Tribology Aspects in Angular Transmission Systems

Part II
Straight Bevel Gears

Dr. Hermann Stadtfeld

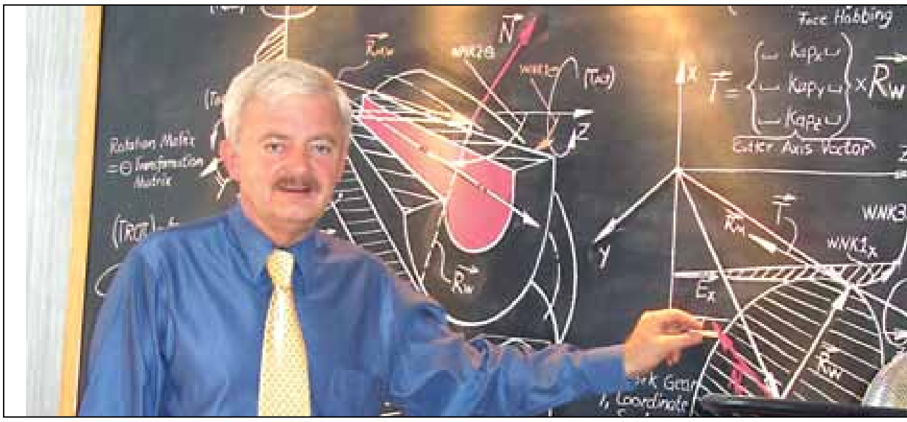
(This is the second of an eight-part series on the tribology aspects of angular gear drives. Each article will be presented first and exclusively by Gear Technology*; the entire series will be included in Dr. Stadtfeld's upcoming book on the subject, which is scheduled for release in 2011.)*

Design. If two axes are positioned in space—and the task is to transmit motion and torque between them using some kind of gears—then the following cases are commonly known:

- Axes are parallel → cylindrical gears (line contact)
- Axes intersect under an angle → bevel gears (line contact)
- Axes cross under an angle → crossed helical gears (point contact)
- Axes cross under an angle (mostly 90°) → worm gear drives (line contact)
- Axes cross under any angle → hypoid gears (line contact)

The axes of straight bevel gears, in most cases, intersect under an angle of 90°. This so-called shaft angle can be larger or smaller than 90°; however, the axes always intersect, which means they have at their crossing point no offset between them (*Author's note: see also previous chapter, "General Explanation*

continued



Dr. Hermann Stadtfeld *received a bachelor's degree in 1978 and in 1982 a master's degree in mechanical engineering at the Technical University in Aachen, Germany. He then worked as a scientist at the Machine Tool Laboratory of the Technical University of Aachen. In 1987, he received his Ph.D. and accepted the position as head of engineering and R&D of the Bevel Gear Machine Tool Division of Oerlikon Buehrle AG in Zurich, Switzerland. In 1992, Dr. Stadtfeld accepted a position as visiting professor at the Rochester Institute of Technology. From 1994 until 2002, he worked for The Gleason Works in Rochester, New York—first as director of R&D and then as vice president of R&D. After an absence from Gleason between 2002 to 2005, when Dr. Stadtfeld established a gear research company in Germany and taught gear technology as a professor at the University of Ilmenau, he returned to the Gleason Corporation, where he holds today the position of vice president-bevel gear technology and R&D. Dr. Stadtfeld has published more than 200 technical papers and eight books on bevel gear technology. He holds more than 40 international patents on gear design and gear process, as well as tools and machines.*

