Optimizing the Operational Behavior of Bevel Gears Using a Tolerance Field-Based Approach

M. Trippe, A. Lemmer, C. Löpenhaus, C. Brecher

Introduction

In the design process of gears, several criteria must be considered, such as a sufficient durability, a low noise excitation, and a robust design in terms of manufacturing and assembly deviations (Ref. 1). Bevel gears present a challenge in the design, due to the complex contact geometry and the operating-dependent contact conditions. In practice, deviations resulting from the manufacturing process as well as the assembly of the gears also influence the operational behavior of the gears. A comprehensive consideration of these expected deviations in the design process for bevel gears by varying the topography parameters as well as the mounting tolerances has not yet been conducted. (Ref. 2)

As Figure 1 shows, the Ease-Off generated in the design is influenced by manufacturing-related flank deviations and assembly related position deviations. In addition, load-related displacements during the operation have a major influence on the operational behavior. Since the reduction of manufacturing and assembly tolerances leads to increasing costs in the production, it is important to generate an optimal micro-geometry regarding deviations in the design. Optimal does not only refer to the quality of the operational behavior but especially to the robustness of the design against deviations (Refs. 3–4).

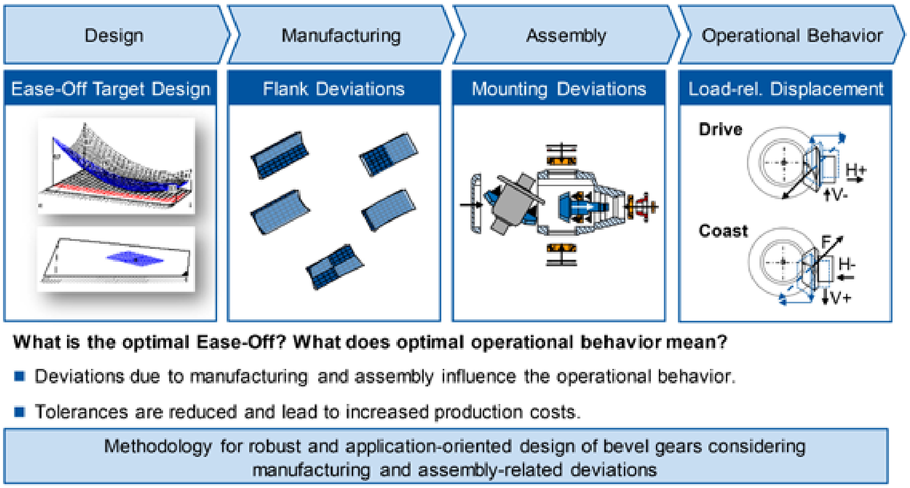

Figure 1 Motivation for robust microgeometries.

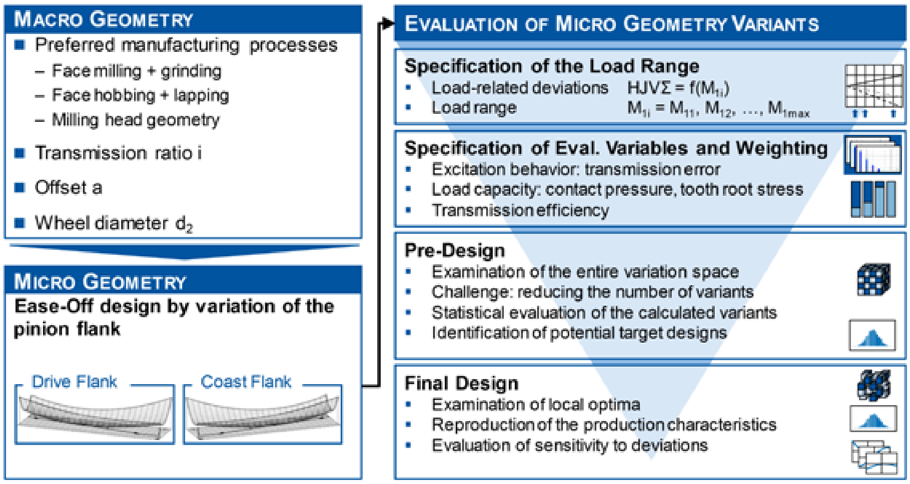

Figure 2 Method for optimizing the microgeometry by variation calculation.

