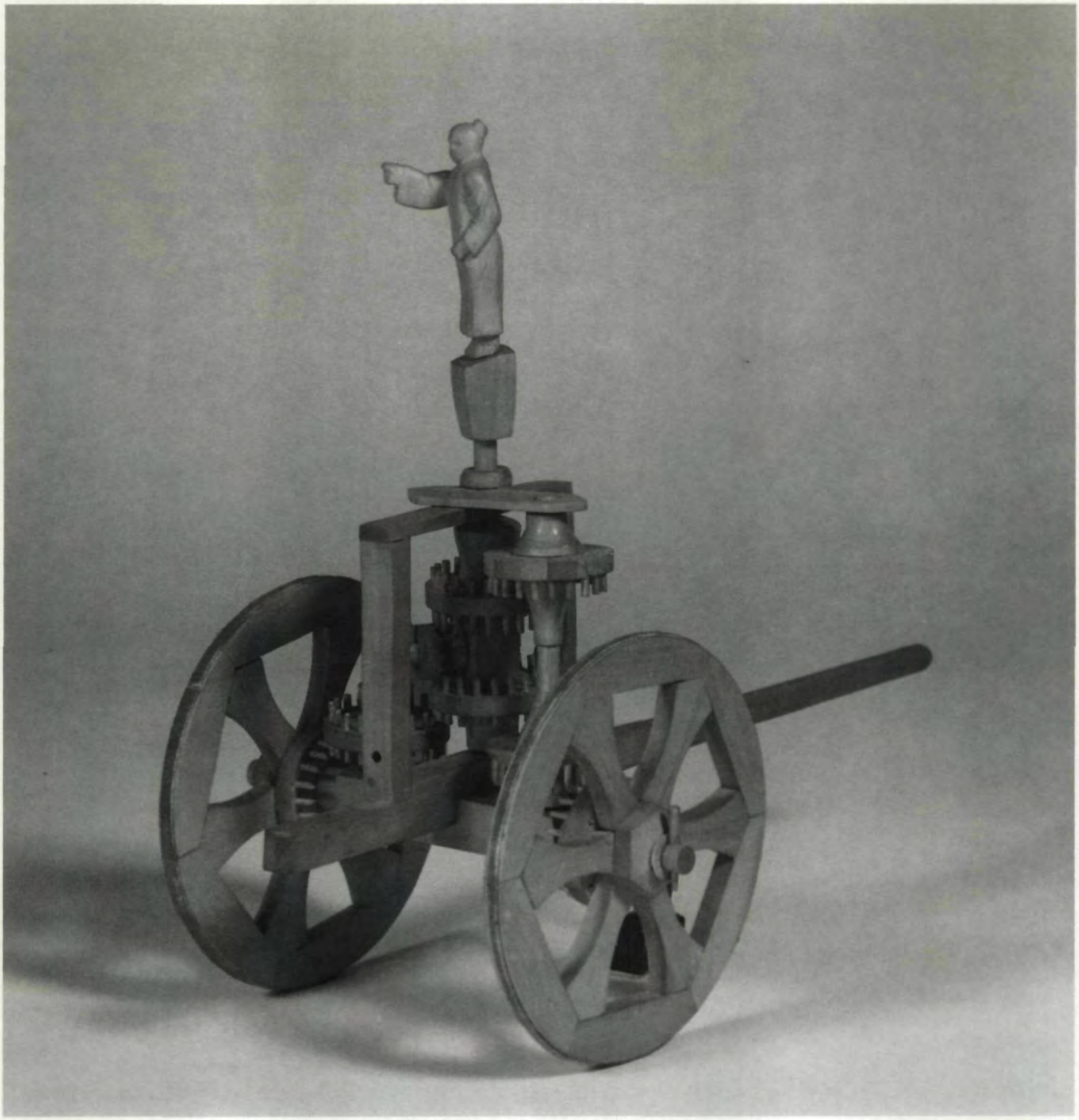

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Curvilinear Cylindrical Gears

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(Photo on left) Wooden model of "South Pointing Chariot". Although the curvilinear gear is not visible on this model, such gears were used in the manufacture of these devices, which may have served as compasses in ancient times.

