Summary:
Our research group has been engaged in the study of gear noise for some nine years and has succeeded in cutting the noise from an average level of some 81-83 dB to 76-78 dB by both experimental and theoretical research. Experimental research centered on the investigation into the relation between the gear error and noise. Theoretical research centered on the geometry and kinematics of the meshing process of gears with geometric error. A phenomenon called "out-of-bound meshing of gears" was discovered and mathematically proven, and an in-depth analysis of the change-over process from the meshing of one pair of teeth to the next is followed, which leads to the conclusion we are using to solve the gear noise problem. The authors also suggest some optimized profiles to ensure silent transmission, and a new definition of profile error is suggested.

Introduction
For some nine years, our research group has been engaging in the study of gear noise and the making of silent gears. Six papers in English have been published on the international conferences and periodicals (see Reference 1-6). Experiments were done on machine tool headstocks' power gears, as they represent the sort of light-load, medium-speed gears for which silent transmission is such a problem. A new single flank total composite error tester (Initiated by Huang Tonglian of China. See Reference 4.) was used to find the relation between gear geometry and the noise. The tester is able to indicate the pitch error, the profile error, the final transmission error and the change-over character (from the meshing of one pair of teeth to the next). Conclusions drawn from these experiments have also been verified by machine testing. When we reached the required accuracy, the gears offer the expected silent transmission. (Under 78 dB measured at 300mm away from pitch point, minimum 73 dB.)

For theoretical analysis, a deep investigation has been made on the meshing process of involute spur gears. A phenomenon called "out-of-bound meshing" of gears was discovered; that is, with the presence of error, the actual contact line of a pair of gears is much discrepant from the theoretical one. It usually goes along a broken line linked by a sector of the addendum circle of one gear and a part of the theoretical contact line. The disclosure of this phenomenon a precise and thorough statement on the change-over process of gear meshing (a geometry and kinematics of the change-over process of gear meshing) is made.

Summarizing from the above mentioned experiments and analysis we formed the following hypothesis concerning gear noise.

Dynamic Behavior and Change-Over Process of Gear Transmission
Out-of-bound meshing and change-over impact of gears. Since gear noise is largely induced from change-over impact, thorough analysis of the change-over process must be made. But before entering the study of it, two things must be mentioned.

a. Out-of-bound meshing (OBM) of gears. When a gear is meshing with its pinion, theoretically speaking, the contact point will move along the common tangent 1-4 (See Table 1a), starting from 1 (the intersecting point of addendum circle of driven gear with g1g2), and ending at 4 (the intersecting point of addendum circle of driving gear with g1g2). On most occasions, there are two adjacent pairs of teeth in mesh, but, actually, there is always a difference in normal pitches of driving and driven gears. Assume t > t, as in the case of Table 1a, the contact point, after passing 4, will continue the meshing and the gap at 1 will be narrowed until at point 4', the next pair of teeth come into meshing at 4' and take over the transmission. So the actual contact line is a broken line e'4'. There is a shock at the instant of take-over, called "leaving impact," of OBM. Table 1b shows the other case, t < t, where the contact line is 2'7'2'e. The impact thus produced is called approaching impact. (4) (5)

b. The single flank total composite gear error tester. It is the main measuring instrument we have been using in this
research work. It is an all-round gear tester developed from the ordinary single flank gear transmission error tester. The result coming out is the transmission error of every individual pair of teeth (from tip to root), but overlapped into a polar coordinate, where \( q \) of the polar coordinate represents the transmission error, while \( \theta \) of the polar coordinate represents the angular position of gear, which determines the position of contact point on the profile, expressed by its generating angle \( \phi_c \). (See Fig. 1a) The greatest merit of this tester is its thorough exposure of the change-over characteristic. Fig. 2 shows the expression of different errors on the tester's record. Fig. 1 indicates the characteristic points on the profile. T and R are the tip and root points of the whole profile. The contact line is \( g_1g_2 \). The pitch point is \( p' \). The generating angle of point \( c \) is \( \phi_c \), which also indicates the angular position of the gear when the contact point passes \( c \). B C is the basic section of profile which offers a contact ratio \( z = \text{Number of teeth} \). Dot lines show the OBM. TB and CR are sections generally considered for tip/root relief.

Now let us see how the change-over takes place. From Table 1a, 1b, one may think that the way of change-over could be either as shown in Table 1a or Table 1b. But remember that both Table 1a and Table 1b are based on the assumption that the profile is an involute curve with error in pressure angle (generated from a base circle with error in diameter), but without tip (root) relief. (TR in Fig. 3a.) Here the change-over point falls on the OBM zone. Now let's see Fig. 3. If the profile has tip relief 1-2 large enough to cover the normal pitch error, then the change-over point \( e' \) will fall within the profile 21R. It is called inside change-over as shown in Table 1c. It is a change-over without OBM. The convex profiles (See Fig. 14, Fig. 15) also belong to this group.