For cylindrical gears, robust microgeometries are designed using variant calculations. This way, production costs can be reduced. For bevel gears, such a design method is presented in this report and applied on a near-series pair of bevel gears. Following, the calculation and design method is described and the optimization potential is shown. The optimization method was developed within the IGF research project 18450 BG/1 (FVA 739 I) and validated by means of test-rig investigations.

Method for Optimizing the Microgeometry

The operational behavior of bevel gears is significantly influenced by deviations to the ideal micro-geometry. With the help of the variant calculation, it is possible to consider and examine the influence of micro-geometry and position deviations in the design of the target geometry. The procedure developed within the scope of the research project for the optimization of the micro-geometry by means of a variant calculation specifying the tolerance and load range is shown in Figure 2. Using a methodological tool, an input data set is created for each variant, a calculation with the bevel gear tooth contact analysis *BECAL* is started and the results are merged and evaluated in the

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methodological tool (Ref. 5).

In the beginning, the macrogeometry is determined based on the preferred manufacturing process, the transmission ratio, and other constructionally specified parameters. In the next step, the microgeometry of the pinion flank is varied and designed. Regarding the design of the microgeometry it is important to ensure the manufacturability and measurability for the production. Therefore, the tooth contact analysis *BECAL* is coupled with a manufacturing simulation to create the geometry data for the calculation (Refs. 5–6). Within the scope of the evaluation, the robustness of the micro-geometry design is evaluated regarding manufacturing and assembly related deviations. The quality is evaluated regarding the operational behavior. The considered tolerance parameters include flank and position deviations as well as load-dependent misalignments, which are specified by the user together with the calculated load range. In addition to the tolerance parameters, evaluation variables (e.g. loaded transmission error, flank pressure) and load-dependent weighting factors for each of these evaluation variables are specified. The excitation behavior is evaluated based on the loaded transmission error, the load-carrying capacity based on the tooth root stresses as well as the flank pressure. The gear efficiency is used to evaluate the performance of the gear set. The introduction of grading scales in combination with the weighting factors makes it possible to summarize different evaluation variables and load levels for each variant (Ref. 7). Based on the overall grades of each variant, the different variants can be evaluated and compared with each other regarding their robustness against deviations and their quality of operational behavior. A full factorial calculation with 14 varied parameters in three steps already results in 4.78 million calculations. Due to a calculation time of ≈10 sec. per variant, this procedure is particularly unsuitable in industrial practice.

Accordingly, the optimization was divided into a pre- and final-design. Simplifications are made for the first assessment of a design in the pre-design to save calculation time. In the subsequent final-design of one or more potential target designs, all topographical and positional deviations are considered.

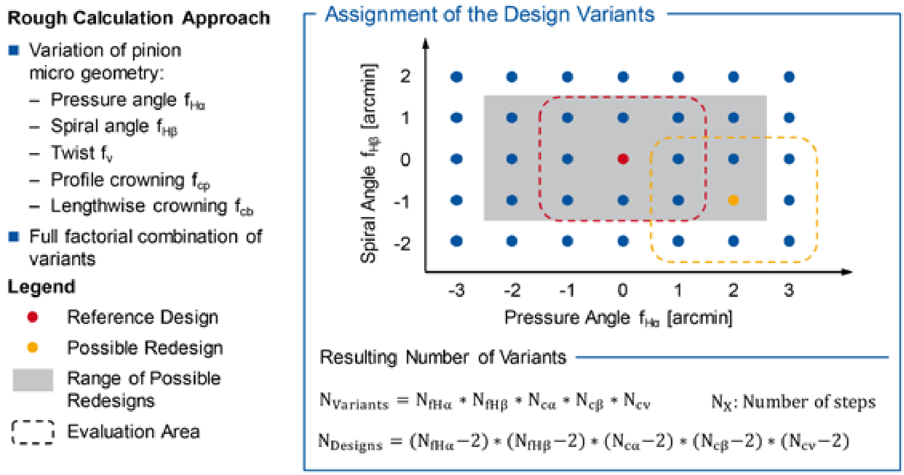

Figure 3 Assignment of the variants to target designs in the pre-design.

