

# Application of Statistical Process Capability Indices in Gear Manufacturing

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This article discusses applications of statistical process capability indices (Cp and Cpk) for controlling the quality of tooth geometry characteristics, including profile and lead as defined by current AGMA-2015, ISO-1328, and DIN-3960 standards. It also addresses typical steps to improve manufacturing process capability for each of the tooth geometry characteristics when their respective capability indices point to an incapable process.

## Introduction

The use of statistical analysis in today's world is omnipresent, inescapable, and vastly beneficial to many human endeavors; e.g. — medicine, weather prediction, government, finance, natural sciences, behavioral science, sports, insurance and — thanks in large part to Dr. W. Edwards Deming — the manufacturing industries. (*Ed's Note: Deming helped develop the sampling techniques still used by the Department of the Census and the Bureau of Labor Statistics. But were you aware: The original notions of Total Quality Management and continuous improvement trace back to a former Bell Telephone employee named Walter Shewhart. One of Deming's former teachers, he preached the importance of adapting management processes to create profitable situations for both businesses and consumers, promoting the utilization of his own creation — the statistical process control (SPC) control chart* Source: Wikipedia).

Manufacturers utilizing machining processes such as turning, milling and grinding have long embraced statistical process control (SPC) as a tool to understand and quantifying their process capability, improving quality, and reducing cost.

Yet some gear manufacturers have only half-heartedly embraced SPC, and many use it only for features such as tooth thickness, diameters, or run-out. Taking full advantage of SPC tools to understand process capabilities and to control the quality of gear tooth profile and lead continues lagging behind.

Indeed — it is difficult to resist the temptation to offer some anecdotal explanations as to why SPC for tooth profile

and lead characteristics remains underutilized.

Perhaps one reason is related to the proud history of gear manufacturers who learned how to precisely machine involute curves long before CNC cutting machines and CMM technology democratized the manufacture and inspection of complex shapes. Once upon a time, gear engineers had to create ingenious mechanical devices in order to precisely machine and measure involute curves. The slow acceptance of modern SPC tools for controlling profile and lead characteristics is somewhat reminiscent of the gear machine tools industry's adaptation of CNC in the 1980s — long after the turning and milling machine makers embraced CNC. The perception was that the earlier controls were neither precise nor fast enough to satisfy gear makers.

Another reason — at least here in the U.S. — is perhaps related to the nature of tolerance band specifications (K-chart) that was not easily conducive to a quantitative, and therefore statistical, analysis.

The final, and possibly least anecdotal, explanation for the reluctance to take full advantage of SPC tools is perhaps a seeming enormity and ambiguity of the task. Consider:

- How many and what specific profile and lead geometry characteristics should be analyzed? Should it be the total, slope, or form errors? Should it be a maximum error or a four-tooth-average?
- Are there differences between the analysis of the slope error and the form error?
- Data collection difficulties; not all inspection technologies have user-friendly means for collecting data auto-

matically and in a format tailored for SPC analysis.

- A lingering concern that one needs to produce a significantly better-than-required quality in order to have a capable process. An informative (included for guidance only, and is not formally a part of the standard) Annex C of AGMA 2015-1-A01 has even attempted to quantify this concern.
- And finally, what should be done with the process capability analysis results? How does one use capability indices to improve quality and reduce scrap cost?

Whatever the reasons for not taking full advantage of modern statistical tools in gear manufacturing, this article is an attempt to address some of the above concerns and provide a few tips for utilization of the process capability indices to assess and, if necessary, improve the process capability for tooth geometry characteristics.

The strategy for improving the process capability is not unlike finding and addressing the root cause of a quality issue based on inspection of a single gear. As gear quality is affected by many overlapping contributing factors (machine, fixture, cutting tools, blanks, machining parameters, set-up, and inspection uncertainty), one needs to navigate all these factors to find and address the dominant contributor responsible for the quality issue. The advantage, however, is that statistical evaluation empowers engineers with the knowledge of multiple data points and a “big picture” perspective. In addition to the specific gear quality issues, engineers are possessed with the ability to know process quality as quantified by the capability indices that help in isolating those specific, contributing factors that require improvement.