Fig. 3b shows the case with profile shape error. It is a typical saddle form profile. The pressure angle is too small, with a relative relief, and the outside change-over will be transformed into inside change-over.

Table 1c shows a profile with normal pressure angle, but with tip relief to obtain inside change-over. Since the tip relief should be large enough to cover the normal pitch error, a fairly large intersecting angle \( \theta \) results, and consequently, little benefit could be gained in reducing noise. Table 1d shows another way of transforming outside change-over into inside change-over. It is done by adapting the pressure angle to obtain correct normal pitch (normal pitch \( t_n = \cos(\alpha) \), varying \( \alpha \), we can get right \( t_n \), expressing on the tester's record, the geometric expression of right \( t_n \) is that the adjacent curves will link up very well. So if the pressure angle can be adapted to make the adjacent curves meet very well, a very small "relative relief" can guarantee the inside change-over.
while intersecting angle \( \theta \), and hence the impact can be kept very small. It is apparent that this way of obtaining inside change-over (Table 1d) is much more favorable than the previous one (Table 1c).

Outside change-over is generally considered as unfavorable. This is because, in the OBM process, the tip of one gear tooth actually does not mesh with the profile of the other gear. There is no common tangent for them. The tip edge of one gear tooth just scrapes the profile of other gear, and what is more, the OBM curve in the tester's record is steep. (See Fig. 2c.) This results in a very large intersecting angle, causing impact. So a gear with approaching OBM is a typically noisy gear.

But leaving OBM acts quite differently. This is due to the phenomenon of losing contact in gear meshing. As shown in Fig. 5, it goes along the dot line. (For detail see Fig. 16c.)

Experiments show that a gear system with normal pitch of the driving gear is a little bit larger than the normal pitch of the driven gear offers fairly silent transmission.

So there are four typical forms of change-over as listed in Table 1. For the outside change-over, the characteristic parameter is the normal pitch difference \( \Delta t_o \); and for inside change-over, it is the intersecting angle \( \theta \) which determines the impact and hence the noise. (See Fig. 4.)

Some discussion on low noise gear profile and the definition of profile error. In the preceding section, we have discussed the gear noise at the instant of take-over. Now let us see what happens during the whole process of meshing of a pair of tooth profiles (from root to tip). They work just like a pair of cams. According to the theory of conjugation, they should be made into accurate involute curves (not concave nor convex on tester's record) to ensure smooth transmission during this period. The characteristic parameter for noise in this case is the reciprocal of the radius of curvature of the curve on the tester's record. (See Reference 4.) But as mentioned above, in order to ensure smooth change-over, pressure angle should be adapted to link up the adjacent curves. So we suggest redefining the profile error, splitting it into two items:

a) Discrepancy from involute curve; that is, draw the two closest involute curves of the same base circle (not necessarily the nominated base circle). The normal distance of them, \( \Delta f \)
is defined as the profile error. (Fig. 6). If the BT/CR section is used for tip/root relief, then only the shape error of BC section will count. Tip/root relief should not be confused with profile error. As shown in Fig. 7, profile error is $\Delta f$ instead of $\Delta f'$.

b) The actual pressure angle in this case is determined by the radius of base circle $R_{base}$; that is, $\alpha = \cos^{-1}(R_{base}/R_{rel})$. (See Fig. 8.)

**Experimental Research on Gear Noise**

Perhaps what interests the gear manufacturers most is the results of experimental research. They wish to know what items of accuracy influence the noise most and how close tolerances must be on errors to ensure silent transmission. The purpose of experimental research was to find the answer for them, but we also did some other experiments with the purpose of investigating the phenomenon of gear noise.

