

# Tribology Aspects in Angular Transmission Systems

## Part VIII: Super-Reduction Hypoid Gears

Dr. Hermann Stadtfeld

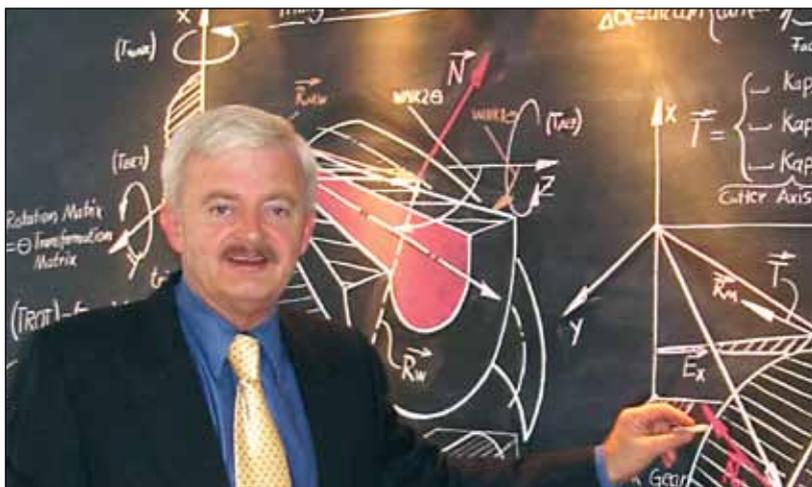
*(This is the final installment of a Gear Technology-exclusive eight-part series on the tribology aspects of angular gear drives. Gear Technology (Randall Publications LLC) is most grateful to Dr. Stadtfeld and the Gleason Corporation for choosing this magazine as the platform for presenting this very informative and most-relevant series of “teach-ins” to its readers.)*

### Design

Super-reduction hypoid gears (SRH) are bevel worm gears with certain differences regarding hypoid gears. If two axes are positioned in space and the task is to transmit motion and torque between them using some

kind of gears with a ratio above 5 and even higher than 50, the following cases are commonly known:

- Axes cross under an angle (mostly  $90^\circ$ ) → Worm Gear Drives (line contact)
- Axes cross under an angle close to  $90^\circ$  → SRH Gears (line contact)



**Dr. Hermann Stadtfeld** received a bachelor's degree in 1978 and in 1982 a master's degree in mechanical engineering at the Technical University in Aachen, Germany. He then worked as a scientist at the Machine Tool Laboratory of the Technical University of Aachen. In 1987, he received his Ph.D. and accepted the position as head of engineering and R&D of the Bevel Gear Machine Tool Division of Oerlikon Buehler AG in Zurich, Switzerland. In 1992, Dr. Stadtfeld accepted a position as visiting professor at the Rochester Institute of Technology. From 1994 until 2002, he worked for The Gleason Works in Rochester, New York—first as director of R&D and then as vice president of R&D. After an absence from Gleason between 2002 to 2005, when Dr. Stadtfeld established a gear research company in Germany and taught gear technology as a professor at the University of Ilmenau, he returned to the Gleason Corporation, where he holds today the position of vice president-bevel gear technology and R&D. Dr. Stadtfeld has published more than 200 technical papers and eight books on bevel gear technology. He holds more than 40 international patents on gear design and gear process, as well as tools and machines.

To establish line contact in SRH gears, the ring gear is defined by Formate-design geometry and then used as a generating gear for the pinion. In order to define a pinion without mutilations and minimum undercut, the pinion pitch diameter and the normal pitch (equal to normal module times  $\pi$ ) are used to calculate the ideal mean lead angle. Figure 1 shows how the SRH pinion with a certain lead angle  $\varphi_e$  is positioned vertically, such that  $90^\circ$  minus the spiral angle  $\beta$  matches the mean lead angle  $\varphi_e$  of the pinion. The last described procedure is done in order to define the hypoid offset. The offset can be changed by increasing or reducing the gear spiral angle. An optical explanation for why bevel worm gears require a different theoretical approach than hypoid gears is illustrated in Figure 2. The normal profile of the hypoid pinion in the top sequence in Figure 2 is easily identified in the top left (front view) or center photos (pinion rotated by the spiral

angle). And yet the normal profile of an SRH pinion is more easily recognized in the side view (right photos). The reason is that the spiral angle in most cases is larger than  $45^\circ$ , which is why the SRH pinion appears like a tapered ACME screw.

