Less Energy Consumption with High-Efficiency Bevel Gears and their Usage in the U.S.

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This presentation introduces a new procedure that—derived from exact calculations—aids in determining the parameters of the validation testing of spiral bevel and hypoid gears in single-reduction axles.

Introduction

The efficiency of a gearbox is the output energy divided by the input energy. It depends on a variety of factors. If the complete gearbox assembly in its operating environment is observed, then the following efficiency influencing factors have to be considered:

Gearset:
- Gear type (straight, spiral bevel, hypoid)
- Manufacturing process (face hobbing, face milling)
- Hard-finishing process (none, lapping, grinding)
- Gear quality and surface roughness
- Characteristic of surface texture
- Gear parameters (spiral angle, pressure angle, profile shift, depth proportion, cutter radius, hypoid offset)
- Contact topography (Ease-Off, motion error, tooth contact)

Gearbox:
- Housing design
- Bearings
- Seal rings
- Kind of lubrication (grease, oil sump, oil circulation)
- Intensity of lubrication present (oil level, volume of circulation)
- Lubrication type (viscosity, additives)
- Lubrication dynamics (oil churning)
- Deflection characteristic under load

Operating Conditions:
- Speed
- Torque
- Direction of rotation (drive or coast)
- Temperature (of gearbox)

Environment:
- Vibration
- Temperature (surrounding)
- Heat transfer properties due to convection, radiation and surrounding fluid movement (e.g. air)

If the gearset is in the center of attention of efficiency and its optimization, then some of the physical relationships that contribute to tooth mesh efficiency have to be observed. While the teeth of a gearset roll in mesh, the flank surfaces have contact zones which move through the allocated contact area. The contact zones start as lines which spread under load to slender ellipses. This contact condition requires the observation of several effects:

Contact Condition Effects
- Relative sliding between the surfaces
- Rolling velocity of one surface relative to the other
- Change of sliding and rolling velocity during the tooth mesh
- Surface texture, roughness and waviness
- Normal forces or normal line force distribution
- Kind of lubrication
- Lubrication gap geometry (reduced curvature in principal directions)
- Lubrication gap kinematics (change of curvature during a mesh)
- Heat transfer properties of gear members, lubrication and surrounding components

Figure 1  Efficiency characteristics versus speed and torque.
Theoretical analysis and practical test rig investigations helped to understand many of the relationships between gear design, manufacturing process, etc., to the power loss during the tooth mesh. A spiral bevel gearset, for example, has a higher efficiency than a straight bevel gearset and a hypoid gearset with moderate offset has an even higher efficiency. Bevel worm gear drives work on an efficiency level that is 10% and more below spiral bevel gears; however, it is higher than the efficiency of worm gear drives.

The investigations about efficiency and the efforts to improve gear efficiency are not based on one particular efficiency number, but an efficiency map. Figure 1 shows the efficiency vs. speed and torque. This typical chart shows there is less dependency from the speed than from the torque. The diagram shows that with low torque, the efficiency is independent from the speed. Doubling the speed means doubling the energy transmitted. If the efficiency remains constant during this speed increase, then the loss of energy in the tooth contact is doubled. Increasing the torque will also increase the efficiency for medium and high speeds, since the lubrication film has a high load-carrying capacity and the load increase will not increase the oil friction by the same amount.

The following sections will elaborate on the effects influencing the efficiency in spiral bevel and hypoid gears, and give hints and guidelines for high-efficiency gear design. The trend here in U.S. regarding the manufacture of more efficient gears is summarized in the last section of this paper.

**Hydrodynamic Friction**

To calculate the precise energy dissipation in the tooth mesh of a bevel gearset, the continuous tooth mesh is analyzed in a certain number of discrete roll positions. The friction factor of each discrete roll position changes, depending on surface conditions, surface kinematics and the dynamics of the lubricant. Also the constant change of the shape of the lubrication gap has an influence upon the elastohydrodynamic friction.