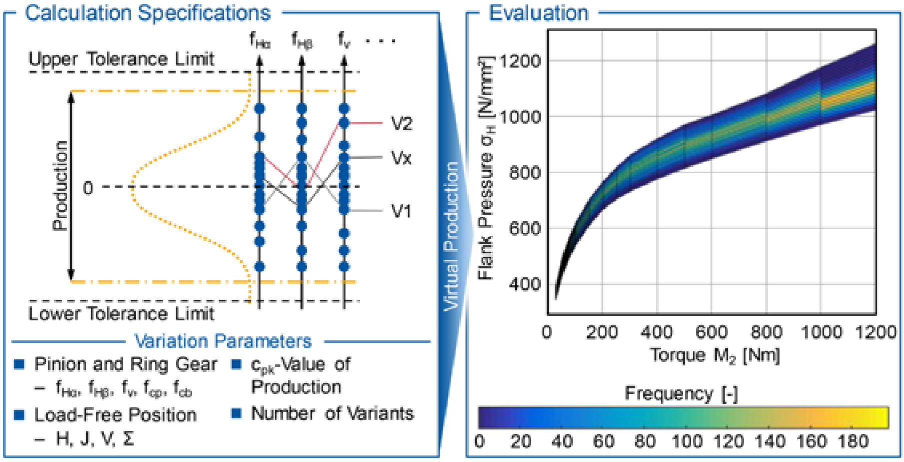

Figure 4 Calculation approach and evaluation of the final-design.

The aim of the pre-design is to identify potential target designs within a given tolerance field, considering all possible variants. For this reason, only the five micro-geometry parameters of the pinion flank shown in Figure 3 are varied to reduce the necessary calculation time. Based on the defined variation values, the microgeometry variants are generated and then calculated in *BECAL*.

The evaluation of the calculated variants is also based on the defined variation steps. For this purpose, the calculated micro-geometries are divided into target designs as it is shown in Figure 3. The maximum number of calculated variants $N_{Variants}$ results from the number of steps of each deviation parameter N_X. The number of designs $N_{Designs}$ results from the reduction by the boundary values of the variation space (N_X-2). In the next step, the variants of the specified evaluation area are assigned to the corresponding target design. This assignment of the microgeometry variants is the basis for the grading, weighting, and evaluation of the calculated variants.

In the final-design calculation, no simplifications are made to represent the influence of manufacturing-related and assembly related deviations of the gear set as precisely as possible. The five topography parameters of the pinion and gear flank are varied as well as the load-free mounting position. These 14 parameters are randomly varied according to the Monte Carlo method using a given process variation and tolerance limits. The user specifies the number of calculated variants and the process variation by the process capability index c_{pk}. For the valuation of the final-design, the algorithm provides the opportunity to evaluate the sensitivity of the calculated variants using statistical parameters. The microgeometry variants of the final-design calculation correspond to a "virtual production." so that the operational behavior of the gear set can be assessed under realistic production and assembly variations.

Application of the Method

The presented method for optimizing the microgeometry was applied to a near-series automotive gear set. The individual work steps are shown in Figure 5. The reference design corresponds to the near-series gear set. The load-related displacements are approved by measurements in the real application and considered in the calculations. In the first step, a pre-design calculation for the five load levels $M_2 = 100$, 200, 300, 400 and 500 Nm was carried out. The parameters $f_{H\beta}$, f_v, f_{cp} and f_{cb} were varied in seven steps. The parameter $f_{H\alpha}$ was constant for the calculations.

From the calculated variants, the optimal redesign regarding the excitation behavior was selected and transferred to a new microgeometry. The redesign then was examined in the final-design regarding the variance of the loaded transmission error in case of realistic manufacturing and assembly deviations. In all calculation steps, only the drive flank was examined, and the loaded transmission error was used to evaluate the variants. The reason for these limitations is the predominant use of the method for optimizing the noise and excitation behavior of bevel gears.

