 50% failures. The slope of the lines on the second probability diagram is the reciprocal of 10% of this load.

A second probability diagram is drawn with the data points along with the mean failure point, and confidence ranges for each data point. The Mean Fit—Load versus Failure Rate is drawn through the mean Failure Point at the slope determined above. The Conservative Fit—Load versus Failure Rate line is drawn at the same slope to encompass all/most of the confidence ranges as previously described. The load corresponding to 50% failures is picked from the mean fit line, the load corresponding to 10% failures is picked from the conservative fit line, and the load corresponding to the desired design condition is selected by fitting a line through these two points and extrapolating to the required normal probability variate, all as described previously.

The loads are converted to stresses. The R equal 0.1 "running gear" stresses corresponding to 50% failures, 10% failures and design condition are adjusted to account for the stress range anticipated with running gears using allowable stress range diagrams as previously described. A stress-cycles diagram is constructed. A STF 50% failures (G50) curve is fit through the stress corresponding to 50% failure at the run-out limit determined in the statistical analysis and the rest of the data points using the best method available. For the limited data in the following examples, this is by eye. If more data were available, Weibull analyses could be conducted at several loads and used to define the finite life portion of the curve more precisely. This curve is moved linearly downward to the adjusted stresses at the run-out limit to represent 50% failures, 10% failures, and design condition for running gears. The full procedure is illustrated with the following two examples.

Example 1—Case Carburized Gears. The STF specimen gears used in the program the first set of sample data was taken from are Gear Research's standard pattern; hence the factor for converting load to stress is 19.3 psi bending stress per pound of load. The test results are summarized in Table 1.

Figure 5 shows the first statistical diagram constructed from this data. This diagram is used to select the load that corresponds to 50% failures; in this case the value is 7,050 pounds, which appears intuitively obvious from an examination of Table 1. The reciprocal of the slope of the fit lines shown in Figure 6 is taken as 10% of this value. Figure 6 shows the statistical step from STF to running gear data. In this case, the selected slope of the fit lines appears to fit the data very well. The translated R = 0.1 running gear G50 maximum load is 5,700 pounds, which corresponds to 110 ksi maximum bending stress. The allowable stress range diagram shown in Figure 7 shows the adjustment of this stress to R = negative 0.2 stress, which is 94.9 ksi.

Figure 8 shows a stress cycles diagram with STF and running gear data. An approximate STF G50 curve was fit by eye judgment to the data, starting with 136 ksi (corresponding to 7,050 pounds load) at 5 million cycles. The other curves were located by calculating the stress at five million cycles, as described above, and moving the entire curve linearly down to that point. The exact shape of the translated running gear curves below five million cycles cannot be accurately determined with the limited data available, so this method was adopted as the simplest expedient. Bending results from running gear tests are shown as hollow squares. These tests were conducted at extremely high overload to ensure that bending failures occurred rather than surface durability failures. STF tests were not conducted at high enough loads to directly compare with these results.

Given the paucity of data, it appears at first blush that the translated stresses are too low. In a later test program, three surface durability tests were conducted with the same grade of carburized steel gears at a load corresponding to the translated running gear G50 shown in Figure 8; one of these tests resulted in a bending failure and is plotted in Figure 8. Also, the result of one of the original running gear bending tests was an unexplained low-side-outlier; this result appears to lie in the region that would be extrapolated from the translated bending strength curves. This additional data tends to confirm the large step predicted between STF results and running gear result. Figure 8 also shows curves adapted from ANSI/AGMA 2001-C95 allowing for 10% and 50% maximum failures rates (KR = 0.85 and 0.70 respectively; all other rating factors set to unity), which fit the running gear data and the translated curves reasonably well.

Example 2 — Induction Hardened Gears. The single-tooth fatigue specimen gears used in the program for the second set of sample data was taken from the same general design as those used to develop the first set. The hob