SRH gears have a length sliding between their teeth which is present in any flank surface point and dominates the small amounts of profile sliding. The axes of SRH gears in most cases cross under an angle of  $90^\circ$ . This so-called shaft angle can be larger or smaller than  $90^\circ$ , where shaft angles above  $90^\circ$  can lead to internal ring gears that are often limited in their manufacturability due to cutter interference. However, the axes do not intersect and the smallest distance between them is called the *hypoid offset*. The shaft angle is defined in a plane perpendicular to the offset direction (Fig. 3, right).

SRH gears have a parallel-depth profile along the face width if they are manufactured in the continuous-face hobbing process, or a tapered-depth profile along the face width if the manufacturing is done using the single-indexing face milling process.

SRH gear teeth follow in face width direction a curve on the conical gear and pinion body that lies under an angle to a cone element (spiral angle). The tooth lead function in face width direction, if unrolled into a plane, is an epicycloid or a circle, depending on the manufacturing method. The tooth profile is a spherical involute in a parallel-depth tooth system; it is an octoid in a tapered-depth tooth system. The tooth form with a spherical involute will result in line contact between two mating flanks in each angular position if no crowning has been applied. With an octoid, there will be an initial “natural” profile crowning and, depending on the machining setup, some flank twist. Both effects are utilized together with certain corrective machine settings in order to generate the desired crowning.

The photo of an SRH gear set (Fig. 3) explains the definition of right-hand and left-hand spiral direction and indicates the coast and drive-side gear flanks. The cross-sectional drawing to the right in Figure 3 illustrates the blank design for parts manufactured in the face milling process (tapered-depth teeth).

### Analysis

Since the cited distortions in tapered-depth tooth systems are detected by comparison to conjugate mating flanks, it is possible to define potential contact lines that would apply should

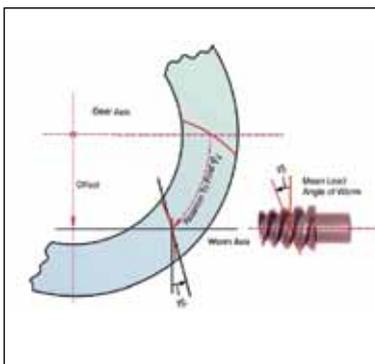


Figure 1—Global SRH design



Figure 2—Difference between face profile and normal profile.

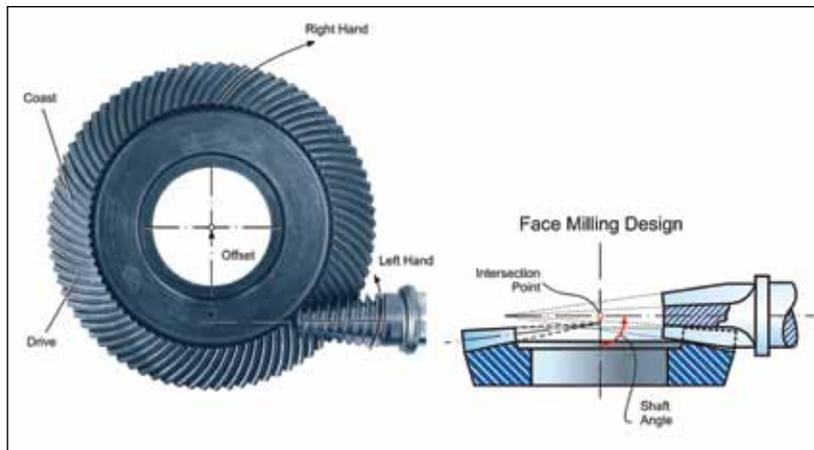


Figure 3—Super-reduction hypoid gear geometry.

the distortions be removed or if load-affected deflections allow for a contact spread. In order to allow for deflections of tooth surfaces, shafts, bearings and gearbox housing—without unwanted edge contact—a crowning in face width and profile direction is applied. A theoretical tooth contact analysis (TCA) previous to gear manufacturing can be performed to observe the effect of the crowning in connection with the basic characteristic of the particular gear set. This also affords the possibility of returning to the basic dimensions in order to optimize them if the analysis results show any deficiencies. Figure 4 shows the result of a TCA of a typical SRH gear set.

