First International Involute Gear Comparison

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Measurement institutions of seven different countries — China, Germany, Japan, Thailand, Ukraine, United Kingdom and the U.S. — participated in the implementation of the first international comparison of involute gear measurement standards. The German metrology institute Physikalisch-Technische Bundesanstalt (PTB) was chosen as the pilot laboratory as well as the organizer. Three typical involute gear measurement standards provided by the PTB were deployed for this comparison: a profile, a helix and a pitch measurement standard. In the final analysis, of the results obtained from all participants, the weighted mean was evaluated as reference value for all 28 measured parameters. However, besides the measurement standards, the measured parameters, and, most importantly, some of the comparison results from all participants are anonymously presented. Furthermore, mishandling of the measurement standards as occurred during the comparison will be illustrated.

Background

International comparisons are required to ensure the compatibility and reliability of measurement results among different countries. In the field of high accurate involute gear metrology which is of enormous economic importance, no international comparison measurement has been conducted so far. Therefore, it was imperative to organize this comparison among five national metrology institutes (Germany (PTB), China (NIM), Japan (AIST), Thailand (NIMT), Ukraine (NSC)), one designated institute (United Kingdom (NGML)) and one competent measurement institutes (U.S. (Y12)). The rules of the comparison following internationally agreed documents published by the Bureau international des Poids et Mesures (BIPM) (Ref. 1) which task is to ensure world-wide conformity of measurements and their traceability to the International System of Units (SI). The BIPM does this with the authority of the Convention of the Metre, a diplomatic treaty between 55 nations. The terminology and symbols used in this paper follow actual documents of the BIPM and the International organization for Standardization (ISO) (Refs. 2–5).

This first comparison was initiated by the PTB. Following the regional meeting in 2007, the Technical Committee of Length (TC-L) of EURAMET (Ref. 6) decided to implement this comparison as regional comparison with the involvement of other non-European participants. The PTB was chosen as pilot laboratory and organizer for the intercomparison. The choice of measurement standards to be used, parameters to be measured, potential participants and time schedule were all decided at the subsequent meetings, while the protocol adopted was later communicated to all participants. Three involute gear measurement standards which are typically used in industry were chosen for this comparison. Each participant was allocated equal amount of specified time to carry out the measurements before the measurement standards had to be sent to the next participant. The measurement comparison was implemented from July 2008 to September 2010.

The measurement standards deployed for this comparison represents the three most typically measurement standards in industry for involute gear metrology: profile, helix and pitch measurement standard (Fig. 1). These measurement standards were developed by the PTB and manufactured from high-alloy steel more than 30 years ago. The choice of helix, profile and pitch measurement standards for this measurement comparison and their suitability were based on a number of factors. Among them are the measurement accuracies and long history of measurement stability which have been observed by the PTB since the time they were manufactured. Furthermore, geometrical parameters of these measurement standards are other important attributes for their choice. Particularly their reference bands and flanks possessed significantly small form and roughness errors. All participants were asked to measure each of the measurement standards and to evaluate their results according to References 7–10.

Profile measurement standard. Figure 1a shows the involute profile measure-

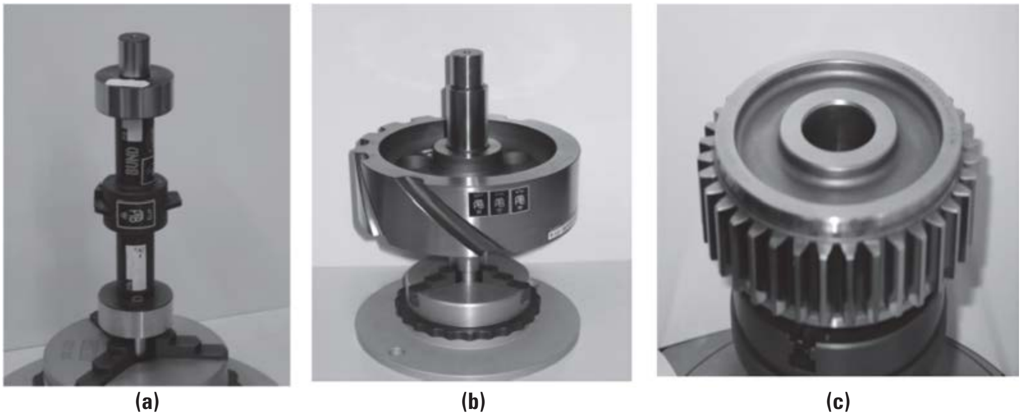
(a) (b) (c)