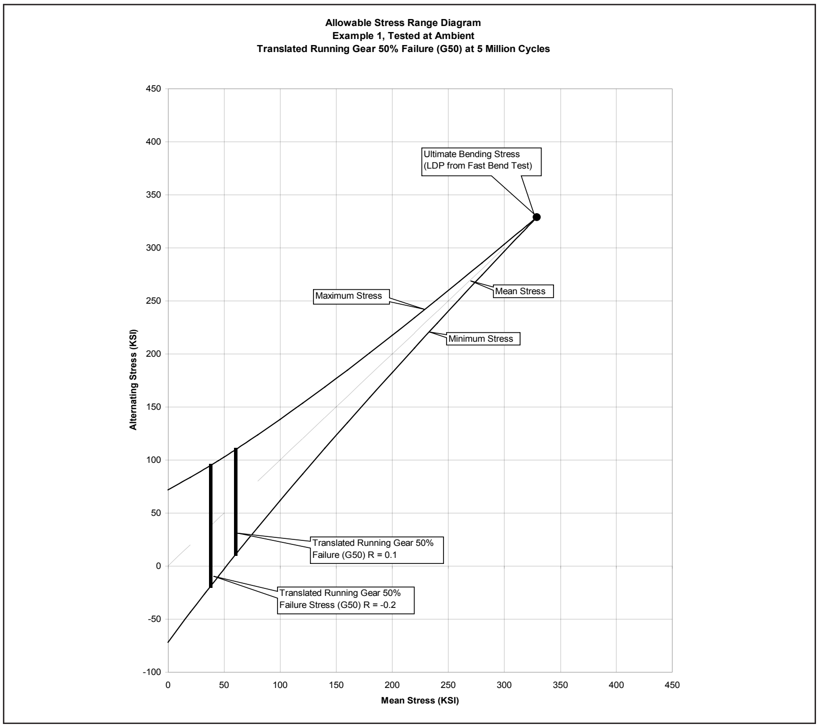

Figure 7—Allowable stress range diagram; Example 1: Tested at ambient; translated running gear 50% failure (G50) at 5 million cycles.

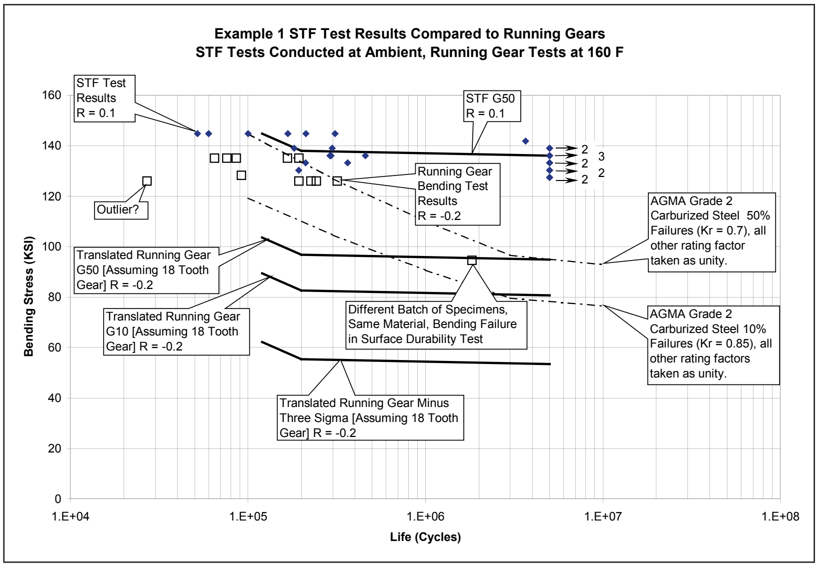

Figure 8—Example 1 STF test results compared to running gears; STF tests conducted at ambient, running gear tests at 160° F.

Table 2 — Example 2 (Induction Hardened) Data			
Maximum Load (Pounds)	Number of Tests	Number of Failures	Failure Rate
9,500	6	6	1.000
9,000	6	4	0.667
8,500	6	3	0.500
8,000	3	1	0.333
7,500	1	0	0.000

