Calculating SAP and TIF

Dan Thurman

Q&A is your interactive gear forum. Send us your gear design, manufacturing, inspection or other related questions, and we’ll put them before our panel of experts. Questions may be mailed to Gear Technology, P.O. Box 1426, Elk Grove Village, IL 60009, USA, faxed to (847) 437-6618 or sent electronically to people@geartechnology.com. An expanded version of Q&A is also available online at www.geartechnology.com.

Question submitted by G. Lueck
Dalton Gear Co., Minneapolis, MN

Q: Are there any simpler formulae for S.A.P. & T.I.F.? Are there any not involving the mating part?

Answer submitted by Dan Thurman

A: Start of Active Profile (SAP) is defined as the lowest point on the gear tooth where contact with the mating gear tooth tip can occur. On gears without tooth tip chamfers, it will be determined by the maximum outside diameter of the mating gear. It is usually expressed in degrees of roll above the base diameter. There is a diameter called the form diameter (Df), associated with this roll angle, which can be calculated.

True Involute Form diameter (TIF) is defined as being the point on the gear tooth where the involute form must begin. It is not necessarily the same diameter as the form diameter determined by the SAP. For example, it is possible for a SAP to occur in an undercut area, but the TIF diameter to be at a higher point on the tooth profile.

SAP is dependent on the mating gear outside diameter and the operating center distance.

Figure 1 contains the equations for calculating SAP, Form Diameters, and mating gear outside diameters. Any combination

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Calculating SAP and TIF

Calculating Transverse Pressure Angle

\[ \phi_t = \arctan \left( \frac{\tan(\phi_0)}{\cos(\psi)} \right) \] (1)

Calculating Operating Transverse Pressure Angle

\[ \phi_0 = \arccos \left( \frac{(N + N_m) \cdot \cos(\phi_1)}{2 \cdot P_n \cdot \cos(\psi)} \right) \] (2)

Calculating Base Circle Diameter of Gear

\[ D_b = \left[ \frac{N \cdot \cos(\phi_1)}{P_n \cdot \cos(\psi)} \right] \] (3)

Calculating Base Circle Diameter of Mating Gear

\[ D_{bm} = \left[ \frac{N_m \cdot \cos(\phi_1)}{P_n \cdot \cos(\psi)} \right] \] (4)

Calculating Pressure Angle at OD/ID of Gear if Mating Gear OD/ID is Given

\[ \phi_{om} = \arctan \left( \frac{(N + N_m) \cdot \tan(\phi_0) - N \cdot \tan(\phi_{om})}{N_m} \right) \] (5)

Calculating Pressure Angle at OD/ID of Mating Gear if SAP is Given

\[ \phi_{om} = \arctan \left( \frac{(N + N_m) \cdot \tan(\phi_0) - N \cdot \tan(\phi_{om})}{N_m} \right) \] (6)

Calculating Start of Active Profile

\[ \phi_{om} = \arctan \left( \frac{(N + N_m) \cdot \tan(\phi_0) - N \cdot \tan(\phi_{om})}{N_m} \right) \] (7)

Calculating OD/ID of Mating Gear

\[ D_{om} = \frac{D_{bm}}{\cos(\phi_{om})} \] (8)

Calculating Pressure Angle at SAP of Gear

\[ \phi_r = \arctan \left( \frac{\pi \cdot SAP}{180} \right) \] (9)

Calculating Form Diameter at SAP of Gear

\[ D_f = \frac{D_r}{\cos(\phi_r)} \] (10)

Calculating Pressure Angle at OD/ID of Gear

\[ \phi_0 = \arccos \left( \frac{D_b}{D_f} \right) \] (11)

Calculating Roll Angle at OD/ID of Gear

\[ \theta_0 = 180 \cdot \frac{\tan(\phi_0)}{\pi} \] (12)

Calculating Profile Contact Ratio

\[ m_T = \frac{\theta_0 - \theta_{om} - \pi N}{360} \] (13)

* Note: quantities enclosed in vertical bars are absolute values.