Figure 1 (a) Profile measurement standard; (b) Helix measurement standard; (c) Pitch measurement standard.

ment standard. In a classical design it consists of two base discs each of d_b 49,997 mm and one centered, involute shape. Table 1 delineates gear parameters that are necessary to measure the profile measurement standard on a coordinate measuring machine (CMM) or other gear measuring machines (GMM). The following typical measurement parameters for the profile evaluation were chosen according to (Refs. 7–9):

- Profile slope deviation $f_{H\alpha}$ in µm
- Profile form deviation $f_{f\alpha}$ in µm
- Profile total deviation F_α in µm

Table 1 Parameters of the three measurement standards

Measurement Standard	Gear Parameter	Value			
Profile	Pressure angle α_n	20°			
	Helix angle β	0°			
	Normal module m_n	2.9559134 mm			
	Face width b	3,2 mm			
	Number of teeth z	18			
Helix	Helix angle β	0°	15°	30°	45°
	Face width b	75 mm	75 mm	75 mm	75 mm
	Transversal module m_t	4 mm	4 mm	4 mm	4 mm
	Number of teeth z	50	50	50	50
Pitch	Pressure angle α_n	20°	20°	20°	20°
	Normal module m_n	4 mm			
	Number of teeth z	37			
	Tip diameter d_a	156 mm			
	Facewidth b	32 mm			
	Pressure angle α_n	20°			

Measurement Procedure

This profile measurement standard was measured along the surface of left flank at the centre of the tooth. The measurement points were selected equidistance over the length of roll. A spherical stylus tip of 8 mm in diameter was chosen for the measurement because it offers guaranteed comparability while being observed since the profile measurement standard has been acquired, and secondly, it enables reduction of the influence of form errors on the flank surface. The evaluated parameters were measured within the following limits:

- Start of profile evaluation (expressed in length of roll): 1 mm
- End of profile evaluation (expressed in length of roll): 18 mm

Measurement References

The reference axis of the measurement standard was numerically determined. For this purpose, the reference bands of approximately 50 mm in diameter of the profile measurement standard were probed in the centre of the discs. In each of the transverse planes at least 36 points at equally spaced distances were measured over the circumference. Through the points, a circle was fitted in accordance with the least squares method and the centre was defined. The axis of the gear measurement standard was defined from the centres of the two circles. The reference point for the height of the profile measurement was determined at the top of the tooth, 2 mm from the tip circle in the direction of the reference axis.

Helix measurement standard. Figure 1b presents a classical helix measurement standard; it embodies four different helix angles (0°, 15°, 30°, 45°), left hand as well as right hand. The measurements were performed only on the right flank. The corresponding gear parameters are listed in Table 1. The following typical measurement parameters for the helix evaluation were chosen according to (Refs. 7–9):

- helix slope deviation $f_{H\beta}$ in µm
- helix form deviation $f_{f\beta}$ in µm
- helix total deviation F_β in µm

Measurement Procedure

The helix measurements were performed on a measurement cylinder at $d_M = 204$ mm. The diameter of the stylus sphere used is approximately 8.0 mm. The evaluation is conducted at the range of $L_\beta = 70$ mm.