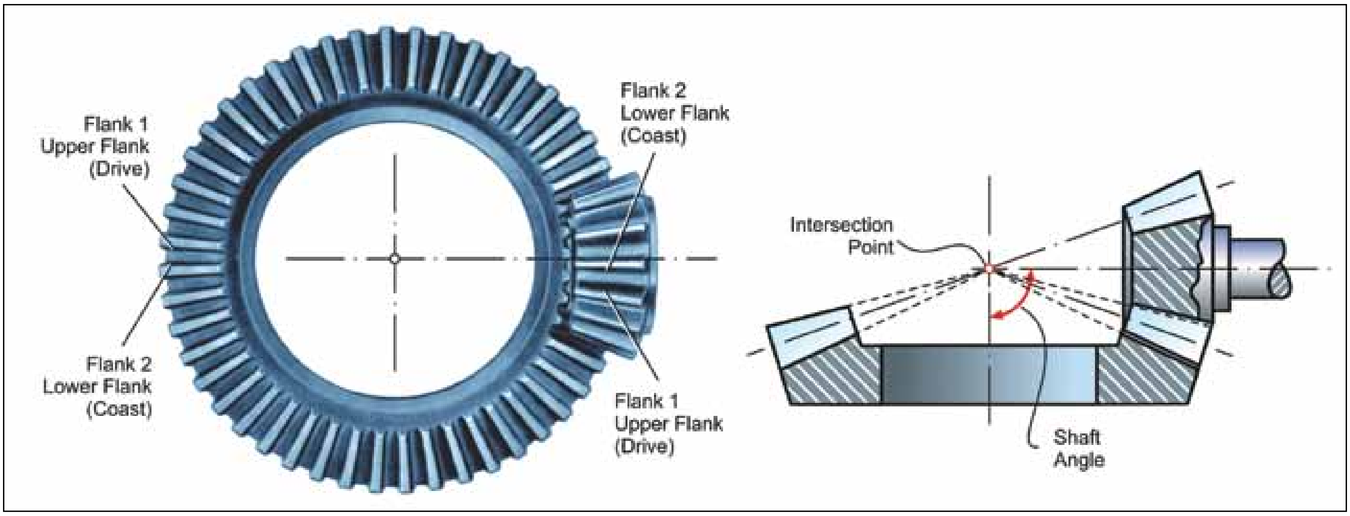


Figure 1—Straight bevel gear geometry.