The two columns in Figure 4 represent the analysis results of the two mating flank combinations (see also Part I, “General Explanation of Theoretical Bevel Gear Analysis”). Using the drive side as the main load transmission direction for SRH gears is a rather binding rule. When the concave gear flanks and the convex pinion flanks are chosen as driving side, then, per definition, this is a “driving-coast side.” A driving-coast side is particularly critical in SRH gearing. In spite of spiral bevel

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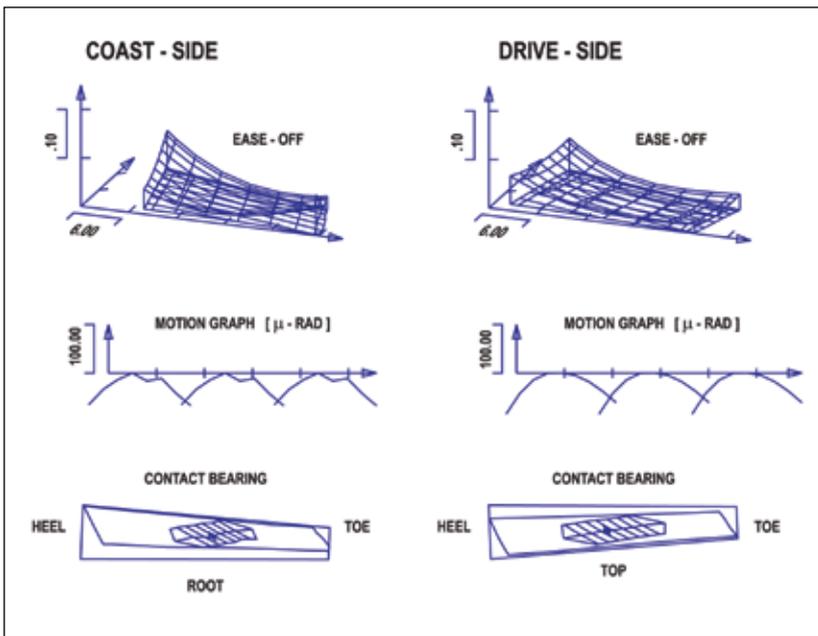


Figure 4—Tooth contact analysis of an SRH gear set.

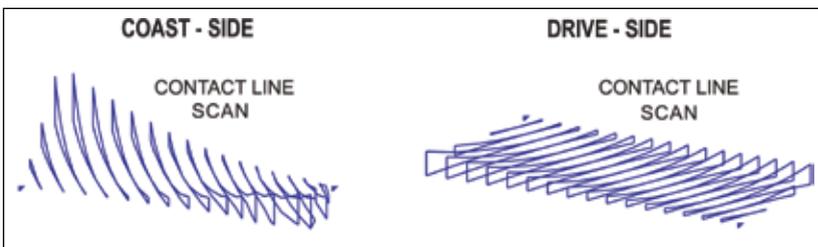


Figure 5—Contact-line scan of an SRH gear set.

and hypoid gears, the axial forces caused by the large-pinion spiral angle dominate and often lead to twice the bending stress in the pinion core diameter at the heel. The fracture risk of SRH gear sets is therefore judged on the tension stress in the pinion core diameter and not on the root of the gear and pinion teeth.

The top graphics show the ease-off topographies. The surface above the presentation grid shows the consolidation of the pinion and gear crowning. The ease-offs in Figure 4 have a combination of length crowning, profile crowning and flank twist, and so a clearance along the boundary of the teeth is established.

Below each ease-off, the motion transmission graphs of the particular mating flank pair are shown. The motion transmission graphs show the angular variation of the driven gear in the case of a pinion that rotates with a constant angular velocity. The graphs are drawn for the rotation and mesh of three consecutive pairs of teeth. While the ease-off requires a sufficient amount of crowning in order to prevent edge contact and allow for load-affected deflections, the crowning in turn causes, in

this example, proportional amounts of angular motion variation of about 50 micro radians.

At the bottom of Figure 4, the tooth contact pattern is plotted inside of the gear tooth projection. These contact patterns are calculated for zero-load and a virtual marking compound film of 6  $\mu\text{m}$  thickness. This in essence duplicates the tooth contact, as one may assume, as in rolling the real version of the analyzed gear set under light load on a roll tester while the gear member is coated with a thin marking-compound layer. The contact lines lie under an angle of  $55^\circ$  to the root direction, depending basically on the spiral angle. The path of contact connects the start and stop of meshing; its orientation is mainly in the face width direction.

The crowning reflected in the ease-off results in a located contact zone inside the boundaries of the gear tooth. A smaller tooth contact area generally results from large magnitudes in the ease-off, and the motion graph, and vice versa.