The Stribeck curve (Fig. 2) is a graphical representation of the friction factor vs. the relative surface velocity of two mating surfaces in a hydrodynamic system. The shape of the Stribeck curve depends on the surface roughness, the reduced curvature and the contact force of the two mating surfaces in the contact zone considering a specific lubricant. The friction factor can be obtained from the Stribeck curve if the relative velocity is known.

The Stribeck curve in Figure 2 shows the conditions along the spectrum of relative velocities (solid body friction, boundary conditions and hydrodynamic friction). Efficiency calculation programs compute “points of the Stribeck curve” a graphical representation of the friction factor vs. the relative surface velocity of two mating surfaces in a hydrodynamic system. The shape of the Stribeck curve depends on the surface roughness, the reduced curvature and the contact force of the two mating surfaces in the contact zone considering a specific lubricant. The friction factor can be obtained from the Stribeck curve if the relative velocity is known.

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**Figure 2** Stribeck curve (Ref. 1).

**Figure 3** Contact gap and contact geometry aspects (Ref. 2).
with equations that consider the “Contact Condition Effects” listed above. The contact gap geometry and kinematic are next to surface roughness, lubrication properties and normal forces, the most important input for a friction factor calculation.

**Contact Gap Analysis**

The basis of a contact gap analysis in gears is the geometric and kinematic understanding of the interaction between the pinion and gear flank surfaces. Figure 3 shows on the left side a pinion flank rolling on a gear flank with a contact zone. The contact zone extends a distance along one pair of corresponding potential contact lines between pinion and gear. While the gearset rotates in mesh, the contact zone will move from its current location for example to the right. The surface curvatures between the two flanks are separated in two principal directions: one along the contact line, one along the path of contact (which is the direction from one contact line to the next).

The curvature in path of contact direction is some magnitudes larger than the curvature in contact line direction, which is reflected by $A \gg B$. However, depending on both the angle of the contact lines and on the direction of the sliding and rolling velocities between both flanks, both directions, contact line direction and the direction perpendicular to that (the latter is not always identical with the path of contact direction) must be considered for hydrodynamic investigation. The right-side graphic in Figure 3 shows the reduced curvatures of 20 discrete contact lines, each in their contact position, plotted above the gear projection plane (contact line scan).

Figure 4 shows the sliding and rolling velocity vectors of a typical hypoid gearset for each path of contact point for the 20 discussed roll positions. Each vector is projected to the tangential plane at the point of origin of the vector. The velocity vectors are drawn inside the gear tooth projection plane. The points of origin of both rolling and sliding velocity vectors (dots) are grouped along the path of contact which is found as the connection of the minima of the individual lines in the contact line scan graphic (Fig. 4, right). An observer who is located in one particular path of contact point on the gear flank surface would notice a momentarily contacting pinion point sliding away in the direction and with the speed the sliding velocity vector represents. The observer could also notice (particularly at the pitch point in straight bevel and spiral bevel gears, where no sliding but only rolling occurs), that the solid body, connected to that point moved in a certain direction by rolling like a wheel rolls on a pavement. The direction of this rolling and the movement—accomplished by the rolling (per time unit)—are represented by the direction and magnitude of the rolling velocity vector. Another possibility to explain the definition of rolling and sliding velocity in bevel and hypoid gears is presented in Figure 5. Disk 1 on top rotates with $\omega_1$ and is in contact with Disk 2 (bottom). The circumferential speed of Disk 1 is called the tangential velocity $V_{\text{tangential}}$. 

![Figure 4 Sliding and rolling velocities of a hypoid gearset along the path of contact (Ref. 2).](image)

![Figure 5 Definition of sliding and rolling velocity.](image)
The component of $V_{\text{Tangential}}$ which points in axial direction of Disk 2 cannot rotate Disk 2; it only causes a sliding $V_{\text{Slide}}$. The component that points in tangential direction of Disk 2 $V_{\text{Roll}}$ causes Disk 2 to rotate with $\omega_2$; it is called the rolling velocity.

It is interesting that the rolling velocities have a relatively consistent direction, whereas the sliding velocities change their direction along the path of contact significantly.