Pre-Design

The simulation parameters used in the pre-design calculation and the results are shown in Figure 6. The range between minimum and maximum values was divided into seven steps for the calculation. Except the profile crowning f_{cp}, the tolerance limits were assumed symmetrical around zero. All five calculated load levels were equally weighted for evaluation. Four varied parameters in seven steps each result in a total number of $N_{Variants} = 7^4 = 2{,}401$ variants and $N_{Designs} = 5^4 = 625$ redesigns. The calculation time for the calculation was ≈33 h. For each new design, the quality of the operational behavior and the robustness of the design against deviations were evaluated. The quality rating evaluates how good the operational behavior of the design is regarding the evaluated parameters. In this case, this is equivalent to the magnitude of the loaded transmission error of a variant. The stability rating evaluates how robust a design is regarding deviations on the pinion tooth flank. In the present case, this means how large the variance of the loaded transmission error is when the pinion micro-geometry is varied. A quality rating of $q = 1$ indicates the lowest or the best loaded transmission error. Whereas, a quality rating of $q = 6$ indicates the highest or worst loaded transmission error. The assignment of the rating scales to the values of the loaded transmission error considers all calculated variants. To determine the overall quality score of a variant, the quality scores of the individual load levels are summarized using the predefined weighting factors.

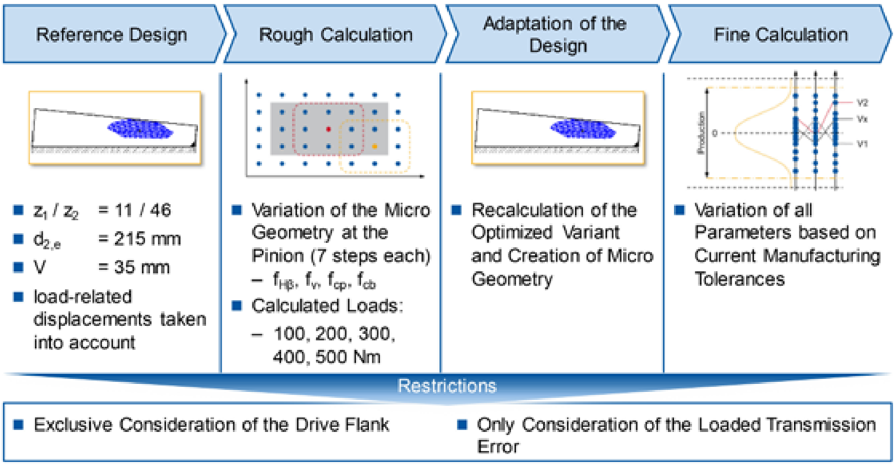

Figure 5 Calculation approach for the optimization of the reference design.

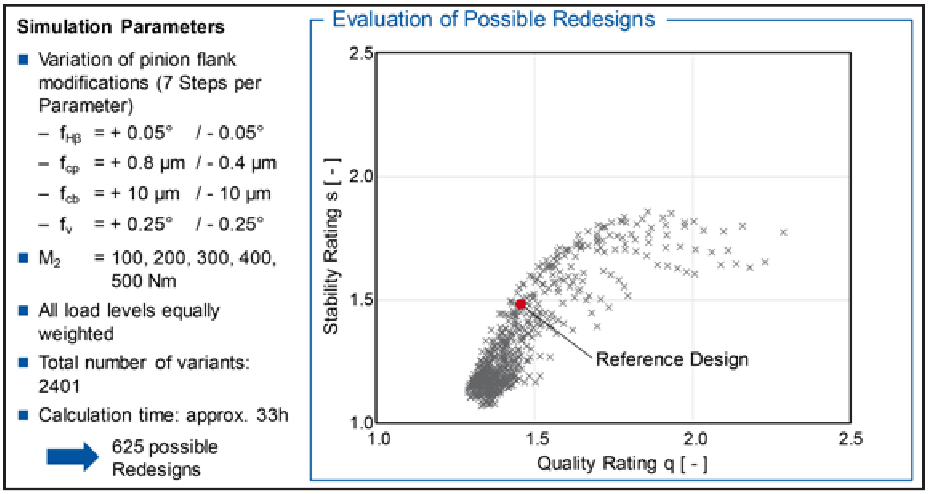

Figure 6 Evaluation of the pre-design calculation.