Figure 5 shows 20 discrete potential contact lines with their individual crowning amounts along their length (contact-line scan). The gap geometry in contact-line direction can be influenced by a change in ease-off topography and optimized regarding the gap kinematic cases (see also Part I, "General Explanation of Theoretical Bevel Gear Analysis," Figure 8). The gap geometry perpendicular to the contact-line direction (not exactly the same as the path-of-contact direction) does not significantly depend on the ease-off topography; rather, it is mainly dominated by the geometry of the mating tooth profiles. Typical of SRH gears is the lubrication gap change from contact line to contact line. Effects such as those discussed in cases 5 and 6 are likely to be applicable in SRH gears and can also be controlled, to some extent, in SRH ease-off developments.

Figure 6 shows the sliding- and rolling-velocity vectors of a typical hypoid gear set for each path of contact point for the 20 discussed roll positions; each vector is projected to the tangential plane at the point-of-origin of the vector. The velocity vectors are drawn inside the gear tooth projection plane. The points-of-origin of both rolling- and sliding-velocity vectors are grouped along the path-of-contact, which is found as the connection of the minima of the individual lines in the contact-line scan graphic (Fig. 5). The rolling- and sliding-velocity vectors in the present example have the same orientation as the contact lines.

Therefore, in order to investigate the hydrodynamic lubrication properties, employing the information from the contact line scan (curvature and curvature change) and the velocities in Figure 6 will be sufficient (see also Part I, “General Explanation of Theoretical Bevel Gear Analysis,” Figure 8, cases 1–6).

With the SRH gear set under discussion, the sliding-velocity vectors between toe and heel—right to left—have about the same direction; they consist of a certain profile component and a larger length component. The reasons for this uniform sliding velocity orientation are the high screw motion that is constant along the face width, and the fact that the pitch line is far above the face of the gear tooth (outside of the flank surface).

The rolling-velocity vectors point down and to the right, and all have basically the same orientation. Opposite directions of sliding and rolling velocities (Fig. 6) deliver the lubrication cases 2 or 4 (see Part I, “General Explanation of Theoretical Bevel Gear Analysis,” Figure 8) that result in an unfavorable negative slippage.

### Manufacturing

Super-reduction hypoid gears are manufactured in a continuous-indexing face hobbing process or in a single-indexing face milling process. In the face milling process the blades are oriented around a circle and pass through one slot (while they plunge or generate the flanks of that particular slot), as illustrated in Figure 7, left (Ref. 2). The work is not performing any indexing rotation. At the blade tip and in equidistant planes (normal to the cutter head axis) the slot width produced has a constant width between toe and heel. In order to achieve a proportionally changing slot width (and tooth thickness), the root line of face milled bevel gears is inclined versus the pitch line (Fig. 7, left). This modification must be implemented in both members, which is why the face angle requires the same modification as the root angle of the mating member.

In face hobbing (Fig. 7, right)—we observe a group of one inside- and one outside-blade passing through one slot—while the work rotates with:

$$\omega_{\text{Work}} = \omega_{\text{Cutter}} \cdot (\text{Number of Cutter Blade Groups}) / (\text{Number of Work Teeth})$$

Due to the relative motion, the following blade group passes through the next slot. The blades in one group are positioned along a spiral, where the sum of the blade groups is oriented around a circle with equal distance to the

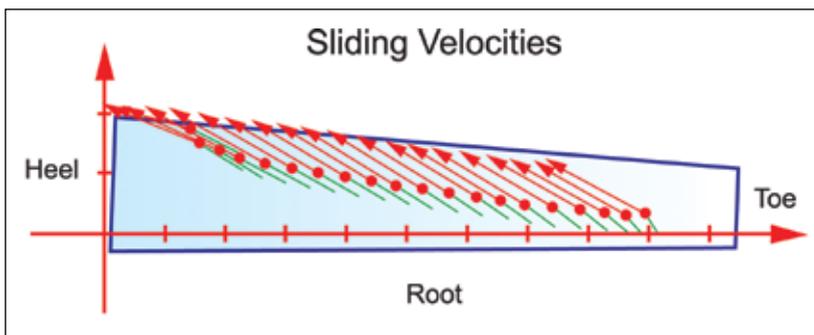


Figure 6—Rolling and sliding velocities of an SRH gear set along the path of contact.