Figure 4 shows the average directions of the contact lines. The sliding and rolling velocities are projected in the contact line direction (example projections at the left and right side, Fig. 4). An analog projection in the direction perpendicular to the contact lines (not identical to the path of contact direction) allows two separate observations of the dynamics along the contact lines and perpendicular to them. The gap geometry change from contact line to contact line (Fig. 3, right) can be considered as an additional aspect. A single observation of the main direction appears unacceptable, given that sliding and rolling velocity have different directions and change along the path of contact significantly.

**Manufacturing Process and Surface Finish**

Bevel and hypoid gears for industrial applications are case-hardened and hard-finished after heat treatment by lapping, grinding or skiving. The chip removing mechanisms of lapping and grinding are quite different.

As demonstrated in Figure 6, (left), in lapping the abrasive grit is flooded with oil into the mesh between a gear and a pinion flank. The relative velocity would only roll the particles between the two surfaces and not lead to any significant material removal. The lapping torque is required in order to press the particles into the surface of one of the two members. If a particle is partially imbedded in the pinion flank, then the relative velocity and the force between the flanks will cause a material removal between this particle and the gear flank surface. Abrasive particles have to get partially imbedded in both mating flanks in order to assure a material removal on both members. Most particles are only slightly imbedded in the surface roughness of a flank and get flushed away after one flank contact ends and the next begins. However, it cannot be avoided that a certain percentage of particles penetrates partially into the surface and remains there after lapping. The surface roughness of lapped gears depends on the lapping grit size and the lapping torque.

Grinding is a cutting process with microscopically small, undefined and randomly distributed cutting edges (Fig. 6, right). The surface roughness of ground gears depends on the grit of the grinding wheel, the in-feed value and the dressing parameters. Independent from those factors, grinding will not leave any abrasive particles imbedded in the flank surfaces.

Since face hobbed gears (continuous index cutting) cannot be ground, but lap very well, and since face milled gears (single index cutting) can be ground but lap with certain obstacles, the common choices are the combinations “face hobbing – lapping” or “face milling – grinding.” The surface finishes produced by the lapping and grinding processes are quite different. Lapped surfaces (left photo, Fig. 7) have a roughness of $Ra$ below 0.5µm, but particles of the lapping grit are pressed into the surface (white spots in Fig. 7, left) where they remain in the later operation, which has a friction increasing effect. Ground surfaces have a surface roughness at or slightly above $Ra = \mu.5m$, with striations that are parallel to the root line. The break-in during the first 1,000 miles of operation of a vehicle, for example, reduces the roughness to $Ra$ values below 0.5µm and reduces the striations. Therefore, the grinding process has significant advantages regarding...
gear efficiency after the flank surfaces are polished due to the natural break-in. It can be stated that the highest surface finishes result in the highest efficiencies of gears. Super-finishing — after grinding— becomes interesting in cases where the best efficiency is required immediately.

The diagram in Figure 8 compares the efficiency of gears with no hard-finishing operation to lapping, skiving and grinding. It seems conclusive that gears with no hard-finishing show low efficiency, where lapping, skiving and grinding deliver gears with high efficiency. After reading the section above, it is also understandable that lapping has some disadvantage due to the remaining lapping grit and therefore delivers lower efficiency than skiving and grinding. Although the skived surface finish has low roughness and excellent potential for high efficiency, it appears that ground surfaces show, after the break-in, the optimal combination of low roughness and oil film-promoting surface texture than skiving. The general experience with modern power transmissions reveals a trend to a higher efficiency of ground gears.

**Efficiency of Different Gear Types**

High- and super-reduction hypoids (HRHs and SRHs) have been systematically tested with respect to efficiency (Ref. 3). Strategies for their optimization have been successfully developed and their limitations are well known. In the comparison with other types of angular gear drives (Fig. 9), the high or super reduction gearsets have the lowest efficiency. In spite of the analogy between cylindrical gears and straight bevel gears, the straight bevel gear efficiency is slightly below that of spur gears, and even a bit more below spiral bevel gears — provided that gear quality and hard-finishing methods are comparable.