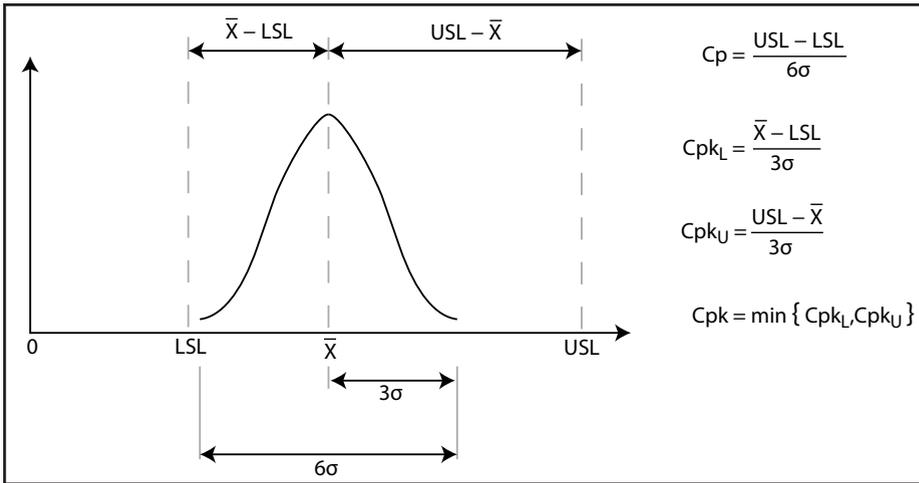


Figure 1 Cp and Cpk determination for a bi-lateral tolerance.

### Basic Process Capability Terminology: Cp and Cpk

Space does not allow covering the basics or definitions of statistical terms. Nevertheless, below are just a few terms for a quick reference.

**USL:** Upper Specification Limit (upper tolerance)

**LSL:** Lower Specification Limit (lower tolerance)

**σ:** Process standard deviation quantifies the data dispersion from the average. A lower σ indicates that the data points tend to be very close to the average, leading to improved process capability. In the absence of specialized software for SPC analysis, an approximation formula in MS Excel spreadsheet, "stdev" can be used. The sample size for evaluating the process capability is typically greater than 25.

**6\*σ:** Statistical process variation – roughly 99.97% of the population will be within this range.

**X̄:** Average of the measured sample population.

**Cp – Capability Index.**  $C_p = (USL - LSL) / (6 * \sigma)$ . This index is a measure of a potential process capability – a ratio between the tolerance range and the process variation. Cp value, however, does not reveal how well the process is centered in relation to the tolerance range.

**Cpk:** Capability Index that takes the centering of the process into account.

For a bilateral tolerance one needs to determine  $C_{pkL}$  and  $C_{pkU}$  and pick the smaller of the two.  $C_{pk} = \min \{ C_{pkU}, C_{pkL} \}$ ;  $C_{pkL} = (USL - \bar{X}) / (3 * \sigma)$ ,  $C_{pkU} = (\bar{X} - LSL) / (3 * \sigma)$  (Fig.1).  $C_{pk} > 1$  provides a statistical assurance that the process is

not only capable, but is also well-centered within the tolerance limits. In the case of a bilateral tolerance, both Cp and Cpk indices provide important insights into the process capability assessment.

For a unilateral tolerance, however, only a Cpk is used for the process assessment, as Cp may have no meaning. For a unilateral tolerance, Cpk is calculated only for the USL:  $C_{pk} = (USL - \bar{X}) / (3 * \sigma)$  (Fig. 2).

Capability indices (Cp and Cpk) greater than unity are a minimum requirement for a capable process. Most companies, however, use more stringent requirements; e.g. – Cp and Cpk must be greater than 1.33, 1.67, or even 2.

**Common cause** variations are random and inherent to the process; these variations come from contributors such as machine, cutting tool, fixture, blanks, set-

up, etc. when the quality of each contributor is in conformance with its respective tolerance limits.

**Assignable cause** variations are non-random and are usually greater than those induced by common causes. An assignable cause variation is frequently induced by the same contributors as common causes; i.e. – machine, cutting tool, fixture, blanks, etc. – when they are damaged, worn out, or, for whatever reason, are outside of their respective tolerance limits. For the process to be in control, all assignable cause variations must be found and eliminated (Ref. 1).

### Preparations and Limitations

Prior to measuring gears, it is important to attain a high confidence level in the inspection process to ensure that reliable data are analyzed. Whenever possible, the inspection fixture should use the same gear datum as the gear cutting fixture. Calibration of the inspection machine and a GR&R (gage repeatability and reproducibility review) should be conducted to determine if the inspection process is compatible with the gear tolerances.

It is also important to note that for extremely precise gear tolerances, when a GR&R results in a P/T (precision/tolerance) ratio greater than 0.3, the measuring system is considered incompatible with the gear tolerances and therefore unacceptable for a process capability study. The P/T ratio shows how much of the gear tolerance would be "eaten-up"