Influence of gear error on the noise level. The experiment was done on $m=2.5$ $z=50$ B=20 lathe headstock gears. They were hobbed, shaved (without shave profile correction), hardened, and then subjected to a "controllable electrolytic honing" with the intention obtaining samples with different magnitudes of errors. The profile of gear samples thus obtained are of typical saddle form, but different in concavity $\Delta f$. It is a pity that in this batch of gear samples, there was no convex profile, and some error in pressure angle was deliberately made. Gear noise and total transmission error were carefully measured. Altogether 242 noise levels were taken. They were listed on noise level sequence. Table 2 shows the two extremities, Nos. 1-26 are silent gears (below March/April 1990 11
80dB). Nos. 223-242 are the most noisy ones, over 84dB. The conclusion is that for medium-module, medium-speed, light-loaded spur gears, the main errors influencing noise are the normal pitch difference and the profile concavity. Positive $\Delta t_n$ (normal pitch of driving gear larger than that of driven gear) offers “leaving impact” change-over (Table 1a), which is apparently more favorable than negative $\Delta t_n$ (approaching impact change-over, Table 1b). From the average value of the extremities, increasing rate of noise could be roughly estimated as every increment of 0.66 microns in negative normal pitch difference (normal pitch of driven gear minus that of driving gear), plus a composite concavity increment of 0.62 microns will cause an increment of 1db in gear noise. The tester's records of the gears involved in first five noise tests are shown in Fig. 9 (1641, 2161, 2251, 0941, 3041), while the tester's records of gears involved in last five noise tests (0151, 0152, 0311, 0331, 1331, 1432, 2031, 2032, 2252) are shown in Fig. 10. Fig. 11 shows gears ground on Chinese worm wheel
Table 3: An index to find tester's record with known number of gear

<table>
<thead>
<tr>
<th>Number of gear</th>
<th>0151</th>
<th>0152</th>
<th>0311</th>
<th>0331</th>
<th>0941</th>
<th>702L</th>
<th>703L</th>
<th>706L</th>
<th>709L</th>
<th>710L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Fig.</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>09</td>
<td>13</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>1432</td>
<td>1641</td>
<td>2031</td>
<td>2032</td>
<td>2161</td>
<td>2251</td>
<td>2252</td>
<td>3041</td>
<td>711L</td>
<td>1331</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>09</td>
<td>10</td>
<td>09</td>
<td>10</td>
<td>09</td>
<td>10</td>
<td>09</td>
<td>12</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 11 – Ground on Chinese worm wheel gear grinder 7232A

Fig. 12 – Ground on Swiss worm wheel gear grinder NZA

Fig. 13 – Ground on Chinese worm wheel gear grinder 7232A plus final free grinding

gear grinder 7232A (703L, 710L). Fig. 12 shows gears ground on Swiss gear grinder NZA (706L, 711L). Fig. 13 shows gears ground on Chinese gear grinder 7232A with final free grinding. (702L, 709L). Table 3 is an index to find the tester's record of known gear number.

Comparison of noise level of gears ground on different gear grinders. Experiments were done mainly on worm abrasive wheel gear grinders because they are productive, and because they show promise for manufacturing silent gears. Table 4 shows the noise level measured at 300mm from the pitch point of the meshing gears. From these test results, one sees that with the addition of a final worm wheel free grinding, the noise level was dropped from an average of 80.7dB to 76.7dB.

Fig. 14 shows the tester's records from Swiss grinder AZA. The machine tool is accurate, but because the wrong diamond dressing wheel was used the profile thus obtained is a convex form.

Some gear makers think that a convex profile may offer silent transmission, perhaps because most gear making methods produce a concave profile and typically noisy gears. Therefore, it is understandable to suppose that a convex profile will offer silent transmission. However, we have shown experimentally that this is not the case. A batch of some 20 gears with profiles as shown in Fig. 15 were made. They turned out to be very noisy (over 85dB).

Experiment for investigating the phenomenon of losing contact in gear meshing. Gear noise researchers have long noticed the phenomenon of losing contact. We did some experiments on this phenomenon as well. The gear noise tester was isolated between driving and driven gears. A signal
Table 4 — Comparison of noise of gears ground on different gear grinders

<table>
<thead>
<tr>
<th>NO.</th>
<th>Type of gear grinder</th>
<th>Way of profile generation and indexing</th>
<th>No. of noise test</th>
<th>Average noise</th>
<th>No. of Fig., of tester’s record</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Chinese horizontal gear grinder 7132A</td>
<td>steel belt and rolling disc with virtual rolling radius adjustable, indexing by change gear and worm wheel</td>
<td>7132A</td>
<td>83dB</td>
<td>(data offered by factory)</td>
</tr>
<tr>
<td>2</td>
<td>Chinese worm wheel gear grinder 7232A</td>
<td>Combine profile generation and indexing just like hobbing</td>
<td>28</td>
<td>80.7dB</td>
<td>Fig. 15</td>
</tr>
<tr>
<td>3</td>
<td>Swiss worm wheel gear grinder NZA</td>
<td></td>
<td>32</td>
<td>78.0dB</td>
<td>Fig. 16</td>
</tr>
<tr>
<td>4</td>
<td>Swiss worm wheel gear grinder AZA</td>
<td></td>
<td>4</td>
<td>81.5dB</td>
<td>Fig. 18</td>
</tr>
<tr>
<td>5</td>
<td>Chinese worm wheel free grinder</td>
<td></td>
<td>34</td>
<td>76.7dB</td>
<td>Fig. 17</td>
</tr>
</tbody>
</table>

