The following excerpt is from the Revised Manual of Gear Design, Section III, covering helical and spiral gears. This section on helical gear mathematics shows the detailed solutions to many general helical gearing problems. In each case, a definite example has been worked out to illustrate the solution. All equations are arranged in their most effective form for use on a computer or calculating machine.

### Given the pitch radius and lead of a helical gear, to determine the helix angle:

<table>
<thead>
<tr>
<th>When,</th>
<th>( R ) = Pitch Radius of Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( L ) = Lead of Tooth</td>
</tr>
<tr>
<td></td>
<td>( \psi ) = Helix Angle</td>
</tr>
</tbody>
</table>

Then,  

\[
\text{TAN} \quad \psi = \frac{2 \pi R}{L}
\]

**Example:**  

\[
R = 3.000 \quad L = 21.000
\]

\[
\text{TAN} \quad \psi = \frac{2 \times 3.1416 \times 3.000}{21.000} = 0.89760 \quad \psi = 41.911^\circ
\]

### The involute of a circle is the curve that is described by the end of a line which is unwound from the circumference of a circle as shown in Fig. 1.

<table>
<thead>
<tr>
<th>When,</th>
<th>( R_b ) = Base Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \theta ) = Vectorial Angle</td>
</tr>
<tr>
<td></td>
<td>( r ) = Length of Radius Vector</td>
</tr>
</tbody>
</table>

Then,  

\[
\theta = \frac{\sqrt{r^2 - R_b^2} - \text{ARC TAN} \ \frac{\sqrt{r^2 - R_b^2}}{R_b}}{R_b}
\]
Given the arc tooth thickness and pressure angle in the plane of rotation of a helical gear at a given radius, to determine its tooth thickness at any other radius:

When,

\[ r_1 = \text{Given Radius} \]
\[ \phi_1 = \text{Pressure Angle at } r_1 \]
\[ T_1 = \text{ARC Tooth Thickness at } r_1 \]
\[ r_2 = \text{Radius Where Tooth Thickness Is To Be Determined} \]
\[ \phi_2 = \text{Pressure Angle at } r_2 \]
\[ T_2 = \text{ARC Tooth Thickness at } r_2 \]

Then,

\[ \cos \phi_2 = \frac{r_1 \cos \phi_1}{r_2} \]
\[ T_2 = 2r_2 \left( \frac{T_1 + \text{INV } \phi_1 - \text{INV } \phi_2}{2r_1} \right) \]

Example:

\[ r_1 = 2.500 \quad T_1 = .2618 \quad r_2 = 2.600 \]
\[ \phi_1 = 14.500^\circ \quad \cos \phi_1 = .96815 \quad \text{INV } \phi_1 = .00554 \]
\[ \cos \phi_2 = \frac{2.500 \times .96815}{2.600} = .93091 \]
\[ \phi_2 = 21.425^\circ \quad \text{INV } \phi_2 = .01845 \]
\[ T_2 = 2 \times 2.600 \left( \frac{.2618 + .00554 - .01845}{5.000} \right) = .2051 \]

![Fig. 2](image)

Given the helix angle, normal diametral pitch and numbers of teeth, to determine the center distance:

When,

\[ \psi = \text{Helix Angle} \]
\[ N_1 = \text{Number of Teeth in Pinion} \]
\[ N_2 = \text{Number of Teeth in Gear} \]
\[ C = \text{Center Distance} \]
\[ P_n = \text{Normal Diametral Pitch} \]

Then,

\[ C = \frac{N_1 + N_2}{2 P_n \cos \psi} \]

Example:

\[ \psi = 30^\circ \quad P_n = 8 \quad N_1 = 24 \quad N_2 = 48 \quad \cos \psi = .86603 \]
\[ C = \frac{24 + 48}{2 \times 8 \times .86603} = 5.1961 \]
Given the arc tooth thickness in the plane of rotation at a given radius, to find the normal chordal thickness and the normal chordal addendum:

When,
- \( T = \text{ARC Tooth Thickness at } R \text{ in Plane of Rotation} \)
- \( T_n = \text{Normal Chordal Thickness at } R \)
- \( Q_n = \text{Normal Chordal Addendum} \)
- \( R_o = \text{Outside Radius} \)
- \( R = \text{Pitch Radius} \)
- \( \psi = \text{Helix Angle at } R \)

Then,
- \( \text{ARC } B = \frac{T \cos^2 \psi}{2R} \)
- \( T_n = \frac{2R \sin B}{\cos \psi} \)
- \( Q_n = R_o - \cos B \)

Example:
- \( T = .2267 \)
- \( R_o = 1.8570 \)
- \( R = 1.7320 \)
- \( \psi = 30^\circ \)
- \( \cos \psi = .86603 \)
- \( \cos^2 \psi = .75000 \)
- \( \text{ARC } B = \frac{.2267 \times .7500}{2 \times 1.7320} = .04908 \)
- \( B = 2.812^\circ \)

- \( \sin B = .04906 \)
- \( \cos B = .99880 \)
- \( T_n = \frac{2 \times 1.7320 \times .04906}{.86603} = .1962 \)
- \( Q_n = 1.8570 - (1.7320 \times .99880) = .1271 \)

Given the circular pitch and pressure angle in the plane of rotation and the helix angle of a helical gear, to determine the normal circular pitch and the normal pressure angle:

When,
- \( \psi = \text{Helix Angle} \)
- \( \phi = \text{Pressure Angle in Plane of Rotation} \)
- \( p = \text{Circular Pitch in Plane of Rotation} \)
- \( \phi_n = \text{Normal Pressure Angle} \)
- \( p_n = \text{Normal Circular Pitch} \)

Then,
- \( p = p \cos \psi \)
- \( \tan \phi_n = \tan \phi \cos \psi \)

Example:
- \( p = .3927 \)
- \( \psi = 23^\circ \)
- \( \phi = 20^\circ \)
- \( \cos \psi = .92050 \)
- \( \tan \phi = .36397 \)
- \( p_n = .3927 \times .92050 = .36148 \)
- \( \tan \phi_n = .36397 \times .92050 = .33503 \)
- \( \phi_n = 18.522^\circ \)
Given the arc tooth thickness and pressure angle in the plane of rotation at a given radius, to determine the radius where the tooth becomes pointed:

When,
\[ r_1 = \text{Given Radius} \]
\[ r_2 = \text{Radius where Tooth Becomes Pointed} \]
\[ T_1 = \text{ARC Tooth Thickness at} \ r_1 \]
\[ \phi_1 = \text{Pressure Angle at} \ r_1 \]
\[ \phi_2 = \text{Pressure Angle at} \ r_2 \]

Then,
\[ \text{INV} \phi_2 = \frac{T_1}{2 \ r_1} + \text{INV} \phi_1 \]
\[ r_2 = \frac{r_1 \ \text{COS} \ \phi_1}{\text{COS} \ \phi_2} \]

Example:
\[ r_1 = 2.500 \quad T_1 = .2618 \quad \phi_1 = 14.500^\circ \]
\[ \text{INV} \phi_1 = .00554 \]
\[ \text{INV} \phi_2 = \frac{.2618}{2 \times 2.500} + .00554 = .05790 \ \text{Radians} \]
\[ \phi_2 = 30.693^\circ \quad \text{COS} \ \phi_2 = .85991 \quad \text{COS} \ \phi_1 = .96815 \]
\[ r_2 = \frac{2.500 \times .96815}{.85991} = 2.8147 \]

Given the normal circular pitch, the normal pressure angle and the helix angle of a helical gear, to determine the circular pitch and the pressure angle in the plane of rotation:

When,
\[ \psi = \text{Helix Angle} \]
\[ \phi_n = \text{Normal Pressure Angle} \]
\[ p_n = \text{Normal Circular Pitch} \]
\[ \phi = \text{Pressure Angle in Plane of Rotation} \]
\[ p = \text{Circular Pitch in Plane of Rotation} \]

Then,
\[ p = \frac{p_n}{\text{COS} \ \psi} \quad \text{TAN} \ \phi = \frac{\text{TAN} \ \phi_n}{\text{COS} \ \psi} \]

Example:
\[ \psi = 25^\circ \quad \phi_n = 20^\circ \quad \text{COS} \ \psi = .90631 \quad \text{TAN} \ \phi_n = .36397 \quad p_n = .5236 \]
\[ p = \frac{.5236}{.90631} = .57772 \quad \text{TAN} \ \phi = \frac{.36397}{.90631} = .40159 \quad \phi = 21.880^\circ \]
Given the tooth proportions in the plane of rotation of a pair of helical gears (parallel shafts), to determine the center distance at which they will mesh tightly:

When,  
\[ r_1 = \text{Given Radius of 1st Gear} \]  
\[ r_2 = \text{Given Radius of 2nd Gear} \]  
\[ N_1 = \text{Number of Teeth in 1st Gear} \]  
\[ N_2 = \text{Number of Teeth in 2nd Gear} \]  
\[ \phi_1 = \text{Pressure Angle at } r_1 \text{ and } r_2 \]  
\[ \phi_2 = \text{Pressure Angle at Meshing Position} \]  
\[ T_1 = \text{ARC Tooth Thickness at } r_1 \]  
\[ T_2 = \text{ARC Tooth Thickness at } r_2 \]  
\[ C_1 = \text{Center Distance for Pressure Angle } \phi_1 \]  
\[ C_2 = \text{Center Distance for Pressure Angle } \phi_2 \]

Then,
\[ \text{INV } \phi_2 = \frac{N_1 (T_1 + T_2) - 2\pi r_1}{2 r_1 (N_1 + N_2)} + \text{INV } \phi_1 \]

\[ C_1 = r_1 + r_2 \]

\[ C_2 = \frac{C_1 \cos \phi_1}{\cos \phi_2} \]

Example:
\[ r_1 = 2.500 \quad T_1 = .2800 \quad N_1 = 30 \quad \phi_1 = 14.500^\circ \]
\[ r_2 = 4.000 \quad T_2 = .2750 \quad N_2 = 48 \quad C_1 = 6.500 \]

\[ \text{INV } \phi_2 = \frac{30 (.2800 + .2750) - 2\pi \times 2.500}{2 \times 2.500 (30 + 48)} + .00545 = .007955 \]

\[ \phi_2 = 16.315^\circ \quad \cos \phi_2 = .95973 \]

\[ C_2 = \frac{6.500 \times .96815}{.95973} = 6.5570 \]

Given the pitch radius and helix angle of a helical gear, to determine the lead of the tooth.

When,  
\[ R = \text{Pitch Radius} \]  
\[ L = \text{Lead of Tooth} \]  
\[ \psi = \text{Helix Angle} \]

Then,
\[ L = \frac{2 \pi R}{\tan \psi} \]

Example:
\[ R = 2.500 \quad \psi = 22.50^\circ \quad \tan \psi = .41421 \]

\[ L = \frac{2 \times 3.1416 \times 2.500}{.41421} = 37.9228 \]
Given the number of teeth, helix angle and proportions of the normal basic rack of a helical gear, to determine the pitch radius and the base radius:

When,  
N = Number of Teeth  
ψ = Helix Angle at R  
P_n = Normal Diametral Pitch  
R = Pitch Radius  
φ_n = Normal Pressure Angle  
φ = Pressure Angle in Plane of Rotation  
R_b = Base Radius