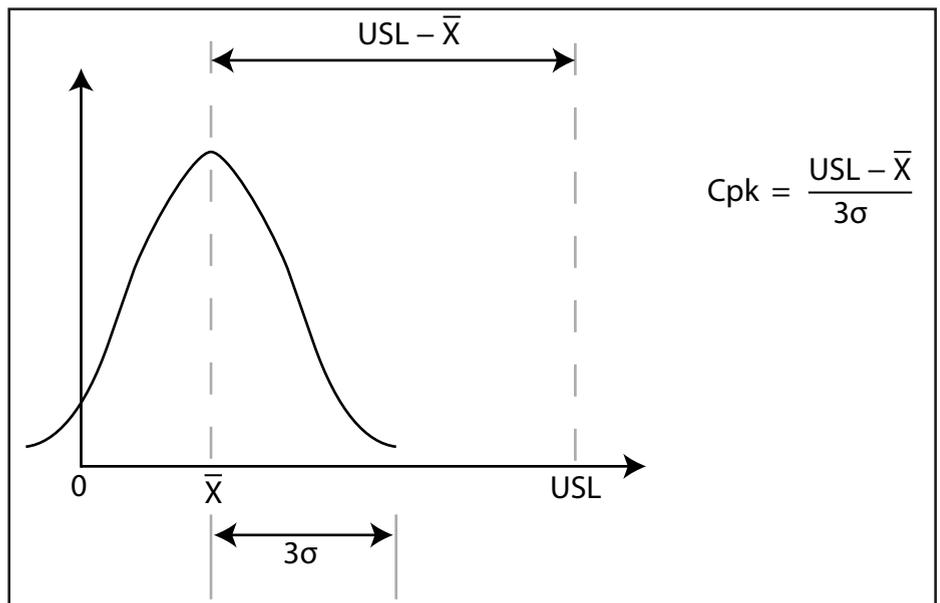


Figure 2 Cpk determination for a unilateral tolerance.

by the measuring system. Generally, a P/T ratio less than 0.1 indicates that the measuring system can reliably determine whether any given part meets the tolerance specification (Ref. 14).

A prudent practice is to study process capability for the blanks' datum features that are used for mounting gears in the gear cutting machine and inspection fixture. This will preempt and reduce some later work of investigating assignable causes if the process is found to be incapable.

### Gear Characteristics: Typical Contributors to Their Process Capability

Since the introduction of AGMA 2015 standard in 2002, the three most widely used gear quality standards — ISO, DIN and AGMA — became conceptually the same. These three standards define tooth profile and tooth lead tolerances for total, slope, and form errors (Fig. 3).

Right and left flanks should be analyzed separately, as they may have different assignable causes for excessive errors and incapable processes.

For the process to be in control, all assignable causes must be found and eliminated (Ref. 1). To determine assignable causes, one must navigate multiple contributors to gear quality; i.e. — gear blanks; cutting/grinding machine; workholding fixture; cutting tool and its resharpening or dressing consistency; setup; cutting conditions; inspection equipment; and inspection fixture. In addition, each manufacturing system may have its own peculiarities, depending on the technology employed. Therefore the typical, assignable causes listed in this section should serve only as a starting point for developing a more comprehensive, customized list. Some hobbing-related examples follow below.

After determining the process capability indices ( $C_p$  and  $C_{pk}$ ) — and finding out that the process is incapable — it would be prudent to start by investigating and addressing assignable root causes for a gear characteristic that has the worst capability index. Frequently, one assignable cause (for example, an excessive blank face run-out in relation to datum bore) adversely affects process capability indices of several gear characteristics.

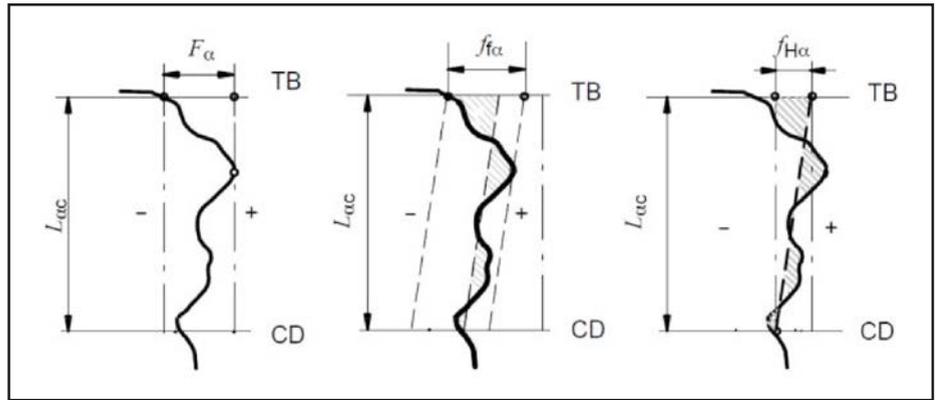


Figure 3 Total, slope, and form errors, AGMA 2015-1-A01.