Fig. 14 — Tester’s record of gear ground on AZA

Fig. 15 — Tester’s record of gear with convex profile

Fig. 16 — Time series curves of losing contact

voltage with some resistors in series was connected to the gears. If the gears were in contact, the signal voltage was short circuited to low voltage. If they lost contact, high voltage appeared. Our experiment not only confirmed the existence of the phenomenon, but also defined some of its characteristics.

Three sorts of losing contact were observed. (See Fig. 16.)

a) Jump over teeth — The driving gear loses contact with the driven gear; then, after crossing several teeth, it comes again into contact.

b) Frequently lose contact for short instant within the period of one tooth, lose contact and come into contact again several times.

c) Jump over teeth and come to frequently losing contact alternatively.

Experiment for investigating the influence of contact ratio on noise level of gears, A pair of non-standard high precision gears were made with very high thin teeth, which could provide a contact ratio of more than two. By varying the center distance on the tester, a relation between the contact ratio and the noise level was found as shown in Fig. 17. Minimum noise
seems to fall on a contact ratio a little bit less than 2 or 1. This indicates that even under the slight load, the influence of torsional stiffness is still of some importance.

A tentative suggestion on the optimized geometry for low noise gears. For the time being, we can suggest only the type (d) in Table 1. That is, doing best to reach better indexing (circular pitch), for the existing error, adapt pressure angle α to ensure correct normal pitch, so that on the tester's record the adjacent curves will meet very well, and a smooth change-over can be realized with slightest tip/root relief. Table 5 shows an example of silent gear pair.

Research on the Making of Silent Gears

From the previous sections, it is clear that gear noise is produced:

a) at the instant of change-over,
b) during the process of meshing of a pair of gear teeth.

The demand for accuracy is also directed at reducing noise, that is:

a) The normal pitch difference Δt, as small as possible. If it is not possible to obtain this figure, a positive Δt, is preferable.
b) The profile form error Δ should preferably be as close to the theoretical involute as possible with minimum relief needed.

From Table 5, it can be seen that to guarantee ideal silent gear transmission, the demand on key items of accuracy is remarkably high.

1) To ensure final accuracy, the key technology is in the finishing method after hardening.

2) To reach high accuracy in normal pitch, free finishing methods are very favorable. They can correct the normal pitch error of the gears with the accurate normal pitch of the tool (gear hone, worm-shaped, abrasive wheel, internal gear hone, etc. In Table 4, it has been shown that an additional free grinding to the worm wheel gear grinder cut down the average noise level by 4dB. Another factory in Dalian succeeded in cutting down the gear noise by some 5-6dB by internal honing. (The gear hone is with a shape of an internal gear.) Further theoretical analysis can prove mathematically that free finishing can guarantee the type of gear shown in Table 1d, while grinding with constrained motion of wheel and job will always lead to the geometry shown in Table 1c.

3) To obtain the ideal profile accuracy, a finishing method with controllable metal removing rate along the full profile (from tip to root) is keenly wished. The field controlled ECH of gears is one of the new technologies we are developing with this intention. (See Reference 3.)

Conclusion

As the introduction of this article has stated, all we are writing here is just an idea formed in our minds by the results of experiments. It can basically explain the phenomenon we can see now, but any disclosure by further experiments may make it necessary to revise the explanation. Besides, there is still a distance between experiment and application in production to cover. But so far as we can see now, a comparatively silent gear (say below 78dB) looks feasible. For further reducing noise, other measure such as damping and absorbing, and sound isolation, should be considered too.

References:


Table 1 – 5 Example of silent gear pair

<table>
<thead>
<tr>
<th>Driving gear</th>
<th>Driven gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear No.</td>
<td>Fig. No.</td>
</tr>
<tr>
<td>702L</td>
<td>Fig. 17</td>
</tr>
<tr>
<td>709L</td>
<td>Fig. 17</td>
</tr>
</tbody>
</table>

Fig. 17 – Relation between contact ratio and noise level

Acknowledgement: The achievement of this research is virtually the sum of the achievements of our research group over the past 9 years. It should be credited to all members and postgraduates of our research group, especially, Y.F. Jia, X.B. Liu, D.L. Wang, W. Li, Y.W. Liu, M.H. Wu, and Y.P. Guo.