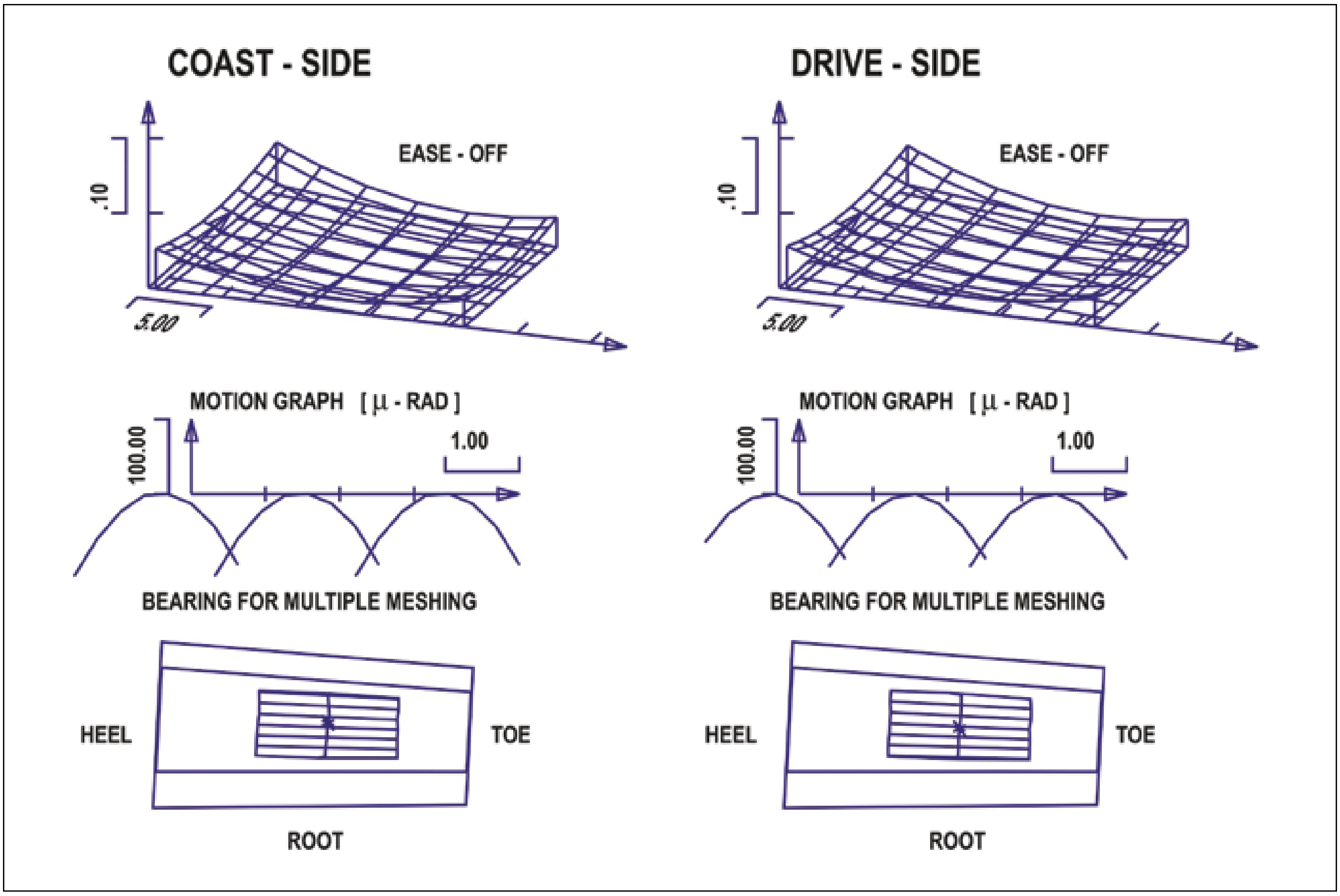


Figure 2—Tooth contact analysis of a straight bevel gear set.

of Theoretical Bevel Gear Analysis" on hypoid gears). The pitch surfaces are cones that are calculated with the following formula:

$$z_1/z_2 = \sin\gamma_1/\sin\gamma_2$$
$$\Sigma = \gamma_2 = 90° - \gamma_1$$

$$\text{in case of } \Sigma = 90° \rightarrow$$
$$\gamma_1 = \arctan(z_1/z_2) \rightarrow$$
$$\gamma_2 = 90° - \gamma_1$$

where:

z_1	number of pinion teeth
z_2	number of gear teeth
γ_1	pinion pitch angle
Σ	shaft angle
γ_2	gear pitch angle

Straight bevel gears are commonly designed and manufactured with tapered teeth, where the tooth cross section changes its size proportionally to the distance of the crossing point between the pinion and gear axes. The profile function of straight bevel gears is a spherical involute, which is the direct analog to the tooth profiles of cylindrical gears.

Figure 1 shows an illustration of a straight bevel gear set and a cross-sectional drawing. Straight bevel gears have no preferred driving direction. Because of the orientation of the flanks during manufacture, the designations "upper" and "lower" flank are used. Per definition, the calculation programs treat the straight bevel pinion like a left-hand member and the straight bevel gear like a right-hand member. Consequently there is a drive side and a cost side designation, which is for proper definition rather than for implying better suitability of torque and motion transmission.

Analysis. The precise mathematical function of the spherical involute will result in line contact between the two mating flanks (rolling without any load). In the case of a torque transmission, the contact lines become contact zones (stripes) with a surface-stress distribution that shows peak values at the two ends of each observed contact line, where the contact line is limited by the inner and outer end of the tooth (toe and heel). In order to prevent this edge contact, a crowning along the face width of the teeth (length crowning) and in profile direction (profile crowning) are introduced into the pinion flanks, the gear flanks or both. A theoretical tooth contact analysis (TCA) previous to gear manufacturing can be performed in order to observe the effect of the crowning in connection with the basic characteristics of the particular gear set. This also affords the possibility of returning to the basic dimensions in order to optimize them if the analysis reveals any deficiencies. Figure 2 shows the result of a TCA of a typical straight bevel gear set.