Let's review one gear characteristic at a time:

### Tooth Profile

#### Profile slope error, $f_{H\alpha}$ .

Figure 4 provides an illustration for calculating the profile slope average (mean) error (Ref. 9) and profile slope variation when three teeth are measured. Assignable causes for the slope average error and the slope variation are different. For example, a gear radial run-out may have a negligible effect on the slope average error, but a dramatic effect on the slope variation. It would therefore be prudent to analyze slope average error and slope variation separately, as it would make it easier to find assignable causes for each respective error.

**Profile slope average error,  $f_{Ham}$  (Ref. 9).** The slope error averaged between four teeth spaced roughly 90° around the circumference can provide insights into a cutting tool; i.e. — hob, shaving cutter, or grinder dressing quality issues as they affect the process capability. The tooth profile slope average feature has a bilateral tolerance, therefore both  $C_p$  and  $C_{pk}$  should be determined. Table 1 covers different  $C_p$  and  $C_{pk}$  scenarios and provides some typical, assignable causes for an incapable process.

**Profile slope variation.** The slope variation between four teeth spaced roughly 90° around circumference can provide insights into fixture and blank quality, or

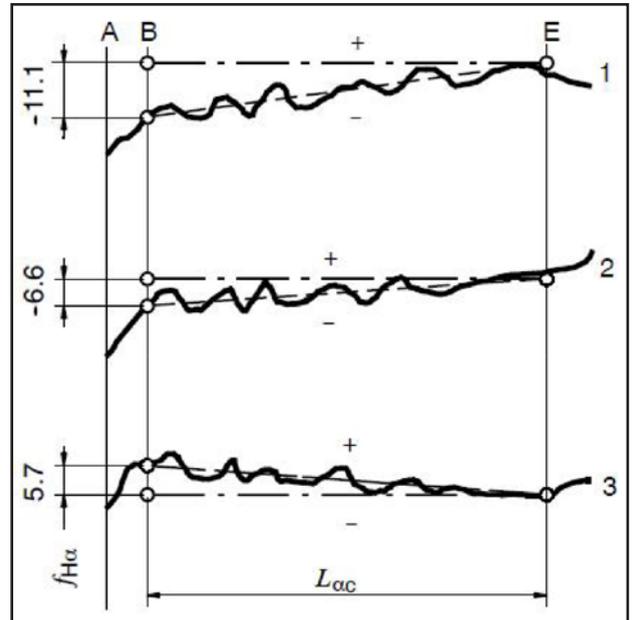


Figure 4 Average and Slope Variation — AGMA 915-1: average =  $(5.7 + (-6.6) + (-11.1)) / 3 = -4\mu\text{m}$ ; variation =  $(5.7 - (-11.1)) / 3 = 16.8\mu\text{m}$ .

other system contributors that create a radial run-out. The tooth slope variation is the difference between max/min slope errors as measured on four teeth of the same flank. Figure 4 shows an example for determining the slope variation error, as measured on three teeth. Note that the four-tooth measurement is a more reliable method for determining the slope variation error. If it is not defined on the drawing, the tolerance for the slope variation can be deduced from the slope tolerance. For example, if the slope tolerance is  $\pm 0.009\text{m}$ , the slope variation tolerance is  $(0.009 - [-0.009]) = 0.018\text{mm}$  (Table 2). Table 2 covers different  $C_p$  and  $C_{pk}$  scenarios, and provides some typical assignable ranges for an incapable process.

**Profile form average error,  $f_{fa}$ .** Profile form error averaged between four teeth spaced roughly 90° around the circum-

Table 1 Profile Slope Average Error					
Cp	Cpk	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of the machining technology.	Recommendations for improvements.
Cp>1	Cpk>1	±0.009mm	The process is capable and well centered.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
Cp>1	Cpk<1	±0.009mm	The process is capable, but not well centered.	<ol style="list-style-type: none"> <li>Hob rake error.</li> <li>Hob profile error.</li> </ol>	<ol style="list-style-type: none"> <li>Re-sharpen the hob.</li> <li>Re-profile the hob.</li> </ol>
Cp<1	Cpk<1	±0.009mm	The process is incapable.	<ol style="list-style-type: none"> <li>Hob cutter issues: excessive wear, rake error, profile error, gash-to-gash index error, thread-to-thread error, excessive cutter runout.</li> <li>Gear blanks issues (excessive runout).</li> <li>Workholding fixture misalignment or inadequate rigidity.</li> <li>Inspection fixture issues.</li> <li>Inspection process issues.</li> </ol>	<ol style="list-style-type: none"> <li>Increase shift frequency or distance, re-sharpen the hob to improve rake and index errors. Reprofile hob to reduce thread-to-thread and profile errors. Indicate the hob proof journals/faces to 0.005/0.007mm</li> <li>Improve blanks face-to-bore runout, improve quality for the datum surface i.e bore size. Use the same datum for fixturing during cutting and inspection.</li> <li>Increase clamping force, reduce radial/axial fixture runout.</li> <li>If possible, use the same datum for inspection and workholding to avoid runout between different datum.</li> <li>Exclude tooth undercut and tip relief from the evaluation zone. Review if the inspection machine needs repair and GR&amp;R.</li> </ol>