The curved tooth cylindrical gear is one of ancient design. Samples which date from the period of the Warring State (475-221 BC) have been excavated from archeological sites in China. One such sample is now on display in the Xi'an Clay Figures of Warriors and Horses Exhibition Hall. This example is about 3/4" in diameter and made of bronze. It was used in the famous model, "Ancient Chinese Vehicle With a Wooden Figure Always Pointing to the South." Although this early gear is handmade and somewhat crude, it is a viable model.

Chinese technologists have been interested in this ancient design, and, after overcoming some technical difficulties, we have successfully designed the special machine tools and manufacturing processes necessary to bring the curvilinear cylindral gear into practical production. The first marketable applications appeared in 1980. Photographs of some of these gears are shown in Fig. 1.

Basic Curvilinear Cylindrical Gear Manufacture

Gear manufacturers and consumers are always searching for more compact or higher capacity gear sets. This design of gearing offers a way to achieve these characteristics. The basic manufacturing process is described below.

First imagine a tooth generating rack made of hard metal. The rack is kept stationery, and the form of its longitudinal tooth line is circular rather than the usual straight line. A plastically compliant gear blank is used and is run along the rack with a pure rolling motion, that is, with no slippage; thus, the blank is squeezed and extended to produce a circular toothed gear conjugate to the rack. (See Fig. 2.) If the tooth line of the rack were not circular, but of another form, the teeth produced on the blank would be of another curved form conjugate to that rack. All such curved tooth forms are called "curvilinear tooth cylindrical gears."

To produce such gears, the combined motion of the gear blank and rack must obey the following rule:

$$V_o = V_p = \pi D_p n \tag{1}$$

where

V_o – Linear translational velocity of the gear blank axis in meters per minute (m/min).

V_p – Circular velocity of the blank on time pitch diameter in m/min.

D_p – Pitch diameter in meters.

π – Pi

n – Rotational speed of the gear blank in rpms.

In practice we use a rotating face mill cutter to produce the gear teeth. The cutter spindle rotates on a fixed or stationary axis while the gear blank is rotated and translated across the turning cutter teeth according to Equation 1. By this rolling generating process the cutter produces one space on the gear. The blank is then indexed one tooth and the

Fig. 1 – Some samples of curvilinear tooth gears.

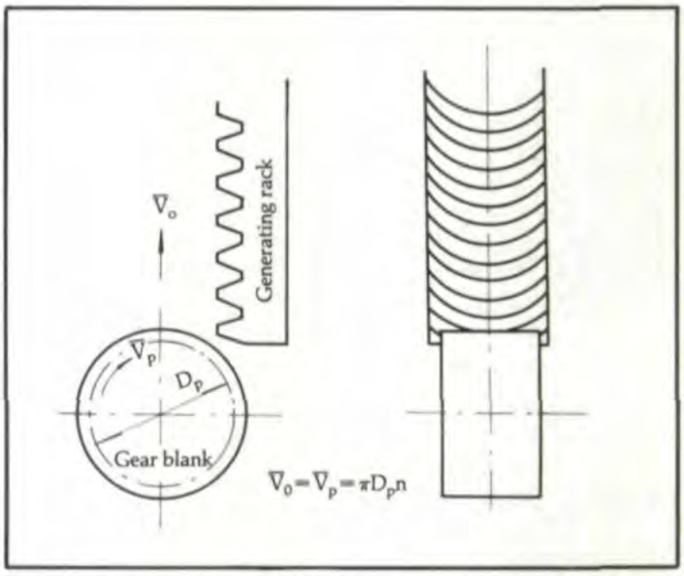

Fig. 2 – The generating principle of a curvilinear tooth cylindrical gear.