The two columns in Figure 2 represent the analysis results of the two mating flank combinations (*see also "General Explanation of Theoretical Bevel Gear Analysis"*). However, the designation "drive" and "coast" are strictly a definition rather than a recommendation. The top graphics show the ease-off topographies. The surface above the presentation grid shows the consolidation of the pinion and gear crowning. The ease-offs in Figure 2 have a combination of length and profile crowning, thus establishing a clearance along the boundary of the teeth.

Below each ease-off, the motion transmission graphs of the particular mating flank pair are shown. The motion transmission graphs show the angular variation of the driven gear in the case of a pinion that rotates with a constant angular velocity. The graphs are drawn for the rotation and mesh of three consecutive pairs of teeth. While the ease-off requires a sufficient amount of crowning—in order to prevent edge contact and allow for load-affected deflections—the crowning in turn causes proportional amounts of angular motion variation of about 90 micro radians in this example.

At the bottom of Figure 2, the tooth contact pattern is plotted inside of the gear tooth projection. These contact patterns are calculated for zero-load and a virtual marking compound film of 6 µm thickness. This basically duplicates the tooth contact; one can observe the rolling of the real version of the analyzed gear set under light load on a roll tester, while the gear member is coated with a marking

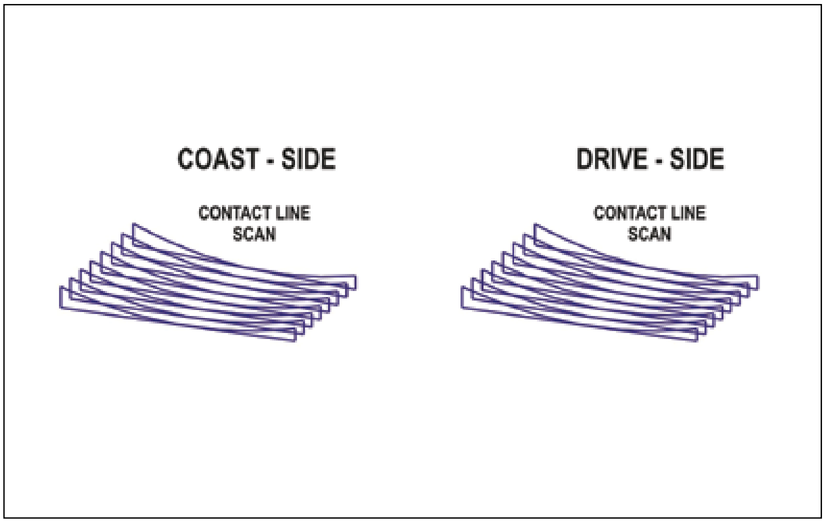


Figure 3—Contact line scan of a straight bevel gear set.