Table 2 Profile Slope Variation					
Cp	Cpk	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of machining technology.	Recommendations for improvements.
Not applicable	Cpk>1	0.009 - (-0.009) = 0.018mm	The process is capable.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
	Cpk<1		The process is incapable.	<ol style="list-style-type: none"> <li>Gear blanks radial runout. For high helix angle gears axial runout could also contribute to profile variation.</li> <li>Workholding fixture misalignment.</li> <li>Inadequate fixture rigidity.</li> <li>Use of unqualified datum for gear inspection.</li> <li>Contamination of mounting surfaces by cutting chips.</li> </ol>	<ol style="list-style-type: none"> <li>Improve the quality of datum surfaces i.e. bore size or bore-to-face runout, or shaft clamping diameter.</li> <li>Reduce axial/radial fixture runout.</li> <li>Increase clamping force.</li> <li>If possible, use the same datum for inspection and workholding to avoid runout between different datum.</li> <li>Improve chip removal process by coolant flushing or other means.</li> </ol>

Table 3 Profile Form Error					
Cp	Cpk	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of the machining technology.	Recommendations for improvements.
Not applicable	Cpk>1	0.013mm	The process is capable.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
	Cpk<1		The process is incapable.	<ol style="list-style-type: none"> <li>Cutting tool issues. Hob runout, hob index error, thread-to-thread error in case of high helix gears and hunting ratio combination of gear teeth/cutter threads. Excessive cutter wear. Hob coating issues. Insufficient number of hob gashes.</li> <li>Cutting conditions. Random gouges as a result of oil contamination. Excessive feed rate for high helix gears.</li> <li>Inspection process issues. Cut-off lines do not exclude tip relief or undercut.</li> </ol>	<ol style="list-style-type: none"> <li>Indicate hob to reduce radial runout. Sharpen the hob to reduce index (gash-to-gash) error. Re-profile hob to reduce thread-to-thread error. Increase shift frequency or amount. Increase frequency of coating stripping prior application of new coating. Use hobs with larger number of gashes.</li> <li>Use cleaner oil. Reduce feed rate in case of high helix angle gears.</li> <li>Review evaluation cut-off lines and exclude tip relief and undercut from the profile evaluation zone.</li> </ol>

Table 4 Helix/Lead Average Slope Error					
Cp	Cpk	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of the machining technology.	Recommendations for improvements.
Cp>1	Cpk>1	±0.011mm	The process is capable and well centered.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
Cp>1	Cpk<1	±0.011mm	The process is capable, but not well centered.	<ol style="list-style-type: none"> <li>1. Machine program cutting angle needs readjustment.</li> <li>2. Fixture misalignment.</li> <li>3. Machine tailstock/center misalignment.</li> </ol>	<ol style="list-style-type: none"> <li>1. Update cutting (or helix) angle based on the average slope error. Angle adjustment amount = atan (average error/evaluation range)</li> <li>2. Check and adjust fixture alignment with the machine centerline.</li> <li>3. Check and adjust machine tailstock/center.</li> </ol>
Cp<1	Cpk<1	±0.011mm	The process is incapable.	<ol style="list-style-type: none"> <li>1-3. Same as above.</li> <li>4. Workholding fixture issues: Excessive axial (and radial for high helix angle gear) runout.</li> <li>5. Gear blanks issues: excessive axial runout and/or perpendicularity.</li> <li>6. Cutting tool issues: thread-to-thread error for hunting ratio of gear teeth/hob threads.</li> <li>7. Rigidity of machine or fixture, or workpiece are inadequate.</li> <li>8. Inspection fixture: inspection fixture and workholding fixture use different datum causing excessive runout.</li> </ol>	<ol style="list-style-type: none"> <li>1-3. Same as above.</li> <li>4. Indicate and reduce runout.</li> <li>5. Improve blank quality. Blanks' datum surfaces should have a capable process.</li> <li>6. Use hobs with improved thread-to-thread error.</li> <li>7. Review system rigidity and make improvements.</li> <li>8. If possible, use the same datum for inspection and workholding to avoid runout between different datum.</li> </ol>