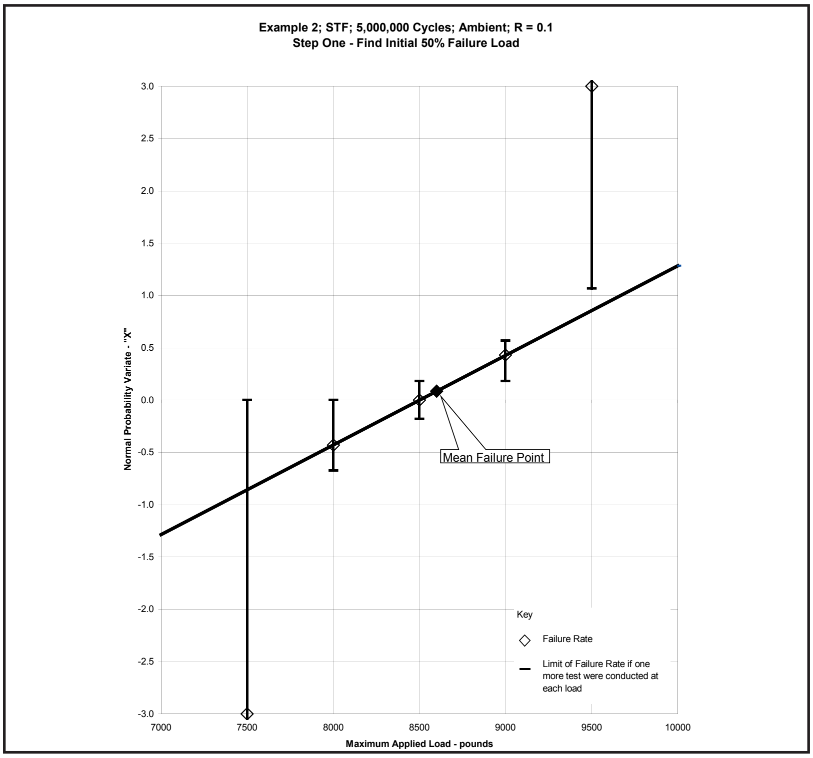

Figure 9—Example 2: STF; 5,000,000 cycles; R = 0.1; Step 1—Find initial 50% failure load.

used to cut these specimens was a short-lead hob that had been sharpened too many times, resulting in a different form in the root fillet. The bending stress was calculated in the manner previously described, with the result that bending stress was 20.3 psi per-pound-load (contrasted to 19.3 psi per-pound- load for the standard root fillet). The running gears used in this program again had 18 teeth. STF test data is summarized in Table 2.

Figures 9 and 10 show the application of Steps One and Two of the analysis method to this data. The slope of the fit lines in Figure 10 appears to be non-conservative (predicting high values of translated running gear bending strength) when compared to the data. Figure 11 is a stress-cycles diagram showing STF results, running gear test results and curves for STF G50, running gear G50, running gear G10 and running gear minus three-sigma. As was the case with the first example set of data, it would have been desirable to conduct more tests and better define the stress-cycles relationship. The STF G50 line is laid in by eye and is a compromise between the four failures below 200,000 cycles and two run-outs at the second highest load, and the six failures below 160,000 cycles at the highest load. The running gear bending results fall very close to the translated running gear's G50 curve. (This particular data set was selected because it comprises the longest cycle running gear bending failure data obtained with the Gear Research Institute's standard specimen gears, giving a better comparison to the portion of the stress-cycles relationship best defined by the STF test.)

All of the running gear bending data points fall above the translated running gear G10 curve, except one outlier run at a lower load in what was intended as a surface durability test. The specimen gears used in these tests were induction hardened. The origin of this outlying failure was at a large inclusion at the case core juncture some 0.050 inches below the root surface, further down the root fillet than the point maximum stress was expected. The material was commercial quality (air melt) cleanliness; however, this inclusion was larger than to be expected in commercial quality material. Thus, this outlying point represents an extreme condition, and it still falls above the translated running gear minus three-sigma curve.

Conclusion and Discussion

The method presented here, while being empirical, makes a reasonable approximation of running gear bending strength based on limited STF bending results. Prior work done in this area by the Gear Research Institute was based on running gear data obtained at very high overloads (as in Example 1), and predicted a smaller difference between STF and running gears. Results such as the bending failure in a surface durability test shown in Figure 8 were considered to be unexplained, low-side-outliers. Using the method presented here, this result fits the predicted trend.

Factors such as residual stress and dynamic loading have not been directly considered here. The STF and running gear specimens used to obtain the data shown in Example 1 were processed in the same manner, which should result in very similar residual stresses. The same was the case for the specimens used in Example 2. Running gear tests were conducted with low-mass gears at low speed in a machine with long shafts (providing torsional springiness, see Figure 12) to recirculate the applied load, resulting in low dynamic stresses. Strain gage measurements with this set-up at standard speed and double standard speed confirm that dynamic loading was minimal.

This area has been examined in the past by other investigators. Seabrook and Dudley (Ref. 7) found that the results of STF tests predicted 30% more strength than was the case with running gear tests using the same materials. This was attributed to a dynamic effect, even though the running gear tests were conducted on a rig designed to minimize dynamic loading. It is interesting to note that this is almost exactly the same difference found here (with gears reflecting four-decades advances in materials and manufacturing processes) that seems to be related to the statistical differences between STF and running gears.