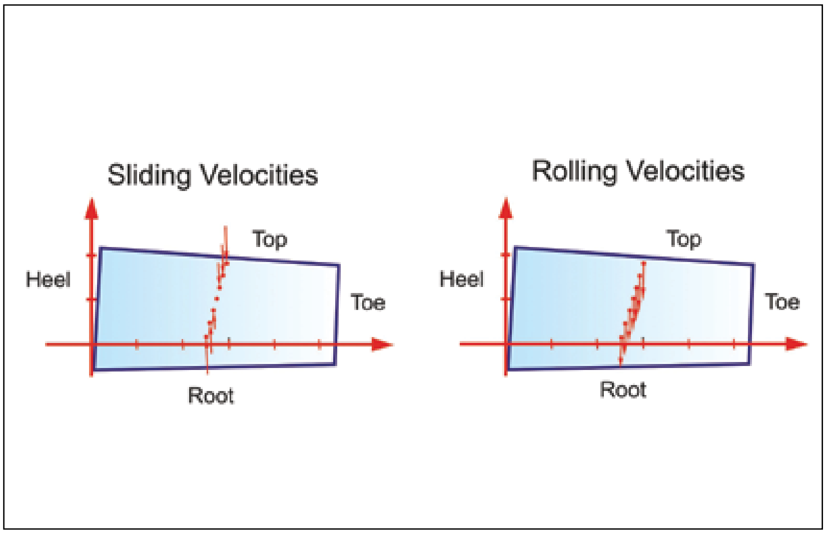


Figure 4—Rolling and sliding velocities of a straight bevel gear set along the path of contact.

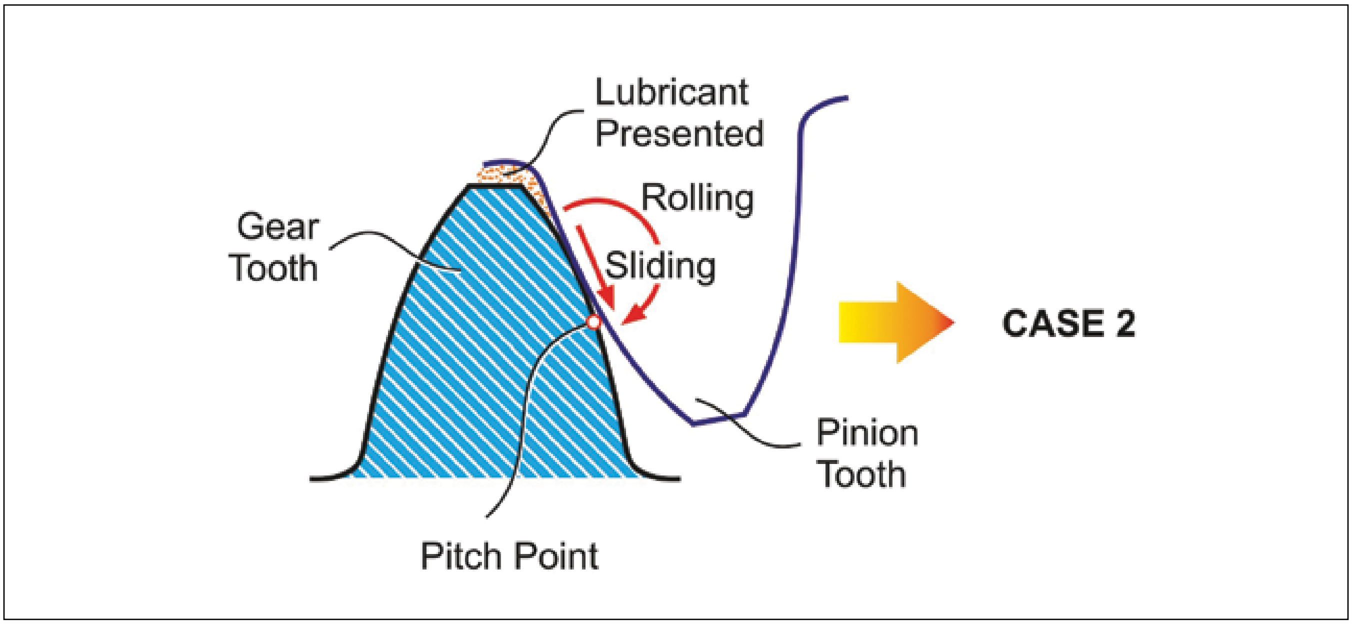


Figure 5—Profile sliding and rolling in straight bevel gears.