Then,  
R = \frac{N}{2 P_n \cos \psi}  
\tan \phi = \frac{\tan \phi_n}{\cos \psi}  
R_b = R \cos \phi = \frac{N \cos \phi}{2 P_n \cos \psi}

Example:  
N = 30  \quad \psi = 25^\circ  \quad P_n = 6  \quad \phi_n = 14.5^\circ  \quad \cos \psi = .90631  \quad \tan \phi_n = .25862

R = \frac{30}{2 \times 6 \times .90631} = 2.7584

\tan \phi = \frac{.25862}{.90631} = .28535  \quad \phi = 15.926^\circ  \quad \cos \phi = .96162

R_b = \frac{30 \times .96162}{2 \times 6 \times .90631} = 2.65256

Given the normal diametral pitch, numbers of teeth and center distance, to determine the lead and helix angle:

When,  
N_1 = Number of Teeth in Pinion  
N_2 = Number of Teeth in Gear  
P_n = Normal Diametral Pitch  
C = Center Distance  
ψ = Helix Angle  
L_1 = Lead of Pinion  
L_2 = Lead of Gear

Then,  
\cos \psi = \frac{N_1 + N_2}{2 P_n C}  
L_1 = \frac{\pi N_1}{P_n \sin \psi}  
L_2 = \frac{\pi N_2}{P_n \sin \psi}

Example:  
P_n = 6  \quad N_1 = 18  \quad N_2 = 30  \quad C = 4.500

\cos \psi = \frac{18 + 30}{2 \times 6 \times 4.500} = .88889  \quad \psi = 27.266^\circ  \quad \sin \psi = .45812

L_1 = \frac{3.1416 \times 18}{6 \times .45812} = 20.5728  \quad L_2 = \frac{3.1416 \times 30}{6 \times .45812} = 34.2880
Given the tooth proportions in the plane of rotation of a helical gear, to determine the position of a mating rack of different circular pitch and pressure angle:

When, 
\[ \psi_1 = \text{Given Helix Angle at } R_1 \]
\[ \psi_2 = \text{Helix Angle for Mating Rack} \]
\[ \psi_0 = \text{Base Helix Angle} \]
\[ \phi_{n1} = \text{Normal Pressure Angle at } R_1 \]
\[ \phi_{n2} = \text{Pressure Angle of Mating Rack} \]
\[ \phi_1 = \text{Pressure Angle at } R_1 \text{ in Plane of Rotation} \]
\[ \phi_2 = \text{Pressure Angle of Mating Rack in Plane of Rotation} \]
\[ R_1 = \text{Given Pitch Radius} \]
\[ R_2 = \text{Pitch Radius with Mating Rack} \]

Then,

\[
\sin \psi_0 = \sin \psi_1 \cos \phi_{n1}
\]
\[
\sin \psi_2 = \frac{\sin \psi_0}{\cos \phi_{n2}} = \frac{\sin \psi_1 \cos \phi_{n1}}{\cos \phi_{n2}}
\]
\[
\tan \phi_2 = \frac{\tan \phi_{n2}}{\cos \psi_2} \quad R_2 = \frac{R_b}{\cos \phi_2}
\]
\[
X = R_2 - a + \frac{1}{2 \tan \phi_2} \left[ 2 R_2 \left( \frac{T_1}{2 R_1} + \frac{\phi_1}{\cos \phi_1} - \frac{\phi_2}{\cos \phi_2} \right) - \frac{\pi R_2}{N} \right]
\]