To determine the stability rating of the redesigns, all variants of the corresponding evaluation are considered. The variance of the evaluation parameters is rated using the standard deviation. The standard deviations of the different redesigns are converted into the stability ratings based on a given rating scale. The evaluation of the pre-design calculation and the selection of the optimal redesign are based on the representation of quality and stability ratings (Fig. 6). In the diagram on the right, the reference design is marked, which was used as input for the pre-design. An equal weighting of all load stages results in numerous redesigns, which, on the one hand, have an improved operational behavior and, on the other hand, are more robust against deviations than the reference design.

The optimal microgeometry found in the pre-design strongly depends on the selected weighting for the evaluation; Figure 7 compares the optimal variants for three different weightings with the reference design. With the same weighting of all load levels, the loaded transmission error of the optimal variant is always lower than the reference design, except for $M_2 = 500$ Nm. If only the two load levels M2 = 100 and 200 Nm are used for the evaluation the loaded transmission error is considerably lower for these two load levels compared to the reference design. However, the loaded transmission error at $M_2 = 400$ and 500 Nm is higher than the loaded transmission error of the reference design.

A comparable result is obtained if only the upper-three load levels are weighted in the evaluation. A load-dependent weighting of the evaluation variables to determine the overall grade of a variant is necessary in practice to meet the various design criteria. On the one hand, the designer must ensure that the gear set has a high efficiency and low noise excitation at low loads. On the other hand, the load carrying capacity of the tooth root and the tooth flank must be ensured under high loads, while efficiency and excitation behavior are of secondary importance.

The comparison of different weightings in the evaluation of the calculation results shows that it is possible to consider different load-depended design criteria in the evaluation method for the pre-design calculation. The optimization potential depends significantly on the selected weighting. Therefore the weighting of the considered load stages and evaluation variables must be tailored exactly to the final application to find an optimal microgeometry design. For the next step of the final-design, the microgeometry variant with an optimum at $M_2 = 100$ and 200 Nm was selected and converted into new geometry data for *BECAL*.

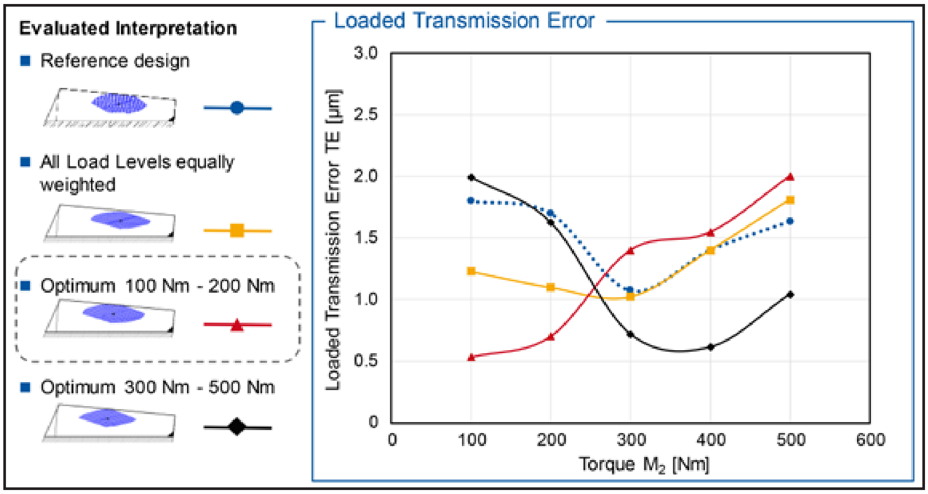

Figure 7 **Comparison of the loaded transmission error for different weightings.**