Face gears are generally below straight bevel gears in an efficiency comparison. Reasons are the low contact ratio and the large difference between operating pitch line and the pinion’s nominal pitch line. The difference in efficiency between face hobbed and face milled spiral bevel and hypoid gears seems surprising at first view. Test rig investigations of face hobbed-lapped and face milled-lapped hypoid gearsets have been conducted. It was assured that spiral angle, pressure angle and tooth contact between the sample gearsets were identical at mid face of pinion and gear. Other basic design parameters like offset, cutter radius and curvature at mid-face, and of course number of teeth, facewidth and module have also been matched as closely as possible. The test gearsets have been lapped to a similar contact size and location. However, the results of an efficiency map show at medium to high torque and speed between 0.5 to 1% lower efficiency of the face hobbed gearset (Ref. 4). It was noticed that the face hobbed gearset operated at about 20°F higher temperature. The developed hydrodynamic friction calculation is unable to capture the efficiency lowering effect of face hobbed gears. This indicated that some effect is not considered in the friction factor calculation. Today, it is supposed that the flank twist of face hobbed teeth is the reason for the efficiency reduction and causes increased friction factors.

![Figure 8](https://www.geartechnology.com)  **Figure 8**  Quantitative comparison of hard-finishing influence with efficiency.

![Figure 9](https://www.geartechnology.com)  **Figure 9**  Quantitative comparison of different gear type efficiency.

![Figure 10](https://www.geartechnology.com)  **Figure 10**  Contact topography: (left) face hobbing; (right) face milling.
twist, the Ease-Off calculation and projection filters out the entire twist. The result is the Ease-Off (left side bottom, Fig. 10). This Ease-Off is identical to the face milled version (Fig. 10, bottom right) which has no flank twist (Fig. 10, top-right). It can be concluded that a contact gap, moving along the facewidth of a twisted flank pair, introduces a different dynamic to the lubricant that has a negative influence on its efficiency, resulting in a different Strubeck curve. The existence and quantification of this relationship has to be verified in future test rig investigations.

The flank twist of face hobbed gears depends on the cutter head size and the number of blade groups. Many blade groups and small cutter diameter result in high amounts of twist. The advantage of the twist is the higher forgiveness of tolerances and housing deflections under load, which means smaller contact movements due to the twisted teeth.

Figure 11  Bearing reaction load calculation.

Figure 12  UNICAL efficiency calculation interface.
Contact Forces Depending on Gear Parameters

The normal force $F_n$, (Fig. 11) is multiplied with the friction factor in order to compute the friction force. Friction-force-times-velocity results in the energy loss that is subtracted from the input energy in order to gain the percentage of energy left for the output shaft. This percentage number is identical to the efficiency. The Gleason efficiency software calculates the normal forces, the velocities and friction factors in 20 discrete roll positions (only one shown in Fig. 11) and uses a numerical integration to gain realistic efficiency values for bevel gearsets (Ref. 5). Multiple teeth in mesh according to the actual load sharing is considered in the Gleason efficiency calculation.

In Figure 11, the force $F_x$ times the radius is equal the torque. Differences between the direction of $F_x$ and $F_n$, due to the spiral angle, cause $F_n$ to become larger than $F_x$. The result is a larger friction force and higher bearing forces — without any visible benefit. However, the face contact ratio depends on a spiral angle and reduces the load-per-tooth. The two effects of $F_n$ larger than $F_x$ and individual $F_n$ lower due to a favorable load sharing, lead to an efficiency optimal spiral angle for every gearset. Efficiency optimal values also exist for the pressure angle, the cutter radius, the offset and other parameters.

Efficiency Calculation Software

Gleason developed an efficiency calculation software that considers all the aspects in gearset efficiency as described in the previous sections (Ref. 4). The software can also compute the energy losses in the surrounding of the gearset caused by bearings and seal rings. Figure 12 shows the input interface of the UNICAL Efficiency module.

In order to generate graphical efficiency maps, UNICAL accepts a range of torques and rpms as well as a number of increments in order to generate an ASCII file with a matrix, which can directly be imported into Excel.