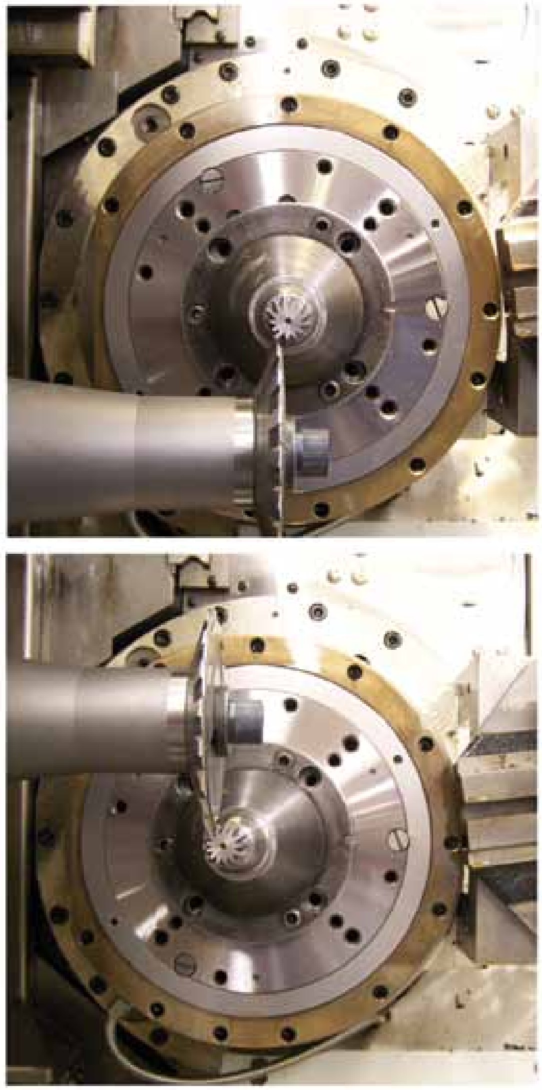


Figure 6—Straight bevel gear cutting with disc cutter (top: lower flank, bottom: upper flank).

compound layer of about 6 µm thickness. The contact lines extend in tooth length direction as straight lines—each of which point to the crossing apex point of face-pitch and root-cone. The path of contact is oriented in profile direction and crosses the contact lines under about 90°.

The crowning reflected in the ease-off results in a located contact zone inside the boundaries of the gear tooth. A smaller tooth contact area generally results from large ease-off and motion graph magnitudes, and vice versa.

Figure 3 shows eight discrete, potential contact lines with their crowning amount along their length (contact line scan). The length orientation of the contact lines, caused by the zero-degree spiral angle, results in a contact line scan with horizontally oriented gap traces. If the gearset operates in the drive direction, then the contact zone (instant contact line) moves from the top of the gear flank to the root. There is no other utilization of the face width than a contact spread under increasing load.

The graph in Figure 4 illustrates the rolling- and sliding-velocity vectors; each vector is projected to the tangential plane at the point-of-origin of the vector. The velocity vectors are drawn inside the gear tooth boundaries (axial projection of one ring gear tooth). The points-of-origin of both the rolling- and sliding-velocity vectors are grouped along the path of contact, which is found as the connection of the minima of the individual lines in the contact line scan graphic (Fig. 3). Figure 4 shows the sliding-velocity vectors with arrow tip, and rolling-velocity vectors as plain lines. Contrary to spiral bevel and hypoid gears, the directions of both—sliding and rolling velocities—are oriented in profile direction. The rolling velocities in all points are directed to the root, while the sliding velocities point to the top above the pitch line and to the root below the pitch line. At the pitch line, the rolling velocity is zero, just like in the case of cylindrical gears.

Straight bevel gears have properties very similar to spur gears. The path of contact moves from top to root (in the center of the face width) and the contact lines are oriented in face width direction (Fig. 2). Sliding- and rolling-velocity vectors are pointing in profile direction (Fig. 4), which will shift the contact lines in Figure 4 exclusively in profile direction. This means the crowning of the contact lines has no significant influence on the lubrication case ("*General Explanation of Theoretical Bevel Gear Analysis*"), but only the involute interaction will define the lubrication case and the hydrodynamic condition.