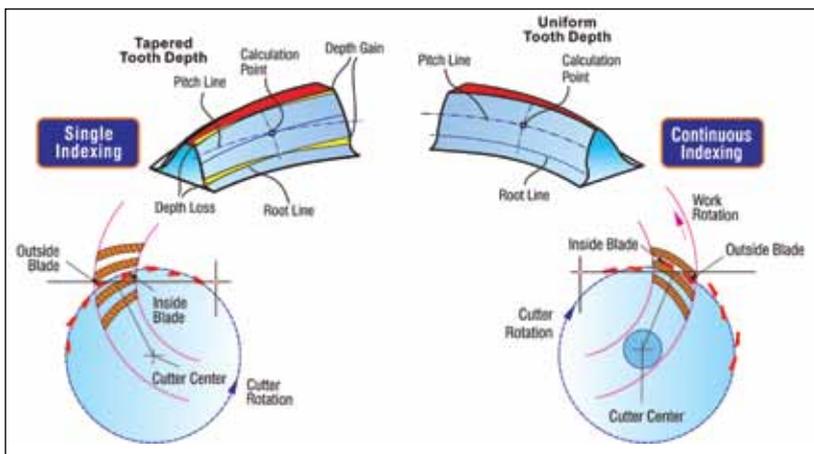


Figure 7—Left: face milling; Right: face hobbing.

cutter head center. Due to the described kinematic, the flank lines of the outer and inner flank are epicycloids that divide slot width and tooth thickness in equal fractions of the circumference at any point along the face width. The result is a “natural” slot width taper, proportional to the distance from the pitch apex. A root angle modification is not required or useful because of the already-existing perfect fit of mating teeth and slots.

Hard-finishing after heat treatment of face milled SRH gears is generally done by grinding. Figure 8 provides a view into the work chamber of a free-form bevel and hypoid gear grinding machine during the grinding of an SRH pinion. The grinding wheel grit material is sintered aluminum oxide that is applied with a surface speed of 25 m/sec. The grinding wheel resembles the cutter head geometry, while the grinding machine uses the same set up geometry and kinematic as the cutting machine for the previous soft-machining. Hard-finishing of face hobbed SRH gears is generally done by lapping. Pinion and gear are rolled under light torque while a lapping compound of silicon carbide oil mixture is

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Figure 8—Super-reduction hypoid pinion grinding.

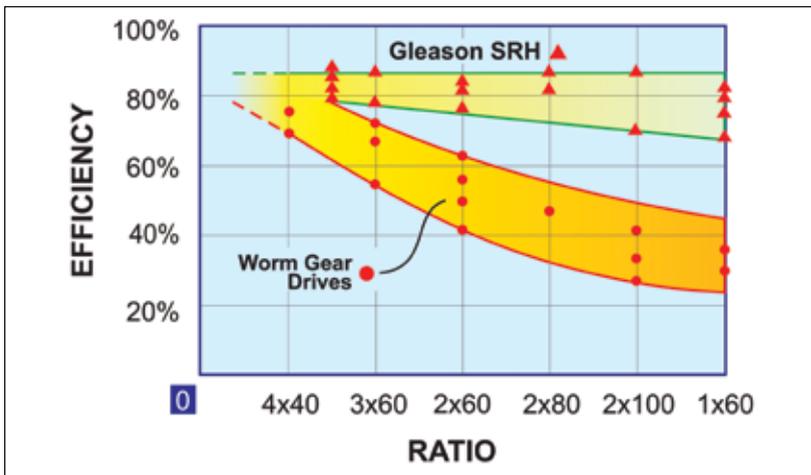


Figure 9—Efficiency comparison.

present between the flanks. Lapping leaves abrasive grain imbedded in the flank surfaces that might lead to several problems such as wear, temperature and lowered efficiency. Lapping of SRH gear sets requires low run-out and spacing errors in the parts before the lapping process and very short lapping times. Due to the much higher number of pinion rotations, the lap removal of the pinion surfaces is a multiple of the material removal on the gear flanks. This results in too much lapping on the pinion and almost no lapping on the gear. A compromise is difficult to find; the best recommendation is a very short lapping with low torque (1 Nm to 3 Nm) with the result of a well lapped pinion and a gear which still shows witness marks of the cutting flats.

The efficiency advantage of SRH gear sets over worm gear drives shown in Figure

9 is only achievable in the case of face milled and ground pairs. Grinding assures precise flank surfaces with consistent surface roughness—an important basis for high efficiency. Additionally, the characteristics of the contact-line scan (and the surface curvature perpendicular to the contact lines), in combination with the sliding- and rolling- velocities, are important factors in order to achieve the lowest possible energy dissipation in the tooth mesh.

#### Application

Most SRH gears are manufactured from carburized steel and are case-hardened to a surface hardness of 60 Rockwell C (HRC) and a core hardness of 36 HRC. Because of the higher pinion revolutions, it is advisable to give the pinion a higher hardness than the ring gear (e.g., pinion 62 HRC, gear 59 HRC). This is more important with increasing ratio and will also reduce the affinity between the pinion and gear flank surfaces and therefore reduce the risk of scoring.