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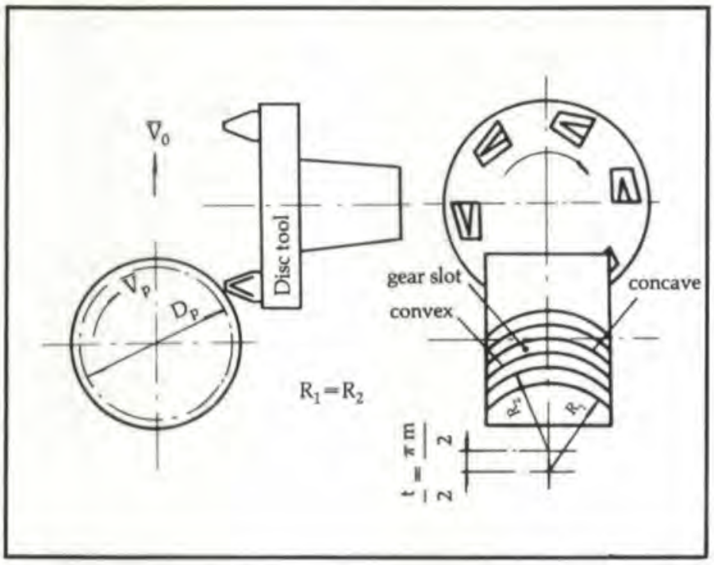

Fig. 3 – Cutting a circular tooth gear with a disc tool.

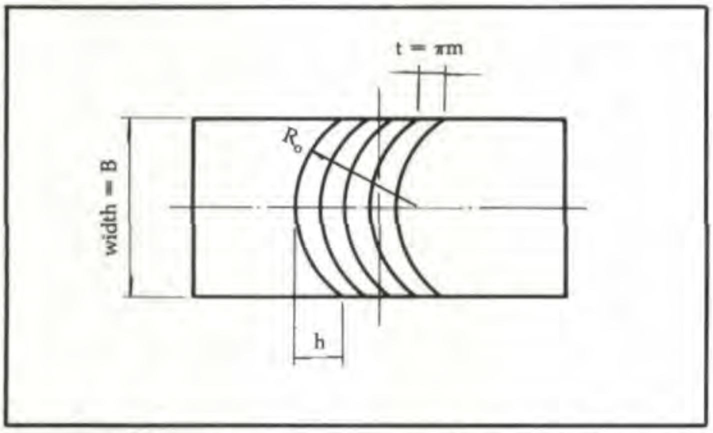

Fig. 4 – The supplementary contact ratio of a circular tooth gear.

generating cycle repeated. This sequence is continued until all the spaces and teeth are formed. (See Fig. 3.)

Since the generating motions are relative, an alternate machine construction would have the gear axis fixed and the cutter translated past the gear blank. The results will be the same.

For ease in manufacturing, we made the profile on the cutter blades in the form of an isoceles trapezoid similar to the tool used in Acme thread turning. The generating action of this tool configuration develops a gear with an involute profile of pressure angle α in the central section and a hyperbolic evoluted profile in the off-center sections. But at any section across the gear face, its concave and convex flanks are conjugate, so any pair of gears can be properly meshed together.

Obviously, the radius of curvature on both flanks of the gear must be alike to achieve proper engagement. The radius of curvature on the outer form of the cutter teeth is larger than that on the inner form. So after the gear is formed by the first cutter, a second cutter is utilized to form the convex faces to the same radius as the concave flank. (See Fig. 3.)

We have now developed a continuous cutting process which provides greater manufacturing efficiency.

Strength Analysis of a Curvilinear Tooth

The Contact Ratio. According to the theory of mechanism, we know that the contact ratio of a spur gear is

$$E_o = E_1 + E_2 - E_A \qquad (2)$$

$$E_o = \frac{\sqrt{(Z_1 + 2)^2 - (Z_1 \cos\alpha)^2}}{2\,\pi\cos\alpha} + \frac{\sqrt{(Z_2 + 2)^2 - (Z_2 \cos\alpha)^2}}{2\,\pi\cos\alpha} - (Z_1 + Z_2)\frac{\tan\alpha}{2\pi} \qquad (2a)$$

where

Z_1 = number of teeth on the pinion
Z_2 = number of teeth on the gear.
α = the pressure angle.

A curvilinear gear tooth possesses the same contact ratio as a spur gear, plus a supplementary contact ratio due to face overlap. (See Fig. 4.)