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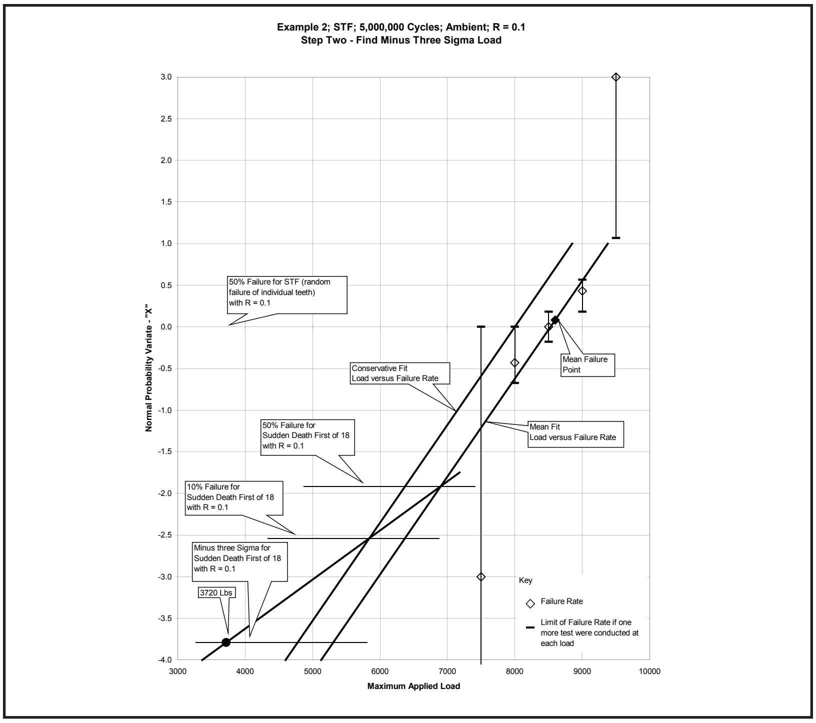

Figure 10—Example 2: STF; 500,000 cycles; ambient; R = 0.1; Step 2—Find minus 3 sigma load.

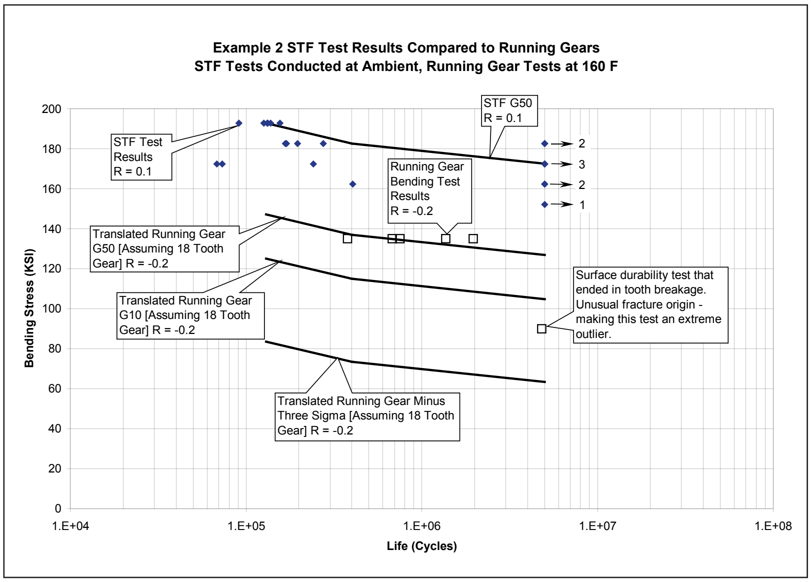

Figure 11—Example 2 STF test results compared to running gears; STF tests conducted at ambient, running gear tests at 160 °F.

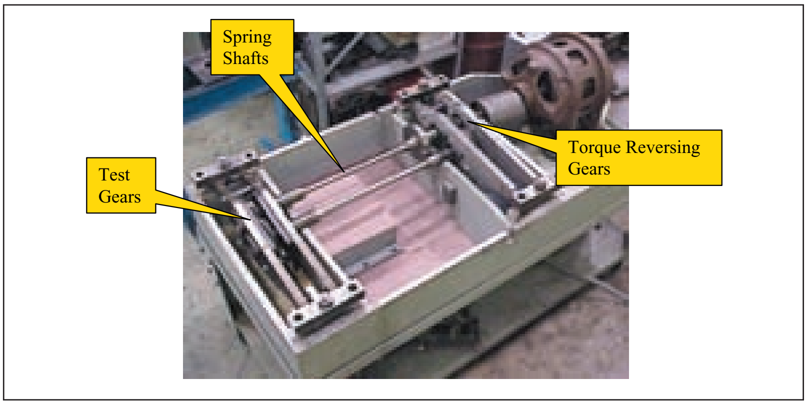

Figure 12—Power re-circulating gear test rig.