Example:
\[ \psi_1 = 25^\circ \quad \phi_{n1} = 14\frac{1}{2}^\circ \quad \phi_1 = 15.926^\circ \quad R_1 = 2.7584 \quad R_b = 2.65256 \]
\[ \phi_{n2} = 20^\circ \quad a = .185 \quad T_1 = .2888 \quad N = 30 \]
\[ \sin \psi_1 = .42262 \quad \cos \psi_1 = .90631 \quad \tan \phi_{n1} = .25862 \quad \tan \phi_{n2} = .36397 \]
\[ p_{n1} = .5236 \quad p_{n2} = .53946 \quad \cos \phi_{n1} = .96815 \quad \cos \phi_{n2} = .93969 \]
\[ [p_{n1} \cos \phi_{n1} = .50692] = [p_{n2} \cos \phi_{n2} = .50692] \]
\[ \sin \psi_2 = \frac{.42262 \times .96815}{.93969} = .43542 \quad \psi_2 = 25.812^\circ \quad \cos \psi_2 = .90023 \]
\[ \tan \phi_2 = \frac{.36397}{.90023} = .40431 \quad \phi_2 = 22.014^\circ \quad \cos \phi_2 = .92709 \]
\[ \tan \phi_1 = .020093 \quad \tan \phi_2 = .007387 \]
\[ R_2 = \frac{2.65256}{.92709} = 2.86117 \]
\[ X = 2.86117 - .185 + \frac{1}{2 \times .40431} \left[ 5.72234 \left( \frac{.2888}{5.5168} + .007387 - .020093 \right) - \frac{.31416 \times 2.86117}{30} \right] = 2.5729 \]
Given the center distance, number of teeth and basic rack proportions (hob proportions) of a pair of helical gears, to determine the hobbing data:

When,

\( \phi_{nc} = \) Pressure Angle of Hob
\( p_{nc} = \) Diametral Pitch of Hob
\( a_c = \) Addendum of Hob
\( C_1 = \) Center Distance with Pressure Angle of \( \phi_1 \)
\( C_2 = \) Given Center Distance of Operation
\( N_1 = \) Number of Teeth in Pinion
\( R_{o1} = \) Outside Radius of Pinion
\( R_{r1} = \) Root Radius of Pinion
\( R_1 = \) Pitch Radius of Pinion
\( b_1 = \) Dedendum of Pinion
\( \psi_1 = \) Helix Angle of Generation

Then, \( \phi_1 = \) Pressure Angle of Generation in Plane of Rotation
\( \phi_2 = \) Pressure Angle of Operation in Plane of Rotation
\( \psi_2 = \) Helix Angle of Generation

Make trial calculation for lead as follows:

\[ \cos \psi_1 = \frac{N_1 + N_2}{2 \ p_{nc} \ C_2} \]
\[ L_1 = \frac{\pi N_1}{p_{nc} \ \sin \ \psi_1} \quad \quad L_2 = \frac{\pi N_2}{p_{nc} \ \sin \ \psi_1} \]

Select values for \( L_1 \) and \( L_2 \) which can be readily obtained on the hobbing machine:

\[ \sin \ \psi_1 = \frac{\pi N_1}{p_{nc} \ L_1} = \frac{\pi N_2}{p_{nc} \ L_2} \quad \quad \tan \phi_1 = \frac{\tan \phi_{nc}}{\cos \ \psi_1} \]
\[ \tan \phi_1 = \frac{C_1}{N_1 + N_2} \quad \quad \cos \ \phi_2 = \frac{C_1 \ \cos \ \phi_1}{C_2} \]
\[ (R_{r1} + R_{r2}) = C_1 - 2 \ a_c + \frac{C_1}{\tan \phi_1} \ \left( INV \ \phi_2 - INV \ \phi_1 \right) \]
\[ b_1 = \frac{C_2 - (R_{r1} + R_{r2})}{1 + \sqrt{\frac{N_2}{N_1}}} \quad \quad b_2 = \frac{C_2 - (R_{r1} + R_{r2})}{1 + \sqrt{\frac{N_1}{N_2}}} \]