Fig. 1 — Equations.
 Quieter Gears

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Extended Example

The following example uses the formulas presented above and a sample gear set (see Fig. 3 for sample specifications) to calculate SAP.

Using formula (1), we can calculate the transverse pressure angle based on the values given for normal pressure angle and helix angle in the example:

\[ \phi_t = \tan^{-1} \left( \tan (\phi_n) \cos (\psi) \right) \]

From this, we can calculate the operating transverse pressure angle using formula (2) and the given values for the center distance, the normal diametral pitch, the helix angle and the number of teeth in gear and mate:

\[ \phi_t' = \text{acos} \left( \frac{(N + N_m) \cos(\phi_t)}{2 \cdot CD \cdot P_n \cdot \cos(\psi)} \right) \]

\[ \phi_t' = \text{acos} \left( \frac{(21 + 18) \cos (20)}{2 \cdot 7.7915 \cdot 2.64 \cdot \cos (0)} \right) \]

\[ \phi_t' = 27.0221166 \text{ deg} \]
We will also need the base circle diameter of the mating gear, which can be determined using formula (4):

\[ D_{bm} = N_m \cdot \frac{\cos (\phi)}{P_n \cdot \cos (\psi)} \]

\[ D_{bm} = 18 \cdot \frac{\cos (20)}{2.64 \cdot \cos (0)} \]

\[ D_{bm} = 6.4069951 \]

Finally, we'll use formula (5) to calculate the pressure angle at the OD of the mating gear based on the given outside diameter:

\[ \phi_{om} = \acos \left( \frac{D_{bm}}{D_{om}} \right) \]

\[ \phi_{om} = \acos \left( \frac{6.4069951}{8.159} \right) \]

\[ \phi_{om} = 38.2546016 \]

Using the calculated and given values, we can now calculate SAP using formula (8):

\[ SAP = 180 \cdot \left[ \left( N + N_m \right) \cdot \tan (\phi) - N_m \cdot \tan (\phi_{om}) \right] \pi \cdot N \]

\[ SAP = 180 \cdot \left[ (21 + 18) \cdot \tan (27.0221166) - 18 \cdot \tan (38.2546016) \right] \pi \cdot 21 \]

\[ SAP = 15.55 \text{ deg} \]

The calculated SAP is given in degrees of roll above the base circle diameter. It can be converted to a corresponding form diameter \((D_f)\) as follows:

\[ D_f = \frac{N \cdot \cos (\phi)}{P_n \cdot \cos (\psi)} \]

\[ D_f = 7.4748277 \]

\[ D_f = D_b \cdot \sqrt{\left( \frac{\pi \cdot \frac{SAP}{180}}{180} \right)^2 + 1} \]

\[ D_f = 7.7451 \]

Many times we will be starting with an existing gear where we already know the form diameter \((D_f)\) or the corresponding roll angle \(\phi_f\) where the involute begins, and we want to find the outside diameter of the mating gear which will reach that point on the involute. In that case, the following equations apply:

Given the Form Diameter \((D_f)\) from the example above:

\[ D_f = 7.7451 \]

\[ SAP = 180 \cdot \frac{\sqrt{(D_f^2 - D_b^2)}}{\pi \cdot D_b} \]

\[ SAP = 15.55 \]
SAP = \frac{360 \cdot m_c}{N} = 15.55 \text{ degrees}

Any roll angle can be converted to a diameter according to the following relationship:

\[ D_x = D_b \cdot \sqrt{\left( \frac{\pi \cdot \theta_x}{180} \right)^2 + 1} \]

For more information about SAP and TIF, see ANSI/AGMA 115.01 Basic Gear Geometry and ANSI/AGMA 1012-F90 Gear Nomenclature, Definitions of Terms with Symbols.

References
1. Fig. 2 extracted from ANSI/AGMA 1012-F90, Gear Nomenclature, Definitions of Terms with Symbols, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, Virginia 22314.

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