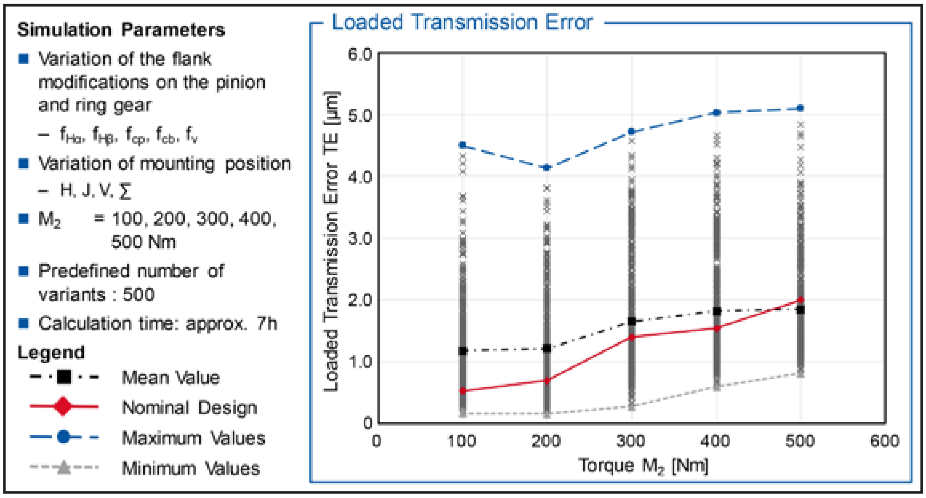

Figure 8 **Statistical evaluation of the final-design calculation.**

Final-Design.

The microgeometry variant optimized in the pre-design calculation was simulated under consideration of realistic process scattering and deviations. The process capability index was set to $c_{pk} = 1.33$. The tolerance limits for the deviation parameters were assumed production-related values. All flank deviations on the pinion and gear as well as the four deviation parameters of the mounting position were considered in the calculation. The calculation in *BECAL* was done for the loads $M_2 = 100$, 200, 300, 400 and 500 Nm as in the pre-design. 500 micro-geometry variants were generated and calculated according to the "Monte Carlo" method assuming a normal distribution (Ref. 8). The research project has shown that a range of 500 variants is sufficient for estimating the spreading of the evaluation variables in the final-design; the calculation time is 7 hours.

Figure 8 shows the calculated loaded transmission errors of all variants over the considered load stages. The maximum transmission error is up to $TE = 5\,\mu m$. The minimal occurring transmission error is $TE = 0.16\,\mu m$. Both the minimum and maximum loaded transmission error increase with the torque and show a local minimum for $M_2 = 200$ Nm. Furthermore, the transmission error of the nominal design without manufacturing and assembly deviations is highlighted in the diagram. A comparison of the nominal design with the minimum and maximum transmission errors shows that the scatter range of the occurring transmission errors is very large.

For the statistical evaluation of the calculation results, the mean value of all variants was calculated for each load level and displayed in the diagram. The mean value of all calculated variants is very close to the values of the nominal design. Most of the variants of the "virtual production" thus show the loaded transmission error defined in the design. The design must be rated as robust against deviations. However, it is also evident that there are "worst case" variants, which have very high transmission errors and lead to the wide range of loaded transmission errors. A further reduction of the resulting scatter of the loaded transmission error is possible on the one hand by selecting a new target design in the pre-design calculation and evaluating it using the final-design calculation. On the other hand, it is possible to use the final-design to identify critical deviation parameters regarding the evaluation parameters. These critical tolerance limits, as well as the process scatter, can be adjusted in a new final-design calculation to analyze and understand the influence of the deviation parameters on the scatter.

One possibility to determine critical tolerance parameters is the multi-variant data analysis (MVDA). The results of the first final-design calculation are statistically examined and the main influencing variables on the respective evaluation parameters are calculated. The corresponding procedure is shown in Figure 9. The results of the first final-design calculation already presented are analyzed with the help of MVDA. The main variables influencing the loaded transmission error are the lengthwise crowning at pinion and gear $f_{H\beta,1}$ and $f_{H\beta,2}$ and the twist at the pinion $f_{v,1}$. The influence of the other deviation parameters on the loaded transmission error is small compared to these three parameters.