Measurement References

The reference axis of the measurement standard was numerically determined. For this purpose, the two reference cylinders of the gear measurement standard were probed. The measurement points were arranged in two end face planes. The end face planes were located at a distance of 43 mm from the lateral surface of the cylinders with 30 mm in diameter. In each of these transversal planes at least 36 points, which were distributed equally spaced over the circumference, were recorded. Through the points, a circle was fitted in accordance with the least squares method. The axis of the gear measurement standard was defined from the centres of the two circles.

Pitch measurement standard. Figure 1c illustrates the pitch measurement standard. The specified gearing parameters embodied in the measurement standard are delineated in Table 1. The following typical measurement parameters for the pitch evaluation were chosen according to (Refs. 7 and 10):

- Cumulative pitch deviation F_P in µm (left and right flank)
- Single pitch deviation f_P in µm (left and right flank)

Measurement Procedure

The pitch measurement standard was mounted on the measuring machine by fixing it with an internal three-jaw chuck at the inner side of the hollow shaft. The pitch was measured in a single-flank mode. The diameter of stylus sphere used was 3.0 mm, while the diameter of the measurement circle was $d_m = 148$ mm.

References

The reference axis of the measurement standard was numerically determined. For this purpose, two circles at two different locations in the bore were measured — one at 10 mm from the reference surface (upper side) of the gear measurement standard, the other at 40 mm. In each case at least 36 points — distributed and equally spaced over the circumference — were recorded. Through these points a circle was fitted in accordance with the least squares method and the center was determined. The axis of the gear measurement standard was defined from the center of each of the two circles.

Measurement and handling instructions. Taking into account the geometrical parameters of each measurement standard, as well as the technical description with all measurement procedures, all valid guidelines and standards for the comparison were prepared and distributed to all participants. Lengths are required to be measured traceable to the latest realization of the meter as set out in the current "Mise en Pratique" (Ref. 11), irrespective of the instrument used. The measurement of the temperature is based on use of the international temperature scale of 1990 (ITS-90). Similarly, the uncertainty of measurement is estimated according to the ISO Guide to the Expression of Uncertainty of Measurement (Ref. 12).

The procedures for the packaging and handling of the measurement standards were also stated in the technical description adopted for the comparison. The measurement standards were sent to the participants in a customized, self-containment case designed for safe transportation. In addition, the case prevents surface scratches and contamination to the measurement standards. The packaging cases are all portable enough to be sent by any courier services.

Similarly, recommendations on physical inspections of the measurement standards both prior to and after measurement were given to the participants. The circulation of the measurement standards was carried out in a loop among the partners. Each partner was given sufficient time to conduct the measurements before sending the measurement standards to the next partner.

Evaluation of reference values and comparison. The reference values must be determined on the basis of the received measurement results. The guidelines — as laid down by the BIPM — allow the use of different methods for the evaluation of reference values. These methods include simple mean, weighted mean, and median. For the sake of consistency one method — the weighted mean $x_{ref,w}$ (Eq. 1) — was chosen as an appropriate method. It considers the n measurement values x_i and the corresponding, expanded measurement uncertainties U_i, which ultimately reflect the measurement condition and competence of each of the participating laboratories.

$$x_{ref,w}=\sum_{i=1}^{n}\frac{x_i}{U_i^2}\cdot\frac{1}{\sum_{i=1}^{n}\frac{1}{U_i^2}} \tag{1}$$

The calculation of the reference value for each measurand (*Ed.'s Note: A physical quantity, property, or condition that is measured.*) was generally based on all submitted measurement results, with the exception of three helix measurands. Following a request by one participant for these measurands, their measurement results were not considered for calculating the corresponding reference values. Nevertheless, according to the regulations of MRA guidelines for CIPM, key comparisons of the measurement results are presented in the final report.

A check for statistical consistency of the results with their associated uncertainties can be made by calculation of the normalized error E_n for each laboratory and for each measurand. The E_n value indicates if the measurement value and its corresponding measurement uncertainty are comparable to the results of the other NMIs. This means that the E_n value is the internationally agreed upon parameter that shows whether the individual value x_i — together with its determined expanded measurement uncertainty U_i, and the expanded measurement uncertainty of the corresponding reference value $U_{ref,\,w}$ — are reliable in comparison with the calculated reference value $x_{ref,\,w}$. The absolute value IE_nI must be less than 1 to meet this quality criterion for indicating that the laboratory is capable of obtaining a qualified result.