The supplementary contact ratio of a curvilinear tooth gear is

$$E_S = \frac{h}{t} = \frac{h}{m\pi}$$

Where

t = Tooth pitch distance

m = Module of the gear

$h = R_o - \sqrt{R_o^2 - (\frac{B}{2})^2}$

R_o = Radius of tooth curvature

B = Gear face width

∴ the total contact ratio of a curvilinear tooth gear is

$$\Sigma E = E_o + E_S \qquad (3)$$

Evidently, ΣE is greater than E_o. That means that a pair of curvilinear tooth gears has a greater number of gear teeth simultaneously in mesh; thus the force exerted on every individual tooth may be reduced materially.

For instance, if E_o of a spur gear equals 1.7, that means at a certain instant, only one tooth pair is meshing. But for a pair of curvilinear toothed gears, ΣE increased to 2.2, at least two tooth pairs are in mesh at all times.

The Moment of Inertia and Section Modulus of the Tooth Root of a Curvilinear Gear. First, we observe the cross section of the tooth root of a spur gear. (See Fig. 5.)

The moment of inertia is

$$I_{spur} = \frac{1}{12} BS_o^3 \qquad (4)$$

The section modulus is

$$W_{spur} = \frac{1}{6} BS_o^2 \qquad (5)$$

Then let us observe the cross section of the tooth root of a circular tooth gear. (Fig. 6.)

By computation, we found that its modulus W_c is bigger than W_{spur}. We tabulate the numerical value of a real example in Table 1.

For instance, if we design a pair of curvilinear gears whose $B/2R_o = 0.6$, from the table we find $W_c/W_{spur} = 1.20$ and $\Sigma E = 2.76$ (ΣE of the spur gear is only 1.7). Then the strength of such gear is $1.20 \times \frac{2.76}{1.7} = 1.95$ times stronger than the spur gear.

Table 1. Moment of Inertia and Section Modulus of Circular Gear Tooth

$B/2R_o$	I_c/I_{spur}	W_c/W_{spur}	ΣE
0.50	2.52	1.059	2.55
0.52	2.64	1.076	2.60
0.54	2.79	1.105	2.64
0.56	2.96	1.139	2.67
0.58	3.14	1.173	2.72
0.60	3.31	1.20	2.76
0.62	3.53	1.236	2.81
0.64	3.74	1.275	2.85
0.66	3.93	1.304	2.90
0.68	4.18	1.344	2.95
0.70	4.48	1.395	3.0

I_c – Moment of inertia of a circular gear tooth
I_{spur} – Moment of inertia of a spur gear tooth
W_c – Section modulus of a circular gear tooth
W_{spur} – Section modulus of a spur gear tooth

ΣE is calculated corresponding to a special case if $Z_1 = 1.9$, and $Z_2 = 60$; whereas E_o of spur gear is only 1.7.

Table 2, The Comparison of Tooth Length Between Spur Gear and Circular Tooth Gear

$B/2R_o$	L/B	$B/2R_o$	L/B	
0.50	1.047	0.60	1.072	
0.52	1.052	0.62	1.079	
0.54	1.056	0.64	1.085	
0.56	1.060	0.66	1.092	
0.58	1.066	0.68	1.10	

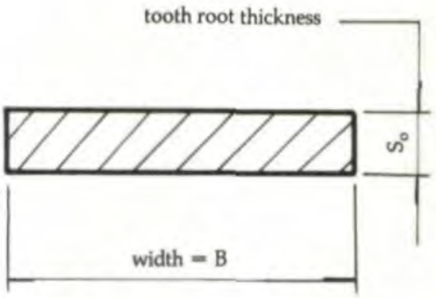

Fig. 5 – Cross section of the tooth root of a spur gear.

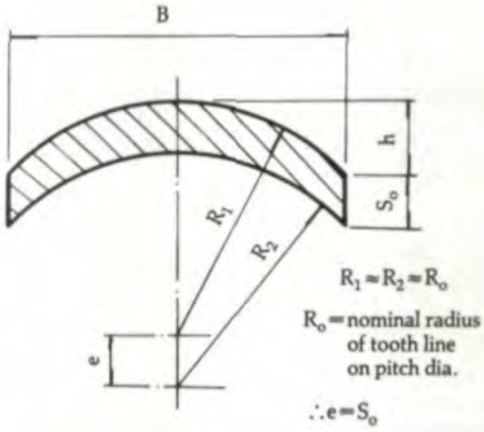

Fig. 6 – Cross section of the tooth root of a circular tooth gear.