Based on these results, the tolerance limits for the lengthwise crowning of the pinion and gear were reduced by 40%. Furthermore, the tolerance limits of the twist on the pinion have been reduced by 30%. The tolerance limits of the other deviation parameters and the process capability index remained unchanged. Using these values, a new final-design calculation with 500 variants was carried out and evaluated. In the lower part of Figure 9 the results of the first and second final-design calculations are compared. The adjustment of the tolerance zones leads to a significant reduction of the scatter of the loaded transmission error. At a load of $M_2 = 100$ Nm, the maximum transmission error is reduced by ≈44%. For $M_2 = 400$ Nm the decrease is ≈27%. The adjustment of the tolerance limits leads not only to a reduced scatter but also to a changed mean value of the occurring transmission errors. The mean transmission errors in the second final-design calculation decrease for all load levels. For the loads $M_2 = 100$ and 200 Nm, the difference between the new mean value and the nominal design is $\approx TE = 0.27\ \mu m$. For the loads $M_2 = 300$ and 400 Nm, the mean values almost exactly match the values of the nominal design. For the load stage $M_2 = 500$ Nm, the mean value of the transmission error is even lower and thus better compared to the nominal design. The consideration of the tolerance limits within the "virtual production" of the final-design calculation shows that a simulative tolerance engineering is possible with the help of the developed procedure. Besides the restriction of tolerance limits to reduce the scatter of the evaluation variables, the method also offers the possibility of estimating the influence of an expansion of individual tolerance limits on the production result. This results in an additional potential to reduce production costs.

Summary and Outlook

Bevel gears have operation-dependent, complex contact conditions that pose a design challenge. Furthermore, in practice, production and assembly related deviations influence the operational behavior. A consideration of the effects of manufacturing and assembly tolerances on the variation of the characteristic values for the operational behavior within a variant calculation has not yet taken place. This deficit was considered in this presented research project IGF 18450 BG/1 and a method for the design of flank topographies robust against deviations for bevel gears was developed, considering manufacturing and assembly related deviations as well as load-related displacements.

Within the scope of this report, the method for optimizing the pinion micro-geometry is presented and applied on a near-series bevel gear set. The method is divided into two sub-calculations. The pre-design calculation makes it possible to optimize the pinion micro-geometry regarding the quality of the operational behavior and the robustness against deviations. In the second calculation step, the final-design, the micro-geometry selected in the pre-design is examined and evaluated in the environment of a "virtual production." Based on the calculation results, the expected variation of the operational behavior can be considered, and the influence of manufacturing and assembly tolerances can be analyzed.

The evaluation of the presented pre-design shows that different load-dependent weightings lead eventually to different excitation characteristics. This allows the designer to adapt the optimization to the specific requirements of the final application. The statistical evaluation of the calculation results of the final-design calculation shows that the mean value of the loaded transmission error of all variants is close to that of the nominal design. However, the scatter of the transmission error is large. A multi-variant data analysis is used to determine the tolerance parameters with the greatest influence on the loaded transmission error. The tolerance limits of the corresponding parameters are then restricted, and the detailed calculation is repeated. The adjustment of the tolerance limits leads to a significant decrease in the scatter of the transmission error and thus to a better result of the "virtual production."

The results of this report show that the developed method makes it possible to optimize the pinion micro-geometry regarding quality and stability as well as tolerance limits in the production. In this way, production costs can be reduced by widening suitable tolerance limits and by an increased material utilization.