According to publications and guidelines, there are slightly different approaches for the calculation of the E_n value; they concern the use of:

Standard measurement uncertainty or the expanded measurement uncertainty

Arithmetic operator in the denominator ("+" or "-")

Due to prior agreement with the EURAMET TC-L (Ref. 6) and other experts and guidelines for measurement uncertainty evaluation (Refs. 13–15), the E_n value was calculated according to the approach shown in Equation 2:

Table 2 Overview of the comparability of the measurement results; grey-colored cells indicate where comparability factor E_n is not fulfilled

		a	b	c	d	e	f	g
profile	$f_{H\alpha}$	0.14	0.06	0.17	0.51	0.32	0.71	1.53
	$f_{f\alpha}$	0.14	0.09	0.16	0.00	0.14	0.07	0.21
	$F\alpha$	0.05	0.02	0.18	0.07	0.09	0.32	0.14
helix 0°	$f_{H\beta}$	0.06	0.46	0.17	0.21	0.38	1.09	no results
	$f_{f\beta}$	0.35	0.03	0.33	0.26	0.43	0.27	
	F_β	0.36	0.01	0.25	0.23	0.32	0.20	
helix 15° left hand	$f_{H\beta}$	0.25	0.50	0.13	0.43	0.30	0.48	
	$f_{f\beta}$	0.07	0.17	0.25	0.14	0.15	0.04	
	F_β	0.39	0.44	0.11	0.22	0.40	0.38	
helix 15° right hand	$f_{H\beta}$	0.36	0.18	0.03	0.41	0.18	0.50	
	$f_{f\beta}$	0.02	0.10	0.32	0.23	0.39	0.52	
	F_β	0.35	0.02	0.08	0.43	0.13	0.29	
helix 30° left hand	$f_{H\beta}$	1.34	0.77	0.34	1.04	0.39	0.60	
	$f_{f\beta}$	0.10	0.10	0.54	0.48	0.21	0.33	
	F_β	0.52	0.69	0.04	0.69	0.62	0.76	
helix 30° right hand	$f_{H\beta}$	1.24	0.18	0.07	0.65	1.18	0.73	
	$f_{f\beta}$	0.14	0.16	0.47	0.19	0.19	0.10	
	F_β	0.25	0.20	0.07	0.45	0.91	0.69	
helix 45° left hand	$f_{H\beta}$	4.06	0.14	0.03	1.63	0.98	1.70	
	$f_{f\beta}$	0.50	0.37	0.43	0.12	0.04	0.24	
	F_β	1.18	0.19	0.09	1.44	1.00	1.57	
helix 45° right hand	$f_{H\beta}$	0.44	0.15	0.09	0.79	0.24	0.56	
	$f_{f\beta}$	0.01	0.36	0.08	0.74	0.22	0.10	
	F_β	0.22	0.33	0.05	0.05	0.35	0.02	
pitch left flank	F_P	1.56	0.13	0.32	0.13	0.39	0.06	2.21
	f_P	0.36	0.08	0.03	0.01	0.20	0.07	1.00
pitch right flank	F_P	0.28	0.08	0.12	0.01	0.16	0.22	1.38
	f_P	0.19	0.12	0.09	0.03	0.16	0.15	0.43
$\sum E_n>1$		5	0	0	3	2	3	(4+21)

$$E_n(k=2)=\frac{1}{k}\,\frac{x_i-x_{ref}}{\sqrt{|u_i^2-u_{ref}^2|}}=\frac{x_i-x_{ref,w}}{\sqrt{|u_i^2-u_{ref,w}^2|}} \tag{2}$$

wherein is

$$U_{ref,w}(k=2)=2\cdot\frac{1}{\sum_{i=1}^{n}\frac{1}{U_i^2}}$$

As recommended by the WG-MRA "Guidance Document" GD-1 (Ref. 15), the calculation of the E_n value was based on the expanded measurement uncertainty. Moreover, in a case of correlation between the participant measurement results and the weighted mean reference value, the measurement uncertainty contributions in the denominator must be subtracted.