Figure 10 shows the two most common surface damages on SRH flank surfaces due to failure by scoring (left photo) and “frosting” (right photo). Frosting is a surface failure in which a crack propagation 10 to 15 microns occurs beneath the surface (Ref. 3). High surface stress—in combination with a surface roughness characteristic as it is generated by grinding—promotes frosting in the flank areas with negative slippage. Negative slippage is present in SRH gearing within the entire flank surface, which leads to lubrication cases 2 and 4 (see Part I, “General Explanations on Theoretical Bevel Gear Analysis,” Figure 8). Micro crack propagation is promoted in lubrication case 2 at the left side of the contact point, demonstrating that SRH gear sets are prone to frosting.

In order to achieve surface durability in SRH gear sets, a high surface finish quality and a good lubrication with special high-pressure hypoid oils are recommended in order to maintain a surface-separating oil film. Synthetic hypoid oils provide the best results regarding the prevention of surface damage and the improvement of efficiency. A large contact pattern is also advantageous because of the reduction of the specific surface stress.

The advantages of super-reduction hypoids are:

- **Potentially higher reduction ratio in a one-stage reduction**
- **Back driving requires high torque or is even impossible**

- **Higher efficiency than worm gear drives**
- **Higher efficiency than multiple-stage reduction with standard hypoids**

SRH gears have axial forces that can be calculated by applying a normal force vector at the position of the mean point at each member (see also Part I, "General Explanation of Theoretical Bevel Gear Analysis"). The force vector normal to the transmitting flank is separated into its X, Y and Z components, from which the force components in those directions are calculated (Fig. 11).

The relationship in Figure 11 leads to the following formulas, which can be used to calculate bearing-force components in a Cartesian coordinate system and assign them to the bearing-load calculation in a CAD system:

$$F_x = -T / (A_m \cdot \sin\gamma)$$

$$F_y = -T \cdot (\sin\gamma \cdot \sin\beta \cdot \cos\alpha + \cos\gamma \cdot \sin\alpha) / (A_m \cdot \sin\gamma \cdot \cos\beta \cdot \cos\alpha)$$

$$F_z = -T \cdot (\cos\gamma \cdot \sin\beta \cdot \cos\alpha - \sin\gamma \cdot \sin\alpha) / (A_m \cdot \sin\gamma \cdot \cos\beta \cdot \cos\alpha)$$

where:

$T$	Torque of observed member
$A_m$	Mean cone distance
$\gamma$	Pitch angle
$\beta$	Spiral angle (in hypoids for pinion)
$\alpha$	Pressure angle
$F_x, F_y, F_z$	Bearing load force components

To achieve correct results it is required to use the pinion spiral angle for the SRH pinion and the gear spiral angle for the mating ring gear. Between pinion and gear spiral angle in SRH the following relationship applies:

$$\beta_{\text{pinion}} = \beta_{\text{gear}} + \arctan(a/A_m);$$

where:  $a$  = shaft offset

In Figure 13 the offset  $a$  is positive for cases 1 and 4 and negative for cases 2 and 3. The pinion spiral angle is positive in all left columns of Figure 13 and negative in the right columns (gear spiral angle has the opposite sign). The bearing-force calculation formulas are based on the assumption that one pair of teeth transmits the torque with one normal force vector in the mean point of the flank pair. The results are good approximations that reflect the real bearing loads for multiple tooth meshing within an acceptable tolerance. A precise calculation is possible with the Gleason bevel and hypoid gear software.

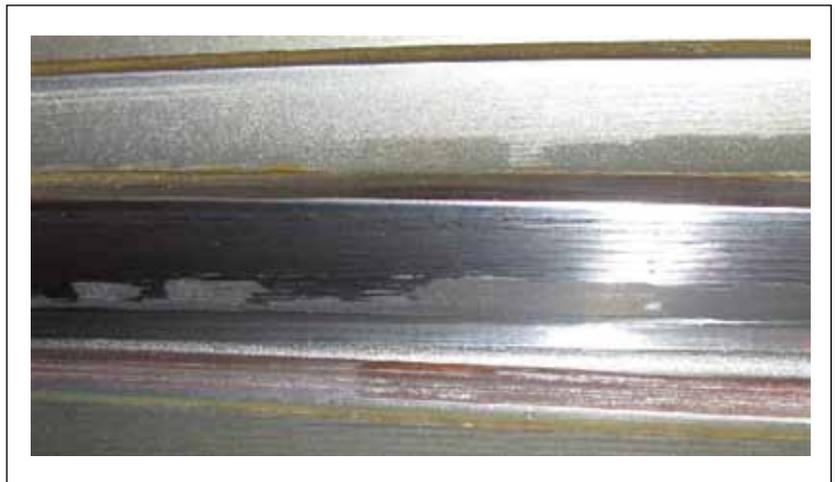


Figure 10—Scoring (left) and frosting (right).