Summary

The different sections of this paper gave the reader hints and explanations that favor grinding (over lapping), face milling over face hobbing, and recommended the reader a small hypoid offset vs. no offset for highest efficiency. This pre-selection is often not possible and doesn’t always apply. It seems equally important to offer some quantitative hints that may help to select efficiency-optimal gear parameters.

The following list includes the major gear parameters of an automotive gearset that was the basis of a parameter variation that delivered the efficiency dependency diagrams shown in Figure 13:

**Basic Gearset Data**
- Ratio: $13 \times 43$
- Face Module: 4.4 mm
- Ring Gear OD: 196 mm
- Facewidth: 33 mm
- Spiral Angle Gear: 31°
- Included Pressure Angle: 40°
- Hypoid Offset: 29 mm

![Figure 13 Efficiency influence on test hypoid gearset.](image)
The tables in Figure 13 cover cutter diameter, pressure angle, spiral angle, length crowning, profile crowning and hypoid offset. In many cases, variation of certain parameters is not an option. However, if the gear engineer conducts a total observation of all parameter influences regarding strength and efficiency, it is often possible to find a way to higher efficiency with equal strength. The trends in the diagrams in Figure 13 are results of test rig efficiency measurements on an automotive hypoid gearset, for average operating conditions (Ref. 4).

Based on the statements in this paper, the highest efficiency is achieved with a face milled ground hypoid gearset with a small offset, a low pressure angle, low spiral angle, a length crowning which is 3% of the facewidth and a profile crowning which is very small (or zero). In order to maximize the efficiency, this gearset should be cut with a small cutter diameter (e.g. ratio involute outer cone 0.75) and show a surface finish with an Ra = 0.5 mm or below.

**Trend in Efficient Gear Manufacturing in the U.S.**

The trend in the U.S. regarding efficiency optimization is significant. The U.S. Government supports manufacturers with grants that have plans to increase the operation efficiency of their products. In many cases those grants are related to geartrains in vehicles, marine and aerospace applications as well as energy producing (or converting) equipment.

The U.S. automotive industry is preparing for smaller vehicles with advanced, energy saving technologies. The customer in the U.S. still likes to buy SUVs, pick-up trucks and rear wheel-drive fun or muscle cars. The automobile producer will still build those lucrative vehicles and uses the additional profits to develop fuel saving technology. As the number of sold large SUVs, trucks and fun cars reduces per year, the smaller cars are built more fuel efficient and the larger cars are equipped with many sophisticated features. This is a well-planned preparation for the sophisticated and fuel efficient cars of tomorrow (Ref. 6).

The low popularity of diesel cars is mostly based on the high diesel tax in the U.S. Certain customers are interested in diesel vehicles with their high gas mileage and their high torque; they regret that Diesel-powered cars are not yet widely commercially available. Many people have a critical but educated opinion about hybrid cars. Combustion engine, plus electrical motor and generator, plus the additional weight and space of the batteries results in complex and expensive systems that require more energy to build and yet have the environmental headache regarding the toxic batteries. The maintenance cost of hybrid vehicles is a multiple of that which non-hybrids require after five years of service. Experts announced that the market in the U.S. and Europe will be saturated with 20% hybrid vehicles in the long term (Ref. 7). The other cars will be small with an increase of manual shift transmissions and mild hybrid features like shift indicator and automatic engine shutoff at traffic lights, etc. The percentage of diesel vehicles will increase as soon as the lawmakers set a new direction with modern emission regulations and adequate diesel taxes.

The speed reduction of a prime mover to the wheels or to propellers is inefficient, with slow running electric motors and electronic control units. The required speed reduction for fast-running small-size motors with a high efficiency rating shows a trend to more gears with high efficiency and high power density in most industries, and particularly in the personal transportation sector. The inefficiency of slow-running prime movers and, in particular, electric motors electric motors—at least as they are built today—is is a physical law which will only be overcome with fundamentally different designs that utilize other physical effects. Since this is not in sight for the coming decades, gears with the highest efficiency will maintain a high demand and the manufacturers that can make them will be very successful.

**References**