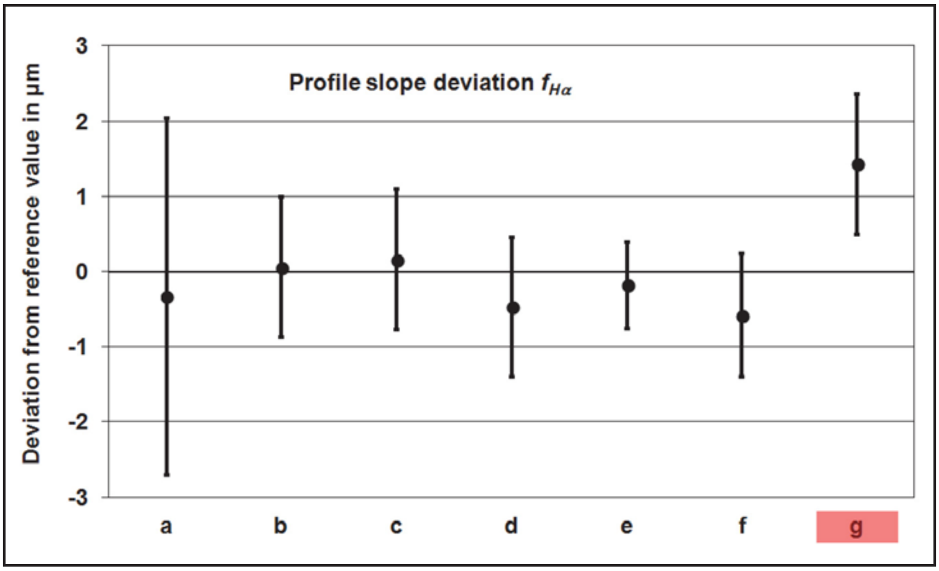

Figure 2 Slope deviations of the profile measurement s.

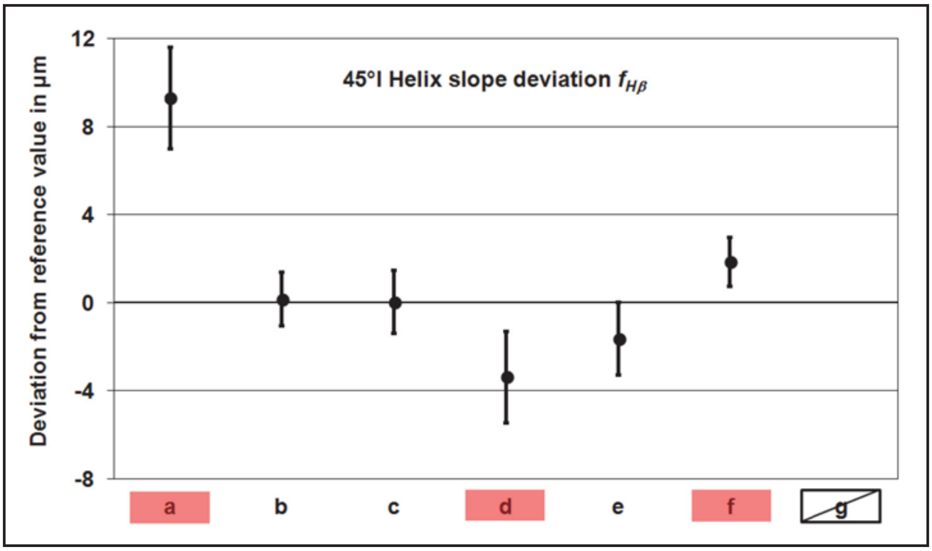

Figure 3 Slope deviations of the helix measurements — 45° left hand, right flank.