If the lubricant were presented, for example, on the top of the gear tooth as in Figure 5, the sliding- and rolling-velocity directions would result in Lubrication Case 2 as previously discussed in "General Explanation of Theoretical Bevel Gear Analysis." As the rolling progresses below the pitch point, the sliding velocity will change its direction and the lubrication case becomes Case 3, which is very unfavorable and reason to assure lubrication is presented on both sides of the contact zone.

Manufacturing. The manufacturing processes of straight bevel gears are planing with two tool generators, milling with two interlocking disk cutters or milling with a single-disk cutter (Gleason Coniflex). The planing and interlocking disk cutter processes are outdated and typically performed on older, not current mechanical machine tools. The single-disk-cutter milling process was developed for modern free-form machines. It enables the use of carbide cutting tools in a high-speed, dry-cutting process.

The blades of the circular cutter disk envelope an axial plane (or slight cone) on the right side of the disk in Figure 6. This plane is oriented in space and simulates one side of a generating rack, analog to a cylindrical, gear-generating rack. Due to the diameter of the cutter disk, the root line of the straight bevel gear

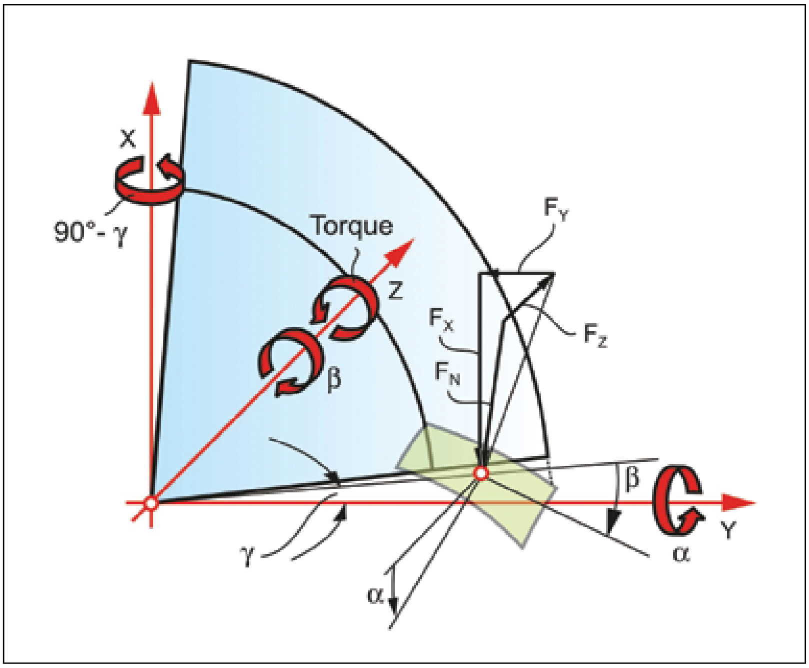


Figure 7—Force diagram for calculation of bearing loads.

cut shown in Figure 6 is curved, rather than straight. The curve in the root is a side effect of this particular process, and has never proven to be of any disadvantage regarding the gear set kinematics or strength. The left photo in Figure 6 shows the cutting of the lower flanks. The opposite flanks of the same slots are cut with the same tool in the upper position, as shown in the right photo in Figure 6.

Hard finishing after heat treatment is possible by grinding with a permanent, CBN-coated grinding wheel, which basically resembles the geometry of the cutter disk. The geometry and kinematics of the grinding process are identical to the cutting in Figure 6.

Application. Most straight bevel gears used in power transmission are manufactured from carburized steel and undergo a case hardening to a surface hardness of 60 Rockwell C (HRC) and a core hardness of 36 HRC. Because of the higher pinion revolutions, it is advisable to provide the pinion a higher hardness than the ring gear (e.g., pinion 62 HRC, gear 59 HRC).