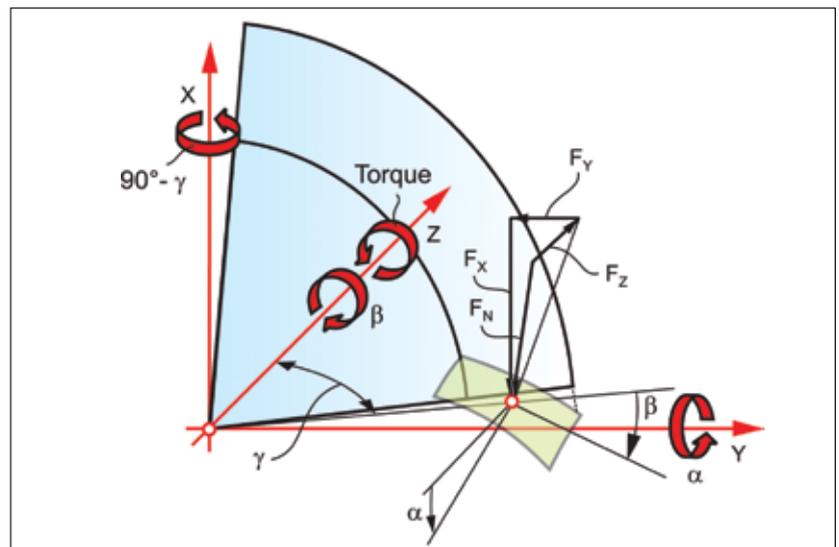


Figure 11—Force diagram for calculation of bearing loads.

Root bending strength is not the criteria for breakage or fracture in SRH gear sets. The weak spot for breakage is the pinion core at the heel. Due to the rather small core diameter—in connection with the notch effect—caused by the high pinion spiral angle, the highest tensile stress concentration is at the end of the teeth (heel) where the square in the drawing of the cantilever beam in Figure 12 is located. The bending moment and the forces are calculated from the force calculation above. Drive operation reduces the tension in the critical fiber of the cantilever beam (Fig. 12) and coast operation increases the tension—which is why in coast direction a significantly lower amount of load than in drive direction can be transmitted in order to assure endurance life.

Hypoid pinions have an advantage if an offset is chosen that increases the pinion spiral angle. Together with the spiral angle, the

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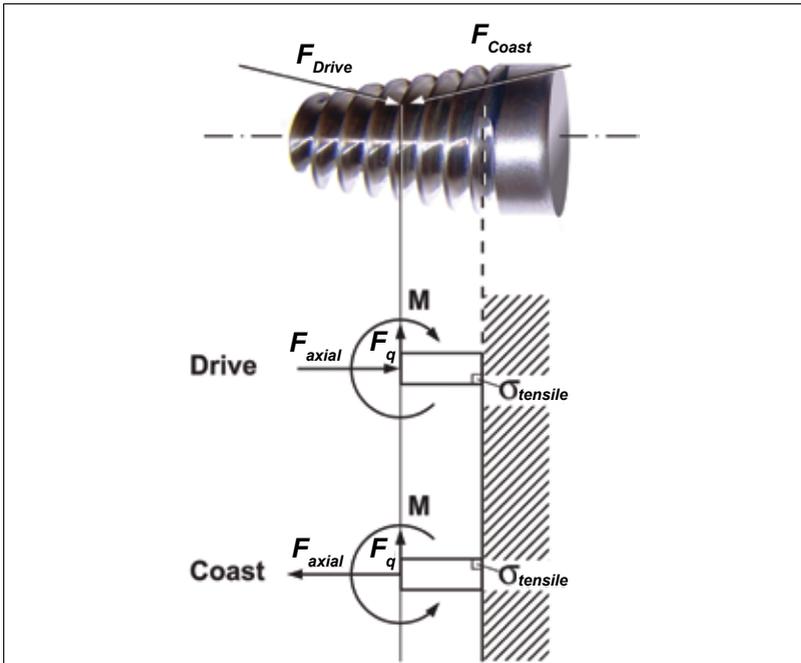


Figure 12—Strength criteria bending stress of pinion core in heel region.