Table 5 Helix/Lead Slope Variation					
Cp	Cpk	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of the machining technology.	Recommendations for improvements.
Not applicable	Cpk>1	0.011-(-0.011) = 0.022	The process is capable.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
	Cpk<1		The process is incapable.	<ol style="list-style-type: none"> <li>1. Gear blanks axial runout. For high helix angle gears the radial runout could also contribute to lead slope variation.</li> <li>2. Workholding fixture runout of datum surfaces.</li> <li>3. Inspection fixture issues. Use of unqualified datum for workholding during inspection.</li> <li>4. Hob thread-to-thread error when cutting gears with hunting ratio combination.</li> <li>5. Contamination of mounting surfaces by cutting chips.</li> </ol>	<ol style="list-style-type: none"> <li>1. Improve the quality of datum surfaces i.e. bore size or bore-to-face runout, or shaft clamping diameter.</li> <li>2. Indicate and reduce fixture axial and radial runout.</li> <li>3. If possible, use the same datum for inspection and workholding to avoid runout between different datum.</li> <li>4. Reduce hob thread-to-thread error.</li> <li>5. Improve chip removal process by coolant flushing or other means.</li> </ol>

Table 6 Helix/Lead Form Error					
Cp	Cpk	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of the machining technology.	Recommendations for improvements.
Not applicable	Cpk>1	0.013mm	The process is capable.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
	Cpk<1		The process is incapable.	<ol style="list-style-type: none"> <li>1. Machine rigidity: Looseness in cutter head or table spindle, worn bearings, worn ways.</li> <li>2. Excessive feed rate.</li> <li>3. Oil contamination causing gouges.</li> <li>4. Excessive hob thread-to-thread error when cutting gears with hunting ratio combination of gear teeth and hob threads.</li> <li>5. Inspection process issues. Cut-off lines do not exclude chamfers.</li> </ol>	<ol style="list-style-type: none"> <li>1. Machine repair is required.</li> <li>2. Reduce feed rate.</li> <li>3. Replace oil.</li> <li>4. Reprofile the hob to reduce thread-to-thread error.</li> <li>5. Review and adjust evaluation cutoff lines to exclude chamfers on both sides.</li> </ol>

Table 7 Single Pitch Deviation					
Cp	Cpk	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of the machining technology.	Recommendations for improvements.
Not applicable	Cpk>1		The process is capable.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
	Cpk<1	0.015mm	The process is incapable.	<ol style="list-style-type: none"> <li>1. Machine: Worn table, excessive backlash in the table wormgear, poor synchronization of hob spindle with machine table.</li> <li>2. Excessive hob thread-to-thread error when cutting gears with non-hunting ratio combination of gear teeth and hob threads.</li> <li>3. Oil contamination causing gouges.</li> </ol>	<ol style="list-style-type: none"> <li>1. Machine repair is required.</li> <li>2. Reprofile the hob to reduce thread-to-thread error.</li> <li>3. Replace oil.</li> </ol>

Table 8 Cumulative Pitch Deviation					
Cp	Cpk	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of the machining technology.	Recommendations for improvements.
Not applicable	Cpk>1		The process is capable.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
	Cpk<1	0.050mm	The process is incapable.	<ol style="list-style-type: none"> <li>1. Gear blanks radial runout.</li> <li>2. Workholding fixture runout.</li> <li>3. Inspection process issues. Use of unqualified datum for inspection.</li> </ol>	<ol style="list-style-type: none"> <li>1. Improve the quality of datum surfaces i.e. bore size or bore-to-face runout, or shaft clamping diameter.</li> <li>2. Improve quality of the fixture.</li> <li>3. If possible, use the same datum for inspection and workholding to avoid runout between different datum.</li> </ol>

ference can provide insights into cutting tool quality and cutting condition. One can study the max form error instead of an average. However, the author recommends evaluation of the profile form *average* error instead, as it would reduce the sometimes confusing effects of random small cutting blemishes/gouges, random inspection machine sensitivities to an external environment, or inspection machine filtering issues. Table 3 provides some typical assignable causes for an incapable process.

**Profile total error,  $F_a$ .** The total error could be regarded as the sum of its component errors, i.e. — slope and form. When the process is capable for the slope and form characteristics, it would typically be capable for total error as well. If the process is incapable for the total error, one could make improvements by studying and improving the process for the components of the total error — slope and form errors — starting with the worst of the two.

### Tooth Lead

**Lead slope error,  $f_{H\beta}$ .** Like causes of profile slope error, assignable causes of lead slope average error and lead slope variation come from different contributors. Gear wobble (axial run-out), for example, may have only a slight effect on slope

average error, but a *considerable* effect on slope variation. Therefore, it would be prudent to analyze lead slope average error and slope variation separately.

**Lead slope average error,  $f_{H\beta m}$ .** The slope error averaged between four teeth spaced roughly 90° around circumference can be affected by various factors such as machining parameters, fixture alignment, or machine tailstock alignment, and others. The tooth lead slope error is a bilateral tolerance, so both Cp and Cpk should be determined (Table 4).