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For more information: Questions or comments regarding this paper? Contact Moritz Trippe at *m.trippe@wzl.rwthaachen.de.*

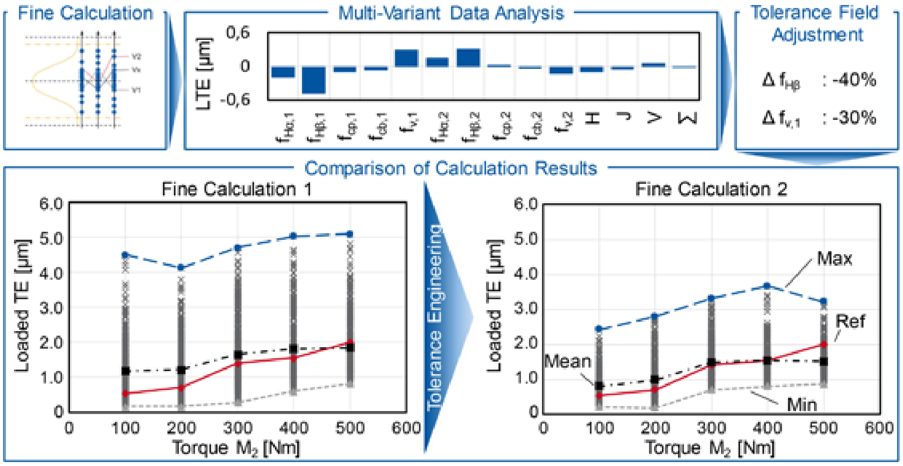

Figure 9 Tolerance engineering using the final-design calculation.

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Moritz Trippe is a research assistant in the Gear Department of the Laboratory of Machine Tools and Production Engineering (WZL) of RWTH Aachen University. His research focus is the load carrying capacity of the tooth root and the influence of dynamics on gear load capacity. Trippe studied mechanical engineering at RWTH Aachen University and has a master´s degree in automotive engineering and transport.

Dr. -Ing. Dipl. -Wirt. -Ing. Christoph Löpenhaus has since 2014 served as Chief Engineer in the Gear Department of WZL, RWTH Aachen / Laboratory of Machine Tools and Production Engineering (WZL), RWTH Aachen. He previously held positions there as (2011–2014) Team Leader, Group Gear Testing Gear Department Chair of Machine Tools Laboratory of Machine Tools and Production Engineering (WZL) RWTH Aachen; (2010–2011) Research Assistant, Group Gear Testing Gear Department Chair of Machine Tools Laboratory of Machine Tools and Production Engineering (WZL) RWTH Aachen: (2007–2009) as Student Researcher, Group Gear Design and Manufacturing Calculation Gear Department Chair of Machine Tools Laboratory of Machine Tools and Production Engineering (WZL) RWTH Aachen; and (2004–2009) as a student in Industrial Engineering RWTH Aachen.

Andreas Lemmer has a diploma in mechanical engineering BA from Mannheim. He has worked as a development engineer (2005–2012) axle drive VAN/Truck, Daimler AG, Kassel; as a quality engineer (2012–2014) in “acoustics” for manual and automatic transmissions, Daimler AG, Untertürkheim; and is presently (since 2014) a development engineer for axle drive VAN/ Truck, Daimler AG, Kassel.

Prof. Dr.-Ing. Christian Brecher has since January 2004 been Ordinary Professor for Machine Tools at the Laboratory for Machine Tools and Production Engineering (WZL) of the RWTH Aachen, as well as Director of the Department for Production Machines at the Fraunhofer Institute for Production Technology IPT. Upon finishing his academic studies in mechanical engineering, Brecher started his professional career first as a research assistant and later as team leader in the department for machine investigation and evaluation at the WZL. From 1999 to April 2001, he was responsible for the department of machine tools in his capacity as a Senior Engineer. After a short spell as a consultant in the aviation industry, Professor Brecher was appointed in August 2001 as the Director for Development at the DS Technologie Werkzeugmaschinenbau GmbH, Mönchengladbach, where he was responsible for construction and development until December 2003. Brecher has received numerous honors and awards, including the Springorum Commemorative Coin; the Borchers Medal of the RWTH Aachen; the Scholarship Award of the Association of German Tool Manufacturers (Verein Deutscher Werkzeugmaschinenfabriken VDW); and the Otto Kienzle Memorial Coin of the Scientific Society for Production Technology (Wissenschaftliche Gesellschaft für Produktionstechnik WGP).