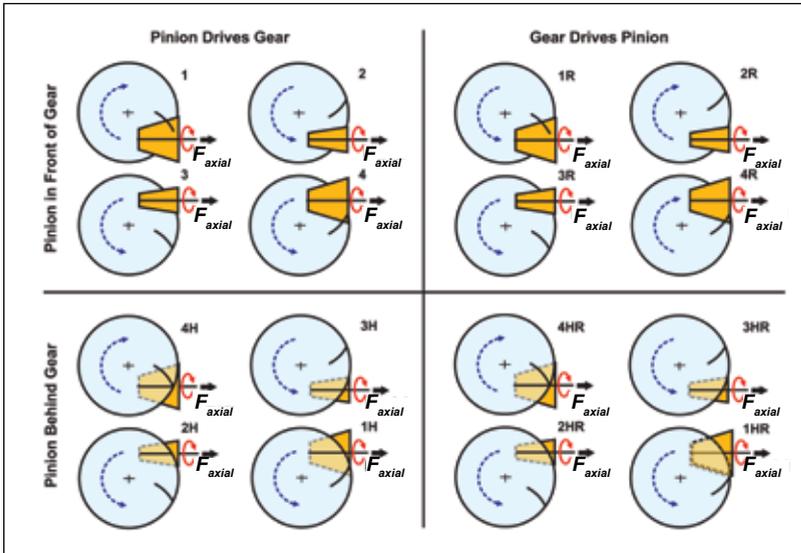


Figure 13—The 16 cases of hypoid offset.

Table 1— Speed limitations for different angular drives							
Gear Type	Straight Bevel	Zerol	Spiral Bevel	Face Gears	Beveloid	Hypoid	SRC
Grease	3m/s	5m/s	5m/s	3m/s	5m/s	N/A	N/A
Oil Sump	30m/s	40m/s	60/s	30m/s	70m/s	60m/s	30m/s

pinion diameter increases. Figure 13 has a summary of the 16 different hypoid cases. The left-side column is for a driving pinion, the right-side column for a driving gear. In the upper section the pinion is in front of the ring gear; in the lower section the pinion is behind the ring gear. The torque transmission in all cases utilizes the drive-side. The vector  $F_{axial}$  points in the opposite direction in cases

of coast-side torque transmission (which will expand the scheme in Figure 13 to 32 cases in total. Cases 1 and 4 (and sub-cases R, H and HR) in Figure 13 are the hypoid cases with a positive offset. The right column in Figure 13 is not practical since back driving is either impossible (self-locking) or results in an extremely low efficiency (e.g., 5% to 10%). Negative offsets (cases 2 and 4 and their sub-cases) are in conflict with the theoretical basics of worm-shaped hypoid pinions and will lead to tooth mutilation. Thus, only cases 1 and 4, along with sub-cases 1H and 4H, are possible configurations for SRH gearing.

SRH gears require—even with low RPMs—a high-pressure oil with additives or special synthetic hypoid oils. A sump lubrication is recommended; the oil level has to cover the face width of the teeth lowest in the sump. Excessive oil causes foaming, cavitations and unnecessary energy loss. The optimal operating direction of SRH gears is the drive-side where the convex gear flank and the concave pinion flank mesh. In the drive-direction (Fig. 11) the forces between the two mating members bend the pinion sideways and axially away from the gear, generating more backlash. Coast-side operation reduces the backlash and also causes higher tensile strength in the pinion core, when compared to a drive operation that can lead to fracture of the pinion core at the heel. Table 1 summarizes the speed limits in connection with various lubricants. ⚙

*Note to readers: The release date for the German-language version of Dr. Stadtfeldt's book—Gleason Kegelaradtechnologie—is September, 2011; published by Expert, Esslingen, Germany; Pages: 500; Price: Euro 51.40; ISBN: 978-3-8169-2983-3. The English-language version—Gleason Bevel Gear Technology—will be released approximately one year later, September, 2012.*

### References

1. Stadtfeld, H.J. "SRH—an Economical and Efficient Replacement of Worm Gear Drives," Gleason Publication, Rochester, New York, June, 2010.
2. Stadtfeld, H.J. *Handbook of Bevel and Hypoid Gears*, Rochester Institute of Engineering, Rochester, New York, March, 1993.
3. Leube, H. "Investigations on Surface Fatigue in Case-Hardened Cylindrical Gears," Dissertation, Technical University of Aachen, February, 1986.