The Length of Tooth Contact Line. The length of the tooth contact line of a spur gear equals its width, but the length of tooth contact line of a circular tooth gear is somewhat longer. We may compare the two in Table 2.

The Stress Induced On a Curvilinear Gear Tooth. From this discussion, we can conclude the following:

- The bending stress of circular gear teeth is reduced by the higher contact ratio and the greater section modulus.
- The contacting stress of the circular gear tooth is also reduced by its higher contact ratio and longer length of tooth contact.
- If an alternate tooth curve is used and is approximately an arc, then it has features similar to a circular gear.

The Characteristics of Curvilinear Tooth Gears

1. Curvilinear tooth gears possess a higher bending and contacting strength; thus, the center distance of gear boxes may be reduced, while maintaining power transmissions.
2. Since there are more teeth simultaneously engaged, these gears run smoothly with low noise.
3. Oil is retained within the concave tooth surface, so there is always an oil film between the two engaging surfaces,

resulting in good lubrication qualities.

4. No axial force is produced during operation, so axial bearing load is negligible.
5. Gears with similar, but not identical, modules can be cut with a single tool set, saving tooling costs.
6. Because of the simplicity of the cutting tool, it is easy to implement carbide cutting, resulting not only in an increase in manufacturing efficiency, but also in the ability to cut gears with much higher hardnesses, producing improved quality.

At present, these gears are in use in steel plants, aluminum rolling mills, cement equipment plants and other places. Their superiority has been proven in practice. The service life of machinery has been improved by anywhere from two to forty times or more.

Editors' Note:

Gearing of this type was available in the U.S. some years ago under the name "Spuricals". The herringbone type gear, in use worldwide, also has the same attributes and would be the first choice by a gear applications engineer today. Some reasons are currently available machine and cutting tools and the availablility of some competitive gear vendors. Some of the variations in the longitudinal form of gear teeth that have been used are shown in Fig 7.

To produce these gears with proper contact, some form of profile and longitudinal adjustments may be necessary. The profile of the blades can be modified to produce tip relief or root relief on the gear tooth, and the radius of curvature of the cutters can be altered to provide a central bearing contact, avoiding tooth edge contact.

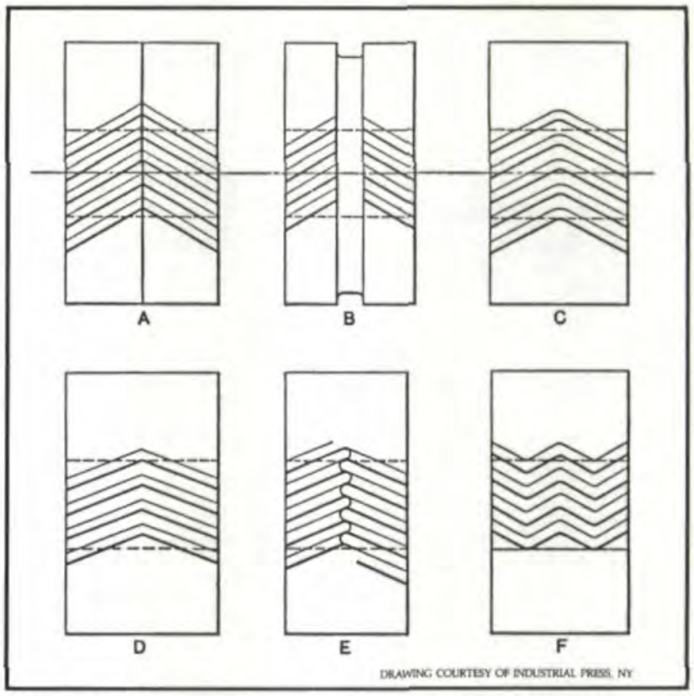

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Fig. 7 – Diagrammatical views of different types of herringbone gears.

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