Introduction

Can a gear profile generated by the hobbing method be an ideal involute? In strictly theoretical terms — no, but in practicality — yes. A gear profile generated by the hobbing method is an approximation of the involute curve. Let's review a classic example of an approximation.

Do regular polygons and circles have anything in common? Yes. One can approximate a circle simply by increasing the number of sides of a polygon.

Let's assume that one has scissors and can cut only a straight line. Let's cut the simplest polygon, a triangle, from a piece of paper. It hardly resembles a circle (Fig. 1). The shaded area shows the variation between a triangle and a circumscribed circle. But if the number of the sides of a polygon is doubled (Fig. 2), the variation between a polygon and a circle is reduced dramatically. Yet, if the number of sides is doubled once again, the variation can hardly be seen (Fig. 3).

By increasing the number of sides of a polygon further, it is possible to get so close to a circle that the variation becomes negligible — the difference cannot be seen or even measured.

The process of generating a gear on a hobbing machine is based on a similar idea of approximation. Hobbing is a process that generates a number of connected lines which approximate an involute curve.
Involute Generation on a Hobbing Machine

A hobbing machine cannot cut curves, but it can cut lots of straight lines in a certain pattern. Therefore, the idea of approximation is utilized in order to generate an involute. Every cutting edge of a hob cuts a straight line. The number of straight lines (enveloping cuts) should be large enough so that the difference between the involute and the combination of straight lines becomes negligible.

Figs. 5 and 6 show gear profile generation as seen by an observer who rotates with the gear.

Fig. 5 shows an approximation of an involute generated by only three cutting edges of a hob. The shaded area illustrates the variation between the involute and the approximating cuts.

If the number of cutting edges is increased, as in Fig. 6, the variation becomes less apparent. The involute variation generated by an ideal hob can be calculated as follows:

\[
\text{Profile Variation} = \frac{\pi^2 \cdot Z_0 \cdot M_n \cdot \sin(NPA)}{(4 \cdot Z_2 \cdot i^2)}
\]

- \(Z_0\) — Number of hob starts
- \(M_n\) — Normal module
- \(NPA\) — Normal pressure angle
- \(Z_2\) — Number of gear teeth
- \(i\) — Number of hob gashes

As one can see from the formula, an exponential reduction in variation can be obtained by increasing the number of gashes. A gear generated by means of an ideal hob, an ideal machine and an ideal fixture will have a profile curve that is an approximation of an involute, in the same way an equilateral polygon approximates a circle. The whole topology of a gear tooth consists of numerous cuts in lead and involute direction (Fig. 7).

A center of every single generating cut lies on the line of action (Fig. 8). The dashed lines depict hob cutting edges from the point of view of an observer rotated the gear.

After a hob with a sufficient number of cutting edges is selected, the hob should be able to generate an ideal involute or at least an involute with a predictable variation. Why does it sometimes fail? Well, because we live in an imperfect world, especially when it comes to a hobbing machine, a work-holding fixture, a blank or a hob, all of which effect

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The ultimate goal of hob inspection is to make sure that during the hobbing process, the cutting edges of a hob have a minimum deviation from their theoretical positions.

There are several hob geometrical characteristics. Some of them, like the line of action, show the direct variation of cutting edges from their theoretical positions at the points where they generate gear involutes. Most characteristics, however, can only show the displacement of cutting edges indirectly.

Commonly accepted characteristics that a hob manufacturer or a hob user might check include the following: Radial runout of proof flanges, face runout, rake, flute index, flute lead, lead and thread-to-thread variation, outside diameter, pressure angle, line of action, radial and axial relief and tooth thickness.

**Radial Runout of Proof Flanges**

Most hobs have ground proof diameters or hubs on both sides (Fig. 9). These diameters are used by operators to indicate a hob when mounting it on the machine (Fig. 10). All hob geometrical characteristics are referenced to proof diameters. Usually proof diameters are checked first.