**Lead slope variation.** The slope variation between four teeth spaced roughly 90° around circumference is affected mainly by fixture and blank quality. The slope variation is the difference between the max/min slope errors, as measured on four teeth of the same flank. This characteristic has a unilateral tolerance that, if not specified on the drawing, can be deduced from the slope error tolerance. For example, if the slope tolerance is  $\pm 0.009$  m, the slope variation tolerance is  $(0.009 - [-0.009]) = 0.018$  m (Table 5).

**Lead average form error,  $f_{\beta}$ .** Lead form error averaged between four teeth spaced roughly 90° around circumference is affected by machine and fixture rigidity, cutting condition, coolant quality, and other factors. One can study the max form error rather than an average.

However, the author recommends evaluation of lead average form error, as it would reduce often confusing effects of random small cutting blemishes/gouges, random inspection machine sensitivities to an external environment, or inspection machine filtering issues (Table 6).

**Lead total error,  $F_{\beta}$ .** Like the profile total error, the lead total error could be regarded as the sum of its component errors — slope and form. When the process is capable for the slope and form errors, it is typically capable for the total error as well. If the process is incapable for the total error, one can make improvements by studying and improving the process for the components of the total error — slope and form errors — starting with the worst of the two.

### Tooth Index

**Single pitch deviation,  $f_{pr}$ .** The single pitch deviation (sometimes referred to as pitch variation) is the difference between theoretical pitch and actual distance between two adjacent teeth. Earlier AGMA and DIN standards also discussed spacing variation, i.e. — the difference between two adjacent pitches. Both pitch deviation (variation) and spacing variation errors are frequently affected by the same contributors (Table 7).

**Cumulate pitch deviation,  $F_p$ .** Cumulative/total pitch deviation. (Table 8) provides some typical assignable causes for an incapable process. More often than not, radial run-out is the main culprit for excessive cumulative pitch deviation.

**Tooth Thickness**

*Tooth thickness* (or dimension over pins or span measurement) has a bilateral tolerance, so both  $C_p$  and  $C_{pk}$  should be determined; average tooth thickness should be analyzed to exclude the effects of run-out (Table 9).

**Practical Applications and Recommendations**

Figure 5 depicts a “real world” example of the multiple trial runs in the process to

improve the capability of a tooth grinding operation, the goal being capability indices greater than 1.33. The first trial determined that the process was incapable —  $C_p$  and  $C_{pk}$  are mostly less than unity — (see red and yellow colors). This example shows that it took two additional trials to fully “de-bug” the process.

The process capability indices for gear characteristics, including profile and lead, can provide multiple benefits; a systematic approach to problem solving is just one of the benefits. Others include:

- Capability indices can qualify and quantify the capability of a new technology to consistently produce gears per required specifications.
- Capability indices can help compare and contrast capabilities of different technologies and/or processes.

- Capability indices can quantify existing processes that can help determine if a new technology is required.
- Processes that have capability indices greater than one can utilize a less-frequent inspection strategy, and therefore can benefit from improved efficiency and inspection cost reduction.
- When a quality issue arises, historical capability indices are powerful references that can help identify root causes with greater efficiency and confidence.
- Capability indices could also help more accurately predict the process cost — both fixed (machines) and variable (cutting tools).

Different gear cutting technologies have much in common. For example, an excessive gear blank “face-to-bore” run-out is the prime suspect for an assignable cause of an excessive lead slope variation,

$C_p$	$C_{pk}$	Tolerance example	Process assessment	Some typical assignable causes for an incapable process. Other assignable causes may exist depending on specifics of the machining technology.	Recommendations for improvements.
$C_p > 1$	$C_{pk} > 1$	$\pm 0.025\text{mm}$	The process is capable and well centered.		Machine, fixture, blanks, cutting tool, & cutting conditions, inspection procedure are capable of making gears of specified quality.
$C_p > 1$	$C_{pk} < 1$	$\pm 0.025\text{mm}$	The process is capable, but not well centered.	Setup issue.	Adjust the center distance between hob and workpiece accordingly.
$C_p < 1$	$C_{pk} < 1$	$\pm 0.025\text{mm}$	The process is incapable.	<ol style="list-style-type: none"> <li>1. Hob cutter issues: flute sharpening error, excessive cutter wear.</li> <li>2. Machine hob shift mechanism and hob axes have excessive parallelism error.</li> </ol>	<ol style="list-style-type: none"> <li>1. Resharpen the hob. Increase hob shift frequency or shift distance.</li> <li>2. Reduce parallelism error between the hob axis and machine shifting mechanism.</li> </ol>