Note: When smallest \( N \) is 30 or more, then, \( b_1 = b_2 = \frac{C_2 - (R_{r1} + R_{r2})}{2} \)

\[ R_1 = \frac{N_1 \ C_2}{N_1 + N_2} \quad \quad R_2 = \frac{N_2 \ C_2}{N_1 + N_2} \quad \quad R_{r1} = R_1 - b_1 \quad \quad R_{r2} = R_2 - b_2 \]
\[ h_1 = .932 \ [C_2 - (R_{r1} + R_{r2})] \quad \quad R_{o1} = R_{r1} + h_1 \quad \quad R_{o2} = R_{r2} + h_1 \]

\[ \tan \ \psi_2 = \frac{2 \ \pi \ R_1}{L_1} = \frac{2 \ \pi \ R_2}{L_2} \]

(Continued on next page)
Example:  \[ N_1 = 20 \quad N_2 = 60 \quad p_{nc} = 5 \quad A_c = .2314 \quad C_2 = 9.00 \]

\[ \phi_{nc} = 14.500 \quad \text{TAN } \phi_{nc} = .25862 \]

Trial Calculation:

\[ \cos \psi_1 = \frac{20 + 60}{2 \times 5 \times 9.00} = .88889 \quad \psi_1 = 27.266^\circ \quad \sin \psi_1 = .45812 \]

\[ L_1 = \frac{20 \pi}{5 \times .45812} = 27.4303 \quad L_2 = \frac{60 \pi}{5 \times .45812} = 82.2909 \]

We will select the following values for \( L_1 \) and \( L_2 \):

\[ L_1 = 27.500 \quad L_2 = 82.500 \]

\[ \sin \psi_1 = \frac{20}{5 \times 27.500} = .45695 \quad \psi_1 = 27.1910 \quad \cos \psi_1 = .88949 \]

\[ \tan \phi_1 = \frac{.25862}{.88969} = .29069 \quad \phi_1 = 16.208^\circ \quad \cos \phi_1 = .96025 \quad \text{INV } \phi_1 = .007796 \]

\[ p_1 = 5 \times .88949 = 4.44745 \]

\[ C_1 = \frac{20 + 60}{2 \times 4.44745} = 8.99392 \]

\[ \cos \phi_2 = \frac{.99392 \times .96025}{.9} = .95960 \quad \phi_2 = 16.3416 \quad \text{INV } \phi_2 = .007994 \]

\[ (R_{11} + R_{22}) = 8.99392 - 2 \times .2314 + \frac{8.99392}{.90969} [1.007994 - .007796] = 8.5372 \]

\[ b_1 = \frac{9.00 - 8.5372}{1 + \sqrt{60/20}} = .16938 \quad b_2 = \frac{9.00 - 8.5372}{1 + \sqrt{20/60}} = .29340 \]

\[ R_1 = \frac{20 \times 9.00}{20 + 60} = 2.250 \quad R_2 = \frac{60 \times 9.00}{20 + 60} = 6.750 \]

\[ R_{11} = 2.250 - .16938 = 2.08062 \quad R_{12} = 6.750 - .29340 = 6.45660 \]

\[ h_1 = .932 [9.00 - 8.5372] = .43133 \]

\[ R_{01} = 2.08062 + .43133 = 2.51195 \quad R_{02} = 6.45660 + .43133 = 6.88793 \]

\[ \tan \psi_2 = \frac{2 \pi \times 2.250}{27.5} = .514079 \quad \psi_2 = 27.207 \]

The specifications for this pair of gears are as follows:

\[ N_1 = 20 \quad R_{11} = 2.08062 \quad L_2 = 82.500 \quad \text{Helix angle for hobbing} = 27.1910 \]

\[ R_{01} = 2.51195 \quad N_2 = 60 \quad R_{12} = 6.45660 \]

\[ R_1 = 2.250 \quad R_{02} = 6.88793 \quad C_2 = 9.00 \]

\[ L_1 = 27.500 \quad R_2 = 6.750 \]

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Given the proportions of an internal helical gear drive, to determine the contact ratio:

When,

\[ R_1 = \text{Pitch Radius of Helical Gear} \quad R_2 = \text{Pitch Radius of Internal Gear} \]
\[ R_{01} = \text{Outside Radius of Helical Gear} \quad R_i = \text{Internal Radius of Internal Gear} \]
\[ R_{b1} = \text{Base Radius of Helical Gear} \quad R_{b2} = \text{Base Radius of Internal Gear} \]
\[ \phi = \text{Pressure Angle in Plane of Rotation} \]
\[ p = \text{Circular Pitch in Plane of Rotation} \]
\[ C = \text{Center Distance} \]
\[ m_p = \text{Contact Ratio} \]

Then,

\[ m_p = \frac{\sqrt{R_{01}^2 - R_{b1}^2} + C \sin \phi - \sqrt{R_i^2 - R_{b2}^2}}{p \cos \phi} \]

Example:

\[ R_1 = 1.250 \quad R_{01} = 1.4375 \quad R_{b1} = 1.1746 \quad \phi = 20^\circ \quad p = .3927 \]
\[ R_2 = 3.500 \quad R_i = 3.4375 \quad R_{b2} = 3.2888 \quad C = 2.250 \]
\[ \sin \phi = .34202 \quad \cos \phi = .93969 \]

\[ m_p = \frac{\sqrt{(1.4375)^2 - (1.1746)^2} + (2.250 \times .34202) - \sqrt{(3.4375)^2 - (3.2888)^2}}{.3927 \times .93969} = 1.62 \]

Given the proportions of a pair of helical gears (external or internal), to determine the face contact ratio:

When,

\[ F = \text{Face Width} \]
\[ p = \text{Circular Pitch in Plane of Rotation} \]
\[ \psi = \text{Helix Angle} \]
\[ m_f = \text{Face Contact Ratio} \]

Then,

\[ m_f = \frac{F \tan \psi}{p} \]

Example:

\[ F = 1.500 \quad p = .3927 \quad \psi = 30^\circ \quad \tan \psi = .57735 \]

\[ m_f = \frac{1.500 \times .57735}{.3927} = 2.20 \]

Given the proportions of a pair of helical gears (external or internal), to determine the total contact ratio:

When,

\[ m_p = \text{Contact Ratio} \]
\[ m_f = \text{Face Contact Ratio} \]
\[ m_t = \text{Total Contact Ratio} \]

Then,

\[ m_t = m_p + m_f \]

Example:

\[ m_p = 1.59 \quad m_f = 2.20 \]
\[ m_t = 1.59 + 2.20 = 3.79 \]
estimated), the amount of crowning should be chosen in such a way that when applying the service load, the lowest root stresses will be the result. This criterion is satisfied when the product

\[ K_c = K_{R_{t}} \cdot Y_\tau \cdot K_{R_{f}} \]

reaches a minimum.

As an example this optimization is performed for the test gears in Fig. 18. One can see that the curve for \( K_c \) has a flat minimum in the area of small crowning values (near gear set B). This result seems to be plausible because of the very stiff test rig.

It should be noted that the optimization method introduced here is only based on the tooth root stresses and should only be used if tooth breakage is the critical failure criterion. An optimization for contact stresses may be quite different and usually provides a guide to higher amounts of crowning.

**Summary**

By strain gauge measurements of spiral bevel gears, the influence of lengthwise crowning and relative displacements between pinion and gear on tooth root stresses was investigated. It was found that the crowning effects the load distribution over the lines of contact and the load sharing between pairs of teeth meshing simultaneously. For both influences a quantitative description could be derived.

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(continued from page 45)

In the case of relative displacements, deviations in pinion mounting distance and in offset have the strongest influence on the root stresses. A method was introduced to determine the increase or decrease of maximum stresses that have to be expected for a combination of certain values of these parameters. Further, an optimization criterion was derived that allows finding the amount of lengthwise crowning producing the lowest root stresses for a certain service condition.

References