Excessive hub runout causes a gear profile error that can be approximately calculated as shown in Fig. 11.

\[
\text{Profile error} = 2 \cdot \text{eccentricity} \cdot \sin(\text{axial pressure angle})
\]

Fig. 12 shows the effect of hob radial runout on gear involute. Fig. 13 shows the least squares method for determination of a circle's center and out-of-roundness. This method allows one to determine concentricity and out-of-round amount very precisely.

A CNC inspection machine will automatically check the hub's runout at a specified position (Fig. 14).

Evaluation should include the determination of total runout, out-of-round and concentricity errors (Fig. 15). The results of inspection and evaluation can also be presented in circular form as shown in Fig. 16. This chart shows a round surface that is magnified 2000 times. The distance to the center of the best fit circle has the
Effect of hob radial runout on gear involute

Fig. 12

Eccentricity

Fig. 13

Distance from cutting edge
For checking upper hub

Fig. 14

Hob Journal Inspection

Fig. 15

Hob Journal Inspection

Fig. 16
Fig. 17
Hob face runout

Fig. 18
Left flank Right flank
Effects of hob face runout on gear involute

Table: Hob Face Inspection

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<th>Hob ID</th>
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Fig. 19
Hob Face Inspection

Fig. 20
Surface Variation

Fig. 21
Surface Variation

Fig. 22
Surface Variation

Fig. 23
Form
Slope
Total
Evaluation range
same magnification; thus, eccentricity can be scaled. All the numerical evaluations are also displayed on the chart. The results of hub inspection on both sides are superimposed so the runout inaccuracies can be compared visually.

The evaluation program may have a built-in AGMA, DIN, and ISO hob tolerance system. If the operator specifies the required quality class, the program should automatically compute the required tolerance. For characteristics which have quality classification, the actual quality may also be automatically determined and displayed.

**Face Runout**

Hob faces are frequently utilized for clamping during mounting a hob on either a hobbing machine or a hob sharpening machine. The hob faces have to be trued (Fig. 17). Excessive face runout can result in involute variation (Fig. 18). Inspection and evaluation of face runout include out-of-flat and eccentricity (Fig. 19).

**Rake**

Some simplification of surface variation may be useful for process analysis and problem solving.

**General Surface Variation Components.**

The variation of any surface from its ideal condition can be simplified as a variation of a mountain terrain in relation to a flat surface. One could ski on a mountain with a steady and even drop (Fig. 20), or on a horizontally undulating terrain (Fig. 21). But frequently mountain terrain is a combination of both (Fig. 22).

The concept of breaking down the total surface variation into several components is widely used in many applications, including hobs and gears. Fig. 23 illustrates the least squares method for the determination of form and slope error components. Frequently, slope and form errors are useful even if not specified by DIN/AGMA/ISO standards. The breakdown of the total value into slope and form components helps to determine the sources of errors and better identify any needed process adjustments.

**Hob Rake Inspection.** Hob rake is a line resulting from the intersection of a tooth face with a plane that is normal to the hob axis. If this line crosses the hob center, it is called a zero rake.

Rake offset is the amount by which the design rake line is distant from the plane of a hob axis (Fig. 24). Hob rake offset is zero if
Fig. 27

Fig. 28

Editor's Note: The second half of this article will appear in our next issue.

Acknowledgements: The author wishes to thank Ed Driscoll for his advice, support, and creativity that inspired and helped write many parts of this paper; John Lange, for co-authoring a paper on a similar subject presented at an AGMA symposium in 1989; Lauren Bromberg for her meticulous and creative editing help; Rachel Haisman, for creating the illustrations that helped simplify the description of surface variation characteristics and Richard Considine for computerizing them, and Esther Munsey for her constructive editorial help. Gear inspection charts are courtesy of Roto-Technology, Inc., Dayton, OH. This article was first presented at the AGMA Gear Manufacturing Symposium, held in Detroit, MI, October 1993.

References:
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