Measurement Results and Analysis

The total measured parameters for all the measurement standards (profile, helix and pitch) — 28 — were analyzed and evaluated. E_n values for each participant were evaluated (Table 2). The cells of the measurement parameters are highlighted in grey when the IE_nI factor is greater than 1.

The most relevant values that indicate the geometrical competence and correct evaluation of each participant are the results of the slope error for profile and helix measurements, and the total error for pitch measurements. The robustness of the slope evaluation for profile and helix measurements is based on the regression algorithm where a single outlier has only a small effect. However, for form and total errors the influence of a single outlier is immense. Yet for pitch measurement evaluation, the probability that an outlier influences the value of the total pitch error is much smaller, as it appears in single pitch error.

Table 2 shows that the magnitude of the evaluated E_n- values based on the participant results is, in some cases, considerably high, meaning the results were either too far from the reference value and/or the estimated measurement uncertainties were underestimated; neither of these cases is acceptable.

However, profile and pitch show more consistency, as compared to helix measurement parameters. The discrepancy of the helix results is more pronounced at higher angle of the helix; this effect cannot be sufficiently explained at this stage, as only one flank (helix angle 45° left) is affected. One possible assumption is that such a discrepancy could be caused by the geometrical errors of the measuring system.

Figures 2 and 3 show results for the profile slope, and the 45° left-hand, helix slope, deviations, respectively. The error bars represent the combined, expanded measurement uncertainties U_i^* based on the quadratic sum of the single standard measurement uncertainty u_i of each participant, and the standard measurement uncertainty of the respective reference value $u_{ref,w}$ (Eq. 3).

$$U_i^*=2\cdot\sqrt{|u_i^2-u_{ref,w}^2|} \tag{3}$$

Participants who don't fulfil the comparability value are highlighted; participants who don't measure the respective measure and are crossed out.

In case of the profile slope deviation, most of the measurement values disperse in the limit of 1 µm with the exception of one particular measurement. Moreover, it shows that the measurement uncertainties from most of the participants were adequately estimated within the optimum range.

Figure 3 shows results of the helix measurements at the right flank of the left-hand 45° helix. It could be seen that the values and the estimated measurement uncertainties do not overlap sufficiently to fulfil the E_n criterion for three participants. The range of results is approximately 12 µm; this value is five times greater that the allowed tolerance accuracy grade, according to ISO 1328-1 (Ref. 7) of the quality requirement.

Damage to the measurement standards. Despite the precautionary measures that emphasized good handling of the measurement standards by all partners, all measurement standards suffered a number of surface damages. Moreover, instead of the customized case provided by the pilot institute, one particular partner used a completely unsuitable package case with the intention of reducing the shipping cost by reducing packaging dimension and weight. The consequences of such negligence are highly visible (Fig. 4). This example should serve to re-emphasize the importance of maintaining good care of a measurement standard as "a master piece."

When the measurement standards were returned to PTB, they were re-measured and evaluated. Fortunately, the results were almost unchanged; most of the damage was found outside the surfaces to be measured. One future recommendation: more attention and emphasis should be given to safety and handling.

Summary and Outlook

- The first international comparison for involute gears — organized by EURAMET — has been successfully implemented.
- The results presented here show that the criteria for comparability were fulfilled; however, discrepancies in the values of the compared measurement parameters of some participants were sizable and fell below the expectations.
- The mishandling of the measurement standards by one participant demonstrated the need to improve the metrological skills of this particular institute.
- In summary, the comparison shows that some participants are able to calibrate gear measurement standards with the required level of competencies. Contrarily, some participants were unable to adequately demonstrate the level of competence required in terms of their measurement values, as well as in the stated measurement uncertainties.
- Caveats aside, the comparison has been accepted for registration as a Supplementary Comparison and will be published on the BIPM key comparison database (KCDB) (Ref. 16).

Figure 4 **Damages inflicted on the measurement standards surface.**

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