**Figure 5 Three sets of the  $C_p$  and  $C_{pk}$  indices show the progress of process improvement efforts.**

Feature	Description	Trial 1				Trial 2				Trial 3			
		Meeting the spec	Fall-out /30 pcs	$C_{pk}$	$C_p$	Meeting the spec	Fall-out /30 pcs	$C_{pk}$	$C_p$	Meeting the spec	Fall-out /30 pcs	$C_{pk}$	$C_p$
1	Tooth Profile Total Average Error, right flank	No	5	0.36		Yes	0	1.68		Yes	0	6.17	
2	Tooth Profile Slope Average Error, right flank	Yes	0	0.68	1.26	Yes	0	0.66	0.69	Yes	0	2.38	3.54
3	Tooth Profile Form Average Error, right flank	No	3	0.43		Yes	0	5.22		Yes	0	7.41	
4	Tooth Lead Total Average Error, right flank	No	1	0.63		Yes	0	7.31		Yes	0	6.49	
5	Tooth Lead Slope Average Error, right flank	No	4	0.29	1.10	Yes	0	5.59	6.51	Yes	0	5.67	6.36
6	Tooth Lead Slope Variation, right flank	Yes	0	1.57		Yes	0	3.04		Yes	0	14.24	
7	Tooth Lead Form Average Error, right flank	Yes	0	1.45		Yes	0	16.45		Yes	0	24.72	
8	Tooth Lead Average Crown, right flank	No	18	-0.22	0.99	Yes	0	0.93	2.79	Yes	0	2.14	2.97
9	Adjacent Pitch Variation, right flank	Yes	0	1.06		Yes	0	1.81		Yes	0	3.94	
10	Adjacent Spacing Variation, right flank	No	2	0.57		Yes	0	2.06		Yes	0	3.22	
11	Cumulative Pitch, right flank	Yes	0	1.33		Yes	0	1.11		Yes	0	1.49	
12	Tooth Profile Total Average Error, left flank	No	2	0.51		Yes	0	3.25		Yes	0	6.42	
13	Tooth Profile Slope Average Error, left flank	Yes	0	0.69	0.72	Yes	0	0.84	1.16	Yes	0	1.92	1.98
14	Tooth Profile Form Average Error, left flank	No	5	0.28		Yes	0	5.20		Yes	0	6.13	
15	Tooth Lead Total Average Error, left flank	No	1	0.48		Yes	0	6.43		Yes	0	6.98	
16	Tooth Lead Slope Average Error, left flank	No	1	0.50	1.33	Yes	0	3.58	5.97	Yes	0	10.20	11.99
17	Tooth Lead Slope Variation, left flank	Yes	0	1.41		Yes	0	3.19		Yes	0	11.99	
18	Tooth Lead Form Average Error, left flank	Yes	0	1.28		Yes	0	16.45		Yes	0	5.15	
19	Tooth Lead Crown Average, left flank	No	29	-0.83	1.23	Yes	0	0.94	2.94	Yes	0	3.83	5.40
20	Adjacent Pitch Variation, left flank	Yes	0	0.81		Yes	0	3.10		Yes	0	4.00	
21	Adjacent Spacing Variation, left flank	No	5	0.23		Yes	0	2.50		Yes	0	4.22	
22	Cumulative Pitch, left flank	Yes	0	1.53		Yes	0	1.15		Yes	0	1.55	
23	Dimension Over Pins	Yes	0	0.59	0.60	Yes	0	0.29	0.54	Yes	0	1.32	2.06
24	Total Composite Error	No	1	0.62		Yes	0	0.80		Yes	0	1.41	
25	Tooth-to-tooth Composite Error	Yes	0	1.27		Yes	0	1.42		Yes	0	5.23	

regardless of whether the gear was produced on a grinding, hobbing, shaving or shaping machine.

However, every technology has its own peculiarities. For example, one needs to address a grinding wheel, a shaving cutter or a shaping cutter when a profile form error needs to be reduced on a grinding, shaving or shaping machine, respectively.

Once again, the (common and assignable) root causes and recommendations for improvements suggested in this article are just starting points that could be expanded and tailored for specific machines, fixture, blanks, cutting tools, set-ups, and experiences.

In rare cases it is possible that after all assignable causes are explored and eliminated, the process becomes in control — but remains incapable. Since both assignable and common causes derive from the same contributors (i.e. — machine, fixture, blanks, cutting tool, set-up, and cutting conditions), one could attempt to find a dominant common cause contributor(s) by following the same process as the one described above. After the dominant, common cause contributor is identified, one could attempt to improve the process by tightening the tolerances of that contributor. For example, one could tighten the tolerances of blanks, or workholding fixture, or use a higher-precision cutting tool, depending on what the dominant contributor is. But if improvement of the contributing factors is not possible, a different gear manufacturing process should be considered — or gear tolerances should be revisited.

In conclusion, customers continuously demand better quality, greater reliability, and lower cost. Application of process capability indices will not only support your process improvement efforts, but will also quantify the improvements with transparency and confidence. ⚙️

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