Regarding surface durability, straight bevel gears are also very similar to spur gears. At the pitch line, the sliding velocity is zero and the rolling velocity, under certain loads, cannot maintain a surface-separating lubrication film. The result is pitting along the pitch line that can destroy the tooth surfaces and even lead to tooth flank fracture. However, it is possible that the pitting can be stabilized if the damage-causing condition is not often represented in the duty cycle.

The axial forces of straight bevel gears can be calculated by applying a normal force vector at the position of the mean point at each member (*see also "General Explanation of Theoretical Bevel Gear Analysis*"). The force vector normal to the transmitting flank is separated in its X, Y and Z components (Fig. 7).

The relationship in Figure 7 leads to the following formulas, which can be used to calculate bearing force components in a Cartesian coordinate system and assign them to the bearing load calculation in a CAD system:

$$F_x = -T / (A_m \bullet \sin\gamma)$$
$$F_y = -T \bullet (\cos\gamma \bullet \sin\alpha) / (A_m \bullet \sin\gamma \bullet \cos\alpha)$$
$$F_z = T \bullet (\sin\gamma \bullet \sin\alpha) / (A_m \bullet \sin\gamma \bullet \cos\alpha)$$

where:
- T torque of observed member
- A_m mean cone distance
- γ pitch angle
- α pressure angle
- F_x , F_y , F_z bearing load force components

The bearing force calculation formulas are based on the assumption that one pair of teeth transmits the torque, with one normal-force vector in the mean point of the flank pair. The results are good approximations, which reflect the real bearing loads for multiple-tooth meshing within an acceptable tolerance. A precise calculation is, for example, possible with the Gleason bevel and hypoid gear software.

Straight bevel gears have lesser axial forces than spiral bevel gears. The axial force component—due to the spiral angle—is zero. Zero-spiral angle minimizes the face-contact ratio to zero, but results in maximal tooth root thickness.

The tooth thickness counts squared in a simplified root-bending-stress calculation using a deflection beam analogy. The thickness reduces by cos (spiral angle). The face-contact ratio increases, simplified by tan (spiral angle). Those formulas applied to a numerical example will always show an advantage of the spiral angle in root-bending strength. However, the crowning of real bevel gears will always cause one pair of teeth to transmit an over-proportionally high share of the load, while the one or two additionally involved tooth pairs will only share a small percentage of the load. Finite element calculations can be useful in finding the optimal spiral angle for maximal root strength. As a rule, bevel gears that are not ground or lapped after heat treatment show the highest root strength with the lowest spiral angles. This explains why—in those cases—the straight bevel gear remains the bevel gear of choice.

Straight bevel gears can operate with regular transmission oil or, in the case of low RPMs, with a grease filling. In case of circumferential speeds above 10 m/min., a sump lubrication with regular transmission oil is recommended. The oil level has to cover the face width of the teeth lowest in the sump. Excessive oil causes foaming, cavitations and unnecessary energy loss. There is no requirement for any lubrication additive. Because the two kinds of flanks in a straight bevel gear (upper and lower) are mirror images of each other, there is no preferred operating direction, which is advantageous for many industrial applications.

(*Ed.'s Note: Next issue—"Zerol Bevel Gears."*)

Corrections

The previous article in this series, "General Explanations on Theoretical Bevel Gear Analysis," which appeared in the August 2010 issue of *Gear Technology*, contained two errors. The corrected or clarified text is highlighted below.
The complete corrected version of the article is available at *http://www.geartechnology.com/issues/0810*

Corrections

Page 49, left middle paragraph:
6 µm instead of 6 mm. (The error appears twice in this paragraph.)

Page 52, Formula 7:
Eliminate absolute value of Fn

$$|Fn| = Fx / (\cos\beta \bullet \cos\alpha) = -T / (A_m \bullet \sin\gamma \bullet \cos\beta \bullet \cos\alpha) \quad (7)$$